



DISCOUNT RATES

The D in the DCF..

ESTIMATING INPUTS: DISCOUNT RATES — EXTENDING THE CONSISTENCY RULE

- While discount rates obviously matter in DCF valuation, they don't matter as much as most analysts think they do.
- At an intuitive level, the discount rate used should be consistent with both the riskiness and the type of cashflow being discounted.
 - **Equity versus Firm:** If the cash flows being discounted are cash flows to equity, the appropriate discount rate is a cost of equity. If the cash flows are cash flows to the firm, the appropriate discount rate is the cost of capital.
 - **Currency:** The currency in which the cash flows are estimated should also be the currency in which the discount rate is estimated.
 - **Nominal versus Real:** If the cash flows being discounted are nominal cash flows (i.e., reflect expected inflation), the discount rate should be nominal

RISK IN THE DCF MODEL

Expectation of cash flows across all scenarios, good and bad. Incorporates all risks that affect the asset / business.

Expected Cash Flows

Risk Adjusted Discount Rate

Discount rate should reflect the risk perceived by the marginal investor in the company

$$\boxed{\text{Risk Adjusted Cost of equity}} = \boxed{\text{Risk free rate in the currency of analysis}} + \boxed{\text{Relative risk of company/equity in question}} \times \boxed{\text{Equity Risk Premium required for average risk equity}}$$

NOT ALL RISK IS CREATED EQUAL...

- Estimation versus Economic uncertainty
 - **Estimation uncertainty** reflects the possibility that you could have the “wrong model” or estimated inputs incorrectly within this model.
 - **Economic uncertainty** comes the fact that markets and economies can change over time and that even the best models will fail to capture these unexpected changes.
- Micro uncertainty versus Macro uncertainty
 - **Micro uncertainty** refers to uncertainty about the potential market for a firm’s products, the competition it will face and the quality of its management team.
 - **Macro uncertainty** reflects the reality that your firm’s fortunes can be affected by changes in the macro economic environment.
- Discrete versus continuous uncertainty
 - **Discrete risk** lie dormant for periods but show up at points in time. (Examples: A drug working its way through the FDA pipeline may fail at some stage of the approval process or a company in Venezuela may be nationalized)
 - **Continuous risks** like changes in interest rates or economic growth occur continuously and affect value as they happen.

RISK AND COST OF EQUITY: THE ROLE OF THE MARGINAL INVESTOR

- **Not all risk counts:** While the notion that the cost of equity should be higher for riskier investments and lower for safer investments is intuitive, what risk should be built into the cost of equity is the question.
- **Risk through whose eyes?** While risk is usually defined in terms of the variance of actual returns around an expected return, risk and return models in finance assume that the risk that should be rewarded (and thus built into the discount rate) in valuation should be the risk perceived by the marginal investor in the investment
- **The diversification effect:** Most risk and return models in finance also assume that the marginal investor is well diversified, and that the only risk that he or she perceives in an investment is risk that cannot be diversified away (i.e, market or non-diversifiable risk). In effect, it is primarily economic, macro, continuous risk that should be incorporated into the cost of equity.

THE COST OF EQUITY: COMPETING “ MARKET RISK” MODELS

Model	Expected Return	Inputs Needed
CAPM	$E(R) = R_f + \beta (R_m - R_f)$	Riskfree Rate Beta relative to market portfolio Market Risk Premium
APM	$E(R) = R_f + \sum \beta_j (R_j - R_f)$	Riskfree Rate; # of Factors; Betas relative to each factor Factor risk premiums
Multi factor	$E(R) = R_f + \sum \beta_j (R_j - R_f)$	Riskfree Rate; Macro factors Betas relative to macro factors Macro economic risk premiums
Proxy	$E(R) = a + \sum \beta_j Y_j$	Proxies Regression coefficients

THE PARAMETERS: COST OF EQUITY

- In the CAPM, the cost of equity:
 - $\text{Cost of Equity} = \text{Riskfree Rate} + \text{Equity Beta} * (\text{Equity Risk Premium})$
- In APM or Multi-factor models, you still need a risk free rate, as well as betas and risk premiums to go with each factor.
- To use any risk and return model, you need
 - A **riskfree rate** as a base
 - A single **equity risk premium** (in the CAPM) or **factor risk premiums**, in the the multi-factor models
 - A **beta** (in the CAPM) or **betas** (in multi-factor models)



DISCOUNT RATES I

The Riskfree Rate

Aswath Damodaran

THE RISK FREE RATE: LAYING THE FOUNDATIONS

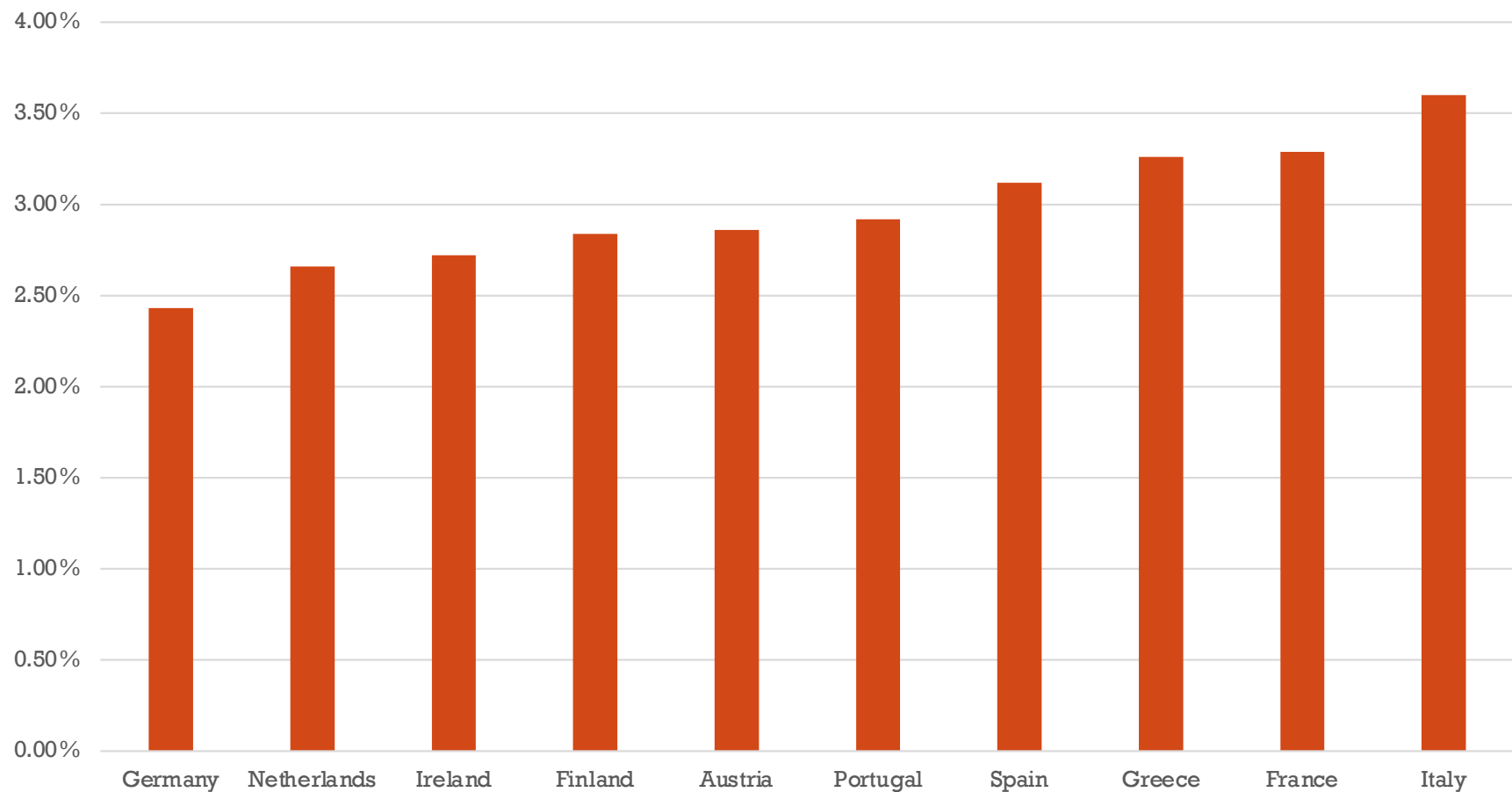
- On a riskfree investment, the actual return is equal to the expected return. Therefore, there is no variance around the expected return.
- For an investment to be riskfree, then, it has to have
 - No default risk
 - No reinvestment risk
- It follows then that if asked to estimate a risk free rate:
 - Time horizon matters: Thus, the riskfree rates in valuation will depend upon when the cash flow is expected to occur and will vary across time.
 - Currencies matter: A risk free rate is currency-specific and can be very different for different currencies.
 - Not all government securities are riskfree: Some governments face default risk and the rates on bonds issued by them will not be riskfree.

TEST 1: A RISKFREE RATE IN US DOLLARS!

- In valuation, we estimate cash flows forever (or at least for very long time periods). The right risk free rate to use in valuing a company in US dollars would be
 - a. A three-month Treasury bill rate (4.36%)
 - b. A ten-year Treasury bond rate (4.58%)
 - c. A thirty-year Treasury bond rate (4.80%)
 - d. A TIPs (inflation-indexed treasury) rate (2.26%)
 - e. The highest of these numbers
 - f. The lowest of these numbers
 - g. Other (Specify)
- What are we implicitly assuming about the US treasury when we use any of the treasury numbers?

TEST 2: A RISKFREE RATE IN EUROS?

Ten-year Euro Government Bond Rates on 1/1/25



TEST 3: A RISKFREE RATE IN INDIAN RUPEES

- The Indian government had 10-year Rupee bonds outstanding, with a yield to maturity of about 6.82% on January 1, 2025.
- In January 2025, the Indian government had a local currency sovereign rating of Baa3. The typical default spread (over a default free rate) for Baa3 rated country bonds in early 2025 was 2.18%. The riskfree rate in Indian Rupees is
 - a. The yield to maturity on the 10-year bond (6.82%)
 - b. The yield to maturity on the 10-year bond + Default spread (4.64%)
 - c. The yield to maturity on the 10-year bond – Default spread (9.00%)
 - d. None of the above

SOVEREIGN DEFAULT SPREAD: THREE PATHS TO THE SAME DESTINATION...

- **Sovereign dollar or euro denominated bonds:** Find sovereign bonds denominated in US dollars, issued by an emerging sovereign.
 - $\text{Default spread} = \text{Emerging Govt Bond Rate (in US \$)} - \text{US Treasury Bond rate with same maturity.}$
- **Sovereign CDS spreads:** Obtain the traded value for a sovereign Credit Default Swap (CDS) for the emerging government.
 - $\text{Default spread} = \text{Sovereign CDS spread (with perhaps an adjustment for CDS market frictions).}$
- **Sovereign-rating based spread:** For countries which don't issue dollar denominated bonds or have a CDS spread, you have to use the average spread for other countries with the same sovereign rating.

APPROACH 1: DEFAULT SPREAD FROM GOVERNMENT BONDS

Country	\$ Bond Rate	Riskfree Rate	Default Spread
	\$ Bonds		
Peru	6.15%	4.58%	1.57%
Brazil	7.75%	4.58%	3.17%
Colombia	6.95%	4.58%	2.37%
Poland	5.35%	4.58%	0.77%
Turkey	13.55%	4.58%	8.97%
Mexico	5.46%	4.58%	0.88%
	Euro Bonds		
Bulgaria	2.86%	2.43%	0.43%

APPROACH 2: CDS SPREADS – JANUARY 2025

Country	CDS Spread	CDS Spread net of US	Country	CDS Spread	CDS Spread net of US	Country	CDS Spread	CDS Spread net of US	Country	CDS Spread	CDS Spread net of US
Abu Dhabi	0.76%	0.35%	Estonia	0.81%	0.40%	Lithuania	0.96%	0.55%	Serbia	1.26%	0.85%
Algeria	1.47%	1.06%	Ethiopia	32.97%	32.56%	Malaysia	0.85%	0.44%	Slovakia	0.55%	0.14%
Angola	6.74%	6.33%	Finland	0.33%	0.00%	Mexico	2.22%	1.81%	Slovenia	0.72%	0.31%
Argentina	10.84%	10.43%	France	0.69%	0.28%	Mongolia	2.89%	2.48%	South Africa	3.00%	2.59%
Australia	0.18%	0.00%	Gabon	9.61%	9.20%	Morocco	1.51%	1.10%	Spain	0.67%	0.26%
Austria	0.26%	0.00%	Germany	0.28%	0.00%	Namibia	3.44%	3.03%	Sri Lanka	NA	NA
Bahrain	2.51%	2.10%	Greece	1.17%	0.76%	Netherlands	0.25%	0.00%	Sweden	0.20%	0.00%
Belgium	0.43%	0.02%	Guatemala	2.33%	1.92%	New Zealand	0.20%	0.00%	Switzerland	0.14%	0.00%
Brazil	3.23%	2.82%	Hong Kong	0.64%	0.23%	Nicaragua	6.57%	6.16%	Thailand	0.70%	0.29%
Bulgaria	1.43%	1.02%	Hungary	1.79%	1.38%	Nigeria	6.44%	6.03%	Tunisia	10.24%	9.83%
Cameroon	7.07%	6.66%	Iceland	0.31%	0.00%	Norway	0.19%	0.00%	Turkey	3.62%	3.21%
Canada	0.38%	0.00%	India	0.95%	0.54%	Oman	1.63%	1.22%	Ukraine	NA	NA
Chile	1.17%	0.76%	Indonesia	1.31%	0.90%	Pakistan	16.49%	16.08%	United Kingdom	0.39%	0.00%
China	0.97%	0.56%	Iraq	4.15%	3.74%	Panama	3.08%	2.67%	United States	0.41%	0.00%
Colombia	3.37%	2.96%	Ireland	0.31%	0.00%	Peru	1.47%	1.06%	Uruguay	1.27%	0.86%
Costa Rica	2.45%	2.04%	Israel	1.44%	1.03%	Philippines	1.21%	0.80%	Venezuela	10.08%	9.67%
Croatia	1.26%	0.85%	Italy	1.18%	0.77%	Poland	1.05%	0.64%	Vietnam	1.65%	1.24%
Cyprus	0.99%	0.58%	Japan	0.33%	0.00%	Portugal	0.58%	0.17%	Zambia	NA	NA
Czech Republic	0.50%	0.09%	Kazakhstan	1.36%	0.95%	Qatar	0.77%	0.36%			
Denmark	0.18%	0.00%	Kenya	5.92%	5.51%	Romania	2.39%	1.98%			
Dubai	1.00%	0.59%	Korea	0.48%	0.07%	Russia	NA	NA			
Ecuador	19.08%	18.67%	Kuwait	0.94%	0.53%	Rwanda	4.55%	4.14%			
Egypt	6.35%	5.94%	Latvia	0.85%	0.44%	Saudi Arabia	1.05%	0.64%			
El Salvador	4.20%	3.79%	Lebanon	NA	NA	Senegal	6.23%	5.82%			

APPROACH 3: TYPICAL DEFAULT SPREADS: JANUARY 2024

S&P Sovereign Rating	Moody's Sovereign Rating	Default Spread
AAA	Aaa	0.00%
AA+	Aa1	0.38%
AA	Aa2	0.46%
AA-	Aa3	0.56%
A+	A1	0.66%
A	A2	0.80%
A-	A3	1.13%
BBB+	Baa1	1.50%
BBB	Baa2	1.79%
BBB-	Baa3	2.07%
BB+	Ba1	2.36%
BB	Ba2	2.83%
BB	Ba3	3.38%
B+	B1	4.24%
B	B2	5.18%
B-	B3	6.12%
CCC+	Caa1	7.06%
CCC	Caa2	8.47%
CCC-	Caa3	9.41%
CC+	Ca1	10.50%
CC	Ca2	11.29%
CC-	Ca3	13.00%
C+	C1	14.50%
C	C2	16.00%
C-	C3	18.00%

GETTING TO A RISK FREE RATE IN BRAZILIAN REAIS ON JANUARY 1, 2024

- The Brazilian government bond rate in nominal reais on January 1, 2025, was 12.30%. To get to a riskfree rate in nominal reais, we can use one of three approaches.
 - **Approach 1: Government Bond spread**
 - Default Spread = Brazil \$ Bond Rate – US T.Bond Rate = 5.75% - 3.88% = 3.17%
 - Riskfree rate in \$R = 12.30% - 3.17% = 9.17%
 - **Approach 2: The CDS Spread**
 - The CDS spread for Brazil, adjusted for the US CDS spread was 2.82%.
 - Riskfree rate in \$R = 12.30% - 2.82% = 9.48%
 - **Approach 3: The Rating based spread**
 - Brazil has a Ba2 local currency rating from Moody's. The default spread for that rating is 2.83%
 - Riskfree rate in \$R = 12.30% - 2.83% = 9.47%

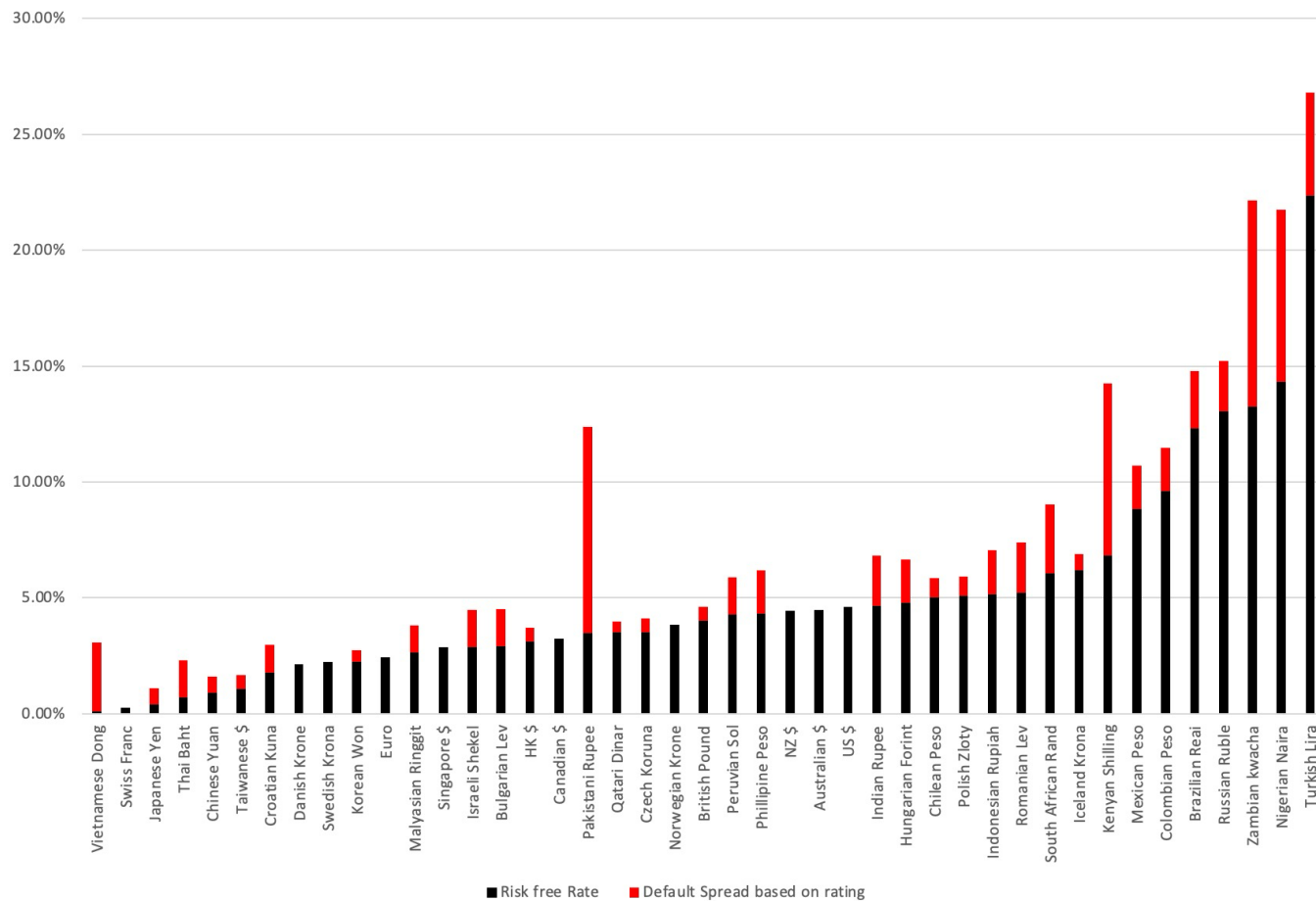
TEST 4: A REAL RISKFREE RATE

- In some cases, you may want a riskfree rate in real terms (in real terms) rather than nominal terms.
- To get a real riskfree rate, you would like a security with no default risk and a guaranteed real return. Treasury indexed securities offer this combination.
- In January 2025, the yield on a 10-year indexed treasury bond was 2.23%. Which of the following statements would you subscribe to?
 - a. This (2.23%) is the real riskfree rate to use, if you are valuing US companies in real terms.
 - b. This (2.23%) is the real riskfree rate to use, anywhere in the world
 - c. Explain.

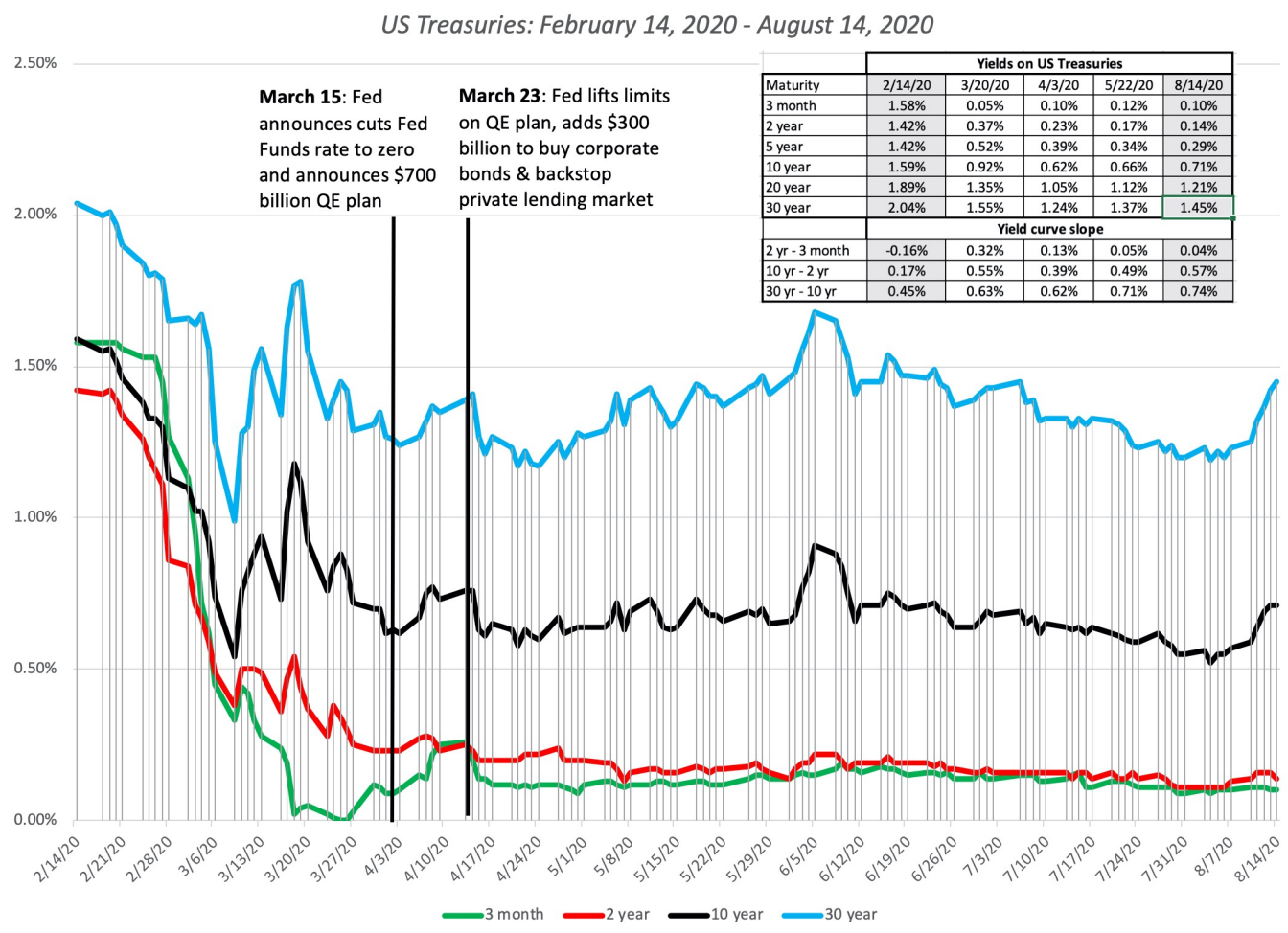
WHY DO RISK FREE RATES VARY ACROSS CURRENCIES?

JANUARY 2025 RISK FREE RATES

Government-bond Based Riskfree Rates in January 2025



OR ACROSS TIME...



RISK FREE RATE: DON'T HAVE OR DON'T TRUST THE GOVERNMENT BOND RATE?

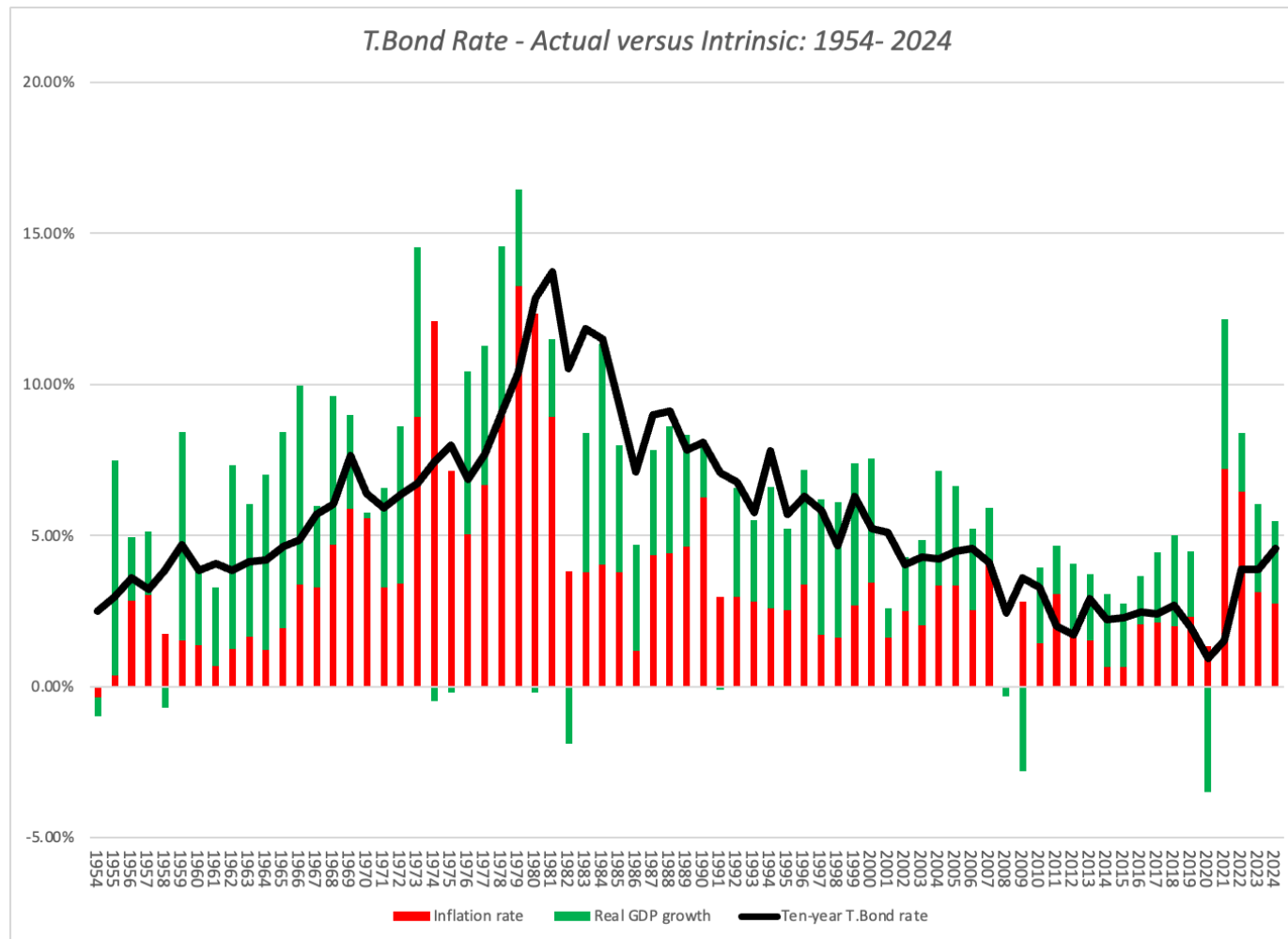
- You can scale up the riskfree rate in a base currency (\$, Euros) by the differential inflation between the base currency and the currency in question. In US \$:
 - Risk free rate_{Currency} = $(1 + \text{Riskfree rate}_{US \$}) \frac{(1 + \text{Expected Inflation}_{\text{Foreign Currency}})}{(1 + \text{Expected Inflation}_{US \$})} - 1$
- Thus, if the US \$ risk free rate is 2.00%, the inflation rate in Egyptian pounds is 15% and the inflation rate in US \$ is 1.5%, the foreign currency risk free rate is as follows:

- Risk free rate = $(1.02) \frac{(1.15)}{(1.015)} - 1 = 15.57\%$

ONE MORE TEST ON RISKFREE RATES...

- On January 1, 2022, the 10-year treasury bond rate in the United States was 1.51%, low by historic standards. Assume that you are valuing a company in US dollars then but are wary about the riskfree rate being too low. Which of the following should you do?
 - a. Replace the current 10-year bond rate with a more reasonable normalized riskfree rate (the average 10-year bond rate over the last 30 years has been about 5-6%)
 - b. Use the current 10-year bond rate as your riskfree rate but make sure that your other assumptions (about growth and inflation) are consistent with the riskfree rate.
 - c. Something else...

SOME PERSPECTIVE ON RISK FREE RATES



NEGATIVE INTEREST RATES?

- In 2022, there were at least three currencies (Swiss Franc, Japanese Yen, Euro) with negative interest rates. Using the fundamentals (inflation and real growth) approach, how would you explain negative interest rates?
 - How negative can rates get? (Is there a lower bound?)
 - Would you use these negative interest rates as risk free rates?
 - a. If no, why not and what would you do instead?
 - b. If yes, what else would you have to do in your valuation to be internally consistent?



DISCOUNT RATES: II

The price of risk

Aswath Damodaran

THE DRIVERS OF EQUITY RISK PREMIUMS

Equity Risk Premium

Risk Aversion

Thesis: As investors become more (less) risk averse, equity risk premiums should rise (fall).

Implication: Markets with aging investors should have higher risk premiums than markets with younger investors.

Economic Uncertainty

Thesis: As uncertainty about the economy increases (decreases), equity risk premiums should increase (decrease).

Implication: Equity risk premiums should rise during economic crises, and be higher in younger & growing economies.

Inflation and Interest Rates

Thesis: As inflation rises (falls), uncertainty about inflation will increase (decrease), pushing up (down) equity risk premiums.

Implication: Equity risk premiums should rise during periods of high and volatile inflation.

Information

Thesis: As corporate disclosures become more (less) informative, equity risk premiums should fall (rise).

Implication: Markets with better disclosure rules and requirements should have lower equity risk premiums than markets without.

Liquidity and Fund Flows

Thesis: As liquidity increases and funds flow into equity markets, equity risk premiums should decrease.

Implication: Events or actions (crises, regulation) that stymie fund flows and liquidity will increase equity risk premiums.

Catastrophic Risk

Thesis: As the likelihood of catastrophic events (low probability events with large consequences) increases, equity risk premiums should rise.

Implication: As investor worries about large consequence events (pandemics, nuclear war) increase, equity risk premiums will go up.

Government Policy

Thesis: Governments that are more capricious, with changing economic rules/policies, will give rise to higher equity risk premiums.

Implication: Equity risk premiums should be higher in countries/markets where there is less continuity in economic policy and regulation.

Central Banks & Monetary Policy

Thesis: Central banks that are less predictable in policy responses and more inconsistent in their actions will push up equity risk premiums.

Implication: As monetary policy becomes more unpredictable, for political reasons or because of inflation, equity risk premiums will rise.

LOOKING BACK: HISTORICAL EQUITY RISK PREMIUMS

- The **historical premium** is the premium that stocks have historically earned over riskless securities.
- While the users of historical risk premiums act as if it is a fact (rather than an estimate), it is sensitive to
 - How far back you go in history...
 - Whether you use T.bill rates or T.Bond rates
 - Whether you use geometric or arithmetic averages.
- For instance, looking at the US:

	<i>Arithmetic Average</i>		<i>Geometric Average</i>	
	Stocks - T. Bills	Stocks - T. Bonds	Stocks - T. Bills	Stocks - T. Bonds
1928-2024	8.44%	7.00%	6.63%	5.44%
Std Error	2.01%	2.12%		
1975-2024	9.25%	7.03%	8.02%	6.22%
Std Error	2.30%	2.67%		
2015-2024	12.34%	13.54%	11.22%	12.71%
Std Error	5.04%	3.84%		

THE PERILS OF TRUSTING THE PAST.....

- **Noisy estimates:** Even with long time periods of history, the risk premium that you derive will have substantial standard error. For instance, if you go back to 1928 (about 90 years of history) and you assume a standard deviation of 20% in annual stock returns, you arrive at a standard error of greater than 2%:

$$\text{Standard Error in Premium} = 20\% / \sqrt{90} = 2.1\%$$

- **Survivorship Bias:** Using historical data from the U.S. equity markets over the twentieth century does create a sampling bias. After all, the US economy and equity markets were among the most successful of the global economies that you could have invested in early in the century.

THE SIMPLEST WAY OF ESTIMATING AN ADDITIONAL COUNTRY RISK PREMIUM: THE COUNTRY DEFAULT SPREAD

- **Estimate default spread for country:** In this approach, the country equity risk premium is set equal to the default spread for the country, estimated in one of three ways:
 - The default spread on a dollar denominated bond issued by the country. (In January 2025, that spread was % for the Brazilian \$ bond) was 1.817%.
 - The sovereign CDS spread for the country. In January 2025, the ten-year CDS spread for Brazil, adjusted for the US CDS, was 3.17%.
 - The default spread based on the local currency rating for the country. Brazil's sovereign local currency rating is Ba2 and the default spread for a Ba2 rated sovereign was about 2.83% in January 2025.
- **Add the default spread to a “mature” market premium:** This default spread is added on to the mature market premium to arrive at the total equity risk premium for Brazil, assuming a mature market premium of 4.33%.
 - Country Risk Premium for Brazil = 2.83%
 - Total ERP for Brazil = 4.33% + 2.83% = 7.16%

AN EQUITY VOLATILITY BASED APPROACH TO ESTIMATING THE COUNTRY TOTAL ERP

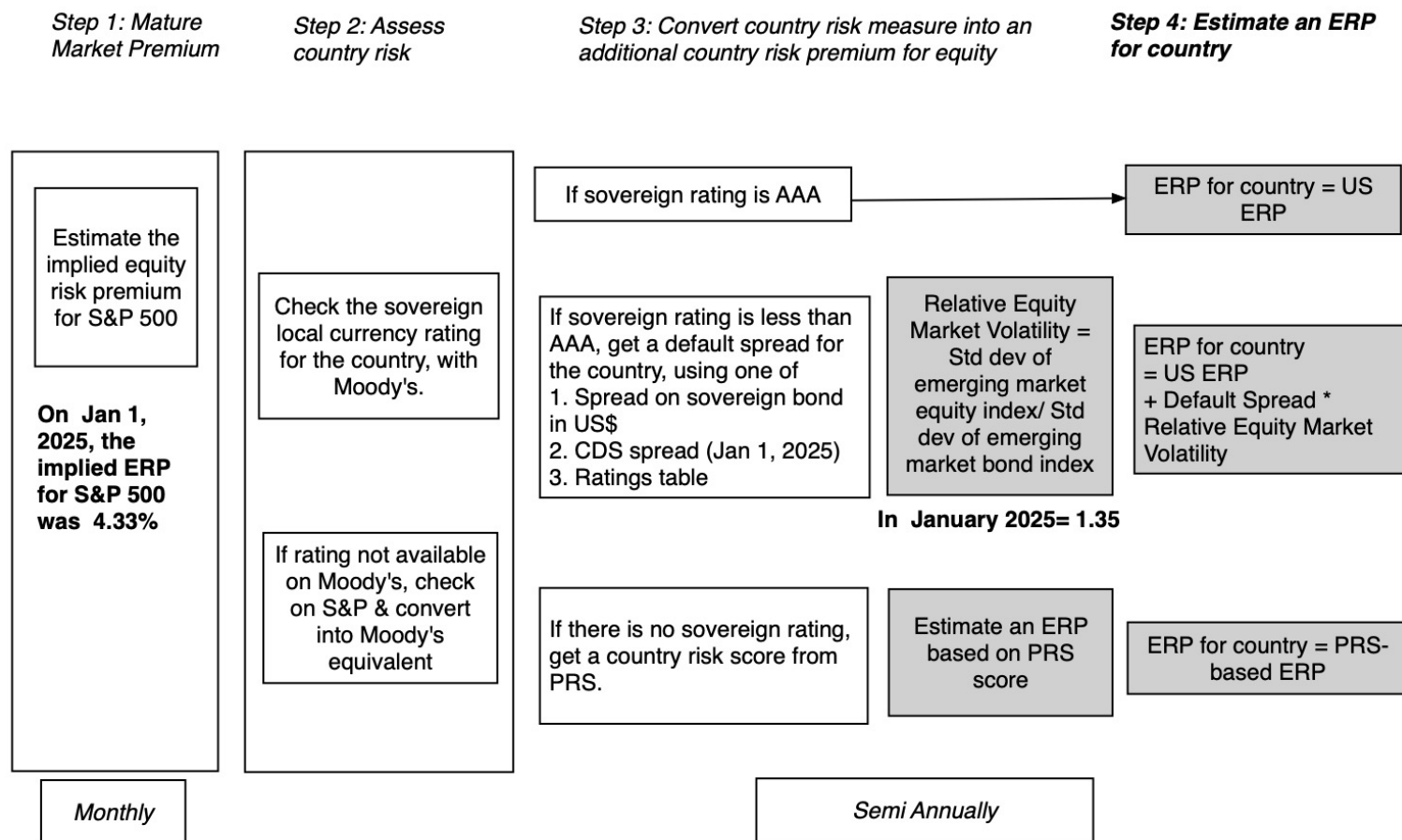
- This approach draws on the **standard deviation of two equity markets**, the emerging market in question and a base market (usually the US). The total equity risk premium for the emerging market is then written as:
 - Equity risk premium = Risk Premium_{US} × ($\sigma_{\text{Country Equity}}$ / $\sigma_{\text{US Equity}}$)
- The country equity risk premium is based upon the **volatility of the market in question relative to U.S market**.
 - Assume that the equity risk premium for the US is 4.33%.
 - Assume that the standard deviation in the Bovespa (Brazilian equity) is 30% and that the standard deviation for the S&P 500 (US equity) is 18%.
 - Total Equity Risk Premium for Brazil = 4.33% (30%/18%) = 7.22%
 - Country equity risk premium for Brazil = 7.22% - 4.33% = 2.89%

A MELDED APPROACH TO ESTIMATING THE ADDITIONAL COUNTRY RISK PREMIUM

- **Country ratings measure default risk.** While default risk premiums and equity risk premiums are highly correlated, one would expect equity spreads to be higher than debt spreads.
- Another is to **multiply the bond default spread by the relative volatility of stock and bond prices in that market.** Using this approach for Brazil in January 2025, you would get:
 - Country Equity risk premium = Default spread on country bond*
 $\sigma_{\text{Country Equity}} / \sigma_{\text{Country Bond}}$
 - Standard Deviation in Bovespa (Equity) = 30%
 - Standard Deviation in Brazil government bond = 20%
 - Default spread for Brazil = 2.83%
 - Brazil Country Risk Premium = 2.83% (30%/20%) = 4.25%
 - Brazil Total ERP = Mature Market Premium + CRP = 4.33% + 4.25% = 8.58%

A TEMPLATE FOR ESTIMATING THE ERP

ERP Estimation Procedure - January 1, 2025



Andorra	Baa1	2.13%	6.46%	Jersey	Aa2	0.66%	4.99%
Austria	Aa1	0.53%	4.86%	Liechtenstein	Aaa	0.00%	4.33%
Belgium	Aa3	0.80%	5.13%	Luxembourg	Aaa	0.00%	4.33%
Cyprus	A3	1.60%	5.93%	Malta	A2	1.13%	5.46%
Denmark	Aaa	0.00%	4.33%	Netherlands	Aaa	0.00%	4.33%
Finland	Aa1	0.53%	4.86%	Norway	Aaa	0.00%	4.33%
France	Aa3	0.80%	5.13%	Portugal	A3	1.60%	5.93%
Germany	Aaa	0.00%	4.33%	Spain	Baa1	2.13%	6.46%
Greece	Ba1	3.34%	7.67%	Sweden	Aaa	0.00%	4.33%
Guernsey	A1	0.94%	5.27%	Switzerland	Aaa	0.00%	4.33%
Iceland	A1	0.94%	5.27%	Turkey	B1	6.01%	10.34%
Ireland	Aa3	0.80%	5.13%	United Kingdom	Aa3	0.80%	5.13%
Isle of Man	Aa3	0.80%	5.13%	Western Europe		1.12%	5.45%
Italy	Baa3	2.93%	7.26%				

Canada	Aaa	0.00%	4.33%
United States	Aaa	0.00%	4.33%
North America		0.00%	4.33%

Caribbean		8.10%	12.43%
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Argentina	Ca	16.02%	20.35%
Belize	Caa1	10.01%	14.34%
Bolivia	Caa3	13.35%	17.68%
Brazil	Ba1	3.34%	7.67%
Chile	A2	1.13%	5.46%
Colombia	Baa2	2.54%	6.87%
Costa Rica	Ba3	4.80%	9.13%
Ecuador	Caa3	13.35%	17.68%
El Salvador	B3	8.67%	13.00%
Guatemala	Ba1	3.34%	7.67%
Honduras	B1	6.01%	10.34%
Mexico	Baa2	2.54%	6.87%
Nicaragua	B2	7.34%	11.67%
Panama	Baa3	2.93%	7.26%
Paraguay	Baa3	2.93%	7.26%
Peru	Baa1	2.13%	6.46%
Suriname	Caa1	10.01%	14.34%
Uruguay	Baa1	2.13%	6.46%
Venezuela	C	23.58%	27.91%
Latin America		4.82%	9.15%

Country	Rating	CRP	ERP
Angola	B3	8.67%	13.00%
Benin	B1	6.01%	10.34%
Botswana	A3	1.60%	5.93%
Burkina Faso	Caa1	10.01%	14.34%
Cameroon	Caa1	10.01%	14.34%
Cape Verde	B2	7.34%	11.67%
Congo (DR)	B3	8.67%	13.00%
Congo (Rep)	Caa2	12.02%	16.35%
Côte d'Ivoire	Ba2	4.02%	8.35%
Egypt	Caa1	10.01%	14.34%
Ethiopia	Caa2	12.02%	16.35%
Gabon	Caa2	12.02%	16.35%
Ghana	Caa2	12.02%	16.35%
Kenya	Caa1	10.01%	14.34%
Mali	Caa2	12.02%	16.35%
Mauritius	Baa3	2.93%	7.26%
Morocco	Ba1	3.34%	7.67%
Mozambique	Caa2	12.02%	16.35%
Namibia	B1	6.01%	10.34%
Niger	Caa3	13.35%	17.68%
Nigeria	Caa1	10.01%	14.34%
Rwanda	B2	7.34%	11.67%
Senegal	B1	6.01%	10.34%
South Africa	Ba2	4.02%	8.35%
Swaziland	B2	7.34%	11.67%
Tanzania	B1	6.01%	10.34%
Togo	B3	8.67%	13.00%
Tunisia	Caa2	12.02%	16.35%
Uganda	B3	8.67%	13.00%
Zambia	Caa2	12.02%	16.35%
Africa		8.31%	12.64%

Albania	Ba3	4.80%	9.13%
Armenia	Ba3	4.80%	9.13%
Azerbaijan	Ba1	3.34%	7.67%
Belarus	C	23.58%	27.91%
Bosnia and Herzegovina	B3	8.67%	13.00%
Bulgaria	Baa1	2.13%	6.46%
Croatia	A3	1.60%	5.93%
Czech Republic	Aa3	0.80%	5.13%
Estonia	A1	0.94%	5.27%
Georgia	Ba2	4.02%	8.35%
Hungary	Baa2	2.54%	6.87%
Kazakhstan	Baa1	2.13%	6.46%
Kyrgyzstan	B3	8.67%	13.00%
Latvia	A3	1.60%	5.93%
Lithuania	A2	1.13%	5.46%
Macedonia	Ba3	4.80%	9.13%
Moldova	B3	8.67%	13.00%
Montenegro	B1	6.01%	10.34%
Poland	A2	1.13%	5.46%
Romania	Baa3	2.93%	7.26%
Serbia	Ba2	4.02%	8.35%
Slovakia	A3	1.60%	5.93%
Slovenia	A3	1.60%	5.93%
Tajikistan	B3	8.67%	13.00%
Ukraine	Ca	16.02%	20.35%
Uzbekistan	Ba3	4.80%	9.13%
Eastern Europe		3.40%	7.73%

Abu Dhabi	Aa2	0.66%	4.99%
Bahrain	B2	7.34%	11.67%
Iraq	Caa1	10.01%	14.34%
Israel	Baa1	2.13%	6.46%
Jordan	Ba3	4.80%	9.13%
Kuwait	A1	0.94%	5.27%
Lebanon	C	23.58%	27.91%
Oman	Ba1	3.34%	7.67%
Qatar	Aa2	0.66%	4.99%
Ras Al Khaimah (Emirate)	A3	1.60%	5.93%
Saudi Arabia	Aa3	0.80%	5.13%
Sharjah	Ba1	3.34%	7.67%
United Arab Emirates	Aa2	0.66%	4.99%
Middle East		2.10%	6.43%

Country	PRS	CRP	ERP
Algeria	69.25	3.52%	7.85%
Brunei	81.75	0.70%	5.03%
Gambia	67.5	5.26%	9.59%
Guinea	57.75	10.52%	14.85%
Guinea-Bissau	63.25	7.60%	11.93%
Guyana	75.75	1.87%	6.20%
Haiti	54.75	14.03%	18.36%
Iran	63.75	7.60%	11.93%
Korea, D.P.R.	51	14.03%	18.36%
Liberia	58.25	10.52%	14.85%
Libya	74.5	1.87%	6.20%
Madagascar	64.5	6.43%	10.76%
Malawi	57.75	10.52%	14.85%
Myanmar	56	11.69%	16.02%
Russia	69.25	3.52%	7.85%
Sierra Leone	59.5	10.52%	14.85%
Somalia	55.5	11.69%	16.02%
Sudan	43.5	20.65%	24.98%
Syria	46.5	20.65%	24.98%
Yemen, Republic	51.5	14.03%	18.36%
Zimbabwe	57.75	10.52%	14.85%

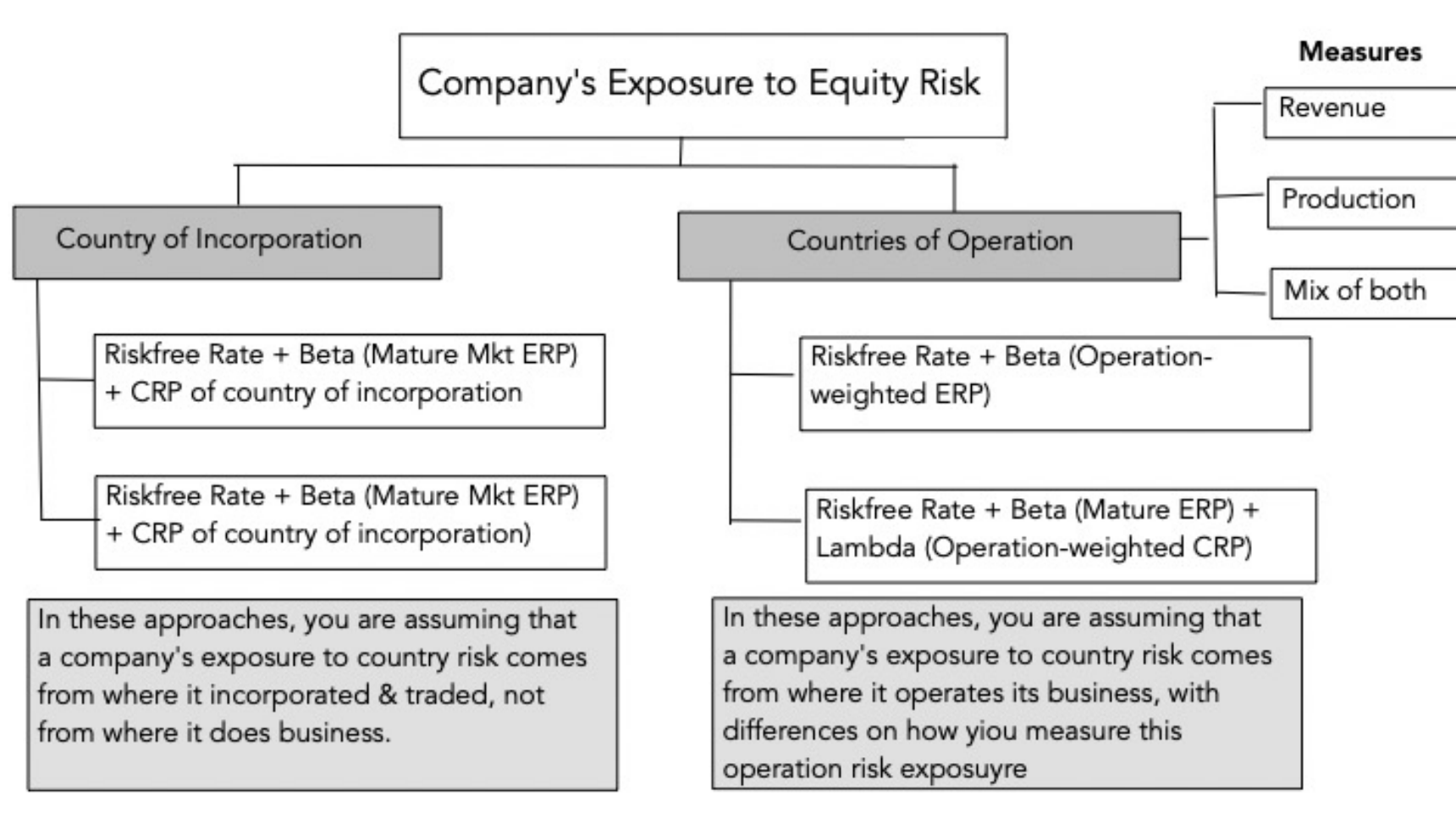
Bangladesh	B2	7.34%	11.67%
Cambodia	B2	7.34%	11.67%
China	A1	0.94%	5.27%
Fiji	B1	6.01%	10.34%
Hong Kong	Aa3	0.80%	5.13%
India	Baa3	2.93%	7.26%
Indonesia	Baa2	2.54%	6.87%
Japan	A1	0.94%	5.27%
Korea	Aa2	0.66%	4.99%
Laos	Caa3	13.35%	17.68%
Macao	Aa3	0.80%	5.13%
Malaysia	A3	1.60%	5.93%
Maldives	Caa2	12.02%	16.35%
Mongolia	B2	7.34%	11.67%
Nepal	Ba3	4.80%	9.13%
Pakistan	Caa2	12.02%	16.35%
Papua New Guinea	B2	7.34%	11.67%
Philippines	Baa2	2.54%	6.87%
Singapore	Aaa	0.00%	4.33%
Solomon Islands	Caa1	10.01%	14.34%
Sri Lanka	Ca	16.02%	20.35%
Taiwan	Aa3	0.80%	5.13%
Thailand	Baa1	2.13%	6.46%
Vietnam	Ba2	4.02%	8.35%
Asia		1.44%	5.72%

Australia	Aaa	0.00%	4.33%
Cook Islands	B1	6.01%	10.34%
New Zealand	Aaa	0.00%	4.33%
Australia & NZ		0.00%	4.33%

FROM COUNTRY EQUITY RISK PREMIUMS TO CORPORATE EQUITY RISK PREMIUMS

- **Approach 1:** Assume that every company in the country is equally exposed to country risk. In this case,
 - $E(\text{Return}) = \text{Riskfree Rate} + \text{CRP} + \text{Beta} (\text{Mature ERP})$
- **Approach 2:** Assume that a company's exposure to country risk is similar to its exposure to other market risk.
 - $E(\text{Return}) = \text{Riskfree Rate} + \text{Beta} (\text{Mature ERP} + \text{CRP})$
- **Approach 3:** Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)
 - $E(\text{Return}) = \text{Riskfree Rate} + \beta (\text{Mature ERP}) + \lambda (\text{CRP})$
 - Mature ERP = Mature market Equity Risk Premium
 - CRP = Additional country risk premium

ESTIMATING COUNTRY RISK PREMIUM EXPOSURE_VARIANTS



OPERATION BASED CRP: SINGLE VERSUS MULTIPLE EMERGING MARKETS

- Single emerging market: Embraer, in 2004, reported that it derived 3% of its revenues in Brazil and the balance from mature markets. The mature market ERP in 2004 was 5% and Brazil's CRP was 7.89%.

	Revenues	Total ERP	CRP
US and other mature markets	97%	5.00%	0.00%
Brazil	3%	12.89%	8%
Embraer		5.24%	0.24%

- Multiple emerging markets: Ambev, the Brazilian-based beverage company, reported revenues from the following countries during 2011.

	Revenues	%	Total ERP	CRP
Argentina	19	9.31%	15.00%	9.00%
Bolivia	4	1.96%	10.88%	4.88%
Brazil	130	63.73%	8.63%	2.63%
Canada	23	11.27%	6.00%	0.00%
Chile	7	3.43%	7.05%	1.05%
Ecuador	6	2.94%	12.75%	6.75%
Paraguay	3	1.47%	12.00%	6.00%
Peru	12	5.88%	9.00%	3.00%
Ambev	204		9.11%	3.11%

EXTENDING TO A MULTINATIONAL: REGIONAL BREAKDOWN

COCA COLA'S REVENUE BREAKDOWN AND ERP IN 2012

<i>Region</i>	<i>Revenues</i>	<i>Total ERP</i>	<i>CRP</i>
Western Europe	19%	6.67%	0.67%
Eastern Europe & Russia	5%	8.60%	2.60%
Asia	15%	7.63%	1.63%
Latin America	15%	9.42%	3.42%
Australia	4%	6.00%	0.00%
Africa	4%	9.82%	3.82%
North America	40%	6.00%	0.00%
Coca Cola	100%	7.14%	1.14%

Things to watch out for

1. Aggregation across regions. For instance, the Pacific region often includes Australia & NZ with Asia
2. Obscure aggregations including Eurasia and Oceania

TWO PROBLEMS WITH THESE APPROACHES..

- **Focus just on revenues:** To the extent that revenues are the only variable that you consider, when weighting risk exposure across markets, you may be missing other exposures to country risk. For instance, an emerging market company that gets the bulk of its revenues outside the country (in a developed market) may still have all of its production facilities in the emerging market.
- **Exposure not adjusted or based upon beta:** To the extent that the country risk premium is multiplied by a beta, we are assuming that beta in addition to measuring exposure to all other macro economic risk also measures exposure to country risk.

A PRODUCTION-BASED ERP: ROYAL DUTCH SHELL IN 2015

Country	Oil & Gas Production	% of Total	ERP
Denmark	17396	3.83%	6.20%
Italy	11179	2.46%	9.14%
Norway	14337	3.16%	6.20%
UK	20762	4.57%	6.81%
Rest of Europe	874	0.19%	7.40%
Brunei	823	0.18%	9.04%
Iraq	20009	4.40%	11.37%
Malaysia	22980	5.06%	8.05%
Oman	78404	17.26%	7.29%
Russia	22016	4.85%	10.06%
Rest of Asia & ME	24480	5.39%	7.74%
Oceania	7858	1.73%	6.20%
Gabon	12472	2.75%	11.76%
Nigeria	67832	14.93%	11.76%
Rest of Africa	6159	1.36%	12.17%
USA	104263	22.95%	6.20%
Canada	8599	1.89%	6.20%
Brazil	13307	2.93%	9.60%
Rest of Latin America	576	0.13%	10.78%
Royal Dutch Shell	454326	100.00%	8.26%

ESTIMATE A LAMBDA FOR COUNTRY RISK

- Country risk exposure is affected by **where you get your revenues and where your production happens**, but there are a host of other variables that also affect this exposure, including:
 - Use of risk management products: Companies can use both options/futures markets and insurance to hedge some or a significant portion of country risk.
 - Government “national” interests: There are sectors that are viewed as vital to the national interests, and governments often play a key role in these companies, either officially or unofficially. These sectors are more exposed to country risk.
- It is conceivable that there is a **richer measure of country risk that incorporates all the variables that drive country risk in one measure**. That way my rationale when I devised “lambda” as my measure of country risk exposure.

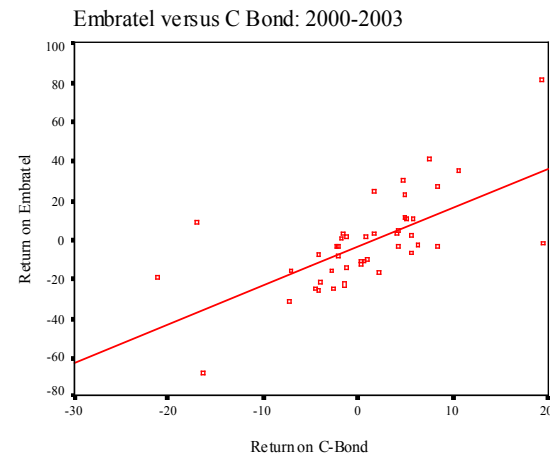
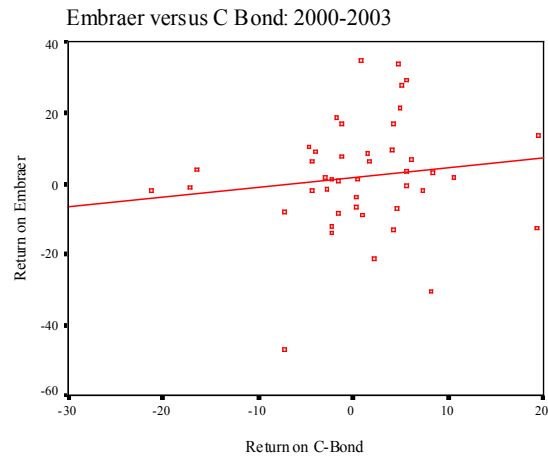
A REVENUE-BASED LAMBDA

- The factor “ λ ” measures the relative exposure of a firm to country risk. One simplistic solution would be to do the following:
 - $\lambda = \frac{\% \text{ of revenues domestically}_{\text{firm}}}{\% \text{ of revenues domestically}_{\text{average firm}}}$
- Consider two firms – Tata Motors and Tata Consulting Services, both Indian companies. In 2008-09, Tata Motors got about 91.37% of its revenues in India and TCS got 7.62%. The average Indian firm gets about 80% of its revenues in India:
 - $\lambda_{\text{Tata Motors}} = 91\%/80\% = 1.14$
 - $\lambda_{\text{TCS}} = 7.62\%/80\% = 0.09$
- There are two implications
 - A company’s risk exposure is determined by where it does business and not by where it is incorporated.
 - Firms might be able to actively manage their country risk exposures

A PRICE/RETURN BASED LAMBDA

$$\text{Return}_{\text{Embraer}} = 0.0195 + \mathbf{0.2681} \text{ Return}_{\text{C Bond}}$$

$$\text{Return}_{\text{Embratel}} = -0.0308 + \mathbf{2.0030} \text{ Return}_{\text{C Bond}}$$



ESTIMATING A US DOLLAR COST OF EQUITY FOR EMBRAER - SEPTEMBER 2004

- Assume that the beta for Embraer is 1.07, and that the US \$ riskfree rate used is 4%. Also assume that the risk premium for the US is 5% and the country risk premium for Brazil is 7.89%. Finally, assume that Embraer gets 3% of its revenues in Brazil & the rest in the US.
- There are five estimates of \$ cost of equity for Embraer:
 - **Approach 1:** Constant exposure to CRP, Location CRP
 - $E(\text{Return}) = 4\% + 1.07 (5\%) + 7.89\% = 17.24\%$
 - **Approach 2:** Constant exposure to CRP, Operation CRP
 - $E(\text{Return}) = 4\% + 1.07 (5\%) + (0.03*7.89\% + 0.97*0\%) = 9.59\%$
 - **Approach 3:** Beta exposure to CRP, Location CRP
 - $E(\text{Return}) = 4\% + 1.07 (5\% + 7.89\%) = 17.79\%$
 - **Approach 4:** Beta exposure to CRP, Operation CRP
 - $E(\text{Return}) = 4\% + 1.07 (5\% + (0.03*7.89\% + 0.97*0\%)) = 9.60\%$
 - **Approach 5:** Lambda exposure to CRP
 - $E(\text{Return}) = 4\% + 1.07 (5\%) + 0.27(7.89\%) = 11.48\%$

VALUING EMERGING MARKET COMPANIES WITH SIGNIFICANT EXPOSURE IN DEVELOPED MARKETS

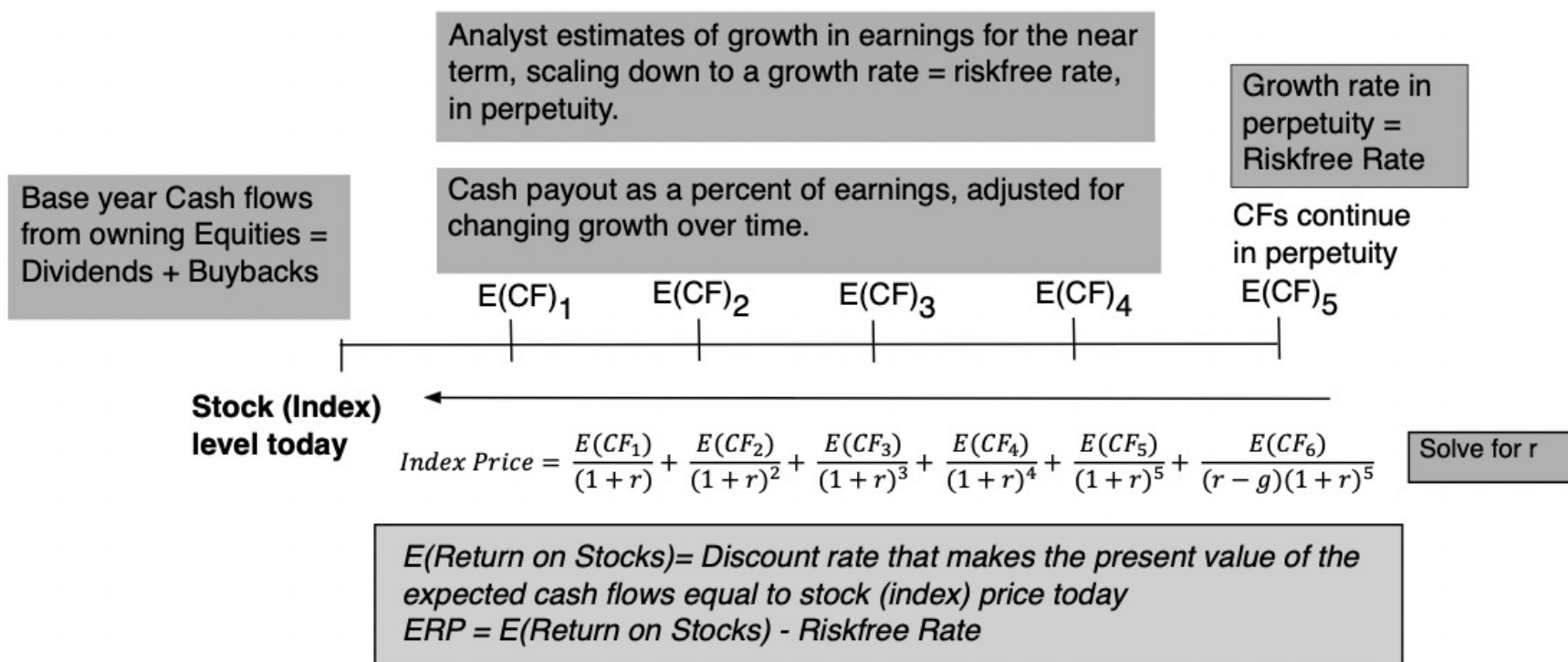
- The conventional practice in investment banking is to add the country equity risk premium on to the cost of equity for every emerging market company, notwithstanding its exposure to emerging market risk.
- Thus, in 2004, Embraer would have been valued with a cost of equity of 17-18% even though it gets only 3% of its revenues in Brazil. As an investor, which of the following consequences do you see from this approach?
 - Emerging market companies with substantial exposure in developed markets will be significantly over valued by analysts
 - Emerging market companies with substantial exposure in developed markets will be significantly under valued by analysts
- Can you construct an investment strategy to take advantage of the mis-valuation? What would need to happen for you to make money of this strategy?

IMPLIED EQUITY PREMIUMS

- **For a start:** If you know the price paid for an asset and have estimates of the expected cash flows on the asset, you can estimate the IRR of these cash flows. If you paid the price, this is your expected return.
- **Stock Price & Risk:** If you assume that stocks are correctly priced in the aggregate and you can estimate the expected cashflows from buying stocks, you can estimate the expected rate of return on stocks by finding that discount rate that makes the present value equal to the price paid.
- **Implied ERP:** Subtracting out the riskfree rate should yield an implied equity risk premium. This implied equity premium is a forward-looking number and can be updated as often as you want (every minute of every day, if you are so inclined).

A FORWARD-LOOKING NUMBER

Implied Equity Risk Premium: Generic Version



The implied equity risk premium is a number backed out from what investors are paying for stocks and their expected cash flows from holding stocks. It is an internal rate of return for equity investors, analogous to a yield to maturity for a bondholder.

EQUITY RISK PREMIUM: JANUARY 2020

Base year cash flow (last 12 mths)

Dividends (TTM): 57.71
 + Buybacks (TTM): 92.80
 = Cash to investors (TTM): **150.50**

Expected cashflow growth in next 5 years

Cash flow growth = Top down analyst estimate of earnings growth for S&P 500 = 3.96%

	Last 12 months	2020	2021	2022	2023	2024	Term Year
Expected Earnings	153.52	159.59	165.90	172.47	179.29	186.38	189.96
Expected Dividends + Buybacks	150.50	156.46	162.65	169.08	175.77	182.73	186.24

Earnings and Cash flows grow @1.92% (set equal to risk free rate) a year forever.

S&P 500 on 1/1/20=
3230.78

$$3230.78 = \frac{156.46}{(1+r)} + \frac{162.25}{(1+r)^2} + \frac{169.08}{(1+r)^3} + \frac{175.77}{(1+r)^4} + \frac{182.73}{(1+r)^5} + \frac{182.73 (1.0192)}{(r - .0192)(1+r)^5}$$

The last term in this equation is the expected index level at the end of year 5 (capturing price appreciation)

Solve for r

r = Implied Expected Return on Stocks = 7.12%

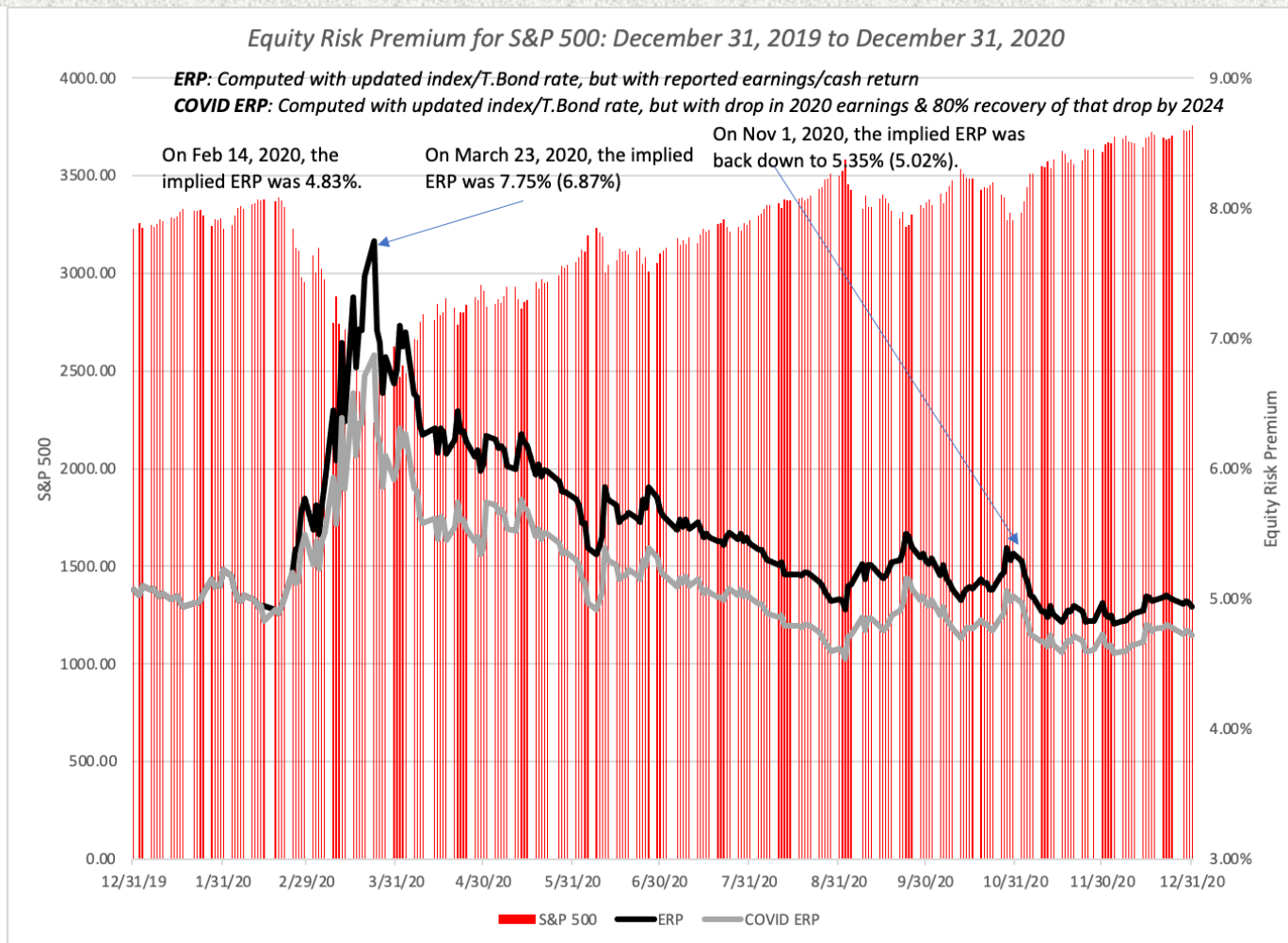
Minus

Risk free rate = T.Bond rate on 1/1/20= 1.92%

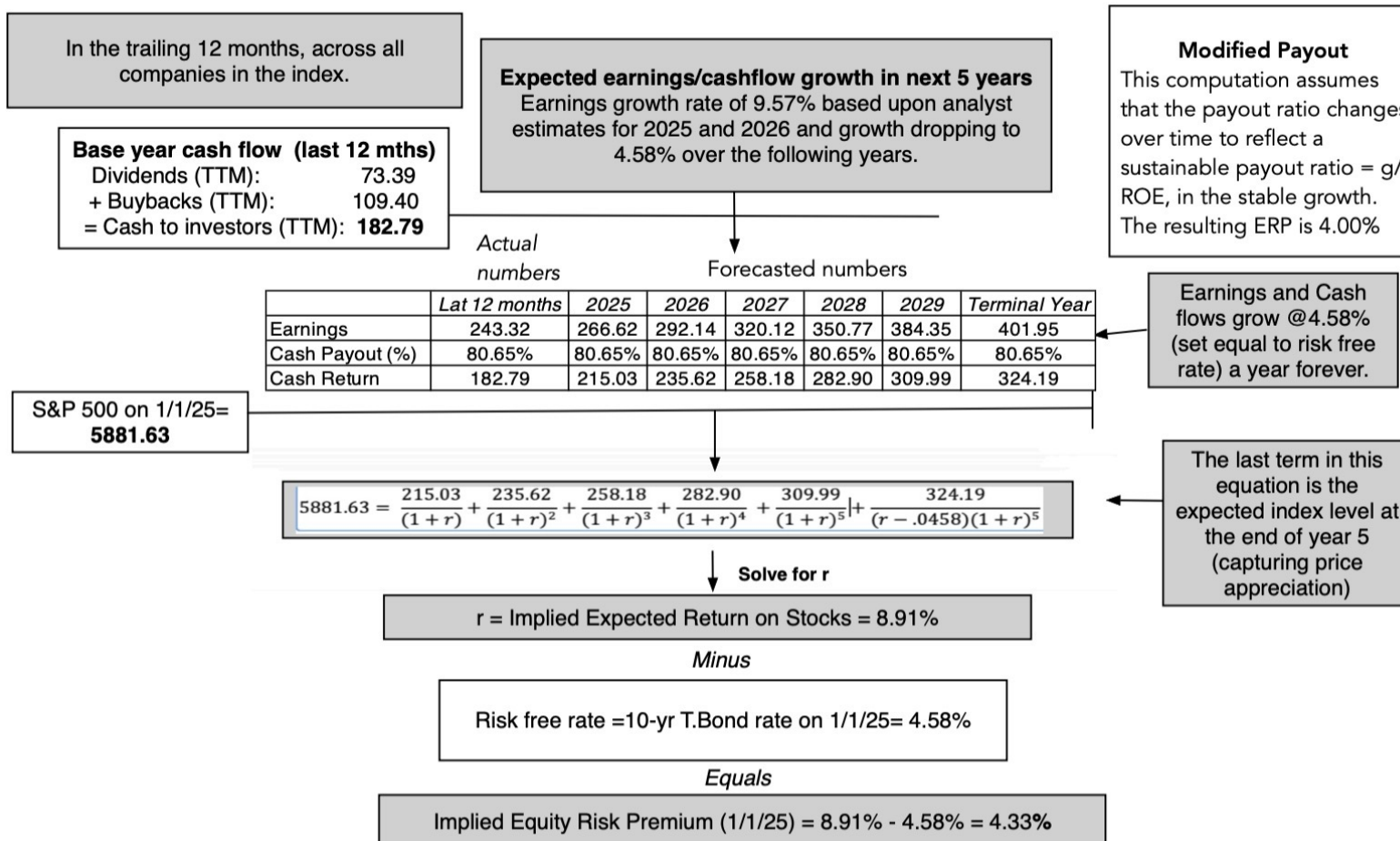
Equals

Implied Equity Risk Premium (1/1/20) = 7.12% - 1.92% = 5.20%

AND IN 2020.. COVID EFFECTS

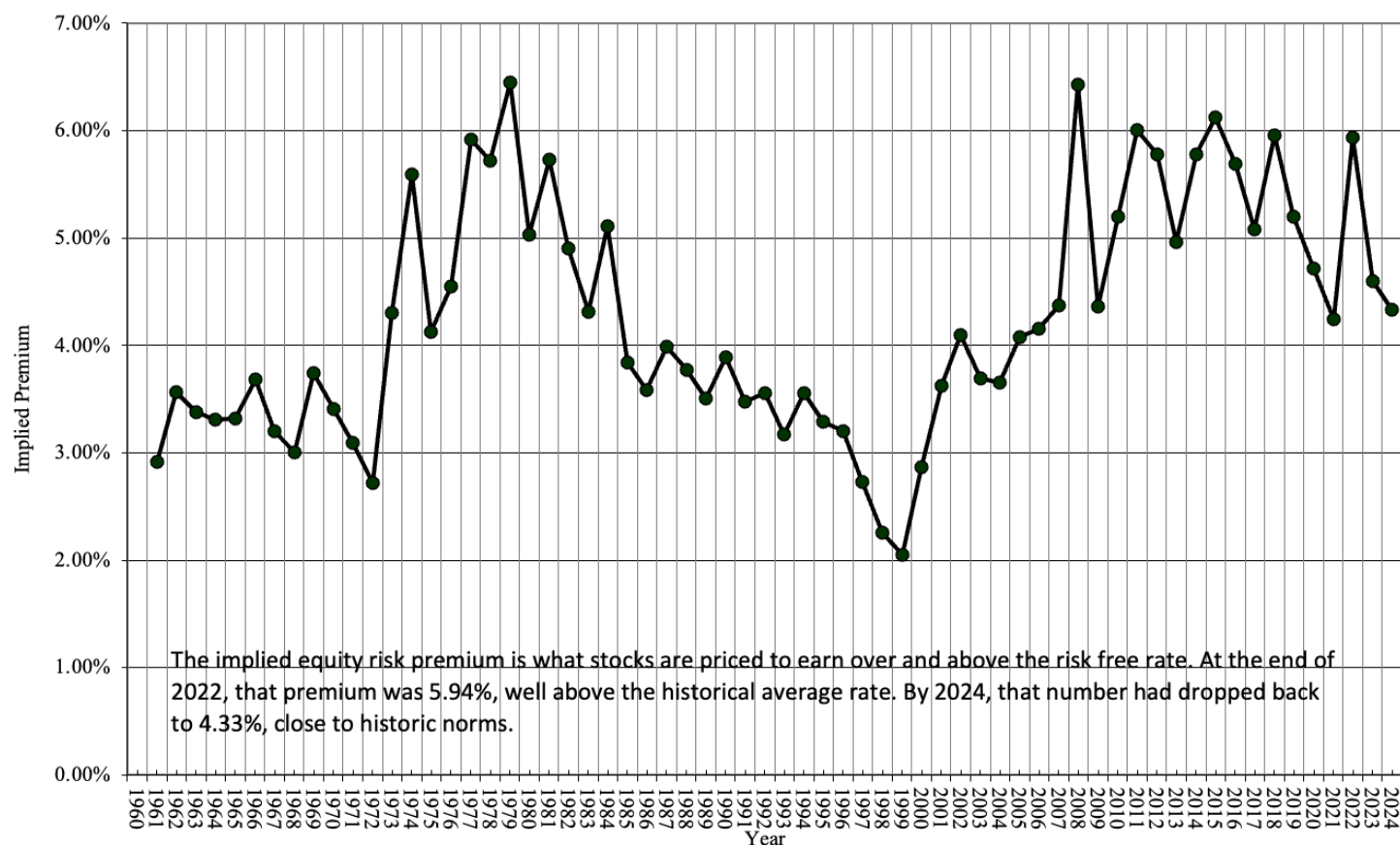


AN UPDATED ESTIMATE: ERP IN 2025

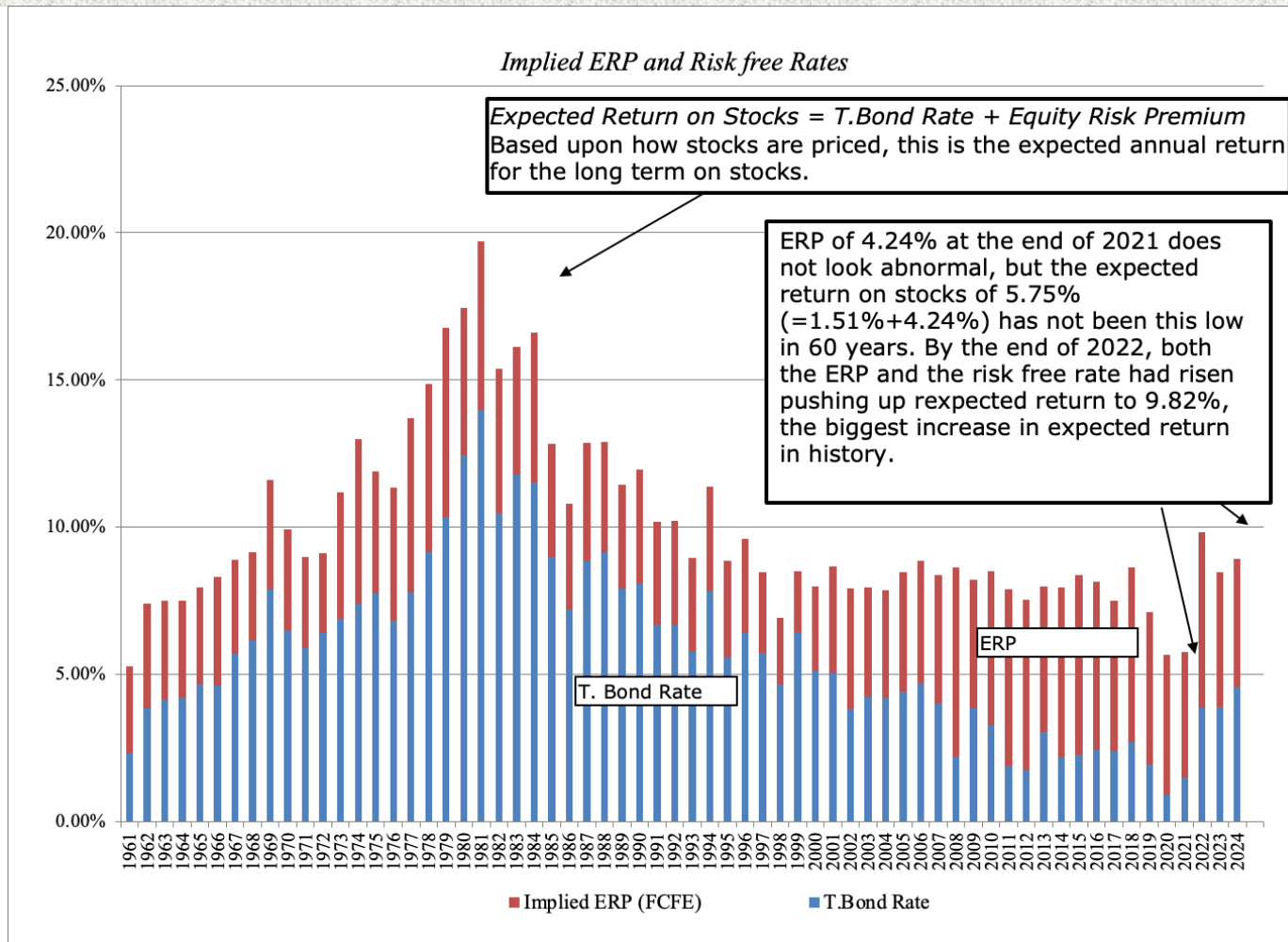


IMPLIED PREMIUMS IN THE US: 1960-2023

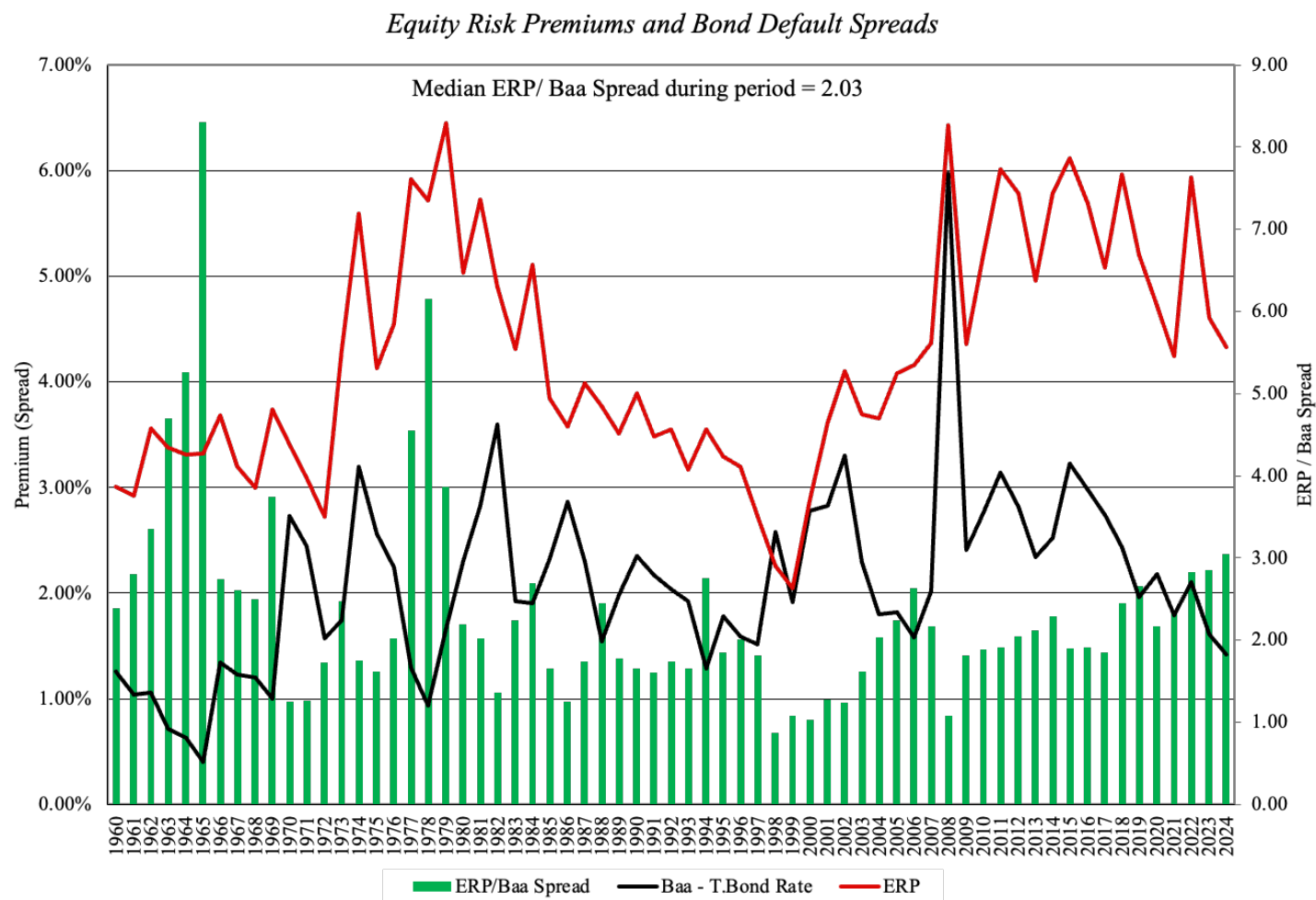
Implied Equity Risk Premium for US Equity Market: 1960-2024



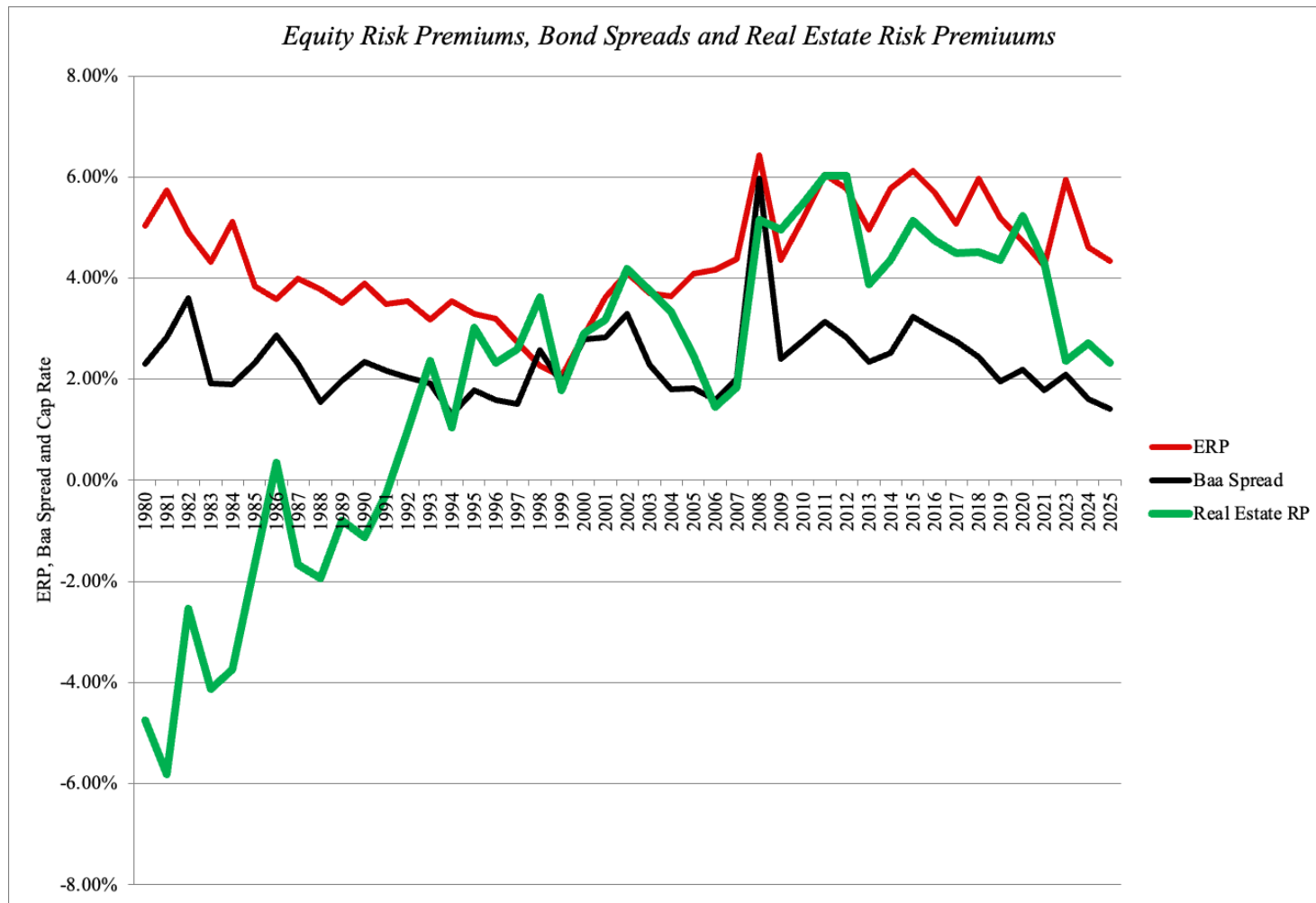
IMPLIED PREMIUM VERSUS RISK FREE RATE



EQUITY RISK PREMIUMS AND BOND DEFAULT SPREADS



EQUITY RISK PREMIUMS AND CAP RATES (REAL ESTATE)



WHY IMPLIED PREMIUMS MATTER?

- In many investment banks, it is **common practice (especially in corporate finance departments) to use historical risk premiums** (and arithmetic averages at that) as risk premiums to compute cost of equity. Often, the defense they offer is that as long as **everyone uses the same premium, there is no cost to being wrong**.
- If all analysts in a group used the arithmetic average premium (for stocks over T.Bills) for 1928-2024 of 8.44% to value stocks in January 2025, given the implied premium of 4.33%, what are they likely to find?
 - a. The values they obtain will be too low (most stocks will look overvalued)
 - b. The values they obtain will be too high (most stocks will look under valued)
 - c. There should be no systematic bias as long as they use the same premium to value all stocks.

WHICH EQUITY RISK PREMIUM SHOULD YOU USE?

If you assume this	Premium to use
Premiums revert back to historical norms and your time period yields these norms	Historical risk premium
Market is correct in the aggregate or that your valuation should be market neutral	Current implied equity risk premium
Market makes mistakes even in the aggregate but is correct over time	Average implied equity risk premium over time.

Predictor	Correlation with implied premium next year	Correlation with actual return- next 5 years	Correlation with actual return – next 10 years
Current implied premium	0.763	0.427	0.500
Average implied premium: Last 5 years	0.718	0.326	0.450
Historical Premium	-0.497	-0.437	-0.454
Default Spread based premium	0.047	0.143	0.160

AN ERP FOR THE SENSEX

- Inputs for the computation
 - Sensex on 9/5/07 = 15446
 - Dividend yield on index = 3.05%
 - Expected growth rate - next 5 years = 14%
 - Growth rate beyond year 5 = 6.76% (set equal to riskfree rate)
- Solving for the expected return:

$$15446 = \frac{537.06}{(1+r)} + \frac{612.25}{(1+r)^2} + \frac{697.86}{(1+r)^3} + \frac{795.67}{(1+r)^4} + \frac{907.07}{(1+r)^5} + \frac{907.07(1.0676)}{(r - .0676)(1+r)^5}$$

- Expected return on stocks = 11.18%
- Implied equity risk premium for India = 11.18% - 6.76% = 4.42%

THE EVOLUTION OF EMERGING MARKET RISK

Start of year	PBV (Developed)	PBV (Emerging)	ROE (Developed)	ROE (Emerging)	US T.Bond Rate	Growth Rate (Developed)	Growth Rate (Emerging)	Cost of Equity (Developed)	Cost of Equity (Emerging)	Differential
2004	2.00	1.19	10.81%	11.65%	4.25%	3.75%	4.75%	7.28%	10.55%	3.27%
2005	2.09	1.27	11.12%	11.93%	4.22%	3.72%	4.72%	7.26%	10.40%	3.14%
2006	2.03	1.44	11.32%	12.18%	4.39%	3.89%	4.89%	7.55%	9.95%	2.40%
2007	1.67	1.67	10.87%	12.88%	4.70%	4.20%	5.20%	8.19%	9.80%	1.60%
2008	0.87	0.83	9.42%	11.12%	4.02%	3.52%	4.52%	10.30%	12.47%	2.17%
2009	1.20	1.34	8.48%	11.02%	2.21%	1.71%	2.71%	7.35%	8.91%	1.56%
2010	1.39	1.43	9.14%	11.22%	3.84%	3.34%	4.34%	7.51%	9.15%	1.64%
2011	1.12	1.08	9.21%	10.04%	3.29%	2.79%	3.79%	8.52%	9.58%	1.05%
2012	1.17	1.18	9.10%	9.33%	1.88%	1.38%	2.38%	7.98%	8.27%	0.29%
2013	1.56	1.63	8.67%	10.48%	1.76%	1.26%	2.26%	6.01%	7.30%	1.29%
2014	1.95	1.50	9.27%	9.64%	3.04%	2.54%	3.54%	5.99%	7.61%	1.62%
2015	1.88	1.56	9.69%	9.75%	2.17%	1.67%	2.67%	5.94%	7.21%	1.27%
2016	1.99	1.59	9.24%	10.16%	2.27%	1.77%	2.77%	5.52%	7.42%	1.89%
2017	1.76	1.48	8.71%	9.53%	2.68%	2.18%	3.18%	5.89%	7.47%	1.58%
2018	1.98	1.66	11.23%	11.36%	2.68%	2.18%	3.18%	6.75%	8.11%	1.36%
2019	1.64	1.31	12.09%	11.35%	2.68%	2.18%	3.18%	8.22%	9.42%	1.19%
2020	2.26	1.64	10.41%	9.10%	1.92%	1.42%	2.42%	5.40%	6.49%	1.10%
2021	2.21	1.77	6.30%	7.31%	0.93%	0.43%	1.43%	3.09%	4.75%	1.67%
2022	2.31	1.67	13.22%	11.99%	1.51%	1.01%	2.01%	6.30%	7.99%	1.69%
2023	2.28	1.44	12.90%	10.93%	3.88%	3.38%	4.38%	7.56%	8.93%	1.37%



DISCOUNT RATES: III

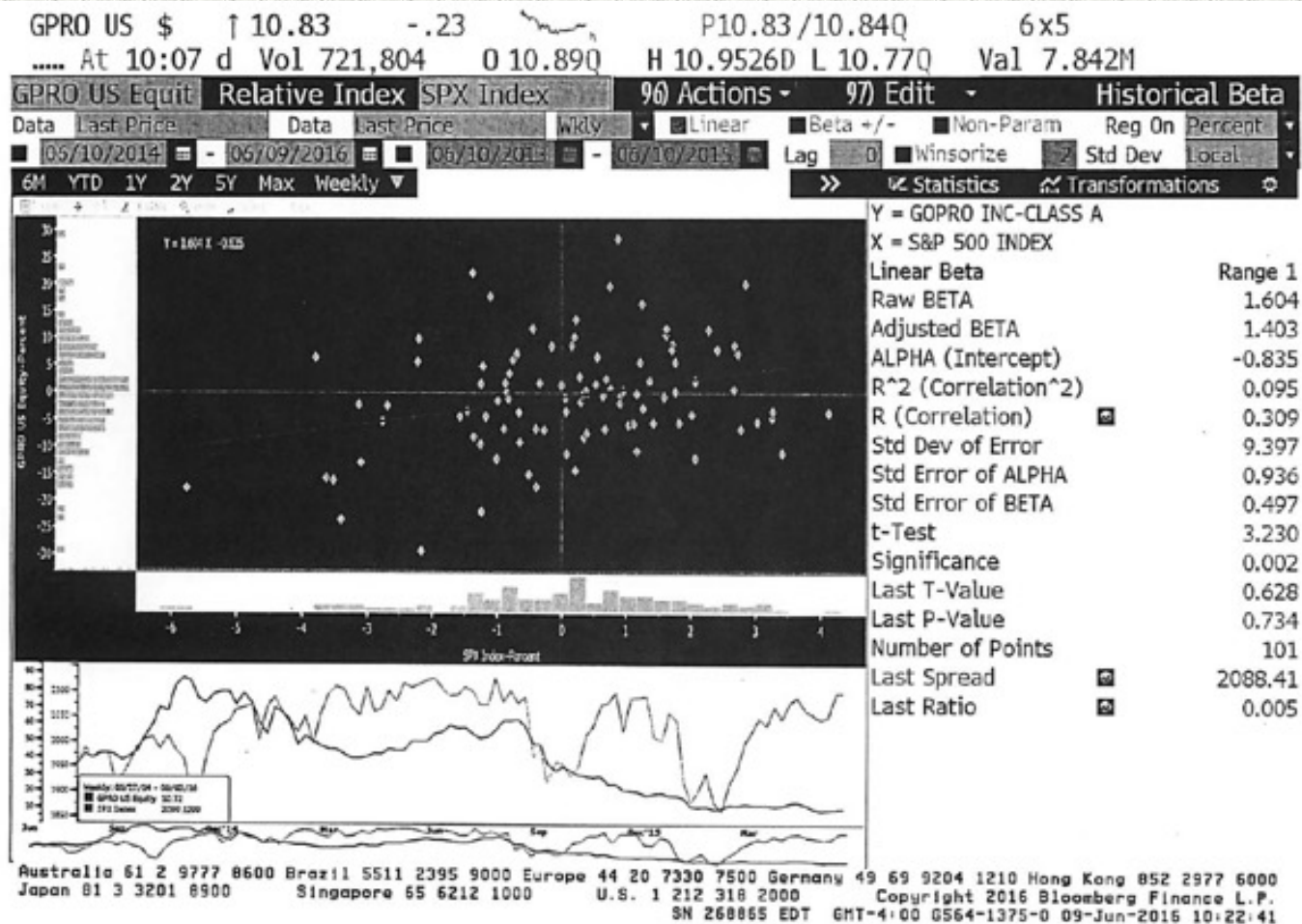
Relative Risk Measures

Aswath Damodaran

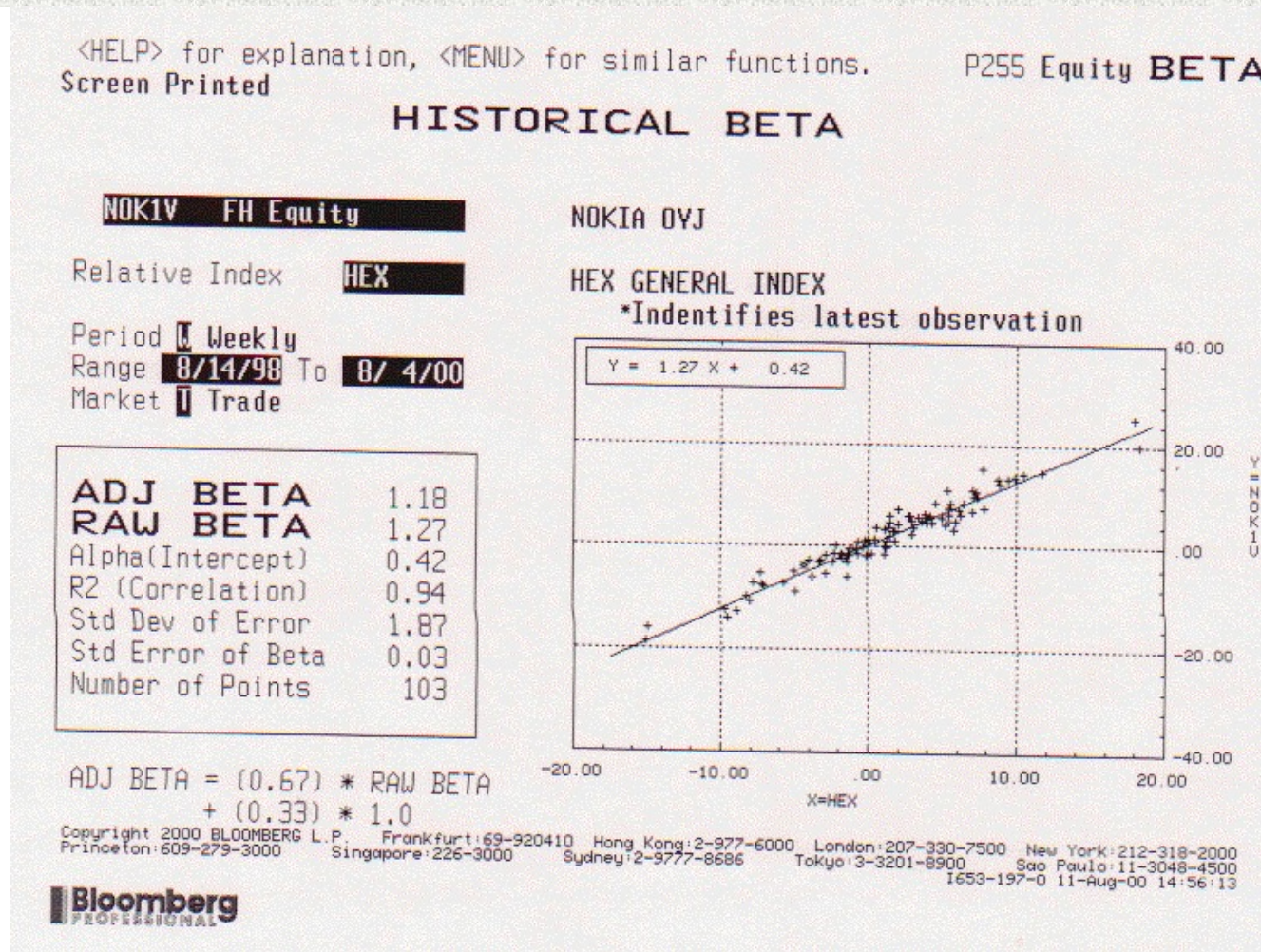
THE CAPM BETA: THE MOST USED (AND MISUSED) RISK MEASURE

- The standard procedure for estimating betas is to regress stock returns (R_j) against market returns (R_m) -
 - $R_j = a + b R_m$
 - where a is the intercept and b is the slope of the regression.
- The **slope of the regression** corresponds to the beta of the stock and measures the riskiness of the stock.
- This beta has three problems:
 - It has high standard error
 - It reflects the firm's business mix over the period of the regression, not the current mix
 - It reflects the firm's average financial leverage over the period rather than the current leverage.

UNRELIABLE, WHEN IT LOOKS BAD..



OR WHEN IT LOOKS GOOD..



ONE SLICE OF HISTORY..

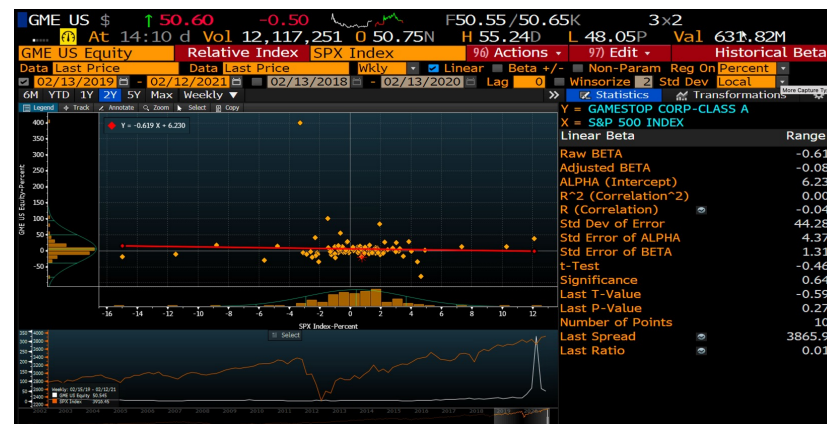
Market Summary > GameStop Corp.
NYSE: GME

+ Follow

50.99 USD -0.11 (0.22%) ↓

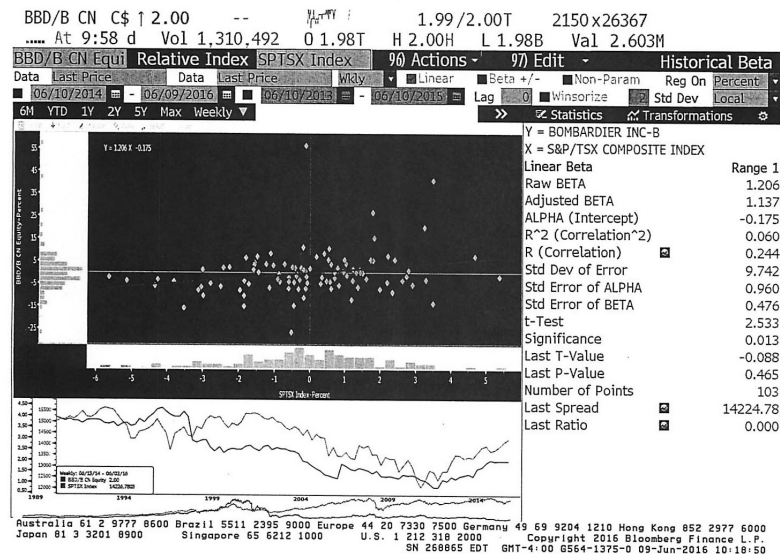
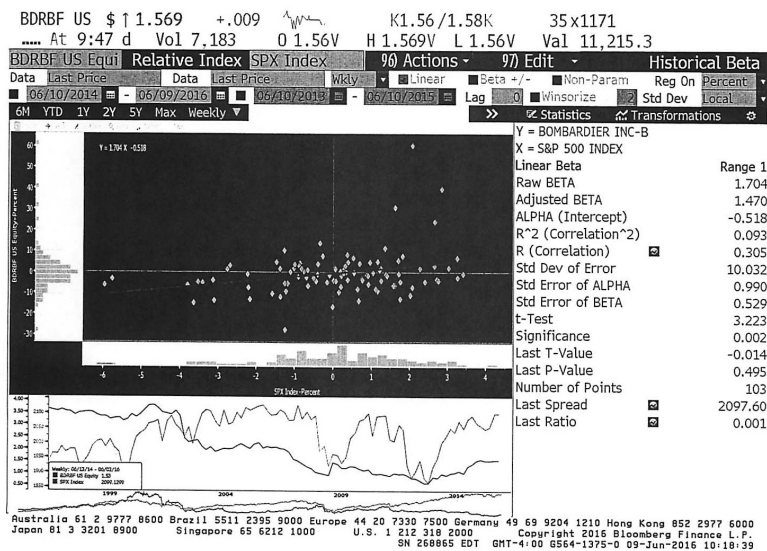
Feb 12, 2:44 PM EST · Disclaimer

1 day 5 days 1 month 6 months YTD 1 year 5 years Max

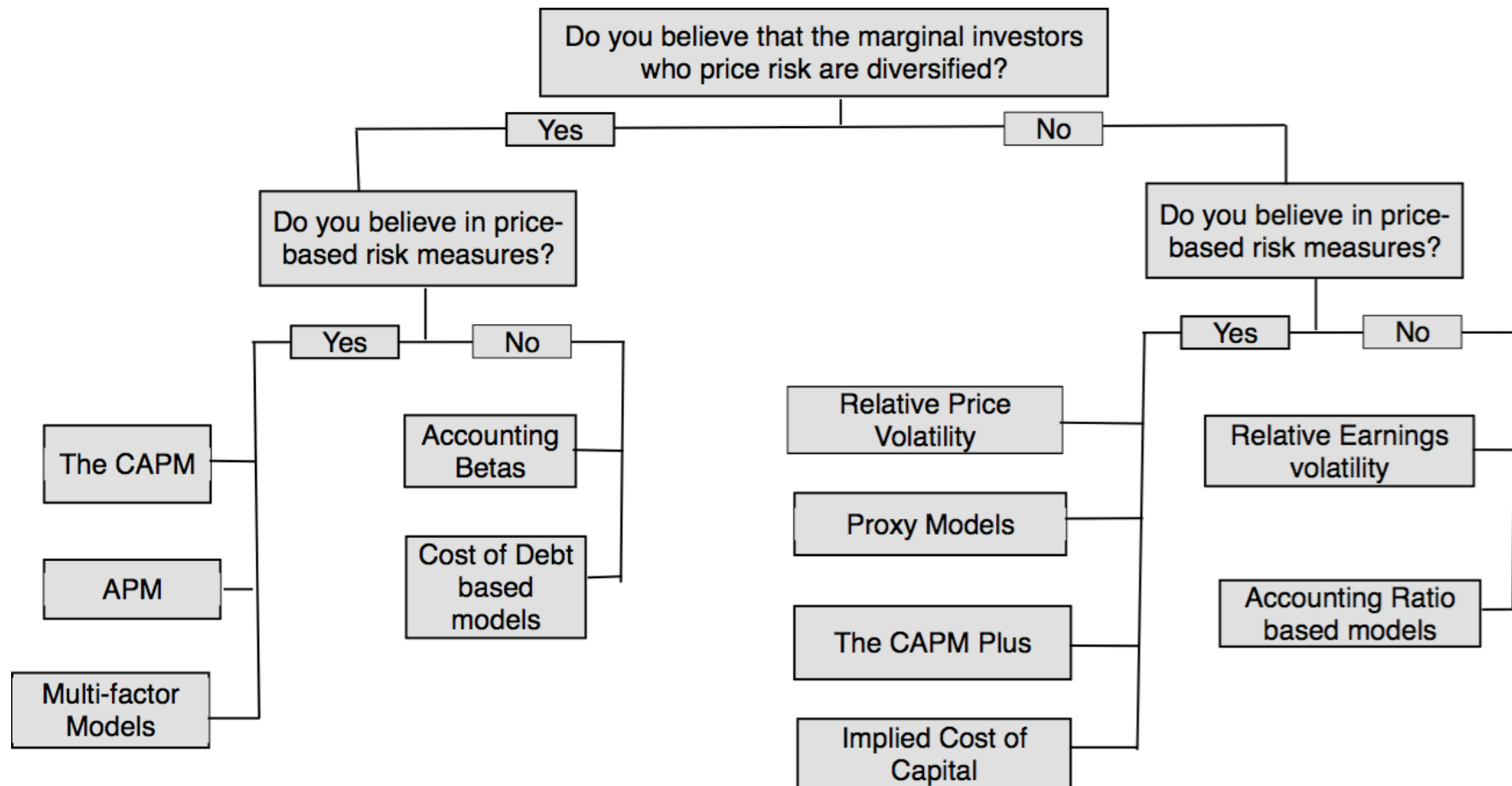


During 2019 and 2020, GME was an extraordinarily volatile stock, as short sellers and long only investors fought out a battle.

AND SUBJECT TO GAME PLAYING



MEASURING RELATIVE RISK: YOU DON'T LIKE BETAS OR MODERN PORTFOLIO THEORY? NO PROBLEM.



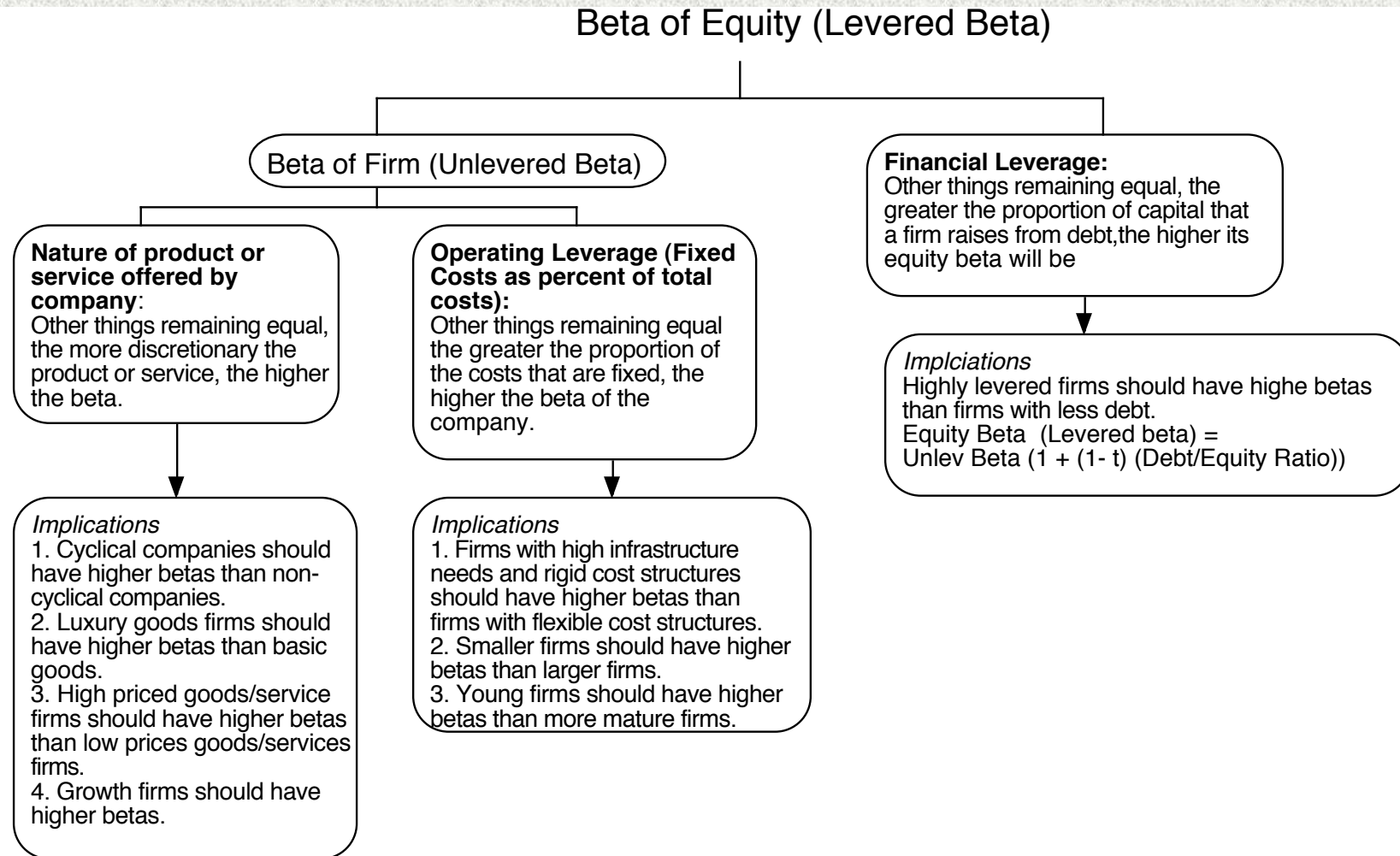
DON'T LIKE THE DIVERSIFIED INVESTOR FOCUS, BUT OKAY WITH PRICE-BASED MEASURES

- Relative Standard Deviation
 - Relative Volatility = Std dev of Stock/ Average Std dev across all stocks
 - Captures all risk, rather than just market risk
- Proxy Models
 - Look at historical returns on all stocks and look for variables that explain differences in returns.
 - You are, in effect, running multiple regressions with returns on individual stocks as the dependent variable and fundamentals about these stocks as independent variables.
 - This approach started with market cap (the small cap effect) and over the last two decades has added other variables (momentum, liquidity etc.)
- CAPM Plus Models
 - Start with the traditional CAPM ($R_f + \text{Beta} (\text{ERP})$) and then add other premiums for proxies.

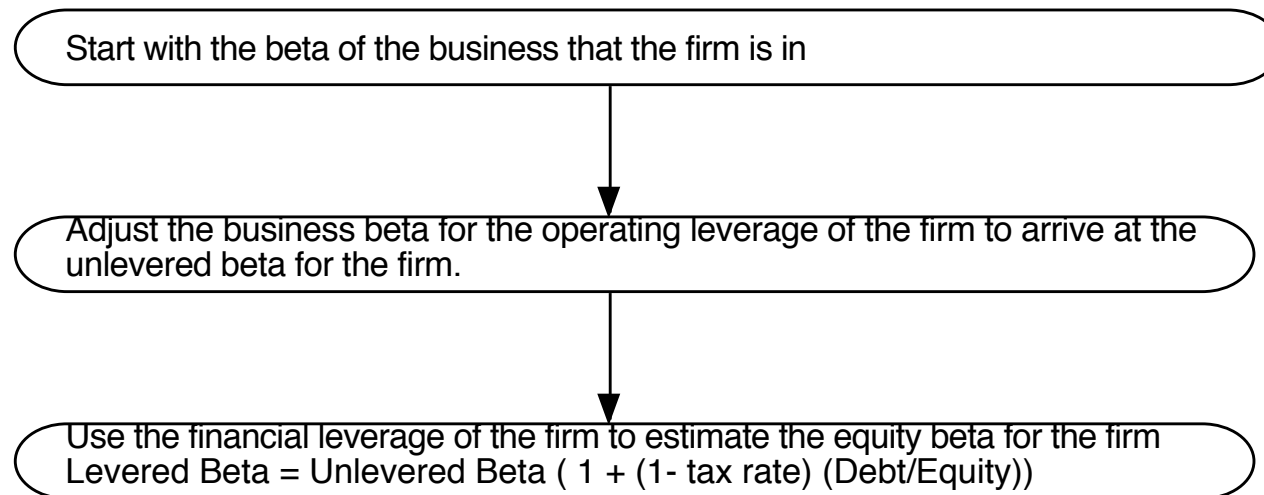
DON'T LIKE THE PRICE-BASED APPROACH..

- **Accounting risk measures:** To the extent that you don't trust market-priced based measures of risk, you could compute relative risk measures based on
 - *Accounting earnings volatility:* Compute an accounting beta or relative volatility
 - *Balance sheet ratios:* You could compute a risk score based upon accounting ratios like debt ratios or cash holdings (akin to default risk scores like the Z score)
- **Qualitative Risk Models:** In these models, risk assessments are based at least partially on qualitative factors (quality of management).
- **Debt based measures:** You can estimate a cost of equity, based upon an observable costs of debt for the company.
 - $\text{Cost of equity} = \text{Cost of debt} * \text{Scaling factor}$
 - The scaling factor can be computed from implied volatilities.

DETERMINANTS OF BETAS & RELATIVE RISK



IN A PERFECT WORLD... WE WOULD ESTIMATE THE BETA OF A FIRM BY DOING THE FOLLOWING



ADJUSTING FOR OPERATING LEVERAGE...

- Within any business, firms with **lower fixed costs (as a percentage of total costs) should have lower unlevered betas**. If you can compute fixed and variable costs for each firm in a sector, you can break down the unlevered beta into business and operating leverage components.
 - $\text{Unlevered beta} = \text{Pure business beta} * (1 + (\text{Fixed costs} / \text{Variable costs}))$
- The biggest problem with doing this is **informational**. It is difficult to get information on fixed and variable costs for individual firms.
- In practice, **we tend to assume that the operating leverage of firms within a business are similar** and use the same unlevered beta for every firm.

ADJUSTING FOR FINANCIAL LEVERAGE...

- **Conventional approach:** If we assume that debt carries no market risk (has a beta of zero), the beta of equity alone can be written as a function of the unlevered beta and the debt-equity ratio
 - $\beta_L = \beta_u (1 + ((1-t)D/E))$
 - In some versions, the tax effect is ignored and there is no $(1-t)$ in the equation.
- **Debt Adjusted Approach:** If beta carries market risk and you can estimate the beta of debt, you can estimate the levered beta as follows:
 - $\beta_L = \beta_u (1 + ((1-t)D/E)) - \beta_{\text{debt}} (1-t) (D/E)$
 - While the latter is more realistic, estimating betas for debt can be difficult to do.

BOTTOM-UP BETAS

Step 1: Find the business or businesses that your firm operates in.

Step 2: Find publicly traded firms in each of these businesses and obtain their regression betas. Compute the simple average across these regression betas to arrive at an average beta for these publicly traded firms. Unlever this average beta using the average debt to equity ratio across the publicly traded firms in the sample.

Unlevered beta for business = $\text{Average beta across publicly traded firms} / (1 + (1 - t) (\text{Average D/E ratio across firms}))$

Step 3: Estimate how much value your firm derives from each of the different businesses it is in.

Step 4: Compute a weighted average of the unlevered betas of the different businesses (from step 2) using the weights from step 3. Bottom-up Unlevered beta for your firm = Weighted average of the unlevered betas of the individual business

Step 5: Compute a levered beta (equity beta) for your firm, using the market debt to equity ratio for your firm. Levered bottom-up beta = $\text{Unlevered beta} (1 + (1 - t) (\text{Debt/Equity}))$

Possible Refinements

If you can, adjust this beta for differences between your firm and the comparable firms on operating leverage and product characteristics.

While revenues or operating income are often used as weights, it is better to try to estimate the value of each business.

If you expect the business mix of your firm to change over time, you can change the weights on a year-to-year basis.

If you expect your debt to equity ratio to change over time, the levered beta will change over time.

WHY BOTTOM-UP BETAS?

- **Less Noisy:** The standard error in a bottom-up beta will be significantly lower than the standard error in a single regression beta. Roughly speaking, the standard error of a bottom-up beta estimate can be written as follows:
 - Std error of bottom-up beta =
$$\frac{\text{Average Std Error across Betas}}{\sqrt{\text{Number of firms in sample}}}$$
- **Updated:** The bottom-up beta can be adjusted to reflect changes in the firm's business mix and financial leverage. Regression betas reflect the past.
- **Don't need prices:** You can estimate bottom-up betas even when you do not have historical stock prices. This is the case with initial public offerings, private businesses or divisions of companies.

ESTIMATING BOTTOM UP BETAS & COSTS OF EQUITY: VALE

<i>Business</i>	<i>Sample</i>	<i>Sample size</i>	<i>Unlevered beta of business</i>	<i>Revenues</i>	<i>Peer Group EV/Sales</i>	<i>Value of Business</i>	<i>Proportion of Vale</i>
Metals & Mining	Global firms in metals & mining, Market cap>\$1 billion	48	0.86	\$9,013	1.97	\$17,739	16.65%
Iron Ore	Global firms in iron ore	78	0.83	\$32,717	2.48	\$81,188	76.20%
Fertilizers	Global specialty chemical firms	693	0.99	\$3,777	1.52	\$5,741	5.39%
Logistics	Global transportation firms	223	0.75	\$1,644	1.14	\$1,874	1.76%
<i>Vale Operations</i>			<i>0.8440</i>	<i>\$47,151</i>		<i>\$106,543</i>	<i>100.00%</i>

Business	Unlevered beta	D/E ratio	Levered beta	Risk free rate	ERP	Cost of Equity
Metals & Mining	0.86	54.99%	1.1657	2.75%	7.38%	11.35%
Iron Ore	0.83	54.99%	1.1358	2.75%	7.38%	11.13%
Fertilizers	0.99	54.99%	1.3493	2.75%	7.38%	12.70%
Logistics	0.75	54.99%	1.0222	2.75%	7.38%	10.29%
Vale Operations	0.84	54.99%	1.1503	2.75%	7.38%	11.23%

EMBRAER'S BOTTOM-UP BETA

<i>Business</i>	<i>Unlevered Beta</i>	<i>D/E Ratio</i>	<i>Levered beta</i>
Aerospace	0.95	18.95%	1.07

$$\begin{aligned}\text{Levered Beta}_{\text{Embraer}} &= \text{Unlevered Beta} (1 + (1 - \text{tax rate}) (\text{D/E Ratio})) \\ &= 0.95 (1 + (1 - .34) (.1895)) = 1.07\end{aligned}$$

- Can an unlevered beta estimated using U.S. and European aerospace companies be used to estimate the beta for a Brazilian aerospace company?

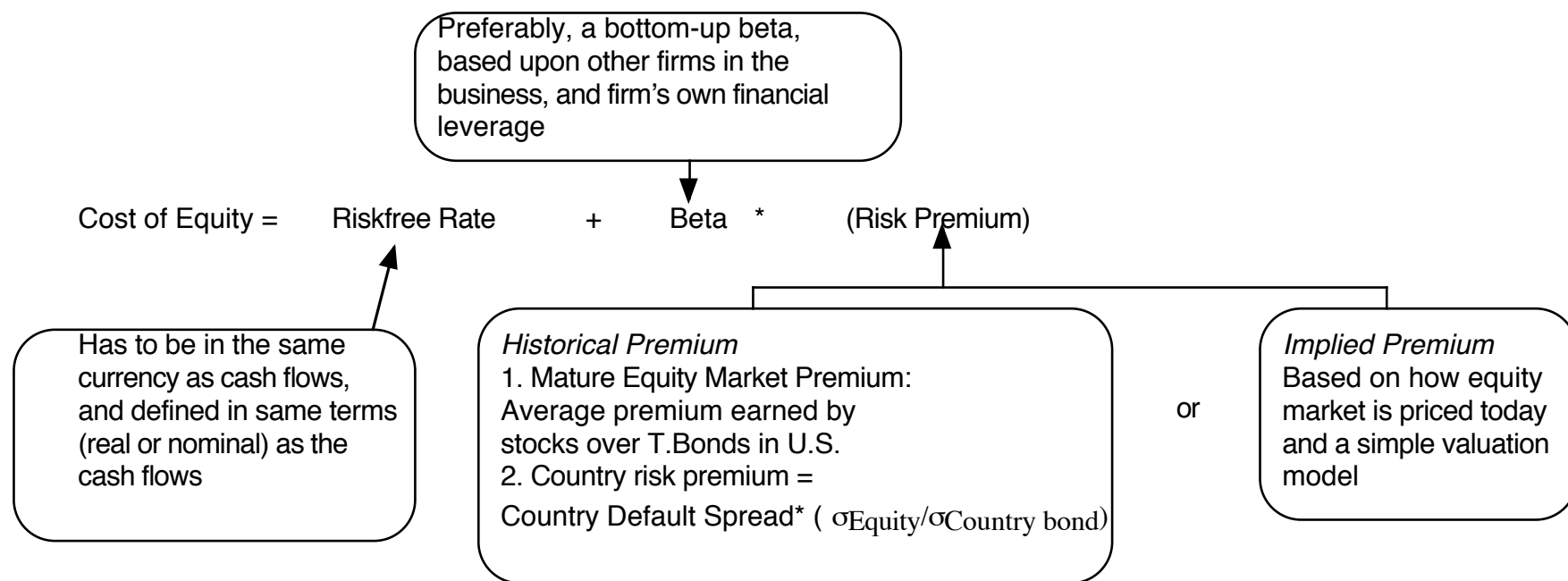
- a. Yes
- b. No

What concerns would you have in making this assumption?

GROSS DEBT VERSUS NET DEBT APPROACHES

- Analysts in Europe and Latin America often take **the difference between debt and cash (net debt)** when computing debt ratios and arrive at very different values.
- For Embraer, using the **gross debt ratio**
 - Gross D/E Ratio for Embraer = $1953/11,042 = 18.95\%$
 - Levered Beta using Gross Debt ratio = 1.07
- Using the **net debt ratio**, we get
 - Net Debt Ratio for Embraer = $(\text{Debt} - \text{Cash}) / \text{Market value of Equity}$
 $= (1953 - 2320) / 11,042 = -3.32\%$
 - Levered Beta using Net Debt Ratio = $0.95 (1 + (1 - .34) (-.0332)) = 0.93$
- The cost of Equity using net debt levered beta for Embraer will be much lower than with the gross debt approach. The cost of capital for Embraer will even out since the debt ratio used in the cost of capital equation will now be a net debt ratio rather than a gross debt ratio.

THE COST OF EQUITY: A RECAP





DISCOUNT RATES: IV

Mopping up

Aswath Damodaran

ESTIMATING THE COST OF DEBT

- The **cost of debt is the rate at which you can borrow money, long term right now**, It will reflect not only your default risk but also the level of interest rates in the market.
- The cost of debt is not the rate at which you have borrowed money in the past or a current book interest rate (interest expense/debt).
- The two most widely used approaches to estimating cost of debt are:
 - Looking up the **yield to maturity on a straight bond outstanding from the firm**. The limitation of this approach is that very few firms have long term straight bonds that are liquid and widely traded
 - Looking up the rating for the firm and **estimating a default spread based upon the rating**. While this approach is more robust, different bonds from the same firm can have different ratings. You have to use a median rating for the firm
- When in trouble (either because you have no ratings or multiple ratings for a firm), estimate a **synthetic rating for your firm** and the cost of debt based upon that rating.

ESTIMATING SYNTHETIC RATINGS

- The rating for a firm can be estimated using **the financial characteristics of the firm**. In its simplest form, the rating can be estimated from the interest coverage ratio
 - Interest Coverage Ratio = $\text{EBIT} / \text{Interest Expenses}$
- For Embraer's interest coverage ratio, we used the interest expenses from 2003 and the **average EBIT from 2001 to 2003**. (The aircraft business was badly affected by 9/11 and its aftermath. In 2002 and 2003, Embraer reported significant drops in operating income)
 - Interest Coverage Ratio = $462.1 / 129.70 = 3.56$

INTEREST COVERAGE RATIOS, RATINGS AND DEFAULT SPREADS: 2004

If Interest Coverage Ratio is		Estimated Bond Rating	Default Spread
> 8.50	(>12.50)	AAA	0.35%
6.50 - 8.50	(9.5-12.5)	AA	0.50%
5.50 - 6.50	(7.5-9.5)	A+	0.70%
4.25 - 5.50	(6-7.5)	A	0.85%
3.00 - 4.25	(4.5-6)	A–	1.00%
2.50 - 3.00	(4-4.5)	BBB	1.50%
2.25- 2.50	(3.5-4)	BB+	2.00%
2.00 - 2.25	((3-3.5)	BB	2.50%
1.75 - 2.00	(2.5-3)	B+	3.25%
1.50 - 1.75	(2-2.5)	B	4.00%
1.25 - 1.50	(1.5-2)	B –	6.00%
0.80 - 1.25	(1.25-1.5)	CCC	8.00%
0.65 - 0.80	(0.8-1.25)	CC	10.00%
0.20 - 0.65	(0.5-0.8)	C	12.00%
< 0.20	(<0.5)	D	20.00%

COST OF DEBT COMPUTATIONS

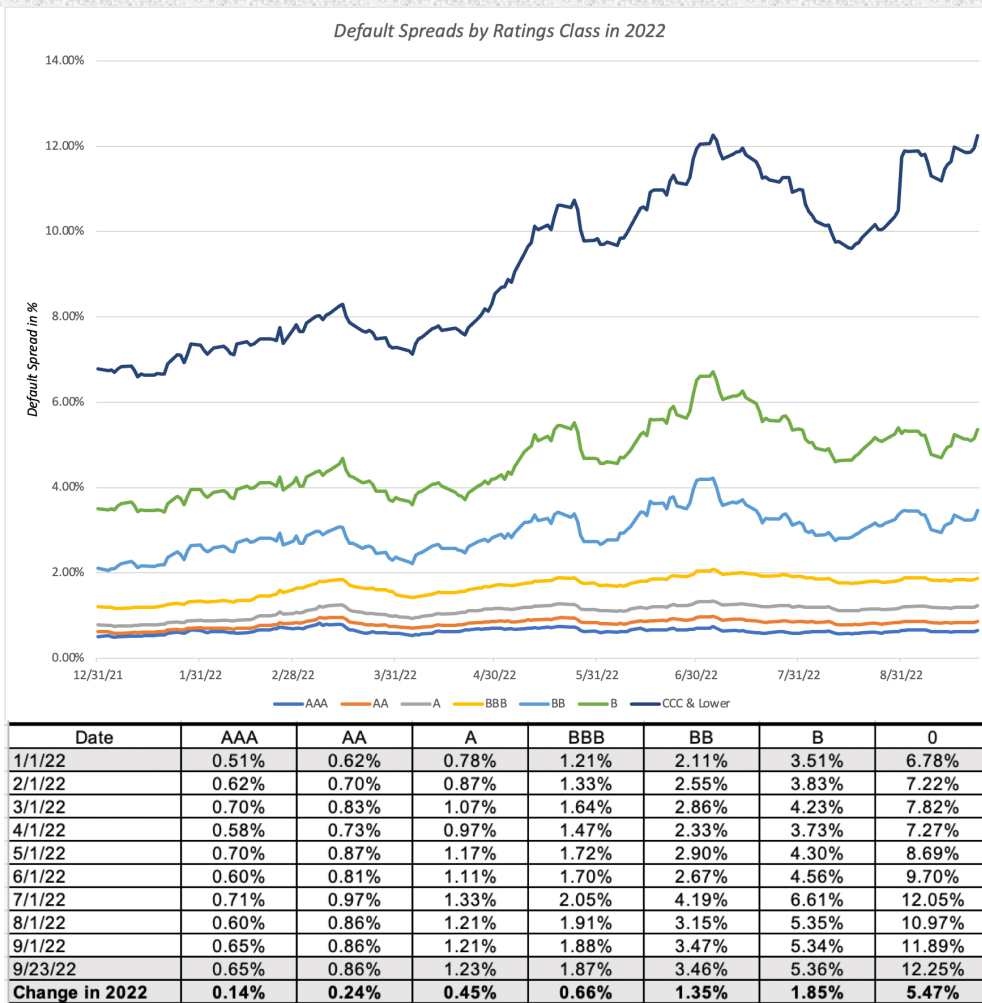
- Based on the interest coverage ratio of 3.56, the synthetic rating for Embraer is A-, giving it a default spread of 1.00%
- Companies in countries with low bond ratings and high default risk **might bear the burden of country default risk**, especially if they are smaller or have all of their revenues within the country.
 - If I assume that Embraer bears all of the country risk burden, I would add on the country default spread for Brazil in 2004 of 6.01%.
 - Larger companies that **derive a significant portion of their revenues in global markets may be less exposed to country default risk**. I am going to add only two thirds of the Brazilian country risk (based upon traded bond spreads of other large Brazilian companies in 2004)

Cost of debt = Riskfree rate + 2/3(Brazil country default spread) +
Company default spread = 4.29% + 2/3 (6.01%) + 1.00% = 9.29%

SYNTHETIC RATINGS: SOME CAVEATS

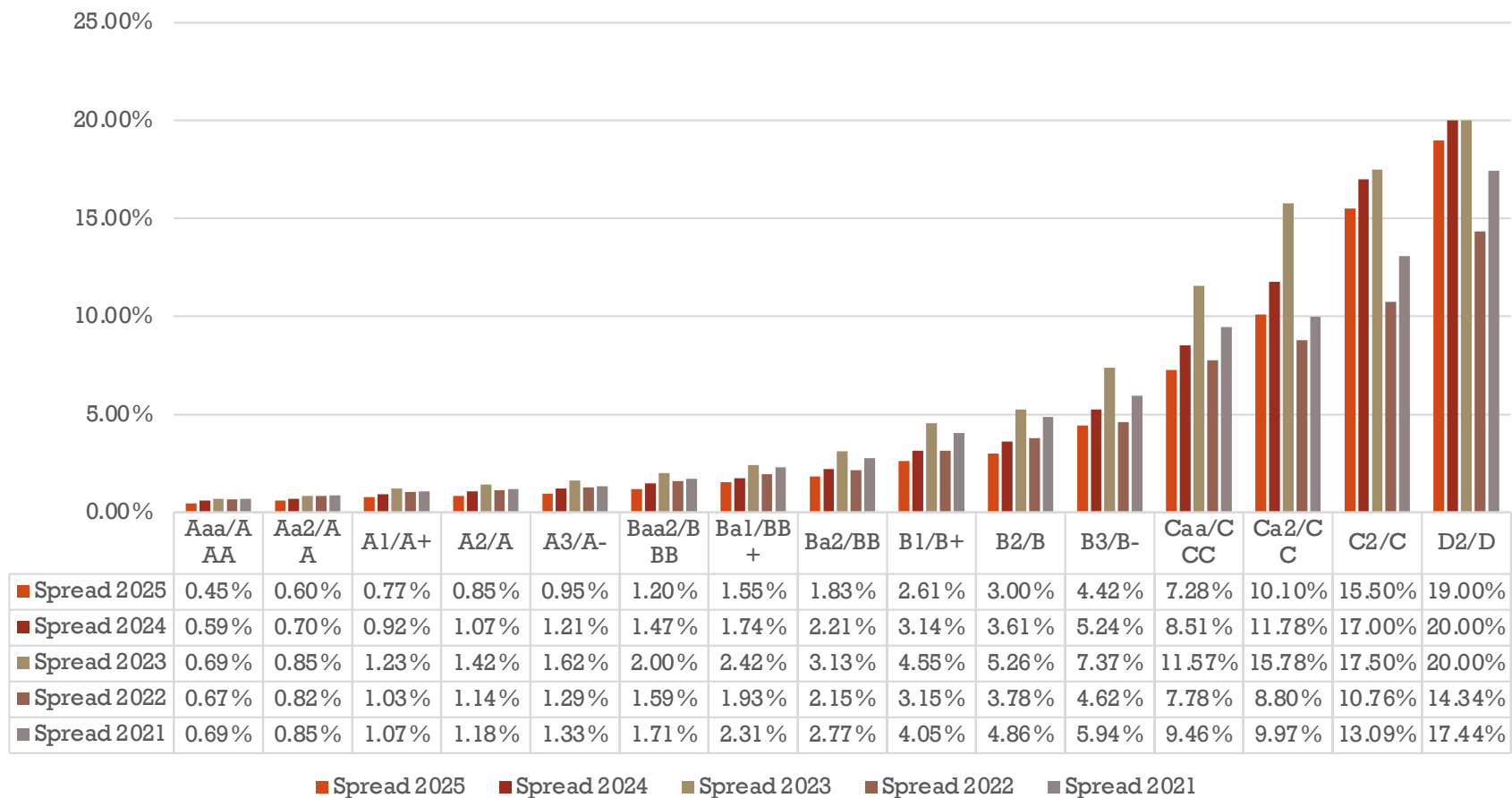
- The relationship between interest coverage ratios and ratings, developed using US companies, **tends to travel well**, as long as we are analyzing large manufacturing firms in markets with interest rates close to the US interest rate
- They are more problematic when looking at smaller companies in **markets with higher interest rates** than the US. One way to adjust for this difference is modify the interest coverage ratio table to reflect interest rate differences (For instances, if interest rates in an emerging market are twice as high as rates in the US, halve the interest coverage ratio).

DEFAULT SPREADS: CHANGE IS A CONSTANT



DEFAULT SPREADS – JANUARY 2025

Corporate Bond Default Spreads on January 1, 2025



SUBSIDIZED DEBT: WHAT SHOULD WE DO?

- Assume that the Brazilian government lends money to Embraer at a subsidized interest rate (say 6% in dollar terms). In computing the cost of capital to value Embraer, should we use the cost of debt based upon default risk or the subsidized cost of debt?
 - a. The subsidized cost of debt (6%). That is what the company is paying.
 - b. The fair cost of debt (9.25%). That is what the company should require its projects to cover.
 - c. A number in the middle.

WEIGHTS FOR THE COST OF CAPITAL COMPUTATION

- In computing the cost of capital for a publicly traded firm, the general rule for computing weights for debt and equity is that you use market value weights (and not book value weights). Why?
 - a. Because the market is usually right
 - b. Because market values are easy to obtain
 - c. Because book values of debt and equity are meaningless
 - d. None of the above
- If a company is not traded, and there is no market value available, would it be reasonable to use book value?
 - a. Yes. There is no choice
 - b. No. There is a choice

If there is a choice, what is it?

ESTIMATING COST OF CAPITAL: EMBRAER IN 2004

- Equity
 - Cost of Equity = $4.29\% + 1.07 (4\%) + 0.27 (7.89\%) = 10.70\%$
 - Market Value of Equity = 11,042 million BR (\$ 3,781 million)
- Debt
 - Cost of debt = $4.29\% + 4.00\% + 1.00\% = 9.29\%$
 - Market Value of Debt = 2,083 million BR (\$713 million)
- Cost of Capital = $10.70\% (.84) + 9.29\% (1 - .34) (0.16) = 9.97\%$
 - The book value of equity at Embraer is 3,350 million BR.
 - The book value of debt at Embraer is 1,953 million BR; Interest expense is 222 mil BR; Average maturity of debt = 4 years
 - Estimated market value of debt = $222 \text{ million (PV of annuity, 4 years, 9.29\%)} + \$1,953 \text{ million} / 1.09294 = 2,083 \text{ million BR}$

IF YOU HAD TO DO IT....CONVERTING A DOLLAR COST OF CAPITAL TO A NOMINAL REAL COST OF CAPITAL

- **Approach 1:** Use a **\$R riskfree rate** in all of the calculations above. For instance, if the \$R riskfree rate was 12%, the cost of capital would be computed as follows:
 - Cost of Equity = 12% + 1.07(4%) + 0.27 (7.89%) = 18.41%
 - Cost of Debt = 12% + 1% = 13%
 - (This assumes the riskfree rate has no country risk premium embedded in it.)
- **Approach 2:** Use the differential inflation rate to estimate the cost of capital. For instance, if the inflation rate in \$R is 8% and the inflation rate in the U.S. is 2%

$$1 + \text{Cost of capital}_{\$R} = (1 + \text{Cost of Capital}_{\$}) \left[\frac{1 + \text{Inflation}_{BR}}{1 + \text{Inflation}_{\$}} \right]$$

$$= 1.0997 (1.08/1.02) - 1 = 0.1644 \text{ or } 16.44\%$$

DEALING WITH HYBRIDS AND PREFERRED STOCK

- When dealing with hybrids (convertible bonds, for instance), **break the security down into debt and equity** and allocate the amounts accordingly. Thus, if a firm has \$ 125 million in convertible debt outstanding, break the \$125 million into straight debt and conversion option components. The conversion option is equity.
- When dealing with **preferred stock, it is better to keep it as a separate component**. The cost of preferred stock is the preferred dividend yield. (As a rule of thumb, if the preferred stock is less than 5% of the outstanding market value of the firm, lumping it in with debt will make no significant impact on your valuation).

DECOMPOSING A CONVERTIBLE BOND...

- Assume that the firm that you are analyzing has \$125 million in face value of convertible debt with a stated interest rate of 4%, a 10-year maturity and a market value of \$140 million. If the firm has a bond rating of A and the interest rate on A-rated straight bond is 8%, you can break down the value of the convertible bond into straight debt and equity portions.
 - Straight debt = (4% of \$125 million) (PV of annuity, 10 years, 8%) + 125 million/1.0810 = \$91.45 million
 - Equity portion = \$140 million - \$91.45 million = \$48.55 million
- The debt portion (\$91.45 million) gets added to debt and the option portion (\$48.55 million) gets added to the market capitalization to get to the debt and equity weights in the cost of capital.

RECAPPING THE COST OF CAPITAL

