1. The random variable \( X \) has mean \( \mu_x \) and standard deviation \( \sigma_x \). The random variable \( Y \) has mean \( \mu_y \) and standard deviation \( \sigma_y \). If \( X \) and \( Y \) are independent, find \( E[XY] \), \( \text{Var}[XY] \) and the standard deviation of \( XY \).

2. The number of auto vandalism claims reported per month at Sunny Daze Insurance Company (SDIC) has mean 100 and standard deviation 20. The individual losses have mean $1,200/claim and standard deviation $80/claim. The number of claims and the amounts of individual losses are independent. Using the normal approximation, calculate the probability that SDIC’s aggregate auto vandalism losses reported for a month will be less than $110,000.

3. Let \( \bar{x} \) be the average of a sample of 26 independent normal random variables with mean 0 and variance 1. Determine \( c \) such that \( \text{Prob}(|\bar{x}| < c) = .5 \). (This is Rice, problem 3, page 198.)

4. Show that if \( X \sim F_{n,m} \), then \( Y = 1/X \sim F_{m,n} \). (This is Rice, problem 5, page 198.)

5. If \( T \sim t_n \), then \( T^2 \sim F_{1,n} \). (This is Rice, problem 6, page 198.)

6. (This is Rice, problem 18 page 316.) Suppose that \( X_1, X_2, \ldots, X_n \) are i.i.d. random variables on the interval \([0,1]\) with density

\[
f(x | \alpha) = \frac{\Gamma(3\alpha)}{\Gamma(\alpha)\Gamma(2\alpha)} x^{\alpha-1}(1-x)^{2\alpha-1}, \quad \alpha > 0, \; 0 \leq x \leq 1.
\]

The parameter \( \alpha \) is to be estimated from the sample. It can be shown that \( E[X] = 1/3 \) and \( \text{Var}[X] = 2/[9(3\alpha+1)] \).

a. How could the method of moments be used to estimate \( \alpha \)?

b. What equation does the MLE of \( \alpha \) satisfy. (I.e., what is the likelihood equation?)

c. What is the asymptotic variance of the MLE of \( \alpha \)?

d. Find a sufficient statistic for \( \alpha \)?

7. (This is Rice, problem 50, page 323.) Suppose that \( X_1, X_2, \ldots, X_n \) are an i.i.d. random sample from a Rayleigh distribution with parameter \( \theta > 0 \),

\[
f(x | \theta) = \frac{x}{\theta^2} \exp[(-x^2/(2\theta^2))], \quad x \geq 0.
\]

a. Find the method of moments estimator of \( \theta \).

b. Find the MLE of \( \theta \).

c. Find the asymptotic variance of the MLE of \( \theta \).
8. In the application of Bayesian estimation of a proportion of defectives that we examined in class and that is detailed in ‘Notes-3,’ we used a uniform prior on \([0,1]\) for the distribution of \(\theta\). Suppose we assume, instead, an informative, beta prior with parameters \(\alpha = 3\) and \(\beta = 7\). Repeat the analysis of the problem to find the Bayesian estimator of \(\theta\) given this prior. (Hint: The beta distribution is a conjugate prior for the likelihood function given in the problem.)

9. This is Rice Problem 8.13. Add part d. Obtain the maximum likelihood estimator of \(\alpha\) (or at least a strategy for finding it).

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**An Angular Distribution**

The angle \(\theta\) at which electrons are emitted in muon decay has a distribution with the density

\[
f(x|\alpha) = \frac{1 + \alpha x}{2}, \quad -1 \leq x \leq 1 \quad \text{and} \quad -1 \leq \alpha \leq 1
\]

where \(x = \cos \theta\). The parameter \(\alpha\) is related to polarization. Physical considerations dictate that \(|\alpha| \leq \frac{1}{3}\), but we note that \(f(x|\alpha)\) is a probability density for \(|\alpha| \leq 1\). The method of moments may be applied to estimate \(\alpha\) from a sample of experimental measurements, \(X_1, \ldots, X_n\). The mean of the density is

\[
\mu = \int_{-1}^{1} x \frac{1 + \alpha x}{2} \, dx = \frac{\alpha}{3}
\]

Thus, the method of moments estimate of \(\alpha\) is \(\hat{\alpha} = 3\overline{X}\). Consideration of the sampling distribution of \(\hat{\alpha}\) is left as an exercise (Problem 13).

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13. In Example D of Section 8.4, the method of moments estimate was found to be \(\hat{\alpha} = 3\overline{X}\). In this problem, you will consider the sampling distribution of \(\hat{\alpha}\).

a. Show that \(E(\hat{\alpha}) = \alpha\)—that is, that the estimate is unbiased.

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b. Show that \(\text{Var}(\hat{\alpha}) = (3 - \alpha^2)/n\). [Hint: What is \(\text{Var}(\overline{X})\)?]

c. Use the central limit theorem to deduce a normal approximation to the sampling distribution of \(\hat{\alpha}\). According to this approximation, if \(n = 25\) and \(\alpha = 0\), what is the \(P(|\hat{\alpha}| > .5)\)?