
OPTION PRICING THEORY AND MODELS
Problem 1

A. The values of the option parameters are as follows:

$$S = \$83$$

$$K = \$85$$

$$t = 0.25$$

$$r = 3.80\%$$

$$\text{Variance} = 0.09$$

$$\text{Value of call} = \$4.42$$

B. To replicate this call, you would have to:

Buy 0.4919 Shares of Stock (this is $N(d_1)$ from the model)

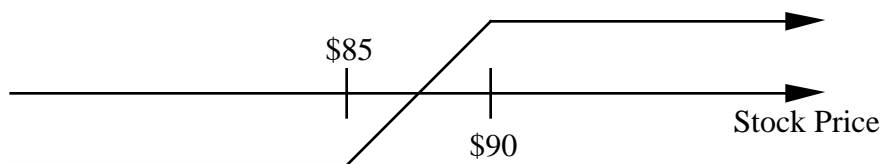
and

$$\text{Borrow } K e^{-rt} N(d_2) = 85 \exp^{-(0.038)(0.25)} (0.4324) = \$36.40$$

C. At an implied variance of 0.075, the call has a value of approximately \$4.00 (the market price).

$$\text{Implied Standard Deviation} = \sqrt{0.075} = 0.2739$$

D.



E.

$$\text{Value of Three-month Put} = C - S + Ke^{-rt} = \$4.42 - \$83 + 85 \exp^{-(0.038)(0.25)} = \$5.62$$

Problem 2

A. $S = \$28.75$

$$K = \$30$$

$$t = 0.25$$

$$r = 3.60\%$$

$$\sigma^2 = 0.04$$

$$\text{PV of Expected Dividends} = \$0.28 / (1.036)^{2/12} = \$0.28$$

$$\text{Value of Call} = \$0.64$$

B. The payment of a dividend reduces the expected stock price, and hence reduces the value of calls and increases the value of puts.

Problem 3

A. First value the three-month call, as above:

$$\text{Value of Call} = \$0.64$$

Then, value a call to the first (and only) dividend payment,

$$S = \$28.75$$

$$K = \$30$$

$$t = 2/12$$

$$r = 3.60\%$$

$$\sigma^2 = 0.04$$

$$y = 0 \text{ (since it assumes exercise before the dividend payment)}$$

$$\text{Value of Call} = \$0.51$$

Since the value of the three-month call is higher, there is no anticipated exercise.

B. If the dividend payment is large enough, it may pay to exercise the call just before the ex-dividend day (before the stock price drops) rather than wait until expiration. This early exercise is more likely for call options:

- (a) the larger the dividend on the stock, and
- (b) the closer the option is to expiration.

Problem 4

A. You would need to borrow $Ke^{-rt} N(d2) = 90 \exp(-0.04)(0.25) (0.4500) = \40.10

B. You would need to buy 0.575 shares of stock.

Problem 5

A. $S = \$4.00$

$$K = \$4.25$$

$$r = 5\%$$

$$t = 1$$

$$\text{Variance} = 0.36$$

$$\text{Value of Warrant} = \$0.93$$

B. Adjusted Stock Price = (Stock Price * Number of Shares Outstanding) + (Warrant price * Number of Warrants Outstanding)/(Number of Shares+Number of Warrants)
 = $(\$4.00 * 11,000,000 + \$0.93 * 550,000)/(11,550,000) = \3.85

(To avoid the circular reasoning problem, the price from the no-dilution case is used.)

Adjusted Exercise Price = \$4.25

$r = 5\%$

$t = 1$

Variance = 0.36

Value of Warrant = \$0.80

(If you are using a spreadsheet with iterations turned on, and are feeding the option prices back to calculate the adjusted stock price, the value of the warrants is still \$0.80.)

C. Dilution increases the number of shares outstanding. For any given value of equity, each share is worth less.

Problem 6

A. $S = 250$

$K = 275$

$t = 5$

$r = 5\%$

$\sigma^2 = (0.15)^2$

$y = 0.03$

 Value of call = \$29.09

B. Value of put with same parameters = \$28.09

C.

(1) The variance will be unchanged for the life of the option. This is likely to be violated because stock price variances do change substantially over time.

(2) There will be no early exercise. This is reasonable and is unlikely to be violated.

(3) Any deviations from the option value will be arbitrated away.

While there are plenty of arbitrageurs eager to exploit deviations from true value, arbitrating an index is clearly more difficult to do than arbitrating an individual stock.

Problem 7

New Security = AT & T stock - Call (K=60) + Put (K=45)
 = $\$50 - \$7.11 + \$3.55 = \46.44

The call with a strike price of \$60 is sold, eliminating upside potential above \$60.
The put with a strike price of \$45 is bought, providing downside protection.