

**VALUATION: PACKET 3
REAL OPTIONS, ACQUISITION VALUATION AND
VALUE ENHANCEMENT**

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Aswath Damodaran

REAL OPTIONS: FACT AND FANTASY

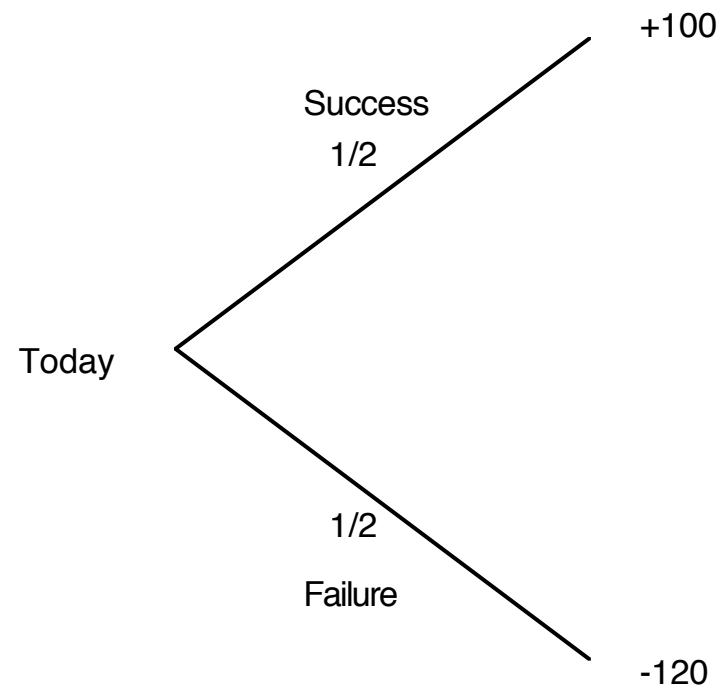
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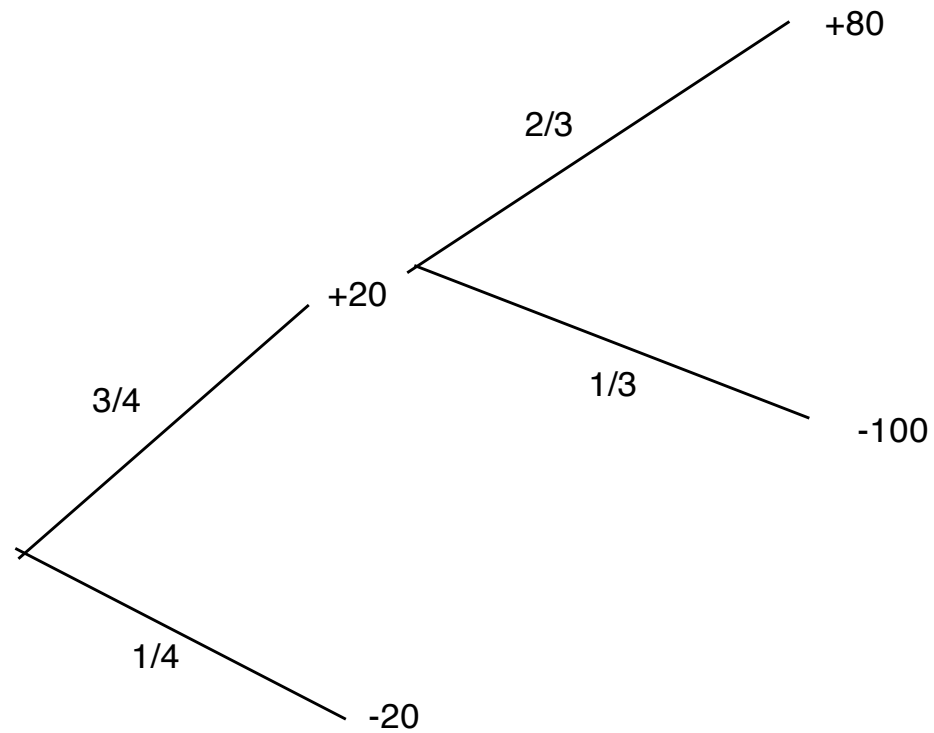
UNDERLYING THEME: SEARCHING FOR AN ELUSIVE PREMIUM

- Traditional discounted cashflow models underestimate the value of investments, where there are options embedded in the investments to
 - Delay or defer making the investment (delay)
 - Adjust or alter production schedules as price changes (flexibility)
 - Expand into new markets or products at later stages in the process, based upon observing favorable outcomes at the early stages (expansion)
 - Stop production or abandon investments if the outcomes are unfavorable at early stages (abandonment)
- Put another way, real option advocates believe that you should be paying a premium on discounted cashflow value estimates.

A BAD INVESTMENT...



BECOMES A GOOD ONE...



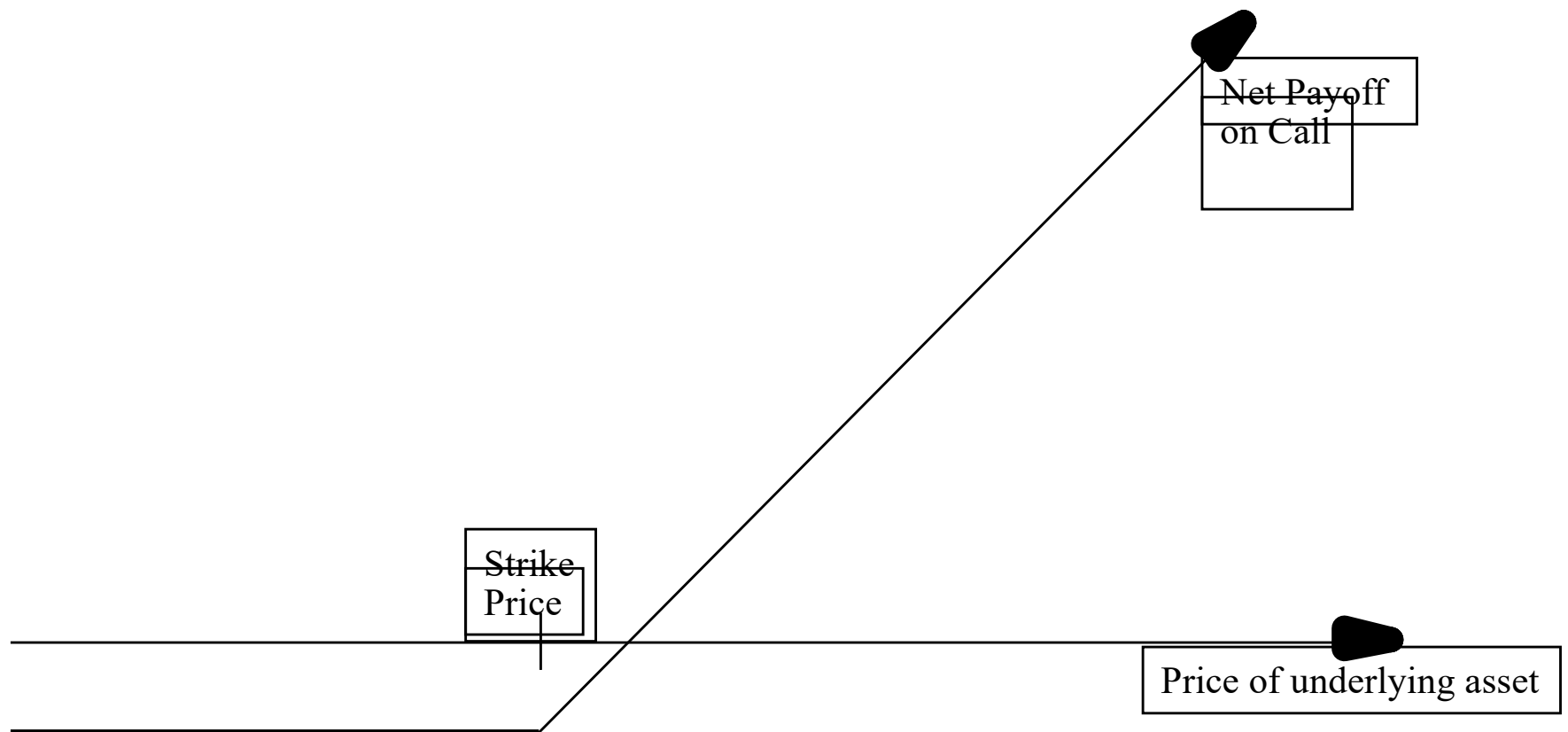
THREE BASIC QUESTIONS

- When is there a real option embedded in a decision or an asset?
- When does that real option have significant economic value?
- Can that value be estimated using an option pricing model?

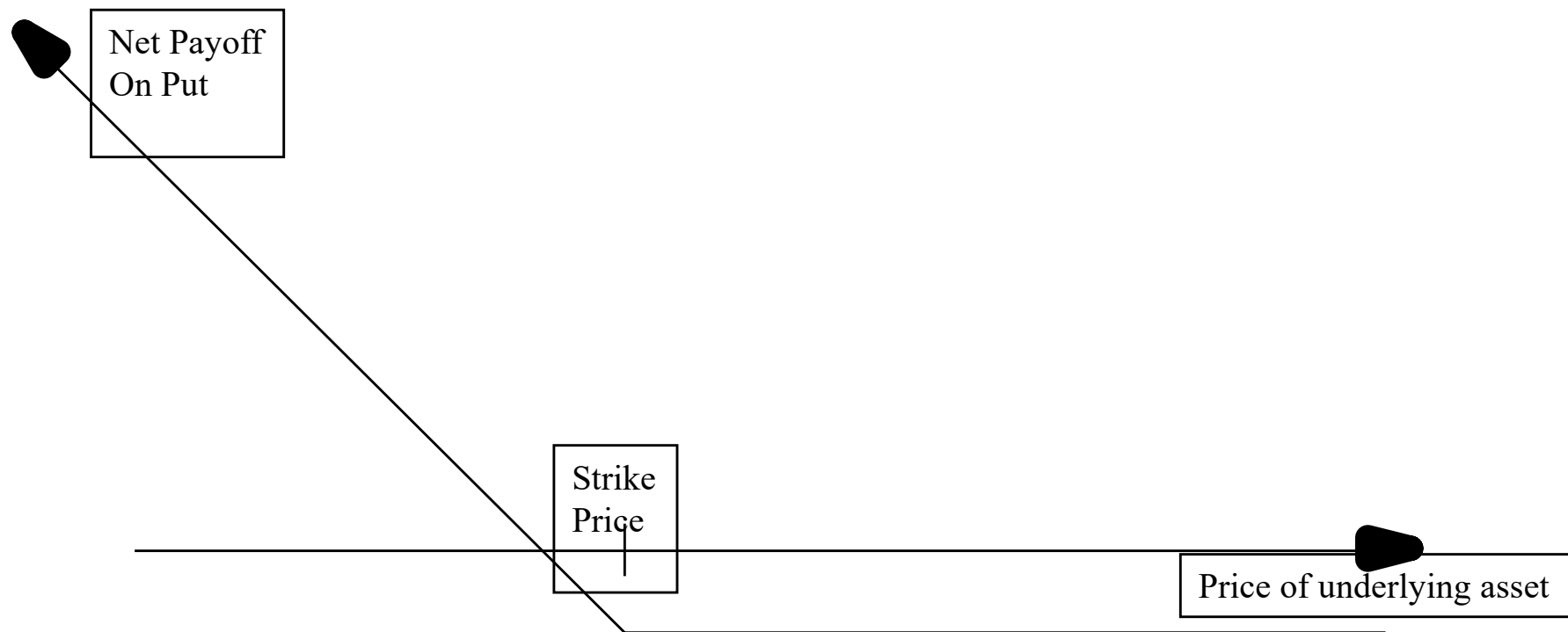
WHEN IS THERE AN OPTION EMBEDDED IN AN ACTION?

- An option provides the holder with the right to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option.
- There has to be a clearly defined underlying asset whose value changes over time in unpredictable ways.
- The payoffs on this asset (real option) have to be contingent on a specified event occurring within a finite period.

PAYOFF DIAGRAM ON A CALL



PAYOFF DIAGRAM ON PUT OPTION



WHEN DOES THE OPTION HAVE SIGNIFICANT ECONOMIC VALUE?

- For an option to have significant economic value, there has to be a **restriction on competition** in the event of the contingency. In a perfectly competitive product market, no contingency, no matter how positive, will generate positive net present value.
- At the limit, real options are most valuable **when you have exclusivity** - you and only you can take advantage of the contingency. They become less valuable as the barriers to competition become less steep.

DETERMINANTS OF OPTION VALUE

- **Variables Relating to Underlying Asset**
 - **Value of Underlying Asset**; as this value increases, the right to buy at a fixed price (calls) will become more valuable and the right to sell at a fixed price (puts) will become less valuable.
 - **Variance in that value**; as the variance increases, both calls and puts will become more valuable because all options have limited downside and depend upon price volatility for upside.
 - **Expected dividends on the asset**, which are likely to reduce the price appreciation component of the asset, reducing the value of calls and increasing the value of puts.
- **Variables Relating to Option**
 - **Strike Price of Options**; the right to buy (sell) at a fixed price becomes more (less) valuable at a lower price.
 - **Life of the Option**; both calls and puts benefit from a longer life.
- **Level of Interest Rates**; as rates increase, the right to buy (sell) at a fixed price in the future becomes more (less) valuable.

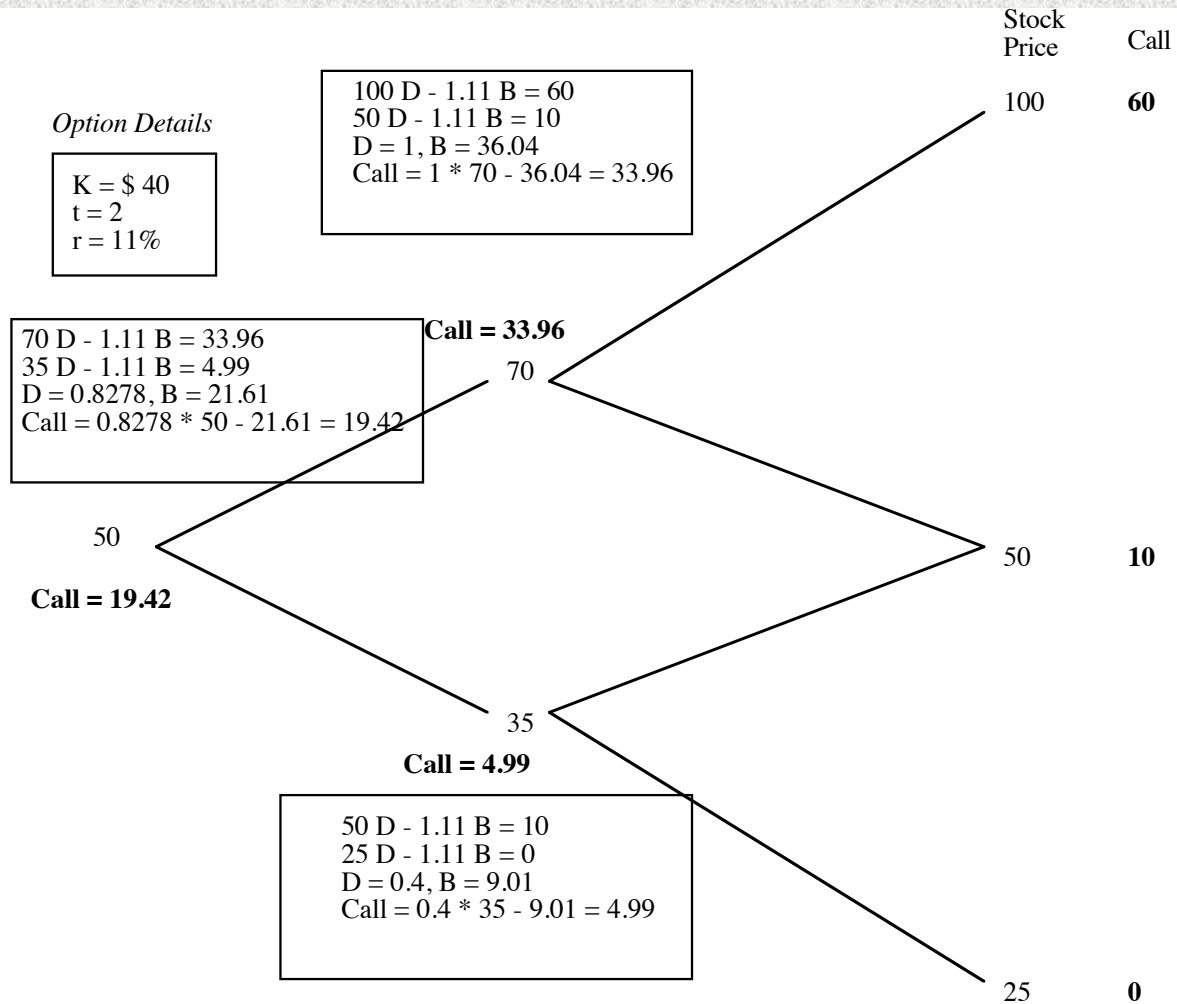
WHEN CAN YOU USE OPTION PRICING MODELS TO VALUE REAL OPTIONS?

- The notion of a replicating portfolio that drives option pricing models makes them most suited for valuing real options where
 - The **underlying asset is traded** - this yields not only observable prices and volatility as inputs to option pricing models but allows for the possibility of creating replicating portfolios
 - An **active marketplace exists for the option** itself.
 - The **cost of exercising the option is known** with some degree of certainty.
- When option pricing models are used to value real assets, we have to accept the fact that
 - The value estimates that emerge will be **far more imprecise**.
 - The value can **deviate much more dramatically from market price** because of the difficulty of arbitrage.

CREATING A REPLICATING PORTFOLIO

- The objective in creating a replicating portfolio is to use a **combination of riskfree borrowing/lending and the underlying asset to create the same cashflows as the option** being valued.
 - Call = Borrowing + Buying D of the Underlying Stock
 - Put = Selling Short D on Underlying Asset + Lending
 - The number of shares bought or sold is called the option delta.
- The **principles of arbitrage** then apply, and the value of the option has to be equal to the value of the **replicating portfolio**.

THE BINOMIAL OPTION PRICING MODEL



THE LIMITING DISTRIBUTIONS . . .

- As the time interval is shortened, the limiting distribution, as $t \rightarrow 0$, can take one of two forms.
 - If as $t \rightarrow 0$, **price changes become smaller**, the limiting distribution is the normal distribution and the price process is a continuous one.
 - If as $t \rightarrow 0$, **price changes remain large**, the limiting distribution is the Poisson distribution, i.e., a distribution that allows for price jumps.
- The Black-Scholes model applies when the limiting distribution is the normal distribution and explicitly **assumes that the price process is continuous** and that there are no jumps in asset prices.

BLACK AND SCHOLES...

- The version of the model presented by Black and Scholes was designed to value European options, which were dividend-protected.
- The value of a call option in the Black-Scholes model can be written as a function of the following variables:
 - S = Current value of the underlying asset
 - K = Strike price of the option
 - t = Life to expiration of the option
 - r = Riskless interest rate corresponding to the life of the option
 - σ^2 = Variance in the $\ln(\text{value})$ of the underlying asset

THE BLACK SCHOLES MODEL

- Value of call = $S N(d_1) - K e^{-rt} N(d_2)$

- where

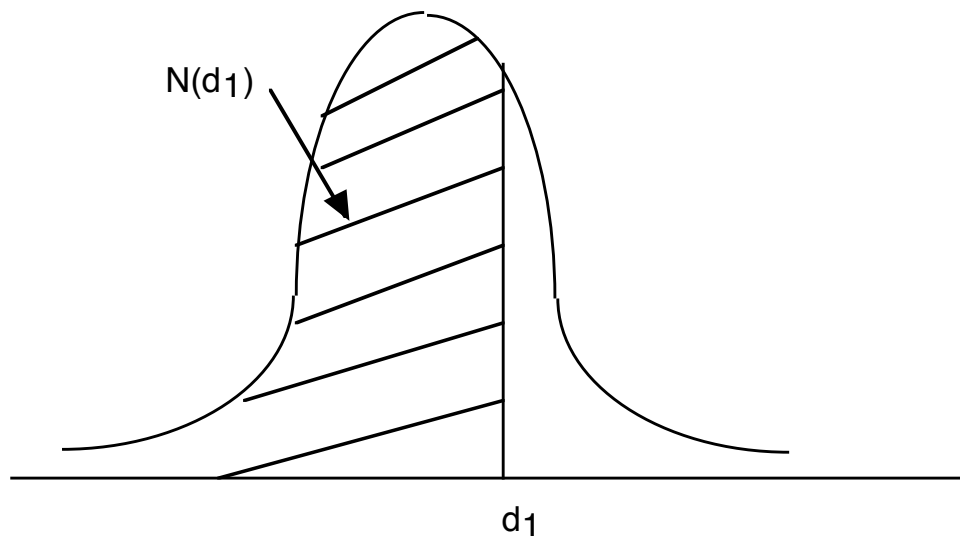
- $$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

- $d_2 = d_1 - \sigma \sqrt{t}$

- The replicating portfolio is embedded in the Black-Scholes model. To replicate this call, you would need to

- Buy $N(d_1)$ shares of stock; $N(d_1)$ is called the option delta
 - Borrow $K e^{-rt} N(d_2)$

THE NORMAL DISTRIBUTION



d	$N(d)$	d	$N(d)$	d	$N(d)$
-3.00	0.0013	-1.00	0.1587	1.05	0.8531
-2.95	0.0016	-0.95	0.1711	1.10	0.8643
-2.90	0.0019	-0.90	0.1841	1.15	0.8749
-2.85	0.0022	-0.85	0.1977	1.20	0.8849
-2.80	0.0026	-0.80	0.2119	1.25	0.8944
-2.75	0.0030	-0.75	0.2266	1.30	0.9032
-2.70	0.0035	-0.70	0.2420	1.35	0.9115
-2.65	0.0040	-0.65	0.2578	1.40	0.9192
-2.60	0.0047	-0.60	0.2743	1.45	0.9265
-2.55	0.0054	-0.55	0.2912	1.50	0.9332
-2.50	0.0062	-0.50	0.3085	1.55	0.9394
-2.45	0.0071	-0.45	0.3264	1.60	0.9452
-2.40	0.0082	-0.40	0.3446	1.65	0.9505
-2.35	0.0094	-0.35	0.3632	1.70	0.9554
-2.30	0.0107	-0.30	0.3821	1.75	0.9599
-2.25	0.0122	-0.25	0.4013	1.80	0.9641
-2.20	0.0139	-0.20	0.4207	1.85	0.9678
-2.15	0.0158	-0.15	0.4404	1.90	0.9713
-2.10	0.0179	-0.10	0.4602	1.95	0.9744
-2.05	0.0202	-0.05	0.4801	2.00	0.9772
-2.00	0.0228	0.00	0.5000	2.05	0.9798
-1.95	0.0256	0.05	0.5199	2.10	0.9821
-1.90	0.0287	0.10	0.5398	2.15	0.9842
-1.85	0.0322	0.15	0.5596	2.20	0.9861
-1.80	0.0359	0.20	0.5793	2.25	0.9878
-1.75	0.0401	0.25	0.5987	2.30	0.9893
-1.70	0.0446	0.30	0.6179	2.35	0.9906
-1.65	0.0495	0.35	0.6368	2.40	0.9918
-1.60	0.0548	0.40	0.6554	2.45	0.9929
-1.55	0.0606	0.45	0.6736	2.50	0.9938
-1.50	0.0668	0.50	0.6915	2.55	0.9946
-1.45	0.0735	0.55	0.7088	2.60	0.9953
-1.40	0.0808	0.60	0.7257	2.65	0.9960
-1.35	0.0885	0.65	0.7422	2.70	0.9965
-1.30	0.0968	0.70	0.7580	2.75	0.9970
-1.25	0.1056	0.75	0.7734	2.80	0.9974
-1.20	0.1151	0.80	0.7881	2.85	0.9978
-1.15	0.1251	0.85	0.8023	2.90	0.9981
-1.10	0.1357	0.90	0.8159	2.95	0.9984
-1.05	0.1469	0.95	0.8289	3.00	0.9987
-1.00	0.1587	1.00	0.8413		

ADJUSTING FOR DIVIDENDS

- If the dividend yield ($y = \text{dividends} / \text{Current value of the asset}$) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

- $$C = S e^{-yt} N(d_1) - K e^{-rt} N(d_2)$$

- where,

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

- $d_2 = d_1 - \sigma \sqrt{t}$

- The value of a put can also be derived:

- $$P = K e^{-rt} (1 - N(d_2)) - S e^{-yt} (1 - N(d_1))$$

DIVIDEND YIELD = COST OF DELAY

- Options have time premiums, and when they are traded, you very seldom get early exercise, with one exception being calls before big ex-dividend dates. The trade off that drives early exercise is:
 - **Loss of the time premium** of the option from **exercising early** (against)
 - **Dividends you will receive**, if you exercise early
 - If the dividend exceeds the time premium, you will see early exercise.
- Put differently, the dividends foregone become the cost of delaying exercise and leaving the option live.
- Thus, having a cost of delay in an option will require that you use a dividend-adjusted version of the option pricing model.

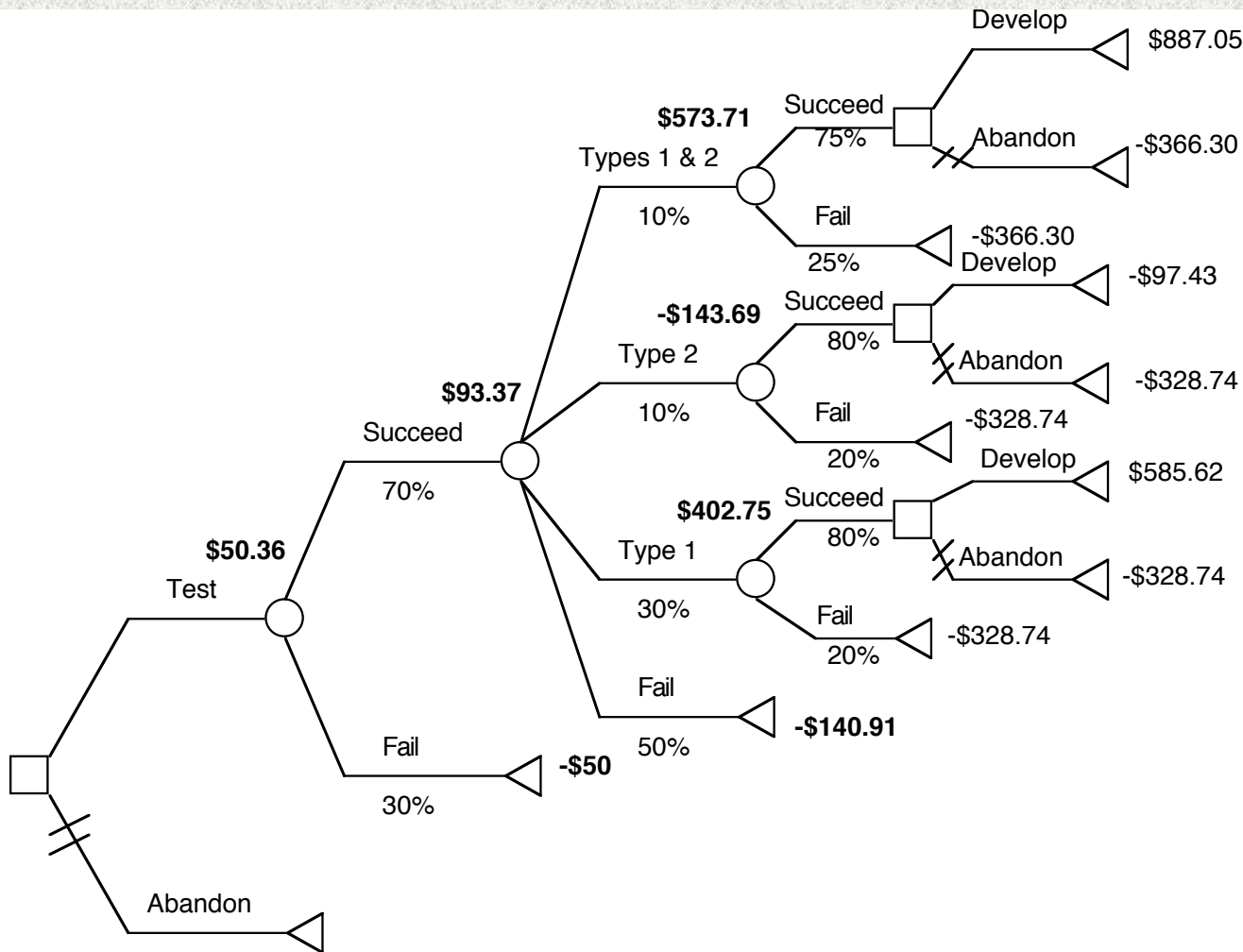
CHOICE OF OPTION PRICING MODELS

- Most practitioners who use option pricing models to value real options argue for the binomial model over the Black-Scholes and justify this choice by noting that
 - Early exercise is the rule rather than the exception with real options
 - Underlying asset values are generally discontinuous.
- If you can develop a binomial tree with outcomes at each node, it looks a great deal like a decision tree from capital budgeting. The question then becomes when and why the two approaches yield different estimates of value.

THE DECISION TREE ALTERNATIVE

- Traditional decision tree analysis tends to use
 - One cost of capital to discount cashflows in each branch to the present
 - Probabilities to compute an expected value
 - These values will generally be different from option pricing model values
- If you modified decision tree analysis to
 - Use **different discount rates at each node** to reflect where you are in the decision tree (This is the Copeland solution) (or)
 - Use the **riskfree rate to discount cashflows in each branch**, estimate the probabilities to estimate an expected value and adjust the expected value for the market risk in the investment
- Decision Trees could yield the same values as option pricing models

A DECISION TREE VALUATION OF A PHARMACEUTICAL COMPANY WITH ONE DRUG IN THE FDA PIPELINE...



KEY TESTS FOR REAL OPTIONS

- Is there an option embedded in this asset/ decision?
 - Can you identify **the underlying asset**?
 - Can you specify **the contingency under which you will get payoff**?
- Is there exclusivity?
 - **If yes**, there is option value.
 - **If no**, there is none.
 - If in between, you have to scale value.
- Can you use an option pricing model to value the real option?
 - Is the underlying asset traded?
 - Can the option be bought and sold?
 - Is the cost of exercising the option known and clear?

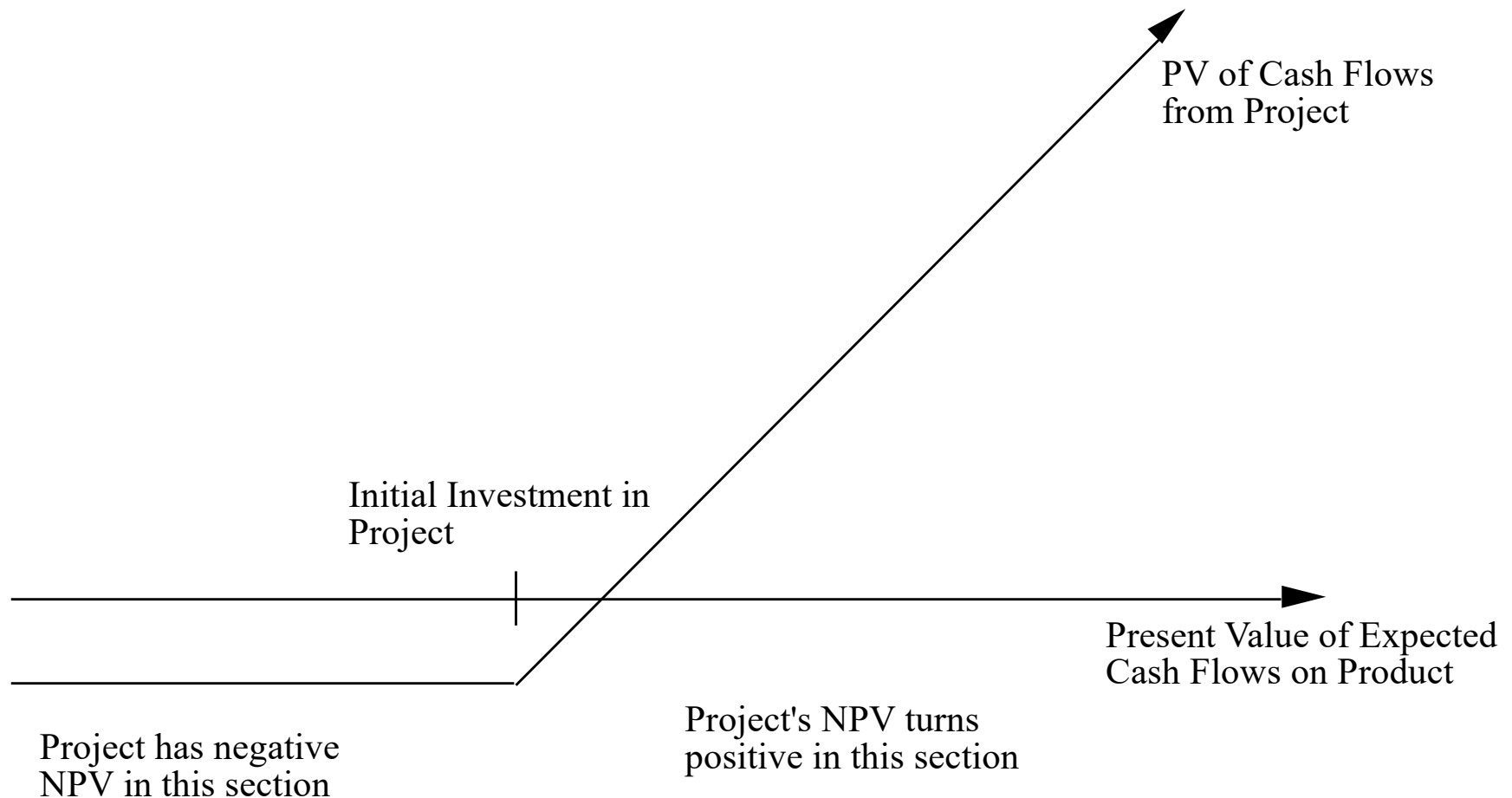
I. OPTIONS IN PROJECTS/INVESTMENTS/ACQUISITIONS

- One of the limitations of traditional investment analysis is that **it is static** and does not do a good job of capturing the options embedded in investment.
 - The first of these options is the **option to delay taking a investment**, when a firm has exclusive rights to it, until a later date.
 - The second of these options is taking one investment may allow us to take **advantage of other opportunities (investments) in the future**
 - The last option that is embedded in projects is the **option to abandon an investment**, if the cash flows do not measure up.
- These **options all add value** to projects and may make a “bad” investment (from traditional analysis) into a good one.

A. THE OPTION TO DELAY

- When a firm has **exclusive rights to a project or product** for a specific period, it can delay taking this project or product until a later date.
- A traditional investment analysis **just answers the question of whether the project is a “good” one if taken today.**
- Thus, the fact that a project **does not pass muster today** (because its NPV is negative, or its IRR is less than its hurdle rate) does not mean that the rights to this project are not valuable.

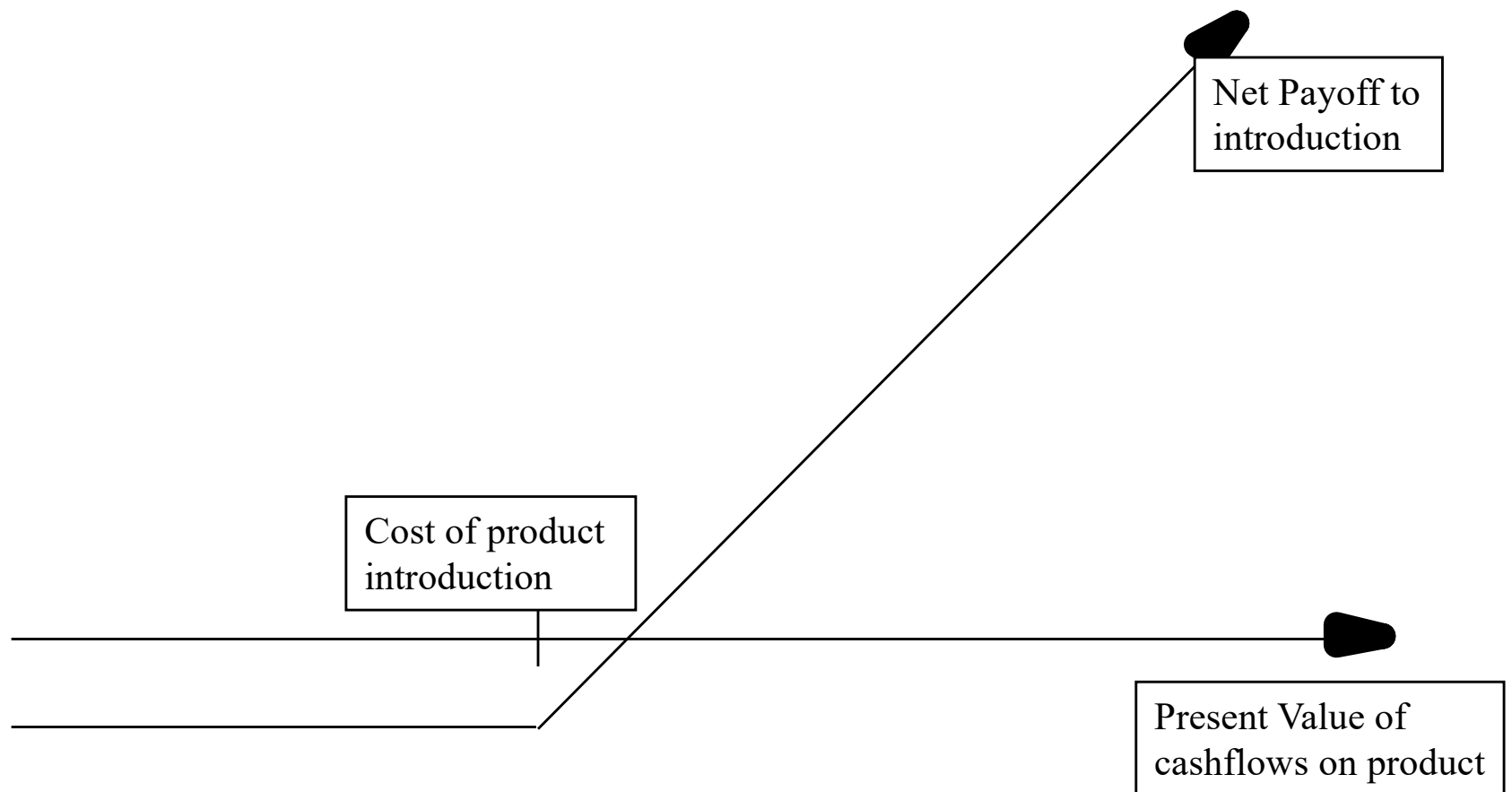
VALUING THE OPTION TO DELAY A PROJECT



EXAMPLE 1: VALUING PRODUCT PATENTS AS OPTIONS

- A product patent provides the firm with the right to develop the product and market it.
 - It will do so only if the present value of the expected cash flows from the product sales exceed the cost of development.
 - If this does not occur, the firm can shelve the patent and not incur any further costs.
- If I is the present value of the costs of developing the product, and V is the present value of the expected cashflows from development, the payoffs from owning a product patent can be written as:
- Payoff from owning a product patent $= V - I$ if $V > I$
 $= 0$ if $V \leq I$

PAYOFF ON PRODUCT OPTION



OBTAINING INPUTS FOR PATENT VALUATION

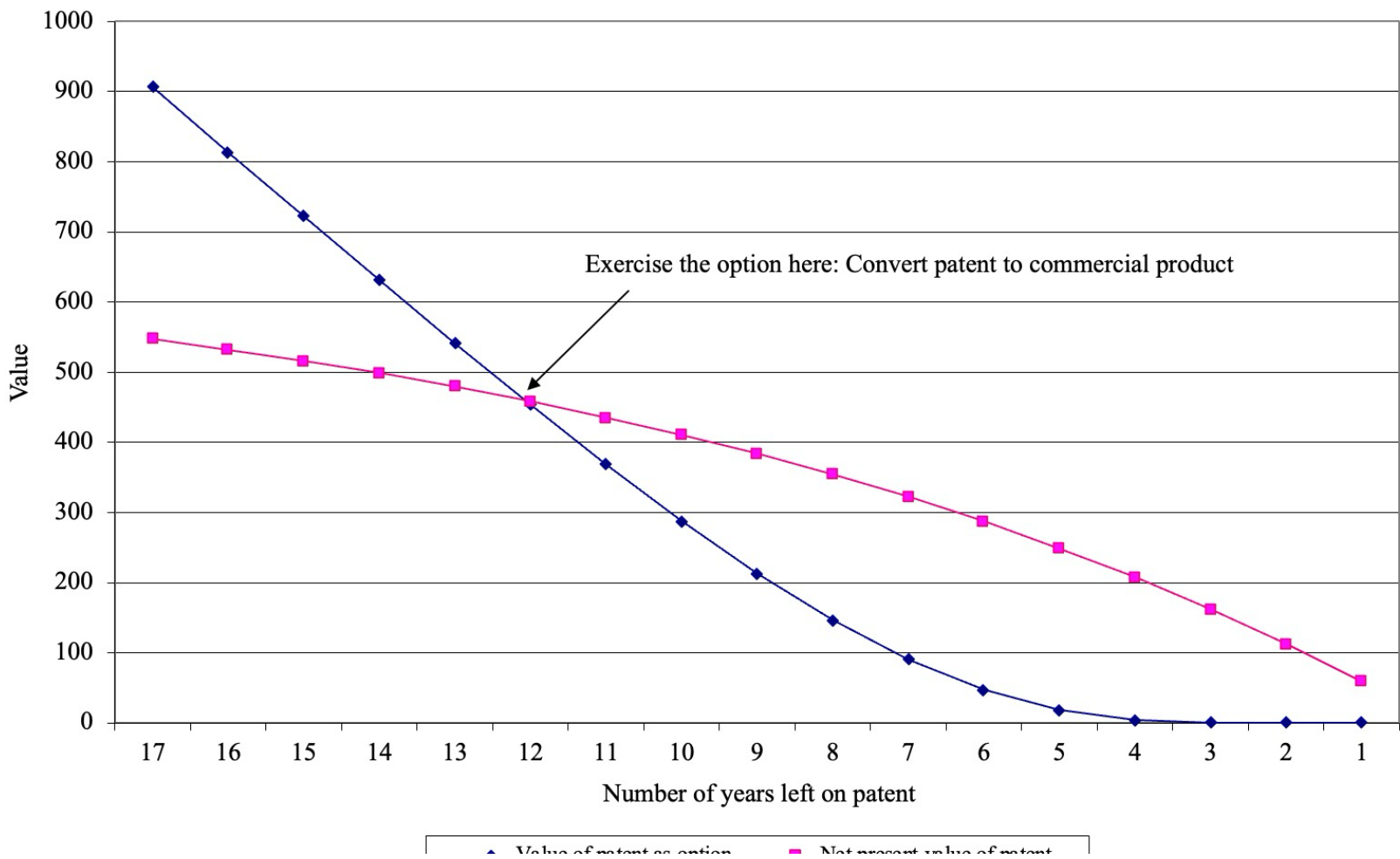
Input	Estimation Process
1. Value of the Underlying Asset	<ul style="list-style-type: none"> • Present Value of Cash Inflows from taking project now • This will be noisy, but that adds value.
2. Variance in value of underlying asset	<ul style="list-style-type: none"> • Variance in cash flows of similar assets or firms • Variance in present value from capital budgeting simulation.
3. Exercise Price on Option	<ul style="list-style-type: none"> • Option is exercised when investment is made. • Cost of making investment on the project; assumed to be constant in present value dollars.
4. Expiration of the Option	<ul style="list-style-type: none"> • Life of the patent
5. Dividend Yield	<ul style="list-style-type: none"> • Cost of delay = Cash flow next year as % of Value of Underlying asset • If cash flows not available, use $1/n$ (one less year or protection from competition).



VALUING A PRODUCT PATENT: AVONEX

- Biogen, a bio-technology firm, has a patent on Avonex, a drug to treat multiple sclerosis, for the next 17 years, and it plans to produce and sell the drug by itself.
- The key inputs on the drug are as follows:
 - PV of Cash Flows from Introducing the Drug Now = S = \$ 3.422 billion
 - PV of Cost of Developing Drug for Commercial Use = K = \$ 2.875 billion
 - Patent Life = t = 17 years Riskless Rate = r = 6.7% (17-year T.Bond rate)
 - Variance in Expected Present Values = s² = 0.224 (Industry average firm variance for bio-tech firms)
 - Expected Cost of Delay = y = 1/17 = 5.89% (since no cash flows are available)
- The output from the option pricing model
 - d1 = 1.1362 N(d1) = 0.8720
 - d2 = -0.8512 N(d2) = 0.2076
 - Call Value = $3,422 \exp^{(-0.0589)(17)} (0.8720) - 2,875 \exp^{(-0.067)(17)} (0.2076) = \$ 907$ million

THE OPTIMAL TIME TO EXERCISE



VALUING A FIRM WITH PATENTS

- The value of a firm with a substantial number of patents can be derived using the option pricing model.
- Value of Firm = Value of commercial products (using DCF value)
 - + Value of existing patents (using option pricing)
 - + (Value of New patents that will be obtained in the future – Cost of obtaining these patents)
- The last input **measures the efficiency of the firm in converting its R&D into commercial products**. If we assume that a firm earns its cost of capital from research, this term will become zero.
- If we use this approach, we should be **careful not to double count** and allow for a high growth rate in cash flows (in the DCF valuation).

VALUE OF BIOGEN'S EXISTING PRODUCTS

- Biogen had two commercial products (a drug to treat Hepatitis B and Intron) at the time of this valuation that it had licensed to other pharmaceutical firms.
- The license fees on these products were expected to generate \$ 50 million in after-tax cash flows each year for the next 12 years.
- To value these cash flows, which were guaranteed contractually, the pre-tax cost of debt of the guarantors was used:
 - Present Value of License Fees = \$ 50 million $(1 - 1.07^{-12})/.07$
= \$ 397.13 million

VALUE OF BIOGEN'S FUTURE R&D

- Biogen continued to fund research into new products, spending about \$ 100 million on R&D in the most recent year. These R&D expenses were expected **to grow 20% a year for the next 10 years**, and 5% thereafter.
- It was assumed that every dollar invested in research would create **\$ 1.25 in value in patents (valued using the option pricing model described above) for the next 10 years**, and break even after that (i.e., generate \$ 1 in patent value for every \$ 1 invested in R&D).
- There was a significant amount of risk associated with this component and the cost of capital was estimated to be 15%.

VALUE OF FUTURE R&D

Yr	Value of Patents PV (at 15%)	R&D Cost	Excess Value	
1	\$ 150.00	\$ 120.00	\$ 30.00	\$ 26.09
2	\$ 180.00	\$ 144.00	\$ 36.00	\$ 27.22
3	\$ 216.00	\$ 172.80	\$ 43.20	\$ 28.40
4	\$ 259.20	\$ 207.36	\$ 51.84	\$ 29.64
5	\$ 311.04	\$ 248.83	\$ 62.21	\$ 30.93
6	\$ 373.25	\$ 298.60	\$ 74.65	\$ 32.27
7	\$ 447.90	\$ 358.32	\$ 89.58	\$ 33.68
8	\$ 537.48	\$ 429.98	\$ 107.50	\$ 35.14
9	\$ 644.97	\$ 515.98	\$ 128.99	\$ 36.67
10	\$ 773.97	\$ 619.17	\$ 154.79	\$ 38.26
				\$ 318.30

VALUE OF BIOGEN

- The value of Biogen as a firm is the sum of all three components – the present value of cash flows from existing products, the value of Avonex (as an option) and the value created by new research:

$$\begin{aligned}\text{Value} &= \text{Existing products} + \text{Existing Patents} + \text{Value: Future R\&D} \\ &= \$ 397.13 \text{ million} + \$ 907 \text{ million} + \$ 318.30 \text{ million} \\ &= \$1622.43 \text{ million}\end{aligned}$$

- Since Biogen had no debt outstanding, this value was divided by the number of shares outstanding (35.50 million) to arrive at a value per share:
 - Value per share = \$ 1,622.43 million / 35.5 = \$ 45.70

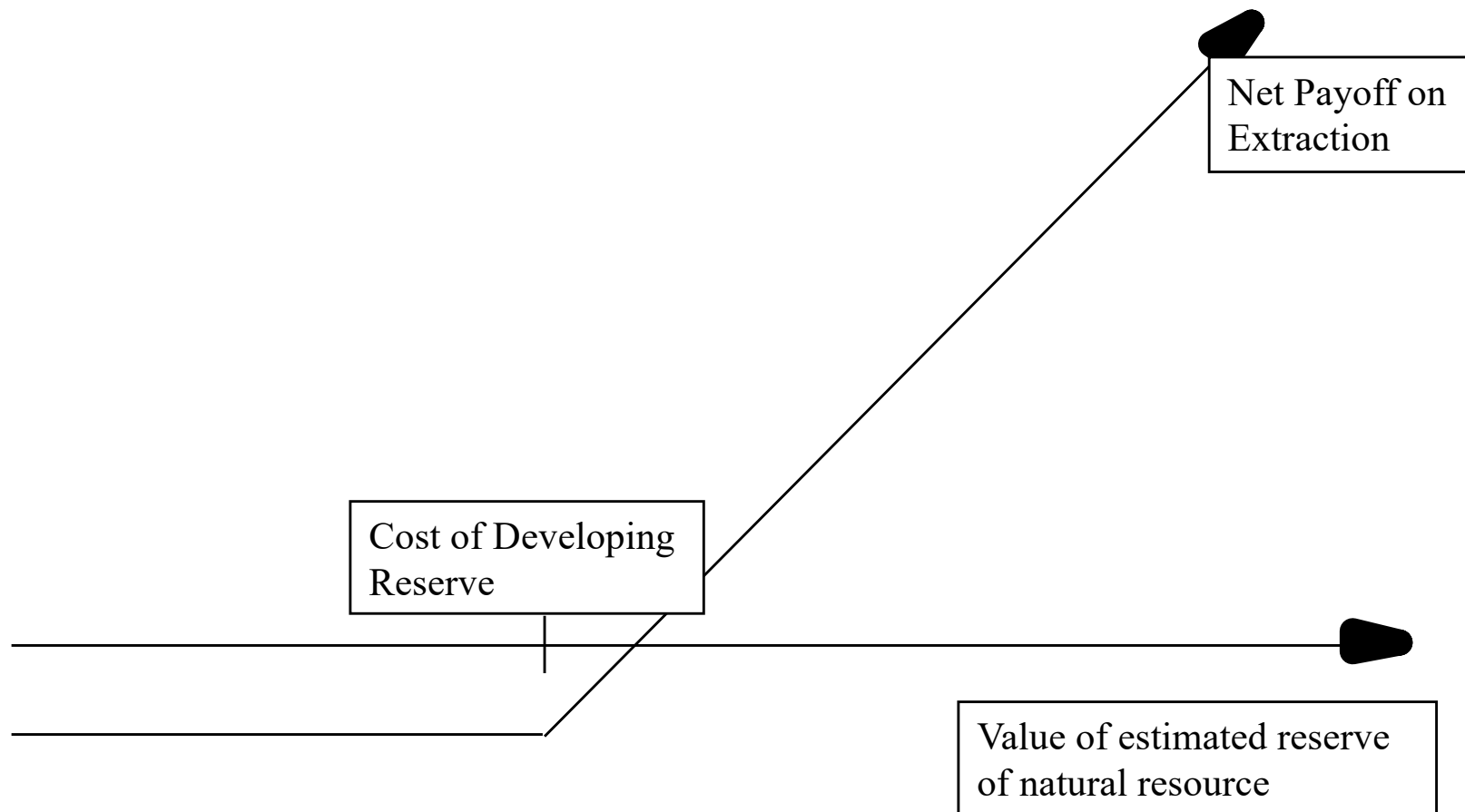
THE REAL OPTIONS TEST: PATENTS AND TECHNOLOGY

- **The Option Test:**
 - Underlying Asset: Product that would be generated by the patent
 - Contingency:
 - If PV of CFs from development $>$ Cost of development: $PV - Cost$
 - If PV of CFs from development $<$ Cost of development: 0
- **The Exclusivity Test:**
 - Patents restrict competitors from developing similar products
 - Patents do not restrict competitors from developing other products to treat the same disease.
- **The Pricing Test**
 - **Underlying Asset:** Patents are not traded. Not only do you therefore have to estimate the present values and volatilities yourself, you cannot construct replicating positions or do arbitrage.
 - **Option:** Patents are bought and sold, though not as frequently as oil reserves or mines.
 - **Cost of Exercising the Option:** This is the cost of converting the patent for commercial production. Here, experience does help and drug firms can make fairly precise estimates of the cost.
- **Conclusion: Option exists but option pricing models are stretched.**

EXAMPLE 2: VALUING NATURAL RESOURCE OPTIONS

- In a natural resource investment, the underlying asset is the resource, and the value of the asset is based upon two variables - **the quantity of the resource** that is available in the investment and the **price of the resource**.
- In most such investments, there is a cost associated with developing the resource, and the **difference between the value of the asset extracted and the cost of the development** is the profit to the owner of the resource.
- Defining the cost of development as X , and the estimated value of the resource as V , the potential payoffs on a natural resource option can be written as follows:
 - Payoff on natural resource investment $= V - X$ if $V > X$
 $= 0$ if $V \leq X$

PAYOFF DIAGRAM ON NATURAL RESOURCE FIRMS



ESTIMATING INPUTS FOR NATURAL RESOURCE OPTIONS

Input	Estimation Process
1. Value of Available Reserves of the Resource	<ul style="list-style-type: none"> • Expert estimates (Geologists for oil.); The present value of the after-tax cash flows from the resource are then estimated.
2. Cost of Developing Reserve (Strike Price)	<ul style="list-style-type: none"> • Past costs and the specifics of the investment
3. Time to Expiration	<ul style="list-style-type: none"> • Relinquishment Period: if asset has to be relinquished at a point in time. • Time to exhaust inventory - based upon inventory and capacity output.
4. Variance in value of underlying asset	<ul style="list-style-type: none"> • based upon variability of the price of the resources and variability of available reserves.
5. Net Production Revenue (Dividend Yield)	<ul style="list-style-type: none"> • Net production revenue every year as percent of market value.
6. Development Lag	<ul style="list-style-type: none"> • Calculate present value of reserve based upon the lag.



VALUING GULF OIL

- Gulf Oil was the target of a takeover in early 1984 at \$70 per share (It had 165.30 million shares outstanding, and total debt of \$9.9 billion).
 - It had **estimated reserves of 3038 million barrels of oil** and the **average cost of developing these reserves was estimated to be \$10 a barrel in present value dollars** (The development lag is approximately two years).
 - The average relinquishment life of the **reserves is 12 years**.
 - The **price of oil was \$22.38 per barrel**, and the production cost, taxes and royalties were estimated at \$7 per barrel.
 - The **treasury bond rate** at the time of the analysis was 9.00%.
 - Gulf was expected to have **net production revenues each year of approximately 5% of the value** of the developed reserves. The variance in oil prices is 0.03.

VALUING UNDEVELOPED RESERVES

- Inputs for valuing undeveloped reserves
 - Value of underlying asset = Value of estimated reserves discounted back for period of development lag = $3038 * (\$ 22.38 - \$7) / 1.052 = \$42,380.44$
 - Exercise price = Estimated development cost of reserves = $3038 * \$10 = \$30,380 \text{ million}$
 - Time to expiration = Average length of relinquishment option = **12 years**
 - Variance in value of asset = **Variance in oil prices = 0.03**
 - Riskless interest rate = 9%
 - Cost of delay = Expected CF next year / Value of developed reserves = 5%
- Based upon these inputs, the Black-Scholes model provides the following value for the call:
 - $d1 = 1.6548$ $N(d1) = 0.9510$
 - $d2 = 1.0548$ $N(d2) = 0.8542$
 - Call Value = $42,380.44 \exp^{(-0.05)(12)} (0.9510) - 30,380 (\exp^{(-0.09)(12)} (0.8542))$
 - = \$ 13,306 million

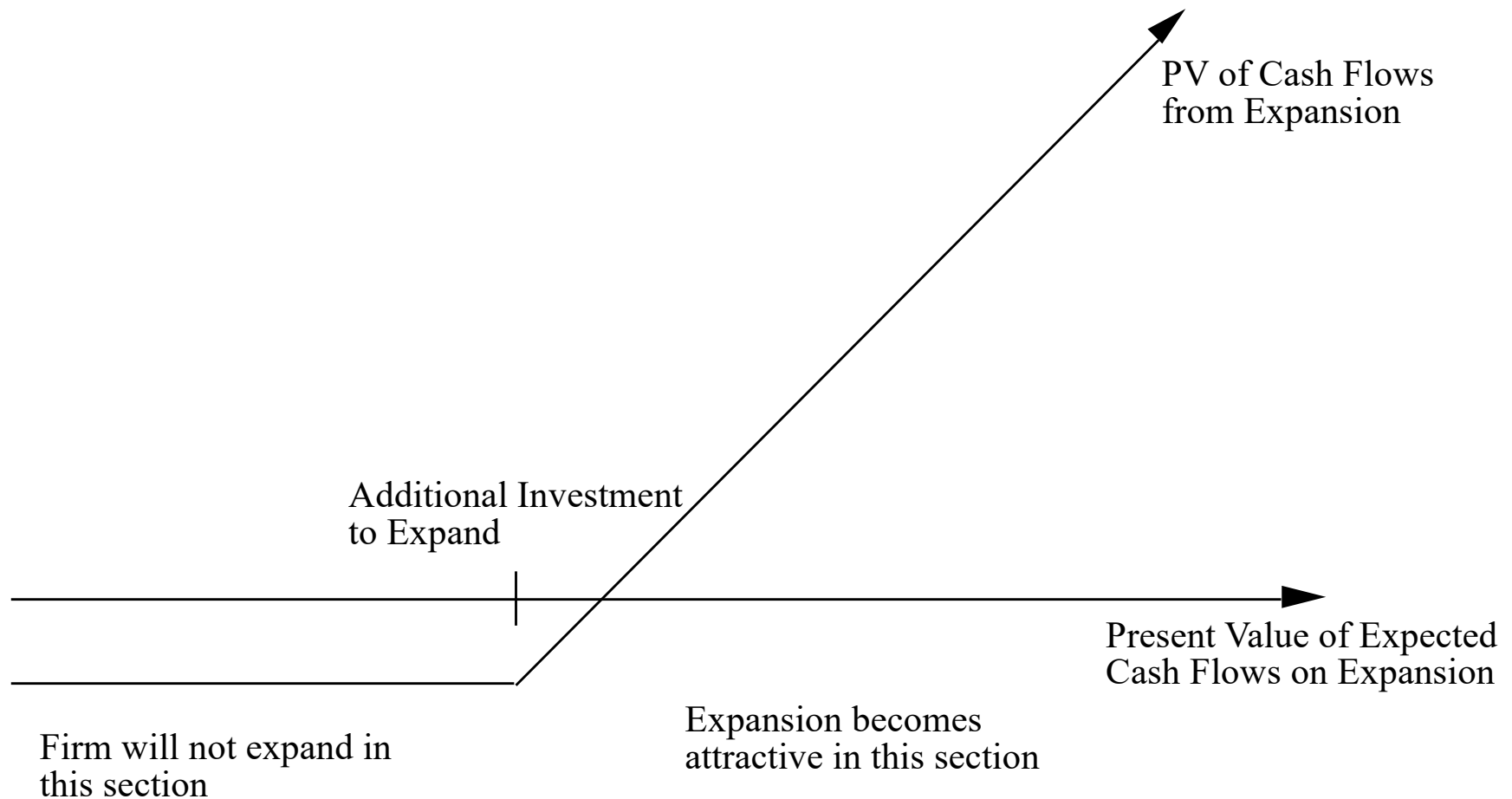
VALUING GULF OIL

- In addition, Gulf Oil had free cashflows to the firm from its oil and gas production of \$915 million from already developed reserves and these cashflows are likely to continue for ten years (the remaining lifetime of developed reserves).
- The present value of these developed reserves, discounted at the weighted average cost of capital of 12.5%, yields:
 - Value of already developed reserves = $915 (1 - 1.125^{-10}) / .125 = \5065.83
- Adding the value of the developed and undeveloped reserves
 - Value of undeveloped reserves = \$ 13,306 million
 - Value of production in place = \$ 5,066 million
 - Total value of firm = \$ 18,372 million
 - Less Outstanding Debt = \$ 9,900 million
 - Value of Equity = \$ 8,472 million
 - Value per share = \$ 8,472 / 165.3 = \$51.25

B. THE OPTION TO EXPAND/TAKE OTHER PROJECTS

- Taking a project today may **allow access to other valuable projects** in the future.
- Thus, even though a project may have a negative NPV, it may be a project worth taking **if the option it provides the firm (to take other projects in the future) provides a more-than-compensating value.**
- These are the options that firms **often call “strategic options”** and use as a rationale for taking on “negative NPV” or even “negative return” projects.

THE OPTION TO EXPAND



THE OPTION TO EXPAND: VALUING A YOUNG, START-UP COMPANY

- You have complete a DCF valuation of a small anti-virus software company, Secure Mail, and estimated a value of \$115 million.
- Assume that there is the possibility that the company could use the customer base that it develops for the anti-virus software and the technology on which the software is based to create a database software program sometime in the next 5 years.
 - It will **cost Secure Mail about \$500 million** to develop a new database program, if they decided to do it today.
 - Based upon the information you have now on the potential for a database program, the company can **expect to generate about \$ 40 million a year in after-tax cashflows for ten years**. The cost of capital for private companies that provide database software is **12%**.
 - The **annualized standard deviation** in firm value at publicly traded database companies is **50%**.
 - The **five-year treasury bond rate is 3%**.

VALUING THE EXPANSION OPTION

- S = Value of entering the database software market
= PV of \$40 million for 10 years @12% = \$226 million
- K = Exercise price
= Cost of entering the database software market = \$ 500 million
- t = Period over which you have the right to enter the market
= 5 years
- s = Standard deviation of stock prices of database firms = 50%
- r = Riskless rate = 3%

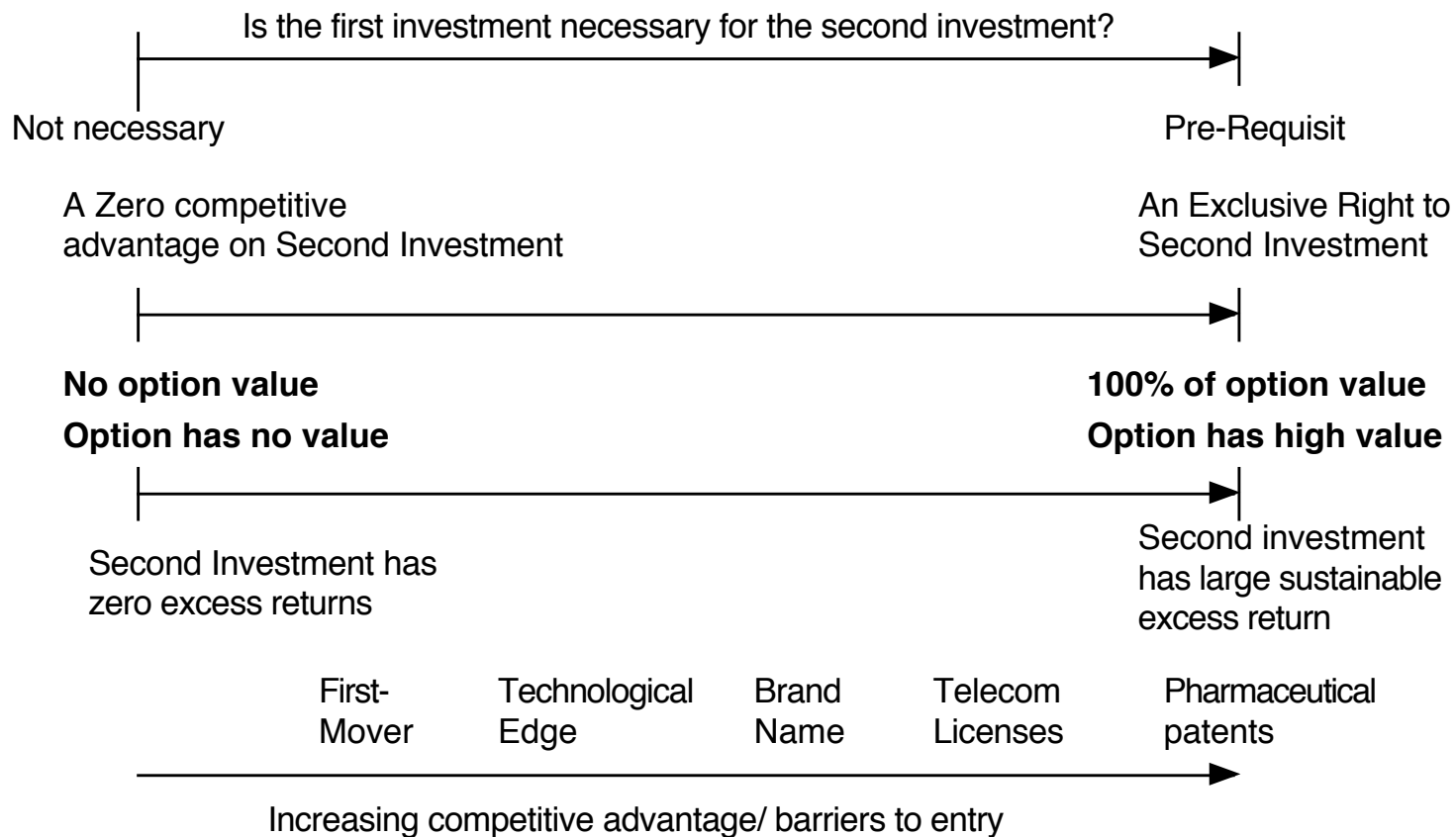
▪ Call Value= \$ 56 Million

DCF valuation of the firm = \$ 115 million

Value of Option to Expand to Database market = \$ 56 million

Value of the company with option to expand = \$ 171 million

A NOTE OF CAUTION: OPPORTUNITIES ARE NOT OPTIONS...

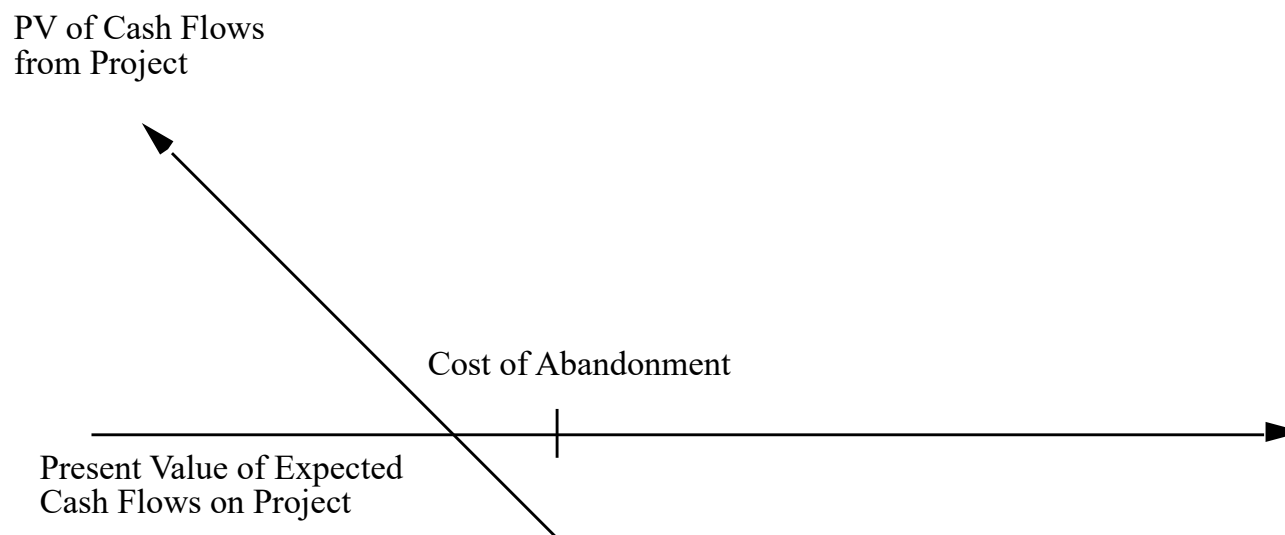


THE REAL OPTIONS TEST FOR EXPANSION OPTIONS

- **The Options Test**
 - Underlying Asset: Expansion Project
 - Contingency
 - If PV of CF from expansion $>$ Expansion Cost: $PV - \text{Expansion Cost}$
 - If PV of CF from expansion $<$ Expansion Cost: 0
- **The Exclusivity Test**
 - Barriers may range from strong (exclusive licenses granted by the government) to weaker (brand name, knowledge of the market) to weakest (first mover).
- **The Pricing Test**
 - Underlying Asset: As with patents, there is no trading in the underlying asset and you have to estimate value and volatility.
 - Option: Licenses are sometimes bought and sold, but more diffuse expansion options are not.
 - Cost of Exercising the Option: Not known with any precision and may itself evolve over time as the market evolves.
- Using option pricing models to value expansion options will not only yield extremely noisy estimates, but may attach inappropriate premiums to discounted cashflow estimates.

C. THE OPTION TO ABANDON

- A firm may sometimes have the **option to abandon a project**, if the cash flows do not measure up to expectations.
- If abandoning the project **allows the firm to save itself from further losses**, this option can make a project more valuable.



VALUING THE OPTION TO ABANDON

- Airbus is considering a **joint venture with Lear Aircraft** to produce a small commercial airplane (capable of carrying 40-50 passengers on short haul flights)
 - Airbus will have to **invest \$ 500 million for a 50% share of the venture**
 - Its share of the **present value of expected cash flows is 480 million.**
- Lear Aircraft, which is eager to enter into the deal, offers to buy Airbus's 50% share of the investment anytime over the next five years **for \$ 400 million**, if Airbus decides to get out of the venture.
- A simulation of the cash flows on this time share investment yields a **variance** in the present value of the cash flows from being in the partnership **is 0.16.**
- The project has a **life of 30 years.**

PROJECT WITH OPTION TO ABANDON

- Value of the Underlying Asset (S)
= PV of Cash Flows from Project = \$ 480 million
- Strike Price (K)
= Salvage Value from Abandonment = \$ 400 million
- Variance in Underlying Asset's Value = 0.16
- Time to expiration = Life of the Project = 5 years
- Dividend Yield = **1/Life of the Project = 1/30 = 0.033**
- The five-year riskless rate is 6%.

SHOULD AIRBUS ENTER INTO THE JOINT VENTURE?

- Value of Put = $Ke^{-rt} (1-N(d2)) - Se^{-yt} (1-N(d1))$
= $400 \exp^{(-0.06)(5)} (1-0.4624) - 480 \exp^{(-0.033)(5)} (1-0.7882)$
= \$ 73.23 million
- The value of this abandonment option has to be added on to the net present value of the project of -\$ 20 million, yielding a total net present value with the abandonment option of \$ 53.23 million.
- While this is what Lear Aircraft wants from the deal, it has to have a large enough net present value of the cost of the put option.

IMPLICATIONS FOR INVESTMENT ANALYSIS/ VALUATION

- Having a option to abandon a project can **make otherwise unacceptable projects acceptable.**
- Other things remaining equal, you would attach more value to companies with
 - **More cost flexibility**, that is, making more of the costs of the projects into variable costs as opposed to fixed costs.
 - **Fewer long-term contracts/obligations** with employees and customers, since these add to the cost of abandoning a project.
- These actions will undoubtedly cost the firm some value, but this has to be **weighed off against the increase in the value of the abandonment option.**

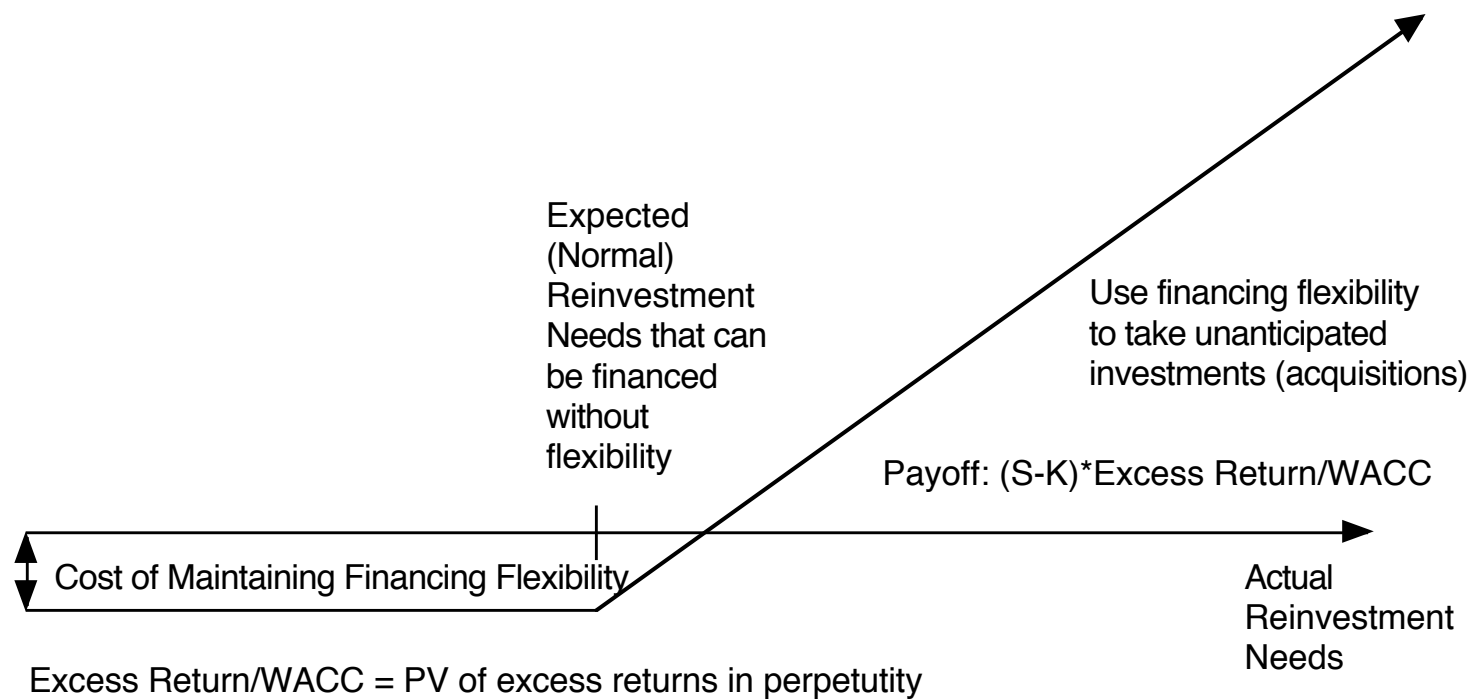
D. OPTIONS IN CAPITAL STRUCTURE

- The most direct applications of option pricing in capital structure decisions is in the **design of securities**. In fact, most complex financial instruments can be broken down into some combination of a simple bond/common stock and a variety of options.
 - If these securities are **to be issued to the public**, and traded, the options must be priced.
 - If these are **non-traded instruments** (bank loans, for instance), they still have to be priced into the interest rate on the instrument.
- The other application of option pricing is in valuing flexibility. Often, firms preserve debt capacity or hold back on issuing debt because they want to maintain flexibility.

THE VALUE OF FLEXIBILITY

- Firms **maintain excess debt capacity or larger cash balances** than are warranted by current needs, to meet **unexpected future requirements**.
- While **maintaining this financing flexibility has value** to firms, it also has a cost; the excess debt capacity implies that the firm is **giving up some value and has a higher cost of capital**.
- The **value of flexibility can be analyzed using the option pricing framework**; a firm maintains large cash balances and excess debt capacity in order to have the option to take projects that might arise in the future.

THE VALUE OF FLEXIBILITY



DISNEY'S OPTIMAL DEBT RATIO

Debt Ratio	Cost of Equity	Cost of Debt	Cost of Capital
0.00%	13.00%	4.61%	13.00%
10.00%	13.43%	4.61%	12.55%
Current: 18%	13.85%	4.80%	12.22%
20.00%	13.96%	4.99%	12.17%
30.00%	14.65%	5.28%	11.84%
40.00%	15.56%	5.76%	11.64%
50.00%	16.85%	6.56%	11.70%
60.00%	18.77%	7.68%	12.11%
70.00%	21.97%	7.68%	11.97%
80.00%	28.95%	7.97%	12.17%
90.00%	52.14%	9.42%	13.69%

INPUTS TO OPTION VALUATION MODEL- DISNEY

Model input	Estimated as	In general...	For Disney
S	Expected annual reinvestment needs (as % of firm value)	Measures magnitude of reinvestment needs	Average of Reinvestment/ Value over last 5 years = 5.3%
s^2	Variance in annual reinvestment needs	Measures how much volatility there is in investment needs.	Variance over last 5 years in $\ln(\text{Reinvestment/Value}) = 0.375$
K	(Internal + Normal access to external funds)/ Value	Measures the capital constraint	Average over last 5 years = 4.8%
T	1 year	Measures an annual value for flexibility	T = 1

VALUING FLEXIBILITY AT DISNEY

- The value of an option with these characteristics is 1.6092%. You can consider this the value of the option to take a project, but the overall value of flexibility will still depend upon the quality of the projects taken.
- Disney earns **18.69% on its projects** has a **cost of capital of 12.22%**. The excess return (annually) is 6.47%. Assuming that they can continue to generate these excess returns in perpetuity:
 - Value of Flexibility (annual)
= 1.6092%(.0647/.1222) = 0.85 % of value
- Disney's cost of capital at its optimal debt ratio is 11.64%. The **cost it incurs to maintain flexibility is therefore 0.58% annually** (12.22%-11.64%). It therefore pays to maintain flexibility.

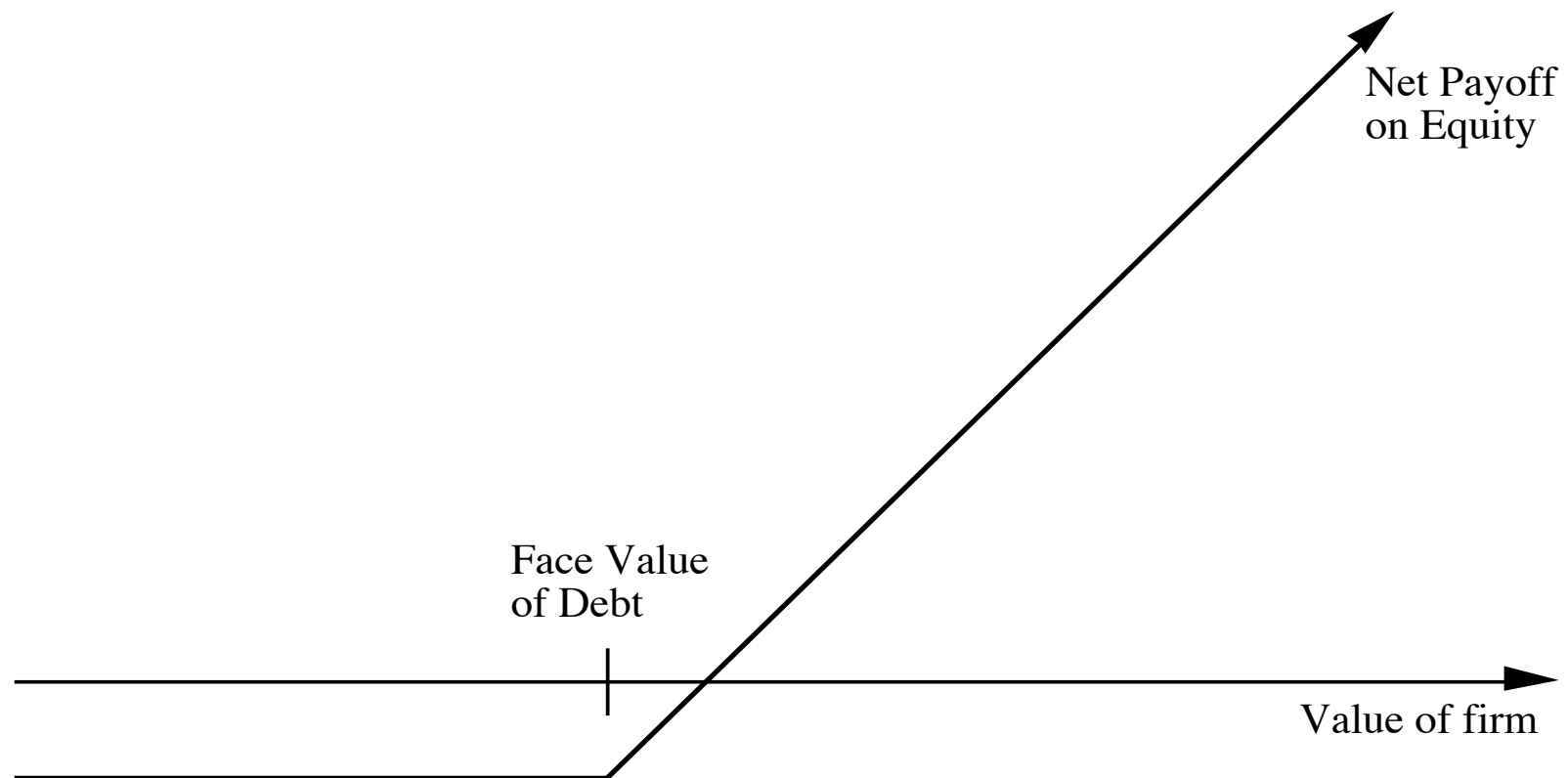
DETERMINANTS OF THE VALUE OF FLEXIBILITY

- **Capital Constraints (External and Internal):** The greater the capacity to raise funds, either internally or externally, the less the value of flexibility.
 - 1.1: Firms with significant internal operating cash flows should value flexibility less than firms with small or negative operating cash flows.
 - 1.2: Firms with easy access to financial markets should have a lower value for flexibility than firms without that access.
- **Unpredictability of reinvestment needs:** The more unpredictable the reinvestment needs of a firm, the greater the value of flexibility.
- **Capacity to earn excess returns:** The greater the capacity to earn excess returns, the greater the value of flexibility.
 - 1.3: Firms that do not have the capacity to earn or sustain excess returns get no value from flexibility.

E. VALUING EQUITY AS AN OPTION

- The **equity in a firm is a residual claim**, i.e., equity holders lay claim to all cashflows left over after other financial claim-holders (debt, preferred stock etc.) have been satisfied.
- If a firm is liquidated, the same principle applies, with **equity investors receiving whatever is left over in the firm** after all outstanding debts and other financial claims are paid off.
- The **principle of limited liability, however, protects equity investors** in publicly traded firms if the value of the firm is less than the value of the outstanding debt, and they cannot lose more than their investment in the firm.

PAYOFF DIAGRAM FOR LIQUIDATION OPTION



APPLICATION TO VALUATION: A SIMPLE EXAMPLE

- Assume that you have a firm whose assets are **currently valued at \$100 million** and that the **standard deviation in this asset value is 40%**.
- Further, assume that the **face value of debt is \$80 million** (It is zero coupon debt with 10 years left to maturity).
- If the ten-year treasury bond rate is 10%,
 - how much is the **equity worth**?
 - What should the **interest rate on debt** be?

MODEL PARAMETERS

- Value of the underlying asset = S
 - Value of the firm = \$ 100 million
- Exercise price = K
 - Face Value of outstanding debt = \$ 80 million
- Life of the option = t
 - Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2
 - Variance in firm value = 0.16
- Riskless rate = r
 - Treasury bond rate corresponding to option life = 10%

VALUING EQUITY AS A CALL OPTION

- Based upon these inputs, the Black-Scholes model provides the following value for the call:
 - $d1 = 1.5994$ $N(d1) = 0.9451$
 - $d2 = 0.3345$ $N(d2) = 0.6310$
- Value of the call = $100 (0.9451) - 80 \exp^{(-0.10)(10)} (0.6310) = \75.94 million
- Value of the outstanding debt = $\$100 - \$75.94 = \$24.06$ million
- Interest rate on debt = $(\$ 80 / \$24.06)^{1/10} - 1 = 12.77\%$

I. THE EFFECT OF CATASTROPHIC DROPS IN VALUE

- Assume now that a catastrophe wipes out half the value of this firm (the value drops to \$ 50 million), while the face value of the debt remains at \$ 80 million. What will happen to the equity value of this firm?
 - a. It will drop in value to \$ 25.94 million [\$ 50 million - market value of debt from previous page]
 - b. It will be worth nothing since debt outstanding > Firm Value
 - c. It will be worth more than \$ 25.94 million

VALUING EQUITY IN THE TROUBLED FIRM

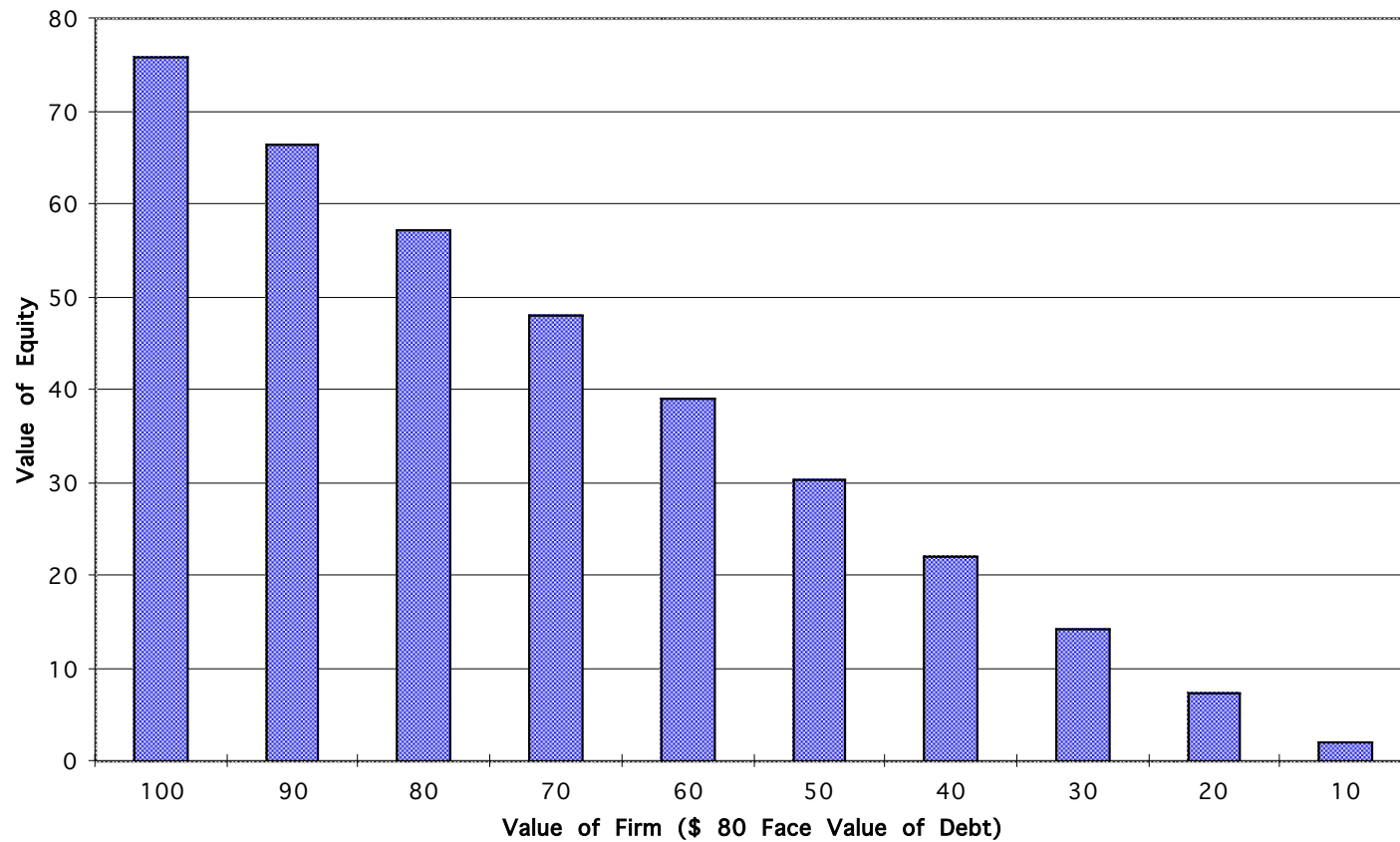
- Value of the underlying asset = S
 - Value of the firm = \$ 50 million
- Exercise price = K
 - Face Value of outstanding debt = \$ 80 million
- Life of the option = t
 - Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2
 - Variance in firm value = 0.16
- Riskless rate = r
 - Treasury bond rate corresponding to option life = 10%

THE VALUE OF EQUITY AS AN OPTION

- Based upon these inputs, the Black-Scholes model provides the following value for the call:
 - $d1 = 1.0515$ $N(d1) = 0.8534$
 - $d2 = -0.2135$ $N(d2) = 0.4155$
- Value of the call = $50 (0.8534) - 80 \exp^{(-0.10)(10)} (0.4155) = \30.44 million
- Value of the bond = $\$50 - \$30.44 = \$19.56$ million
- The **equity in this firm drops by \$45.50 million**, less than the overall drop in value of \$50 million, because of the option characteristics of equity.
- This might explain why stock in firms, which are in Chapter 11 and essentially bankrupt, still has value.

EQUITY VALUE PERSISTS ..

Value of Equity as Firm Value Changes



II. THE CONFLICT BETWEEN STOCKHOLDERS AND BONDHOLDERS

- Consider again the firm described in the earlier example , with a value of assets of \$100 million, a face value of zero-coupon ten-year debt of \$80 million, a standard deviation in the value of the firm of 40%. The equity and debt in this firm were valued as follows:
 - Value of Equity = \$75.94 million
 - Value of Debt = \$24.06 million
 - Value of Firm == \$100 million
- Now assume that the stockholders have the opportunity to take a project with a **negative net present value of -\$2 million**, but assume that this project is a **very risky project that will push up the standard deviation in firm value to 50%**. Would you invest in this project?
 - Yes
 - No

VALUING EQUITY AFTER THE PROJECT

- Value of the underlying asset = S
 - Value of the firm = \$ 100 million - \$2 million = \$ 98 million (The value of the firm is lowered because of the negative net present value project)
- Exercise price = K
 - Face Value of outstanding debt = \$ 80 million
- Life of the option = t
- Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2
 - Variance in firm value = 0.25
- Riskless rate = r
 - Treasury bond rate corresponding to option life = 10%

OPTION VALUATION

- **Option Pricing Results for Equity and Debt Value**
 - Value of Equity = \$77.71
 - Value of Debt = \$20.29
 - Value of Firm = \$98.00
- The value of equity rises from \$75.94 million to \$77.71 million, even though the firm value declines by \$2 million. The increase in equity value comes at the expense of bondholders, who find their wealth decline from \$24.06 million to \$20.19 million.

EFFECTS OF AN ACQUISITION

- Assume that you are the manager of a firm and that you buy another firm, with a fair market value of \$ 150 million, for exactly \$ 150 million. In an efficient market, the stock price of your firm will
 - Increase
 - Decrease
 - Remain Unchanged

EFFECTS ON EQUITY OF A CONGLOMERATE MERGER

- You are provided information on two firms, which operate in unrelated businesses and hope to merge.

	Firm A	Firm B
Value of the firm	\$100 million	\$ 150 million
Face Value of Debt (10 yr zeros)	\$ 80 million	\$ 50 million
Maturity of debt	10 years	10 years
Std. Dev. in value	40 %	50 %
Correlation between cashflows	0.4	

- The ten-year bond rate is 10%.
- The variance in the value of the firm after the acquisition can be calculated as follows:
 - Variance in combined firm value
$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho^{12} \sigma_1 \sigma_2$$

$$= (0.4)^2 (0.16) + (0.6)^2 (0.25) + 2 (0.4) (0.6) (0.4) (0.4) (0.5)$$

$$= 0.154$$

VALUING THE COMBINED FIRM

- The values of equity and debt in the individual firms and the combined firm can then be estimated using the option pricing model:

	<i>Firm A</i>	<i>Firm B</i>	<i>Combined firm</i>
Value of equity in the firm	\$75.94	\$134.47	\$ 207.43
Value of debt in the firm	\$24.06	\$ 15.53	\$ 42.57
Value of the firm	\$100.00	\$150.00	\$ 250.00

- The combined value of the equity prior to the merger is \$ 210.41 million and it declines to \$207.43 million after.
- The wealth of the bondholders increases by an equal amount.
- There is a **transfer of wealth from stockholders to bondholders, as a consequence of the merger**. Thus, conglomerate mergers that are not followed by increases in leverage are likely to see this redistribution of wealth occur across claim holders in the firm.

OBTAINING OPTION PRICING INPUTS - SOME REAL WORLD PROBLEMS

- The examples that have been used to illustrate the use of option pricing theory to value equity have made some simplifying assumptions. Among them are the following:
 - 1) There were **only two claim holders** in the firm - debt and equity.
 - 2) There **is only one issue of debt** outstanding, and it can be retired at face value.
 - 3) The debt has a **zero coupon** and no special features (convertibility, put clauses etc.)
 - 4) The **value of the firm and the variance in that value can be estimated.**

REAL WORLD APPROACHES TO VALUING EQUITY IN TROUBLED FIRMS: GETTING INPUTS

Input	Estimation Process
Value of the Firm	<ul style="list-style-type: none"> • Cumulate market values of equity and debt (or) • Value the <u>assets in place</u> using FCFF and WACC (or) • Use cumulated market value of assets, if traded.
Variance in Firm Value	<ul style="list-style-type: none"> • If stocks and bonds are traded, $\sigma^2_{\text{firm}} = w_e^2 \sigma_e^2 + w_d^2 \sigma_d^2 + 2 w_e w_d \rho_{ed} \sigma_e \sigma_d$ where σ_e^2 = variance in the stock price w_e = MV weight of Equity σ_d^2 = the variance in the bond price w_d = MV weight of debt • If not traded, use variances of similarly rated bonds. • Use average firm value variance from the industry in which company operates.
Value of the Debt	<ul style="list-style-type: none"> • If the debt is short term, you can use only the face or book value of the debt. • If the debt is long term and coupon bearing, add the cumulated nominal value of these coupons to the face value of the debt.
Maturity of the Debt	<ul style="list-style-type: none"> • Face value weighted duration of bonds outstanding (or) • If not available, use weighted maturity



VALUING EQUITY AS AN OPTION - EUROTUNNEL IN EARLY 1998

- Eurotunnel has been a financial disaster since its opening
 - In 1997, Eurotunnel had earnings before interest and taxes of -£56 million and net income of -£685 million
 - At the end of 1997, its book value of equity was -£117 million
- It had £8,865 million in face value of debt outstanding
 - The weighted average duration of this debt was 10.93 years

Debt Type	Face Value	Duration
Short term	935	0.50
10 year	2435	6.7
20 year	3555	12.6
Longer	1940	18.2
Total	£8,865 mil	10.93 years

THE BASIC DCF VALUATION

- The value of the firm estimated using projected cashflows to the firm, discounted at the weighted average cost of capital was £2,312 million.
- This was based upon the following assumptions –
 - Revenues will grow 5% a year in perpetuity.
 - The COGS which is **currently 85% of revenues will drop to 65% of revenues in yr 5** and stay at that level.
 - Capital spending and depreciation **will grow 5% a year in perpetuity.**
 - There are no working capital requirements.
 - The debt ratio, which is **currently 95.35%, will drop to 70%** after year 5. The cost of debt is 10% in high growth period and 8% after that.
 - The beta for the stock **will be 1.10 for the next five years**, and drop to 0.8 after the next 5 years.
 - The long term bond rate is 6%.

OTHER INPUTS

- The stock has been traded on the London Exchange, and the **annualized std deviation based upon ln (prices) is 41%.**
- There are Eurotunnel bonds, that have been traded; the annualized std deviation in ln(price) for the bonds is 17%.
 - The correlation between stock price and bond price changes has been 0.5. The proportion of debt in the capital structure during the period (1992-1996) was 85%.
 - Annualized variance in firm value
 - $= (0.15)^2 (0.41)^2 + (0.85)^2 (0.17)^2 + 2 (0.15) (0.85)(0.5)(0.41)(0.17) = 0.0335$
- The 15-year bond rate is 6%. (I used a bond with a duration of roughly 11 years to match the life of my option)

VALUING EUROTUNNEL EQUITY AND DEBT

- Inputs to Model
 - Value of the underlying asset = S = Value of the firm = £2,312 million
 - Exercise price = K = Face Value of outstanding debt = £8,865 million
 - Life of the option = t = Weighted average duration of debt = 10.93 years
 - Variance in the value of the underlying asset = s^2 = Variance in firm value = 0.0335
 - Riskless rate = r = Treasury bond rate corresponding to option life = 6%
- Based upon these inputs, the Black-Scholes model provides the following value for the call:
 - $d_1 = -0.8337$ $N(d_1) = 0.2023$
 - $d_2 = -1.4392$ $N(d_2) = 0.0751$
- Value of the call = $2312 (0.2023) - 8,865 \exp^{(-0.06)(10.93)} (0.0751) =$
£122 million
- Appropriate interest rate on debt = $(8865/2190)(1/10.93)-1=$
13.65%

IN CLOSING...

- There are **real options everywhere**.
- **Most of them have no significant economic value** because there is no exclusivity associated with using them.
- When options have significant economic value, the inputs needed to value them in a binomial model can be used in more traditional approaches (decision trees) to yield equivalent value.
- The real value from real options lies in
 - Recognizing that **building in flexibility and escape hatches** into large decisions has value
 - Insights we get on understanding how and why companies behave the way they do **in investment analysis and capital structure choices**.