Options

Concepts and Buzzwords

- Put-Call Parity
- Volatility Effects
- Call, put, European, American, underlying asset, strike price, expiration date

Readings

- Tuckman, Chapter 19
- Veronesi, Chapter 6

Call Option

- A *European* call option is a contract that gives the owner the *right* but not the *obligation* to buy
- an underlying asset
  - at a pre-specified *strike price*
  - on a pre-specified *expiration date.*
- An *American* call option gives the owner the right to buy the asset at the strike price any time on or *before* the expiration date.

Put Option

- A European put option is a contract that gives the owner the right but not the obligation to *sell*
- an underlying asset
  - at a pre-specified strike price
  - on a pre-specified expiration date.
- An American put option gives the owner the right to sell the asset at the strike price any time on or before the expiration date.

Call Payoff

- Let V_T represent the value of the underlying asset on the expiration date T.
- Consider the payoff of a European call option with strike price K.
  - If the underlying is worth more than K at expiration, the option holder should exercise the option and buy the asset for K, for a net payoff of

    $V_T - K$.
  - If the underlying is worth less than K, the option holder should leave the option unexercised, for a net payoff of 0.
- To summarize, call payoff = $\max(V_T - K, 0)$

Put Payoff

- Now consider the payoff of a European put option with strike price K.
  - If the underlying is worth less than K at expiration, the option holder should exercise the option and sell the asset for K, for a net payoff of

    $K - V_T$.
  - If the underlying is worth more than K, the option holder should leave the option unexercised, for a net payoff of 0.
- To summarize, put payoff = $\max(K - V_T, 0)$

+ Put-Call Parity: Payoffs

- Consider a European call and a European put on the same underlying asset with the same strike price and the same expiration date.
- Math identity: $\text{Max}(V_T - K, 0) = \text{Max}(K - V_T, 0) + V_T - K$
- I.e., Call payoff = Put payoff $+ V_T - K$
- Verify:
  - If $V_T > K$, then the call is *in the money*, with a payoff of $V_T - K$, and the put is *out of the money* with a payoff of zero, so the payoff equation is satisfied:

    $V_T - K = 0 + V_T - K.$
  - If $V_T < K$ then the call is out of the money, the put is in the money, and the equation still holds:

    $0 = K - V_T + V_T - K$

+ Put-Call Parity: Prices

- Suppose the underlying asset pays no cash flows before the option expiration date.
- Then the payoff of the call is the same as the payoff of a portfolio consisting of
  - the put
  - the underlying asset
  - a short position in the riskless zero with par value K.
- Therefore, in the absence of arbitrage, the current price of the call must equal the current value of the portfolio:

  $C = P + V - d_T K$

+ Example: Options on a Zero

- Consider European call and put options on \$100 par of a zero maturing at time 2.
- The options expire at time 1.
- The strike price of the options is \$95.
- We'll use the bond market model developed last lecture to price these options.
- At each node below, zero prices are listed in order of maturity.

Time 0	Time 0.5	Time 1
		0.966581 0.933802
	0.970857 0.941787 0.913180	
0.973047 0.947649 0.922242 0.897166		0.973533 0.947382
	0.976941 0.953790 0.930855	
		0.979071 0.958270

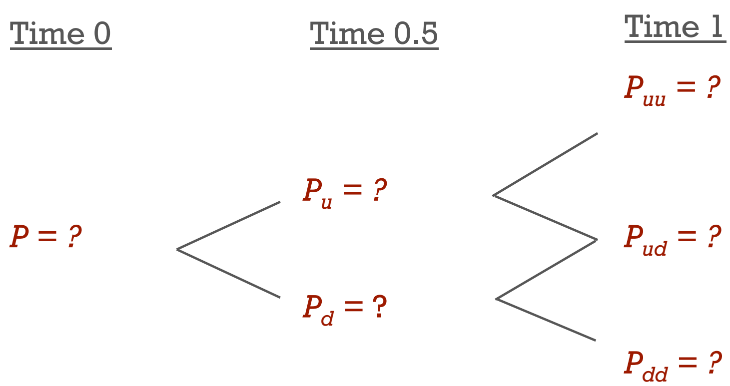
+ Class Problem: Call Valuation

Fill in the tree of prices of the call option.

Time 0	Time 0.5	Time 1
		C_{uu} = ?
	C_u = ?	
C = ?		C_{ud} = ?
	C_d = ?	
		C_{dd} = ?

+ Class Problem: Put Valuation

Fill in the tree of prices of the put option.



+ Class Problem: Put-Call Parity

1) Verify that put-call parity holds at time 0.

2) Verify put-call parity at time 0.5, up.

3) Verify put-call parity at time 0.5, down.

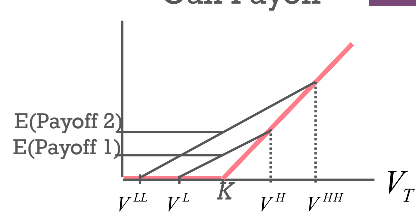
+ Option Prices and Volatility

- Consider changing the volatility of the bond market, holding the term structure constant.
- The higher the volatility of the underlying asset, the higher the value of both call and put options.
- Why? We can see more extreme bond prices will make the payoff of the (mostly out-of-the-money) call uniformly higher, so it's risk-neutral expected value will be greater.
- By put-call parity, the put value must therefore be higher, too.

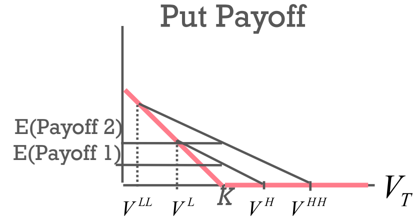
+ Graphical Illustration of the Volatility Effect

- Suppose that under distribution 1, the future asset price will be VL or VH with equal probability.
- Under the higher volatility distribution 2, the future asset price will be either VLL or VHH with equal probability.
- Then the expected option payoff will be higher under distribution 2.

Call Payoff



Put Payoff



+ Example: Increasing Volatility to σ=0.25

- Suppose we increase the volatility parameter σ from 0.17 to 0.25, and recalibrate the model to match the original term structure (m_1=-0.0955, m_2=0.0273, and m_3=0.003).
- The resulting tree of prices of zeroes out to 2 years is below.

Time 0

- 0.973047, 0.947649, 0.922242, 0.897166

Time 0.5

- 0.969448, 0.939007, 0.909090
- 0.978350, 0.956569, 0.934945

Time 1

- 0.963274, 0.927351
- 0.973926, 0.948128
- 0.981548, 0.963142

Time 1.5

- 0.9564
- 0.9690
- 0.9780
- 0.9845

+ Call and Put Prices with σ=0.25

At each node, the top number is the call price and the bottom number is put price. Notice that both option prices are higher with higher volatility.

Time 0

- 0.3128 / 0.6228

Time 0.5

- 0 / 1.1886
- 0.6429 / 0.0916

Time 1

- 0 / 2.2649
- 0 / 0.1872
- 1.3142 / 0