Decision Support System for Pollution Control in the Copper Industry, Including a Model for the Sulfuric Acid Market

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Abstract

In this paper we develop a decision support system for pollution control investment and operational decisions in the copper industry. The system consists of (i) a non-linear integer model to optimize smelters operations, including the investment decisions relating to smelting capacity and pollution control plants to comply with environmental regulations, and (ii) a network flow model to describe the economic behavior of the sulfuric acid market, which considers the sulfuric acid produced at the pollution abatement stages in the smelting process. This second model solves for equilibrium among spatially separated markets, to determine the price and distribution of acid in each demand and supply region.

The two models interact through the input each receives from the other. Thus, the smelter model uses the sulfuric acid price at each smelter to find optimal operational and investment decisions, whereas the sulfuric acid market model considers sulfuric acid output at the smelters as part of the supply input to find the price of this product at each smelter location. The solution given by the decision support system is the global equilibrium obtained when this iterative process between the two models converges. Thus, the price of the sulfuric acid, which is the central component when deciding when and where to locate a sulfuric acid plant, is determined endogenously, rather than assumed exogenous as in most models of this type.

Computational experiments show that expected profits associated with the copper industry can increase significantly when the problem is solved in aggregate, as compared with the smelters making their decisions independently. Several applications of the decision support system are described.

1 Introduction

In this paper we develop a decision support system for pollution control investment and operational decisions in the Chilean copper industry. This research, mainly motivated by environmental

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issues, was supported by the Chilean Copper Commission (Cochilco) as part of a set of initiatives that the Chilean government has been promoting to improve efficiency in the mining sector (Cochilco’s main mission is to develop policies, strategies, and actions to promote permanent and sustainable development of the mining sector). The model presented in this paper is currently used by Cochilco and has become an important decision making tool. For instance, the decision support system is extensively used to revise new investment projects, analyze the economic impact of different environmental standards, and anticipate future infrastructure requirements related to the commercialization of copper and sulfuric acid. The goal of this paper is to describe how operations research tools can be applied to efficiently incorporate the environmental regulations into the copper production process, using the Chilean case as a motivating example.

Copper is one of Chile’s most important industries, accounting for approximately 38% of the country’s total exports. Over the last hundred years copper production has increased significantly, growing from 21 tons/year in 1897 to 828,300 in 1975 and 3,115,800 tons/year in 1996. Despite such significant industry growth over the years, it is only in the 1980s that the Chilean government began to study the environmental effects of the copper production process. The smelting process, that transforms concentrate into refined copper, contributes significantly to the air pollution through the emissions of sulfur dioxide, particulate matter, and arsenic. For example, in 1989 stationary sources produced 99.3% of all sulfur emissions in the country, being 91% of this emitted by the copper smelters (a total of 874.400 tons of sulfur per year were produced by the copper smelters, see Solari(1992)). In 1991 this led to new environmental regulations governing emissions of sulfur dioxide and particulate matter. A new environmental regulation on arsenic emissions was introduced in 1999. One of the most significant environmental challenges faced by the public-sector mining industry today is how to solve the pollution problem in copper smelters.

There are currently seven copper smelters in the country, two private and five state-owned. The state-owned smelters are Chuquicamata, Caletones, Potrerillos, Ventanas, and Paipote. These receive concentrate for transformation into refined copper from 22 treatment plants, seven of which are state-owned. Currently, none of the state-owned smelters fully satisfies the new environmental regulations, although most of them have already submitted clean-up plans, the so-called base clean-up plans, that will eventually ensure compliance with the new regulatory standards. These plans are already being implemented and have completed some of the stages successfully. Each smelter developed its own clean-up plan individually, without considering the effects of the other smelters’ decisions. The total expected cost of these plans is approximately US$1 billion, most of it to be spent on building sulfuric acid plants and electrostatic precipitators — the most commonly used technologies for eliminating the pollutants emitted into the atmosphere during the smelting process. Sulfuric acid plants transform sulfur dioxide into sulfuric acid, making this is one of the most important by-products of the copper smelting process.

The huge monetary investments required by the state-owned smelters’ base clean-up plans led
the Cochilco in 1994 to develop a preliminary model to optimize investment decisions in pollution abatement plants, considering the five state-owned smelters as a single system. There are four main features that make the solution considering the state-owned smelters as a single system better than that obtained by solving the problem individually for each smelter: (i) the volume economies of scale when building sulfuric acid plants, electrostatic precipitators, and smelting facilities, (ii) the redistribution of concentrates among smelters in order to minimize pollution (the sulfur and arsenic content of the concentrates vary according to origin, and thus also does the pollution it causes when processed), (iii) the joint decision on sulfuric acid distribution to the demand regions, and (iv) the effect of total sulfuric acid production on prices.

The objective function of this mixed-integer linear-programming model is to maximize total expected profit from the public-sector copper production process, discounted over the planning horizon, subject to technical, environmental, and market constraints. A complete description of the model can be found in Mondschein and Schilkrut (1997). As described in there, due to the significant differences between the solutions provided by this preliminary model and those of the initial clean-up plans submitted by the smelters for the authorities' approval, Cochilco commissioned the authors of this paper to develop a new model, in order to include several important features not considered in the first decision support system.

In this paper we develop a new mathematical model which includes the three main features not considered in the preliminary model: (i) an economic model for sulfuric acid prices as a function of the supply of this product, (ii) arsenic emission constraints, and (iii) explicit consideration of the heterogeneity of concentrate mineralogical composition, in terms of sulfur and arsenic, which largely depends on the concentrate's origin. These new features significantly increase the model's complexity. Instead of a mixed-integer linear-programming we now have a system with a nonlinear integer model to describe smelter operations including the investment decisions in pollution abatement plants, and a network flow model to describe the economic behavior of the sulfuric acid market. The main sources of nonlinearity come from the consideration of different mixes of copper concentrate in terms of copper, sulfur and arsenic. The sulfuric acid market model is solved for equilibrium among spatially separated markets, to determine the price and the distribution of acid in each demand and supply region. The two models interact through the input that each receives from the other.

The remainder of this paper is organized as follows. In Section 2 we provide a brief description of the copper production process and the sulfuric acid market. In Section 3 we describe the mathematical models for smelter operations and the sulfuric acid market, and in Section 4 we describe the models’ resolution and main results. Finally, in Section 5 we present conclusions.
2 System Description

2.1 Copper production process

In this subsection we briefly describe the copper production process, specially stressing aspects directly involved in the formulation of our problem. A more complete description of the process can be found in The Copper Manual (1976). Figure 1 shows the sequence of stages in the copper production process.

![Diagram of copper production process]

**Extraction:** This is the first stage where the ore is extracted from the mines. The ores are roughly divided into sulfide and oxide ores. The sulfide ores are complex mixtures of sulfides of copper and iron, combined with compounds of other metals such as arsenic, zinc, silver, and gold. The oxide ores originate from the decomposition and oxidization of sulfide ores. The extraction of sulfide or oxide ores depends mainly on the stage of the mine’s exploitation. Typically, oxide ores are found in the first layers, while sulfide ores are present in the deeper ones. Oxide and sulfide ores differ widely regarding their susceptibility to different concentration methods to produce refined copper, mainly due to the chemical properties of the ores. These methods are described below. The treatment of ores’ mixtures highly depends on its composition; for example, if the amount of oxides is low, these are treated similarly to pure sulfide ores. The grade of copper (percentage of copper) in the large Chilean deposits can vary from 1% to 1.8%.

Historically, the copper mining industry in Chile has been classified in three categories: large, medium, and small industry, according to the production volume. Before 1964, most of the mines were owned by private investors. During the period from 1964 to 1967 the large copper mines in Chile, owned by American investors, reached an agreement with the government regarding property and production aspects. Thus, the government held 51% of the shares since 1967 until 1973. In 1973, the nationalization of the large copper mines took place, and from that moment onwards the government had total control of these mines. Nowadays, there are seven State-owned and fifteen private copper mines that belong to the large and medium mining industry; the rest are small private mines. The State-owned mines are: Andina, Chuquicamata, El Salvador, El Teniente, Taltal, Matta, and Vallenar. The private mines are: Candelaria, Cerro Negro, Collahuasi, El Indio,
El Soldado, Los Bronces, Escondida, Las Luces, Los Pelambres, Mantos Blancos, Michilla, Punta del Cobre, Las Cenizas, Tocopilla, and Carola.

Most of the large copper mines have been operating since the beginning of the century, due mainly to their size and also to the development of new technologies that allow to exploit ores with lower grades of copper. The lifetime of the medium size mines is typically from 15 to 20 years. Small mines operate only for a few years.

**Treatment of sulfide ores**

**Flotation:** This consists of the extraction of ore particles containing copper in combination with sulfur. The mineralized rock is crushed and broken into microscopic particles, then air is made to bubble up from the bottom of the cell, the sulfides stick to the bubbles and rise with them to the top where they are removed. In this way, a froth is obtained containing copper concentrate with a purity varying from 20% to 50%. Next, the copper concentrate passes to a flotation process for the recovery of molybdenite sulfide, and finally, the concentrate is dried to obtain a final humidity of approximately 8%.

This process is carried out in treatment plants that are located at the site of the mines, and thus, they are private or State-owned according to the mines’ property. Very small mines do not have treatment plants, and therefore, send their minerals to some State-owned treatment plants to be processed.

The copper concentrate produced at this stage can be either exported to other countries or processed in the country to produced refined copper. The main factors that influence this decision are concentrate and refined copper prices, available capacity to process further the concentrate, and refined copper production costs, which includes the costs to satisfy the environmental regulations.

**Smelting:** At this stage, by melting the concentrates in a furnace (e.g. Flash Furnace or “El Teniente” Converter) and subsequent treatment in converters (e.g. Pierce-Smith Converter), the sulfur, iron and other unwanted elements are removed to produce blister copper with a grade of at least 99.5%. Blister copper can be either sold directly or treated further in refineries to produce a higher grade copper known as copper cathodes.

The smelting process is carried out in the copper smelters and produces essentially all the sulfur dioxide emissions, which also contain significant amounts of arsenic and particulate matter. The gases are either discharged directly into the atmosphere or else treated in abatement plants to eliminate the pollutants before emission into the air.

Electrostatic precipitators are commonly used to reduce the amount of particulate matter in the pollutant gases, with efficiencies in excess of 90%. A significant amount of arsenic is attached to the particulate matter, so the reduction of particulate matter also reduces the amount of arsenic in the gases. 99.99% Sulfuric acid plants are commonly used to eliminate the sulfur dioxide contained
in the gases. These plants transform sulfur dioxide into sulfuric acid with high levels of efficiency. The acid is used in the leaching process in the treatment of oxide ores, as explained below, or else sold for use in other industries. In this way, the costs involved in pollution abatement are partly offset by income from sulfuric acid sales.

The dimensions of the precipitators and sulfuric acid plants are determined by the volume of gases to be processed, which in turn depends on copper production and the smelting-conversion technology used in the smelter. Therefore, it is necessary to evaluate the trade-offs between more expensive technology that produces more concentrated gases versus cheaper technology that produces more diluted gases. Similarly, it is important to decide the mix of gases to process in the abatement plants, as the gases produced at the smelting and conversion stages contain different concentrations of pollutants: poorer gases (low pollutant concentration) produced at the converters are more expensive to process. See Subsection 3.1.3 for details of how the mix of flows (gases and products) are modeled.

There are seven smelters; five of them are State-owned: Chuquicamata, Caletones, Potrerillos, Ventanas, and Paipote. These smelters were built considering the proximity to the treatment plants, access to ports, and availability of energy and water. Most of the State-owned smelters were built at the beginning of the century and no environmental considerations were included in the location decisions. The melting-conversion process is the main focus of the environmental problem studied in this paper. Figure 2 shows the smelters' operations (melting-conversion) where pollutant gases are produced. We remark that, at the moment the decision support system was developed, only some smelters had installed electrostatic precipitators and sulfuric acid plants to treat part of their emissions.

Refining: This is the final phase in obtaining high-grade copper. There are two types of refining: (i) refining by fire and (ii) electrolytic refining. The aim of refining by fire is the elimination of the maximum amount of impurities from blister copper using pyro-metallurgical processes, whereas in electrolytic refining the same goal is achieved by electrolytic precipitation.

Currently there are three electrostatic refineries in the country, located at the site of the State-owned smelters at which they belong: Chuquicamata, Potrerillos, and Ventanas. These refineries process the blister copper produced in the corresponding smelter, but also sell the service to the other private and State-owned smelters. There is only one fire refinery located at the State-owned smelter Caletones.

Treatment of oxide ores

In the leaching process where copper ores are attacked with acid solutions, the copper content is extracted, resulting in a solid phase, poor in copper (residues), and a liquid copper-rich phase (copper sulfate). The leaching process is one of the biggest consumers of the sulfuric acid produced
in copper smelters.

2.2 Characteristics of the sulfuric acid market

World production of sulfuric acid is approximately 150 million tons per year. This production comes from two main sources: sulfur toasting (80%) and the treatment of gases produced in the smelters (20%). The main uses of sulfuric acid are in the production of fertilizers, as a raw material in the chemistry industry, and in the production of copper in the leaching process.

The sulfuric acid market is mainly local, with less than 5% of the output traded on international markets. Most of the international trade is generated by the surplus sulfuric acid from the smelters, which cannot be sold directly in the region. This is because of the product’s high transportation and storage costs, due to its corrosive and toxic properties. These costs are so high, that in certain cases producers make losses when they are unable to sell the product locally and have to trade it on the international market. For example, recently Japan sold the product to Chile at 50 US$/TM; yet transportation cost, incurred by Japan, were over 55 US$/TM.

Chilean sulfuric acid production accounts for approximately 1.6% of the total world output (2.5 million tons per year), and is consumed mainly in the leaching processes to produce refined copper; only a small percentage is used in other industrial processes. As a result of the new environmental regulations on emissions of sulfur dioxide into the air, the production of sulfuric acid in copper smelters is growing significantly. This supply growth arising from copper production, has become a
significant problem for the smelters, due to the need to sell the product with its high transportation and handling costs, preferably in regional markets.

More details about the sulfuric acid market can be found in Metal Economic Group et. al (1994), Fertecon (1990), and Cochilco (1994).

3 Model Description

The mathematical formulation consists of two models that interact to obtain a general equilibrium solution. The first of these, The smelter planning and operation model, solves to optimize smelter operations and investment decisions on smelting capacity and pollution abatement plants. The second model, The sulfuric acid market model solves for equilibrium among spatially separated markets, thereby determining the price and distribution of acid in each demand or supply region. In what follows we describe the two models and the interaction between them.

![Diagram](image)

Figure 3: System under consideration.

3.1 Smelter planning and operation model

The smelter planning and operation model optimizes the investment decisions on pollution abatement plants and smelting capacity, as well as operational decisions regarding the distribution of concentrate among the state-owned smelters, the production of copper and sulfuric acid, and the treatment of pollutant gases. The model considers the production of concentrate as an exogenous variable. The model has a non-linear integer formulation, with the main sources of nonlinearity
arising from its consideration of different concentrate compositions in terms of copper, sulfur and arsenic. The model’s integer structure is due to investment decisions on smelting capacity and abatement plants. Below we present a qualitative description of the model, and in the Appendix we provide the detailed mathematical formulation.

3.1.1 Objective function

The objective function is the maximization of the total discounted profit associated to the public copper industry. It includes the sales of concentrate from state–owned treatment plants (public concentrate) as well as the production of refined copper and sulfuric acid, over the planning horizon. Revenues and costs are as follows.

1. Operational income.
   - Exports of public concentrate.
   - Sales of refined copper produced from public concentrate.
   - Smelting services provided to private treatment plants.
   - Sales of sulfuric acid.

2. Operational costs.
   - Transportation costs of concentrate from state–owned treatment plants to the smelters.
   - Refined copper production costs. These costs depend on the smelting technology used, the mineralogical composition of the concentrate and the smelter’s location.
   - Sulfuric acid production costs.
   - Electrostatic precipitator operating costs.

3. Investments.
   - Sulfuric acid plant and electrostatic precipitator installation costs.
   - New smelting capacity installation costs.
   - Technology upgrade costs. This item includes investments to modernize plant so as to be more efficient or less polluting. This is an alternative for some items of equipment only.

3.1.2 Decision Variables

The decision variables can be divided into investment and operating decisions.
1. Investment decisions: when, where, and how much to invest in sulfuric acid plants, electrostatic precipitators and smelting capacity. Due to the fixed costs, there are binary decision variables regarding whether or not to install a new plant or equipment.

2. Operational decisions:

- Distribution of concentrate from the treatment plants to the state-owned smelters. As the mineralogical composition of the concentrate varies according to its origin, the final mix of concentrates to be processed in a smelter is also a decision variable. Concentrate exports are determined when deciding the amount of concentrate to be processed in the country.

- Copper and sulfuric acid production in each state-owned smelter.

- Emissions of pollutant gases to be discharged directly into the air. These decisions determine the volume of gases treated in electrostatic precipitators and sulfuric acid plants, and hence the amount of sulfur, arsenic, and particulate matter discharged into the atmosphere.

3.1.3 Constraints

We next describe the main technical, environmental and economic constraints at each stage of the process.

1. Treatment plants

- Concentrate production balance

2. Smelters

- Smelting capacity constraint for each plant and smelter.

- Upgrading of smelting capacity as a result of new investment.

- Copper production and pollutant gases balance. These constraints include: (i) mass flow conservation for each substance at any division or combination of product or gas streams, and (ii) concentration conservation at the division of any stream in a liquid or gas phase. These latter constraints are the source of nonlinearity in our model. Since the division of a stream of gases or liquids must be such that each output stream must have the same concentration as the input stream, and since the composition of any stream is a decision variable, the type of constraints required to model this concentration conservation turns out to be quadratic.

For example, let us consider a stream of gases (stream A) containing three different substances: (1) sulfur dioxide, (2) arsenic, and (3) particulate matter (Figure 4). Stream
A is divided into two streams B and C. Stream B is treated in a sulfuric acid plant while Stream C is emitted directly to the atmosphere through the smelter chimney. Let $X_i^j$ be the mass of substance $i$ ($i = 1, 2, 3$) contained in stream $j$ ($j = A, B, C$).

\[
\begin{align*}
\text{Stream A} & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \frac{X_i^A}{\sum_k X_k^A} = \frac{X_i^j}{\sum_k X_k^j} \quad i = 1, 2, 3 \quad \text{and} \quad j = B, C.
\end{align*}
\]

Thus, for each output stream and component we obtain a quadratic constraint (see equation (26) in the Appendix). In this paper we solve the mathematical formulation using nonlinear optimization. However linear programming approximation techniques are also available. For more details on this latter approach we refer the reader to Shapiro (1993) and the references therein.

- Technical constraints on the composition of the concentrate mix to be processed in a smelter. Due to technical characteristics, some of the smelting plants can only process concentrates with certain specific compositions.
- Lower bound for the smelter utilization. Due to economic considerations, operations below a certain minimum utilization rate are considered non-feasible.

3. Abatement plants

- Sulfur, particulate matter, and arsenic production balance at each stage of the smelting process.
- Balance between gases directly emitted into the air without treatment and those treated in electrostatic precipitators. This balance determines the amount of sulfur, particulate matter and arsenic in each gas flow.
- Balance between the volume of gases emitted into the atmosphere and those treated in sulfuric acid plants after treatment in electrostatic precipitators. This determines the sulfur dioxide concentration in the gases treated in the acid plants. The concentration must be within the range for which the acid plant was designed.
• Capacity constraints for the sulfuric acid plants.
• Upgrading of sulfuric acid plants capacity as a result of new investments.
• Capacity constraints for the electrostatic precipitators.
• Upgrading of electrostatic precipitators capacity due to new investments.
• Sulfuric acid production balance.

4. Emissions of pollutant gases

• Upper bound for sulfur emissions.
• Upper bound for emissions of particulate matter.
• Upper bound for arsenic emissions.

3.2 Sulfuric acid market model

As we mentioned in Subsection 2.2 transportation costs are a crucial factor in the sulfuric acid market, and hence also are the geographical distribution of consumers and suppliers. The model developed in this subsection is based on the economic literature dealing with equilibrium among spatially separated markets, where the price paid by consumers equals the price received by producers plus the transportation costs associated with moving the product from its place of production to the demand area. In particular, we are interested in the Competitive Equilibrium, i.e., the equilibrium obtained when agents act as price takers. Since the work by Enke (1951), the spatial price equilibrium problem has been widely studied in the economic literature. Our model takes Samuelson (1952) as its starting point (the base model), and therefore it is briefly described below (for more details on Spatial Price Equilibrium, see Harker (1984)).

3.2.1 Base Model

For simplicity we present the Samuelson’s model for the case of two regions; the extension to the general case of $n$ regions follows directly.

Each zone $i$ ($i = 1, 2$) is characterized by an upward inverse supply curve $S_i(Q_i)$ and a downward inverse demand curve $D_i(Q_i)$, where $Q_i$ represents the level of production or consumption in the $i^{th}$ zone respectively. In general, the two regions are geographically separated, which implies that there is a transportation cost incurred when moving a unit from one region to the other. We denote by $T_{ij} \geq 0$ the unit transportation cost from region $i$ to region $j$.

In the absence of trade between zones, the competitive equilibrium within zone $i$ is given by a supply and demand level $Q^c_i$ and a price level $P^c_i$ such that $P^c_i = S_i(Q^c_i) = D_i(Q^c_i)$. However, the output $\{(Q^c_i, P^c_i) : i = 1, 2\}$ is a stable competitive equilibrium for the two-locality case only if the difference in price between the two regions is smaller than the corresponding transportation
cost, i.e., if and only if $P_2^e - P_1^e \leq T_{12}$ and $P_2^e - P_2^e \leq T_{21}$. If one of these conditions is violated, for example $P_2^e - P_1^e > T_{12}$, then producers in region 1 will be tempted to send some of their production to region 2. This flow of product from region 1 to region 2 will rise the price in region 1 and decrease the price in region 2, reaching an equilibrium when $P_2 - P_1 = T_{12}$.

The competitive equilibrium in this two-locality case can be solved graphically. First, we define the excess-supply curve $ES_i(P)$ as the horizontal subtraction of the supply and the demand in region $i$ at price $P$ (see Figure 5a). Thus, $ES_i(P)$ represents the amount that region $i$ is willing to export at a price $P$. The net excess-supply curve for the two markets (curve $N$ in Figure 5b) is defined as the vertical subtraction of $ES_2$ and $ES_1$, and by construction $N(Q)$ represents the difference in price between region 1 and 2 that will be observed if the flow from region 1 to region 2 is $Q$ units. As we mentioned above, as long as this difference is larger than the transportation cost, there exists an incentive to increase the flow of product from one region to the other. Thus, the equilibrium in this two-locality case can be found intersecting the net excess-supply curve, $N(Q)$, and the transportation-cost curve, $T(Q)$ (dashed line $a - b - c - d$), where

$$T(Q) = \begin{cases} 
T_{12} & \text{if } Q > 0, \\
-T_{21} & \text{if } Q < 0, \\
(-T_{21}, T_{12}) & \text{if } Q = 0.
\end{cases}$$

If this intersection belongs to region $a - b$, then there is a flow from region 2 to region 1 ($Q_{21} > 0$). If the intersection belongs to region $b - c$, then there is no flow of product between regions. Finally, if the intersection falls in region $c - d$ (which is the case in Figure 5b), then there is a positive flow from region 1 to region 2 ($Q_{12} > 0$).

**Figure 5:** Spatial equilibrium for the two-region case.

Samuelson noted that the optimality condition $N(Q) = T(Q)$ for the two-locality case is also the optimality condition for the following optimization problem $\max_Q \left\{ \int_0^Q (N(x) - T(x))dx \right\}$, which can be rewritten as follows:

\footnote{If $ES_i(P) < 0$, then it represents the amount that region $i$ is willing to import at price $P$.}
\[
\max_{Q_{12},Q_{21}} \left\{ -\int_0^{Q_{12}} ES_1(x)\,dx - \int_0^{Q_{21}} ES_2(x)\,dx - T_{12} Q_{12}^+ - T_{21} Q_{21}^+ \right\},
\]

where \( Q_{ij} \) is the level of exports from region \( i \) to region \( j \) and \( Q^+ = \max\{0,Q\} \). Using Samuelson’s nomenclature, we call \( SB_i(Q_i) = -\int_0^{Q_i} ES_i(x)\,dx \) the Social Benefit for region \( i \) when exporting \( Q_i \) units. We note that this Social Benefit is an artificial construction introduced by Samuelson that simplifies the formulation but it is not directly related to any agents’ profit. For the two-locality case, we have that \( Q_1 = Q_{12} = -Q_{21} = -Q_2 \) and problem (1) becomes:

\[
\max_{Q_{12},Q_{21}} \left\{ SB_1(Q_1) + SB_2(Q_2) - T_{12} \cdot Q_{12}^+ - T_{21} \cdot Q_{21}^+ \right\},
\]

s.t. \( Q_1 = Q_{12} = -Q_{21} = -Q_2 \).

Samuelson’s major observation was that formulation (2) is also valid for the general \( n \)-locality case, i.e., he showed that the spatial equilibrium in the general case exists, and it can be found solving the following optimization problem:

\[
(PS) \quad \max_{Q_{ij}} \left\{ \sum_{i=1}^{n} SB_i(Q_i) - \sum_{i=1}^{n} \sum_{j=1}^{n} T_{ij} \cdot Q_{ij}^+ \right\}, \tag{3}
\]

s.t. \( Q_i - \sum_{j} Q_{ij} = 0, \quad Q_{ij} + Q_{ji} = 0. \)

Problem (PS) is, in general, a nonlinear programming problem. For example, Takayama and Judge (1964), assuming linear demand and supply curves, reformulated (3) as a quadratic programming problem. Another special case occurs with perfectly inelastic demand and supply curves. In this case, problem (3) is equivalent to a linear programming problem that minimizes the total transportation cost (provided total production equals total demand among all the regions). As we will show in the next subsection, this latter case is appropriate for modeling the Chilean sulfuric acid market, when certain special market features are combined with the iterative algorithm developed for solving our model.

### 3.2.2 The Model Applied to the Chilean Case

In this subsection we apply Samuelson’s formulation to the Chilean acid market. We first define the commercial regions, then determine the supply and demand curves, and finally apply the model for this particular case.

(i) **Definition of Commercial Regions**

According to the formulation presented in subsection (3.2.1), we define the regions that conform the Chilean sulfuric acid market as the different production and/or consumption centers for the
product. Therefore, a region does not necessarily identify a geographical area but rather a producer and/or consumer.

The commercial regions that produce sulfuric acid can be clearly defined in the Chilean case: there are only 10 plants in the country that currently produce sulfuric acid. Five of them are the state-owned smelters and the rest are private plants. In addition to the domestic production, we consider the international market that is able to supply sulfuric acid to the country in case of local deficit.

On the other hand, consumers of sulfuric acid are numerous; most of them are small mining projects that use sulfuric acid to treat their production of copper oxide ores in the leaching process. Due to this atomized structure of the market, we aggregate the small consumers according to their geographic proximity. For this purpose, we use the Chilean political-administrative division of the country to define four macro-regions: Regions I, II, III, and an aggregate zone combining Regions IV, V, VI, and XIII. For each of these macro-regions, we aggregate all the consumers in a single demand curve and compute an average vector of transportation costs. Since we are particularly interested in studying the operation of the five state-owned smelters, they are treated as independent consumer regions, i.e., their consumption of acid is not incorporated on the demand of the macro-region where they belong. Finally, we assume that the international market is able to absorb any excess of production.

Thus, we define a total of fifteen commercial regions for the sulfuric acid market (see Figure 6). Some of them are consumers (4 macro-regions), others are only producers (5 private-producer plants), and some are both (5 state-owned smelters plus the international market). In what follows we determine the supply and demand curves for each them.

Figure 6: Chilean sulfuric acid market.
(ii) REGIONAL SULFURIC ACID SUPPLY

The production regions are divided as follows: \( I \) represents the international market, \( P \) is the set of Chilean private producers (5 regions), and \( J \) is the set of state-owned smelters (5 regions). For the international market we assume perfectly elastic supply, i.e., if there is a deficit nationally, the international market will satisfy it at a price that is independent of the amount sold. Thus, if \( Q_N(t) \) and \( D_N(t) \) represent the aggregate national production and consumption respectively in year \( t \), international supply is given by \( S_I(t) = (D_N(t) - Q_N(t))^+ \). We denote by \( P^S_I(t) \) the international unit price in year \( t \) in Chilean ports. This is independent of Chilean consumption, and assumed known for all \( t \). This is a reasonable assumption, considering that the Chilean sulfuric acid market accounts for less than 2% of the total world acid trade (Metal Economic Group et. al (1994)). In addition to this fact, we have conducted a sensitivity analysis to measure the impact of this assumption on the estimates of the national prices. It turns out that an error of \( \pm 5\% \) on the estimation of the international price produces a variation of less than \( \pm 0.4\% \) in the average national price. Thus, from an empirical point of view, this assumption is not critical.

For private producers, we assume perfectly inelastic supply, i.e. if \( Q_i(t) \) represents the output of private producers \( i \in P \), we assume known the set \( \{Q_i(t)|i \in P, \ t \geq 1\} \). There are three main reasons for considering private production as independent of price. First, in the range of prices at the product is sold, private-sector production decisions depend mainly on installed productive capacity since the variable production costs are low. Second, private production of sulfuric acid accounts for less than 15% of the total supply. Thus, any variability on the private sector has a small impact at a national level. Finally, according to Chilean experts on the sulfuric acid market, private productive capacity is not expected to grow in the next decades, because private producers are anticipating the increase of acid production by the state-owned smelters as a result of the new environmental regulations.

Finally, for the state-owned smelters, sulfuric acid production is mainly determined by the refined copper production, environmental regulations and the sulfuric acid price. Refined copper production determines the volume of sulfur dioxide produced at the smelters. Hence, if \( V(c) \) represents the volume of sulfur dioxide generated when \( c \) tons of copper are produced, and \( \hat{V} \) represents the maximum volume of sulfur dioxide that may emitted directly into the atmosphere, in compliance with environmental regulations, then sulfuric acid supply from a smelter producing \( c \) tons of copper is given by:

\[
S(c, p) = \alpha \cdot \left[ \left( \frac{V(c) - \hat{V}}{|a|} \right)^+ + \xi \left( \frac{\hat{V} - (\hat{V} - V(c))^+}{|b|} \right) \right],
\]

where \( p \) represents the price of the sulfuric acid and \( \alpha \) is the mass transformation factor between sulfur dioxide and sulfuric acid. Expression \( |a| \) represents the minimum volume of gases to be treated in the smelter for compliance with environmental regulations, and is independent of the
price. On the other hand, \(|b|\) represents that part of the sulfuric acid offer that is price sensitive, i.e. \(\hat{V} - (\hat{V} - V(c))^+\) represents the amount of sulfur dioxide that the smelter can either discharge directly into the air or transform into sulfuric acid for sale. How much the smelter will process of the amount available in \(|b|\) will depend on the observed price, according to the production function \(\xi(V, p)\). This function \(\xi(\cdot)\) relates sulfuric acid production costs and its selling price.

As described at the beginning of this section, our methodology solves the smelter operation model and the sulfuric acid market model separately. Thus, for the acid market model the state-owned smelters production of acid is an input, i.e., it is assumed fixed at each iteration. Hence, the function \(\xi(\cdot)\) is endogenously determined through successive iterations of the two models. We note that the use of this iterative procedure to find the solution of the global model does not require the explicit functional form of \(\xi(\cdot)\).

**(iii) Regional Sulfuric Acid Demand**

The consumer regions are divided into three groups: the international market and the set of state-owned smelters denoted by \(I\) and \(J\) as above, and the set containing the four Chilean macro-regions denoted by \(Q\).

We assume that the international market demand is given by the aggregate Chilean supply surplus and is not price sensitive. Thus, international sulfuric acid demand in year \(t\), \(D_I(t)\), is given by \(D_I(t) = (Q_N(t) - D_N(t))^+\), where \(Q_N(t)\) and \(D_N(t)\) are Chile’s aggregate production and consumption in year \(t\) respectively. As Chilean sales of sulfuric acid do not alter international prices, we assume these to be known and denote them by \(\{P^D_I(t)\}_{t \geq 1}\). This assumption of perfect elasticity is supported by the relative small size of the Chilean sulfuric acid market with respect to the world market.

On the other hand, domestic demand is heavily dependent on leaching projects undertaken within the country to process oxide ores. In this sense, Chilean sulfuric acid demand is a derived demand arising out of the production of copper from oxide ores. In practice, the international copper price is the main deciding factor in leaching projects, with the international price of imported sulfuric acid almost irrelevant in the observed range. We reach this conclusion after interviewing and discussing this issue with the state-owned smelters managers in charge of the sulfuric acid production and commercialization. Accordingly, we assume that Chilean demand for sulfuric acid in mining projects is independent of its price, and the model takes the consumption levels in the state-owned smelters and the four macro regions as given.

In addition, since the state-owned smelters are simultaneously producers and consumers of sulfuric acid, their effective demand is simply their output deficit, i.e., if \(S_j\) is sulfuric acid production and \(D_j\) is sulfuric acid requirement at smelter \(j \in J\), then its effective demand is given by \((D_j - S_j)^+\). Similarly, effective supply of sulfuric acid at state-owned smelter \(j \in J\) is given by \((S_j - D_j)^+\).
(iv) Sulfuric Acid Price Models

Our previous discussion about the national supply and demand of sulfuric acid reveals an important feature of the Chilean market: both national supply and demand are price insensitive. This conclusion, that partly relies on our iterative solving procedure to find the general equilibrium solution, has strong consequences in the structure of problem (PS). In fact, in this situation Samuelson’s model is equivalent to a linear programming problem that minimizes the total transportation cost. Moreover, a minimum flow formulation can be used to find efficiently the distribution of sulfuric acid from producers to consumers. The selling prices, on the other hand, are not directly available as an output of the flow problem. However, they can be computed from the flow solution using Samuelson’s optimality conditions, i.e., if the flow between two regions is positive then the difference in price between these regions has to be equal to the corresponding transportation cost. These conditions define the selling prices in relative terms, since the increase or decrease of all prices by the same amount would produce a new feasible solution. In order to get the absolute prices, we use the international market for which we know in advance the selling (export) or buying (import) prices. We summarize this result in the following proposition.

**Proposition 1** Consider the case where all the regions but one (let say region \( \mathcal{I} \)) have perfectly inelastic supply and demand curves. That is,

\[
S_i(P_i) = Q_i^S \quad \text{and} \quad D_i(P_i) = Q_i^D \quad \forall \ P_i \geq 0 \quad \text{and} \quad i \neq \mathcal{I}.
\]  

Consider also that region \( \mathcal{I} \) behaves as a buffer, supplying or consuming any deficit or surplus of the other regions at a fixed price \( P_\mathcal{I} \). In addition, assume that the transportation cost matrix \( T = [T_{ij}] \) satisfies the triangular inequality condition (i.e., \( T_{ik} \leq T_{ij} + T_{jk}, \forall i, j, k \)). Then, the Spatial Price equilibrium can be found as follows:

**Step 1:** For each region compute \( \Delta Q_i = Q_i^S - Q_i^D, \ i \neq \mathcal{I} \) and \( \Delta Q_\mathcal{I} = -\sum_{i \neq \mathcal{I}} \Delta Q_i \). Define the market network \( G = (N, A) \) such that the set of nodes \( N \) represents the set of regions in the market and each arc \( (i, j) \in A \) reflects the existence of a direct path from region \( i \) to region \( j \). The unit transportation cost along arc \( (i, j) \) is \( T_{ij} \). Let \( M \) be the node-arc incidence matrix for \( G \), \( F = [F_{ij}] \) a matrix of flows, and \( \Delta = (\Delta Q_i) \) the supply-demand vector.

**Step 2:** Solve the network flow problem \( \min_{F \geq 0} \{ TF \text{ s.t. } MF = \Delta \} \) and let \( F^* \) be an optimal solution and \( B \) the basis matrix associated to \( F^* \).

**Step 3:** Let \( \pi = (B^{-1})^T T_B \), where \( T_B \) is the restriction of \( T \) to the basis \( B \). Then, the price vector \( P^* \) defined as:

\[
P_i^* = P_\mathcal{I} - \pi_i + \pi_\mathcal{I}
\]  

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together with \( F^* \) represent a competitive equilibrium for the market. In the particular case when \( F^* \) is non degenerate we can find \( P^* \) using the optimality condition:

\[
P^*_j = P^*_i + T_{ij} \quad \text{for all } (i,j) \quad \text{s.t. } F^*_{ij} > 0
\]

and the border condition for \( P^*_z = P^*_z \).

**Proof:** We need to prove that the pair \((F^*, P^*)\) of flows and prices defined in proposition 1 represents a competitive equilibrium. That is, given \( P^* \) we need to show that there is no supplier nor consumer that can be strictly better off by selling or buying the product differently than the solution \( F^* \) proposed. Since \( T \) satisfies the triangular inequality condition, then this condition is equivalent to:

\[
P^*_i \geq P^*_j - T_{ij} \quad \text{for all } (i,j) \in A.
\]

Given the optimality of \( F^* \), we have that \( T - M' \pi \geq 0 \) (reduced costs are nonnegative), thus:

\[
T_{ij} - \pi_i + \pi_j \geq 0 \iff T_{ij} + (P_z - \pi_i - \pi_Z) - (P_Z - \pi_j + \pi_Z) \geq 0 \iff P^*_i \geq P^*_j - T_{ij}.
\]

Finally, when \( F^* \) is non degenerated it corresponds to a spanning tree for \( G \) and (7), which is the complementary slackness condition for optimality, fully characterizes \( P^* \) up to an additive constant. In this case, this constant is determined using \( P^*_z = P^*_z \).

As we mentioned before, steps 2 and 3 can be efficiently computed (see, for example, Ahuja *et al.* (1993) for details). We also notice that if the solution \( F^* \) is degenerated, then \( P^* \) is not necessarily unique and the particular solution will depend on the bargaining power of the different agents on the market.

A difficulty, however, arises when we apply this approach to compute the regional prices for the Chilean acid market. As we mentioned before, sulfuric acid is a extremely toxic substance with high storage costs. These undesirable properties force producers to sell their acid production as quickly as they can without necessarily caring about prices and profits. This particular feature of the sulfuric acid market is not captured by Samuelson’s model and therefore the optimality conditions (7) are not always satisfied in this case. For this reason, we replace step 3 in the above procedure by a set of empirical models based on historical data that implicitly captures these imperfections of the Chilean sulfuric acid market.

The empirical data used in the regressions is a collection of 140 different contracts signed by Codelco or Enami during the period 1992-1998. Each contract specifies the origen and destination, the quantity sold, and the selling price of the sulfuric acid production. The models that we use to compute regional prices are ad-hoc variations of the traditional *Gravity Model* frequently encountered in the empirical economic literature (see, for example, Erlander (1979) and Evenett and Keller (1998) for more details).
Model for Regional Prices. For each of the four macro-regions in Chile we develop a model that relates the average price in the region to the average national price and to the deficit or surplus in that region and in the country. The model is defined by:

\[ P_i(t) - P_N(t) = \beta_i \cdot (ED_i(t) - ED_N(t)) \text{ for all } i \in \mathcal{Q} \]

where, 
- \( P_i(t) \) = average buying price in region \( i \in \mathcal{Q} \) in year \( t \).  
- \( P_N(t) \) = average buying price in the country in year \( t \).  
- \( ED_i(t) \) = percentage demand surplus of region \( i \in \mathcal{Q} \) in year \( t \) (the quotient between demand and supply in region \( i \) in year \( t \)).  
- \( ED_N(t) \) = percentage demand surplus nationally in year \( t \).  
- \( \beta_i \) = factor for region \( i \in \mathcal{Q} \).

The higher the acid deficit in a region, the higher its purchase price from other regions. This price is defined in terms of its difference from the average national price; thus if the regional deficit is larger than the national deficit, then the average regional price is higher than the average national price (\( \beta_i \geq 0 \)). Using standard econometric statistics \( (t\text{-test}, R^2) \), we found that the models we propose fit quite accurately the historical data. For each of the four macro-regions, the \( \beta_i \) coefficients are statistically significant at a 95% level of confidence, while the \( R^2 \) ranges from 0.41 to 0.93 depending on the region.

Each state-owned smelter is located within one of the four macro-regions. Thus, if a state-owned smelter has a sulfuric acid deficit in a period \( (S_j - D_j < 0, \text{ some } j \in \mathcal{J}) \), the price it pays to meet this deficit is the same as the price in the zone it belongs to. Hence, equation (9) not only defines the set of prices \( \{P_i(t) : i \in \mathcal{Q}\} \), but also the set of prices \( \{P_j(t) : j \in \mathcal{J}\} \). Finally, we remark that the regional prices represent the commercial value of sulfuric acid within each macro region and do not explicitly include the transportation costs.

Model for National Prices. In this model we determine the average national price as a function of the international price and the sulfuric acid deficit (surplus) in a given year. The national price is given by:

\[ P_N(t) = \delta + \epsilon \cdot P^S(t) + \gamma \cdot (D_N(t) - Q_N(t)), \]

where
- \( P_N(t) \) = average national price in year \( t \),  
- \( P^S(t) \) = average international price in year \( t \),  
- \( D_N(t) \) = aggregate national demand in year \( t \),  
- \( Q_N(t) \) = aggregate national supply in year \( t \), and  
- \( \delta, \epsilon, \gamma \) = parameters estimated by ordinary least squares using historical data.
The average national price depends on the international price through the parameter \( \epsilon \geq 0 \); increases (decreases) in the international price cause increases (decreases) in the national price. On the other hand, deficits (surpluses) in Chilean acid supply provoke increases (decreases) in the national price \( (\gamma \geq 0) \). The constant \( \delta \) captures the effect of transportation costs between Chile and the rest of the world. Using the empirical data, and according to the \( t \)-test, the parameters on (10) are statistically significant at a 95\% level of confidence and the goodness of fit of the model \( (R^2) \) is 0.52.

(v) Mathematical Formulation

Finally, we present the mathematical formulation to determine the average selling prices of sulfuric acid for the five state-owned smelters. We notice at this point that since the sulfuric acid model assumes fixed acid production of the state-owned smelters, the solution below does not correspond to equilibrium prices. The equilibrium prices for the sulfuric acid market will be obtained when solving simultaneously both the sulfuric acid market model and the smelter model in Section §3.3.

Given the analysis in (ii), (iii), and (iv), demand and supply curves are assumed perfectly inelastic. Thus, Samuelson’s model is equivalent to a minimum flow problem that minimizes the total transportation cost. For each region \( i \) different than the international market, we compute the total deficit or surplus of the product in year \( t \), \( \Delta Q_i(t) \), as the difference between the supply and the demand for that region, i.e., \( \Delta Q_i(t) = S_i(t) - D_i(t) \). For the international market, the value of \( \Delta Q_I(t) \) is determined by \( \Delta Q_I(t) = - \sum_{i \neq i} \Delta Q_i(t) \). That is, the international market acts as a buffer absorbing any surplus of local production or supplying any local deficit.

Under the assumption of perfectly inelastic supply and demand curves, the mathematical formulation (PS) can be rewritten as the following flow problem:

\[
(PSS) \quad \min_{Q_{ij}(t)} \sum_{i=1}^{15} \sum_{j=1}^{15} T_{ij} \cdot Q_{ij}(t), \quad (11)
\]

s.t.

\[
\sum_{j=1}^{15} Q_{ij}(t) = (\Delta Q_i(t))^+ \quad \forall i \text{ and } t, \quad (12)
\]

\[
\sum_{i=1}^{15} Q_{ij}(t) = (\Delta Q_j(t))^- \quad \forall j \text{ and } t, \quad (13)
\]

\[
Q_{ij}(t) \geq 0 \quad \forall i, j, \text{ and } t. \quad (14)
\]

This model determines sulfuric acid flows, \( \{Q_{ij}(t) : i, j = 1, \ldots, 15\} \), between the fifteen regions defined in (i). Finally, combining these flows with the regional prices \( \{P_{ij}(t) : i \in Q \cup J\} \) obtained from (9) and (10) and the data for the international price, \( \{P^D_I(t)\} \), we can compute the average

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solving price for each of the five state-owned smelters as follows:

\[
P_j(t) = \frac{\sum_{i \in \mathcal{I}} [P_i(t) - T_{ji}] \cdot Q_{ji}(t) + (P_i^D(t) - T_{jX}) \cdot Q_j(t)}{\sum_{i \in \mathcal{I}} Q_{ji}(t) + Q_j(t)} \quad \forall j \in \mathcal{J}.
\]  

(15)

The average selling price at smelter \( j \in \mathcal{J} \) in period \( t \) (\( \bar{P}_j(t) \)) represents the average net profit that smelter \( j \) receives for each ton of sulfuric acid it sells. That is, \( P_i(t) - T_{ji} \) is the net profit that smelter \( j \) gets by selling one ton of sulfuric acid in region \( i \) and \( P_i^D(t) - T_{jX} \) is the net profit that smelter \( j \) gets by selling one ton of acid on the international market.

### 3.3 Interaction between the two models

The interaction between the two models presented above is as follows. The smelter model considers the sulfuric acid price at each smelter to find optimal investment and operational decisions for the smelter’s operations, including the production of sulfuric acid. On the other hand, the sulfuric acid market model considers sulfuric acid production at the smelters as part of the supply input to find the price for this product at each smelter location. The model is solved iteratively until it converges to an equilibrium solution (prices and productions). Figure 7 illustrates this interaction.

![Sulfuric Acid Production and Prices](image)

Figure 7: Interaction between the Smelter planning and operation model and the Sulfuric acid market model.

The algorithm to find the equilibrium solution between the two models is as follows:

- **STEP 1:** Initialization.
  
  \( P_0 \) = vector of initial sulfuric acid prices at each smelter and period of time.
  
  \( k = 1 \)
• **STEP 2:** Resolution of smelter planning and operation model.

\[ Q_k(P_{k-1}) = \text{optimal sulfuric acid production levels at the smelters for the price vector } P_{k-1}. \]

• **STEP 3:** Sulfuric acid market model solution.

\[ P_k = \text{vector of sulfuric acid prices for production levels } Q_k(P_{k-1}). \]

• **STEP 4:** IF \( \| P_k - P_{k-1} \|_1 \leq \varepsilon \), then

STOP; the algorithm has found an \( \varepsilon \)-approximate solution

ELSE, \( k = k + 1 \), GOTO to **STEP 2**

The stop criterion in Step 4 is based on the variation in sulfuric acid prices computed in two consecutive iterations. The algorithm iterates until the difference between these two consecutive price vectors is less than or equal to a tolerance of \( \varepsilon \). For example, if the model optimizes operations for five smelters over a 20-year horizon and we choose \( \varepsilon = 1 \), on average this criterion means that the algorithm ends when the price at each smelter and in each year differs by less than US$0.01 between two consecutive iterations.

From an economic point of view, the algorithm seeks a solution corresponding to competitive equilibrium. Given its iterative structure, the smelter planning and operation model takes the sulfuric acid price as an input and maximizes the smelter's discounted profit, where one set of decision variables corresponds to sulfuric acid production. On the other hand, the sulfuric acid market model receives the output from the state-owned smelters as an input and generates the prices at which those production levels will be consumed. Hence, the type of equilibrium obtained using this algorithm represents a competitive equilibrium for the sulfuric acid market.

Under appropriate conditions for the demand and supply curves (see Mas-Colell, Whinston and Green (1995)), it can be shown that competitive equilibrium exists and is unique. These conditions are not satisfied in our formulation due mainly to the existence of binary investment variables that determine production levels at the state-owned smelters. However, from an empirical point of view, the presence of these irregularities in the supply curve has not led to multiple competitive equilibria. All computational experiments have shown our problem to be very stable, with the same optimal solution being reached for a large set of starting points.

The presence of binary variables also prevents us from guaranteeing the convergence of the algorithm. Depending on the sensitivity of the supply curves to the sulfuric acid price, it is possible that the algorithm iterates, without converging, in such a way that the binary variables alternate their values from one iteration to the next. Fortunately, our data shows that this situation is not likely to happen in the Chilean sulfuric acid market. As we described in Subsection 3.2.2, the production of sulfuric acid is mainly determined by the price of blister copper, with the sulfuric acid price being less relevant given the bounds that the international market of sulfuric acid sets for the export and import of the product. Moreover, the algorithm has converged in all computational experiments performed.
Although the sulfuric acid price does not affect significantly the smelters’ production of sulfuric acid, it is still important to have a model for the sulfuric acid market. In fact, this model provides valuable information about how to efficiently distribute the smelters’ acid production among the different consumers. This information allows managers to (i) estimate average selling price according to relation (15) above and (ii) anticipate future infrastructure requirements as we describe in next section.

4 Model Implementation

For the smelter planning and operation model we formulated a non-linear integer programming model with approximately 18,000 variables (600 binary variables) and 12,000 constraints for a planning horizon of 20 years. It was run on a PC, using GAMS, DICOPT (this manages the master problem), CPLEX (linear and integer programming) and CONOPT (non-linear programming). The sulfuric acid market model was also run in GAMS and CPLEX, and has approximately 2,000 variables and 500 constraints. The iterative process between the two models was written in GAMS.

Resolution time depends heavily on the size of the problem and its parameters. We solved five problems using actual operational data from the smelters, varying the maximum emissions into the atmosphere allowed for the pollutants under analysis. For a 10-year planning horizon, running time ranged from 1150 to 11920 seconds. In these cases the size of the problem was approximately 3,500 constraints and 4,000 variables (180 binary).

The decision support system described in this paper has become an important tool for the decision making process in the copper industry. Although the model was originally conceived as a tool to determine the optimal investment policy for pollution control, it has also been used as a support system in a variety of decisions concerning the copper smelters and related systems; examples are given below. This wide scope of applications is mainly due to its integrated view of operational and investments decisions in the state-owned copper smelters. This integrated view makes it possible to evaluate a set of feasible solutions including alternatives that are difficult to visualize when optimizing each smelter’s operations independently.

In what follows we present five applications of the decision support system. In some cases the model is used on a continuous basis while in others was used for a one time decision.

1. One of Cochilco’s main responsibilities is to revise new investment projects proposed by the state-owned smelters, and give recommendations to the Budget Director, who finally decides whether or not to assign the money required to go ahead with the new project. Since in Chile only the executive branch can propose legislation involving expenditures. This makes the Budget Director a very powerful position, since he can veto initiatives by any ministry or government agency. The decision support system has provided a tool for Cochilco to
economically evaluate the projects on an independent basis; it allows for a fast and easy evaluation, and therefore, is a useful audit tool. For example, a decision has to be made when a smelter presents a project to invest in new smelting capacity with its corresponding discounted net value. Using the decision support system, it is possible to evaluate the difference in the objective functions under the scenarios with and without the investment. If this difference does not match the discounted net value presented by the smelter, then further evaluation is required.

2. At the time the model was conceived, there was an important issue being debated: the installation of a sulfuric acid plant to reduce the sulfur dioxide emissions at the Paipote smelter (the possibility existed of buying a second hand acid plant from a private Chilean smelter). Cochilco and the Planning Ministry (MIDEPLAN) had to evaluate the investment of US$15 million dollars, however there were discordant opinions on whether or not it was a profitable decision. Furthermore, the Finance Ministry and the Budget Director were more drastic and thought that the best decision was to close Paipote, due to financial considerations. Finally, all the parties involved in the decision process agreed to wait until the model was built and implement the decision proposed by it. With the data available at that time, the model suggested to continue Paipote's operations. As a result, Paipote continued its operations and the investment was authorized.

3. The decision support system has been used to estimate the cost of government policies, and the impact of other exogenous factors, such as changes in the price of copper and the incorporation of new smelting technologies. It has also been used by the government to evaluate the economic impact of different environmental standards. For example, while preparing the Arsenic Emission Standard Draft, at the request of Conama (National Environmental Commission), the model was run to evaluate the economic impact of different levels of emissions at each smelter.

From an environmental point of view, the lower the maximum amount of arsenic emissions allowed into the atmosphere, the better. However, more restrictive upper bounds for maximum emissions implies more expensive investment in pollutant abatement plants. In some extreme cases, the smelter's operations could not be profitable and they would have to close. The model showed that the initial arsenic emission standards proposed by Conama were too stringent; with those standards some smelters had to close their operations. Considering the trade-off between environmental and economic impacts, a less restrictive standard was set. The following figure shows how the total income and its composition (for a 20 years horizon) change with different arsenic emission standards. In particular, we vary the total amount of arsenic that the five state-owned smelters are allowed to emit each year.
Figure 8: Impact of the maximum level of arsenic emissions on the operational income and its composition.

Figure 8 plots 5 different scenarios ranging from the restrictive case of 900 (tons./year) to the much flexible 2700 (tons./year) case. Each bar describes the distribution of the total income into the three main components (i) sales of refined copper, (ii) exports of concentrate, and (iii) sales of sulfuric acid. The total income (in billions of dollars) appears on top of each bar. Varying the maximum arsenic emissions from 900 to 2700 (tons./year) changes the total income from 19.5 to 20.5 (billion US$), that is a 5.13% change. This one billion dollars change is quite significant for a country like Chile having a 70 billion US$ GDP. More significant, however, is the effect on the composition of the total income. At the 900 level only a 69% of the income comes from the copper production and 25% is the result of concentrate exports. At the 2700 level copper production accounts for more than 83% of the system income with only 11% coming from the exports of concentrate.

4. To the best of our knowledge, our model is the first to incorporate the behavior of the sulfuric acid market. Thus, the sulfuric acid price, which is a central component in deciding when and where to locate a sulfuric acid plant, is determined endogenously, and not assumed exogenous as in most models of this type. We strongly believe this to be an important contribution; our experience has shown that even experts trading in sulfuric acid have no formal tool for integrating the market and predicting the evolution of its price and distribution in the country. In terms of production and local distribution, this type of result is important to anticipate future infrastructure requirements (storage terminals, roads, railways, and ports) to support commercialization of the product. As reported by Cochinco’s experts, this module has been used to forecast the national sulfuric acid market.

5. The decision support system also makes an important contribution when solving problems
of capacity allocation; the installation of a new smelter has been under debate for a long time, with no agreement reached to date. In order to contribute to the decision on this issue, our model includes the option of installing a new smelter at any year during the 20-year planning horizon. The results we have obtained suggest that it would not be economically advantageous for the State to install a new smelter within this horizon.

5 CONCLUSIONS

In this paper we have developed an investment and operational decision support system for pollution control in the copper industry. The system consists of: (i) a non-linear integer programming problem to describe the state-owned smelters’ operations and (ii) a network flow model to describe the sulfuric acid market. The two models interact through the input each receives from the other, and the system iterates until a competitive equilibrium is reached. Empirical evidence, using current technical, economic and environmental data, shows that an equilibrium can be reached in reasonable computational time; moreover this is unique: different starting points have led to the same equilibrium in all experiments performed.

One of the most novel features of the model is that the distribution and price of sulfuric acid are determined endogenously within the model, and not considered exogenous as in most of the models that analyze the copper industry. The incorporation of the sulfuric acid market in our formulation is a substantial improvement on previous work, considering that one of the main sources of investment during the coming decade relates to the decision on when and where to locate new sulfuric acid plants.

The system described in this paper provides the first formal analytical tool for analyzing investment decisions in pollution control in the copper industry. We believe that one of the main contributions of the decision support system is its current use in analyzing environmental and technical policies, and it has been able to provide decision makers with interesting new investment alternatives.

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A  APPENDIX

In this appendix, we present the mathematical formulation of the two models described in Sections 3.1 and 3.2.

A.1  Sets of Indices

- $I = \{ \text{treatment plants} \}$.
- $J = \{ \text{state-owned smelters} \}$.
- $K = \{ \text{smelting, converting or gas treatment technologies} \}$.
  - $L = \{ \text{smelting technologies} \} \subset K$.
  - $M = \{ \text{converting technologies} \} \subset K$.
  - $N = \{ \text{smelting or converting technologies} \} \subset K$.
  - $O = \{ \text{gas treatment technologies} \} \subset K$.
- $P = \{ \text{national private producers of sulfuric acid} \} \cup \{ \text{international sulfuric acid market} \}$.
- $Q = \{ \text{national consumer regions of sulfuric acid} \} \cup \{ \text{international sulfuric acid market} \}$.
- $S = \{ \text{copper, sulfur, arsenic, particulate matter} \}$.
  - $ST = \{ \text{sulfur, arsenic, particulate matter} \}$.
- $T = \{ \text{time periods} \}$.

A.2  Parameters

A.2.1  Objective Function Parameters

- $p_{\text{cobre}_{ji}}$: price of one ton of copper produced by smelter $j \in J$ during period $t \in T$.
- $p_{\text{conc}_{ia}}$: price of one ton of concentrate produced by treatment plant $i \in I$ during period $t \in T$.
- $p_{\text{acid}}_{ji}$: price of one ton of sulfuric acid produced by smelter $j \in J$ during period $t \in T$.
- $c_{\text{conf}_{uij}}$: transportation cost of one ton of concentrate between treatment plant $i \in I$ and smelter $j \in J$.
- $c_{\text{coop}_{jk}}$: cost of processing one ton using technology $k \in K$ at smelter $j \in J$.
- $c_{\text{insfijo}_{jk}}$: fixed cost of expanding capacity of technology $k \in K$ at smelter $j \in J$. 

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• $cinsvar_{jk}$: variable cost of expanding capacity of technology $k \in K$ at smelter $j \in J$.
• $cinsnf$: installation cost of a new smelter.
• $dclobi$: discount rate in period $t \in T$.

### A.2.2 Concentrate Parameters

• $pr_{id}$: production (tons) of treatment plant $i \in I$ during period $t \in T$.
• $conci_{ist}$: concentration of product $s \in S$ in concentrate from plant $i \in I$ during period $t \in T$.

### A.2.3 Smelter Parameters

• $concobre_{jl}$: copper concentration in the matte produced by technology $l \in L$ at smelter $j \in J$.
• $\alpha_{jls}$: fraction of product $s \in S$ retained in the matte produced by technology $l \in L$ at smelter $j \in J$.
• $\beta_{jns}$: fraction of pollutant $s \in ST$ emitted in the gas produced by technology $n \in N$ at smelter $j \in J$.
• $\nu_{jn}$: sulfur concentration in the gas produced by technology $n \in N$ at smelter $j \in J$.
• $fugity_{jns}$: fraction of pollutant $s \in ST$ emitted as fugitive gas by technology $n \in N$ at smelter $j \in J$.
• $PM10_{prod_{jn}}$: tons of PM10 produced per ton processed by technology $n \in N$ at smelter $j \in J$.
• $efiprec_{js}$: fraction of pollutant $s \in ST$ abated in an electrostatic precipitator at smelter $j \in J$.
• $conminpa_{j}$: minimum sulfur concentration for the gas to enter an acid plant at smelter $j \in J$.
• $conmaxpa_{j}$: maximum sulfur concentration for the gas to enter an acid plant at smelter $j \in J$.
• $pracido_{j}$: tons of sulfuric acid produced per ton of sulfur fed into acid plant at smelter $j \in J$.
• $capin_{jk}$: initial capacity of technology $k \in K$ at smelter $j \in J$.
• $deltamair_{jk}$: maximum capacity expansion of technology $k \in K$ at smelter $j \in J$ during period $t \in T$. 

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\textbf{A.2.4 Environmental Parameter}

- \textit{eminax}_{jst}: maximum emission of pollutant \(s \in ST\) allowed at smelter \(j \in J\) during period \(t \in T\).

\textbf{A.2.5 Sulfuric Acid Market Parameters}

- \(jjt_{j_1j_2}\): transportation cost of one ton of sulfuric acid from smelter \(j_1 \in J\) to smelter \(j_2 \in J\).
- \(ojt_{pj}\): transportation cost of one ton of sulfuric acid from private producer \(p \in P\) to smelter \(j \in J\).
- \(jdt_{jq}\): transportation cost of one ton of sulfuric acid from smelter \(j \in J\) to consumer region \(q \in Q\).
- \(odt_{pq}\): transportation cost of one ton of sulfuric acid from private producer \(p \in P\) to consumer region \(q \in Q\).
- \(jds_j\): sulfuric acid consumption by smelter \(j \in J\) during period \(t \in T\).
- \(dd_{qt}\): sulfuric acid demand of consumer region \(q \in Q\) during period \(t \in T\).
- \(jop_j\): sulfuric acid production at smelter \(j \in J\) during period \(t \in T\).
- \(oop_p\): sulfuric acid production of private producer \(p \in P\) during period \(t \in T\).
- \(precio_q\): average buying price of sulfuric acid in region \(q \in Q\) during period \(t \in T\).
- \(pventa_{qj}\): average selling price of sulfuric acid produced at smelter \(j \in J\) during period \(t \in T\).

\textbf{A.3 Decision Variables}

- \(F_{ijlt}\): tons of concentrate sent from treatment plant \(i \in I\) to smelter \(j \in J\) processed by technology \(l \in L\) during period \(t \in T\).
- \(X_{jnst}\): tons of product \(s \in S\) fed to technology \(n \in N\) at smelter \(j \in J\) during period \(t \in T\).
- \(Y_{jnst}\): tons of pollutant \(s \in ST\) in the gas produced by technology \(n \in N\) at smelter \(j \in J\) during period \(t \in T\).
- \(TR_{ijlmt}\): tons of product \(s \in S\) sent from technology \(l \in L\) to technology \(m \in M\) at smelter \(j \in J\) during period \(t \in T\).
- \(Y_{prec_{mt}}\): tons of pollutant \(s \in ST\) produced by technology \(n \in N\) and treated in an electrostatic precipitator at smelter \(j \in J\) during period \(t \in T\).
\[ Yac_id_{jnt}: \text{tons of pollutant } s \in ST \text{ produced by technology } n \in N \text{ and treated in a sulfuric acid plant at smelter } j \in J \text{ during period } t \in T. \]

\[ VR_{prej}: \text{total volume of gases treated in an electrostatic precipitator at smelter } j \in J \text{ during period } t \in T. \]

\[ Vacid_{j}: \text{total volume of gases fed to a treatment plant at smelter } j \in J \text{ during period } t \in T. \]

\[ ACIDO_{j}: \text{sulfuric acid production at smelter } j \in J \text{ during period } t \in T. \]

\[ COBRE_{jmnt}: \text{copper production produced by technology } m \in M \text{ at smelter } j \in J \text{ during period } t \in T. \]

\[ EMI_{jnt}: \text{emissions of pollutant } s \in ST \text{ at smelter } j \in J \text{ during period } t \in T. \]

\[ CAPAC_{jkt}: \text{capacity of technology } k \in K \text{ at smelter } j \in J \text{ during period } t \in T. \]

\[ AUM_{jkt}: \text{1 if there is an expansion in capacity of technology } k \in K \text{ at smelter } j \in J \text{ during period } t \in T; 0 \text{ otherwise.} \]

\[ OPER_{j}: \text{1 if smelter } j \in J \text{ operates during period } t \in T; 0 \text{ otherwise.} \]

\[ INUEV_{j}: \text{1 if a new smelter is installed during period } t \in T; 0 \text{ otherwise.} \]

\[ JJX_{j1j2}: \text{sulfuric acid sent from smelter } j_1 \in J \text{ to smelter } j_2 \in J \text{ during period } t \in T. \]

\[ OJX_{pj}: \text{sulfuric acid sent from private producer } p \in P \text{ to smelter } j \in J \text{ during period } t \in T. \]

\[ JDX_{jq}: \text{sulfuric acid sent from smelter } j \in J \text{ to consumer region } q \in Q \text{ during period } t \in T. \]

\[ ODX_{pq}: \text{sulfuric acid sent from private producer } p \in P \text{ to consumer region } q \in Q \text{ during period } t \in T. \]

A.4 Objective Functions and Constraints

A.4.1 Smelter’s Operational Model

Objective Function

\[
\begin{align*}
\text{Max}\{FO1\} & = \sum_{i \in I} dc_{oi} \cdot \left( \sum_{j \in J} \sum_{m \in M} [(pco_{b}e_{j} - co_{op_{j}m}) \cdot COBRE_{jmnt}] + \sum_{j \in J} \sum_{o \in O} [(paco_{d}o_{j} - co_{op_{j}o}) \cdot ACIDO_{j}] + \sum_{i \in I} [pconc_{i} \cdot pr_{i}] - \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} [(pconc_{i} + ctconfu_{ij}) \cdot F_{ij}u_{ij}] \right)
\end{align*}
\]
\[- \sum_{j \in J} \sum_{l \in L} [\text{coop}_{jl} \cdot \sum_{s \in S} X_{jlst}] \]
\[- \sum_{j \in J} \sum_{o \in O} [\text{coop}_{jo} \cdot VRprec_{jl}] \]
\[- \sum_{j \in J} \sum_{k \in K} [\text{cinsf}_{j} \cdot AUM_{jkl} + \text{cinsvar}_{jk} \cdot (\text{CAPAC}_{jkl} - \text{CAPAC}_{j,k,t-1})] \]
\[- \text{cinsfn} \cdot \text{INUEV}_{l} \]  

(16)

Constraints

1.- Concentrate Production:

\[ \sum_{j \in J} \sum_{l \in L} \sum_{i \in I} F_{ljlt} \leq pr_{il} \quad \forall i \in I, \forall l \in T \]  

(17)

\[ X_{jlst} = \sum_{i \in I} \text{conc}_{ist} \cdot F_{ljlt} \quad \forall j \in J, \forall l \in L, \forall s \in S, \forall t \in T \]  

(18)

2.- Mass Flow Conservation

\[ \sum_{m \in M} \text{TR}_{jmlst} = \alpha_{jls} \cdot X_{jlst} \quad \forall j \in J, \forall l \in L, \forall s \in S, \forall t \in T \]  

(19)

\[ X_{jmlst} = \sum_{l \in L} \text{TR}_{jmlst} \quad \forall j \in J, \forall m \in M, \forall s \in S, \forall t \in T \]  

(20)

\[ Y_{jnst} = \beta_{jns} \cdot X_{jnst} \quad \forall j \in J, \forall n \in N, \forall s \in ST, \forall t \in T \]  

(21)

\[ Y_{\text{precjnst}} \leq (1 - \text{fugity}_{jns}) \cdot Y_{jnst} \quad \forall j \in J, \forall n \in N, \forall s \in ST, \forall t \in T \]  

(22)

\[ Y_{\text{acidjnst}} \leq (1 - \text{efiprec}_{jns}) \cdot Y_{\text{precjnst}} \quad \forall j \in J, \forall n \in N, \forall s \in ST, \forall t \in T \]  

(23)

3.- Gas Flow Conservation

\[ VRprec_{jl} = \sum_{n \in N} [\nu_{jn} \cdot Y_{\text{precjnst}}] \quad \forall j \in J, \forall t \in T, s = \text{sulfur} \]  

(24)

\[ V\text{acid}_{jl} \geq \sum_{n \in N} [\nu_{jn} \cdot Y_{\text{acidjnst}}] \quad \forall j \in J, \forall t \in T, s = \text{sulfur} \]  

(25)
\[
\frac{Y_{\text{prec}_{jn1}}}{(1 - \text{fugity}_{jn}) \cdot Y_{\text{jns1}}} = \frac{Y_{\text{acid}_{jn1}}}{Y_{\text{acid}_{jns1}}} \quad \forall j \in J, \forall n \in N, \forall s \in ST, \forall t \in T, \ s_1 = \text{sulfur}
\]

(26)

4.- Production

\[
\text{COBRE}_{jn} = a_{jms} \cdot X_{jn1} \quad \forall j \in J, \forall m \in M, \forall t \in T, \ s = \text{copper}
\]

(27)

\[
\text{ACIDO}_{jt} = \sum_{n \in N} [\text{pracido}_j \cdot Y_{\text{acid}_{jn1}}] \quad \forall j \in J, \forall t \in T, \ s = \text{sulfur}
\]

(28)

5.- Capacity

\[
\sum_{i \in I} R_{ijt} \leq \text{CAPAC}_{jlt} \quad \forall j \in J, \forall l \in L, \forall t \in T
\]

(29)

\[
\sum_{i \in L} \frac{TR_{jn1}}{\text{concobre}_{ji}} \leq \text{CAPAC}_{jn1} \quad \forall j \in J, \forall m \in M, \forall t \in T, \ s = \text{copper}
\]

(30)

\[
VR_{\text{prec}j1} \leq \text{CAPAC}_{jot} \quad \forall j \in J, \forall t \in T, \ o = \text{elec. preci.}
\]

(31)

\[
V_{\text{acid}jt} \leq \text{CAPAC}_{jot} \quad \forall j \in J, \forall t \in T, \ o = \text{treat. plant}
\]

(32)

\[
\text{CAPAC}_{jk0} = \text{capin}_{jk} \quad \forall j \in J, \forall k \in K
\]

(33)

\[
\text{CAPAC}_{jkt} \leq \text{deltamax}_{jkt} \quad \forall j \in J, \forall k \in K, \forall t \in T
\]

(34)

6.- Treatment Plant Operation

\[
\sum_{n \in N} Y_{\text{acid}_{jn1}} \geq \text{conminpa}_j \cdot V_{\text{acid}jt} \quad \forall j \in J, \forall t \in T, \ s = \text{sulfur}
\]

(35)

\[
\sum_{n \in N} Y_{\text{acid}_{jn1}} \leq \text{conmaxpa}_j \cdot V_{\text{acid}jt} \quad \forall j \in J, \forall t \in T, \ s = \text{sulfur}
\]

(36)
7.- Emissions

\[
EMI_{jst} = \sum_{n \in N} Y_{jnst} - \sum_{n \in N} [efiprec_{jst} \cdot Yprec_{jnst}] - \sum_{n \in N} Y酸id_{jnst} \quad \forall j \in J, \ \forall s \in ST, \ \forall t \in T
\]

\[
EMI_{jst} \leq emimax_{jst} \quad \forall j \in J, \ \forall s \in ST, \ \forall t \in T
\]  

(37)

(38)

A.4.2 Sulfuric Acid Market Model

Objective Function

\[
Min\{FO2\} = \sum_{t \in T} \sum_{j_1 \in J} \sum_{j_2 \in J} [jj_{j_1 j_2} \cdot JJX_{j_1 j_2}] + \sum_{j \in J} \sum_{q \in Q} [jdt_{j} \cdot JDX_{j}] + \sum_{p \in P} \sum_{j \in J} [ojt_{pj} \cdot OJX_{pj}] + \sum_{p \in P} \sum_{q \in Q} [odt_{pq} \cdot ODX_{pq}]
\]

(39)

Constraints

1.- Demand

\[
\sum_{j_1 \in J} JJX_{j_1 j} + \sum_{p \in P} OJX_{pj} = jd_{jt} \quad \forall j \in J, \ \forall t \in T
\]

(40)

\[
\sum_{j \in J} JDX_{j} + \sum_{p \in P} ODX_{pq} = dd_{ql} \quad \forall q \in Q, \ \forall t \in T
\]

(41)

2.- Production

\[
\sum_{j_1 \in J} JJX_{j_1 j} + \sum_{q \in Q} JDX_{jq} = jo_{jt} \quad \forall j \in J, \ \forall t \in T
\]

(42)

\[
\sum_{j \in J} OJX_{pj} + \sum_{q \in Q} ODX_{pq} = oo_{pq} \quad \forall p \in P, \ \forall t \in T
\]

(43)

\[
pventa_{jt} = \frac{\sum_{q \in Q} [precio_{q} \cdot JDX_{jq}]}{\sum_{q \in Q} JDX_{jq}} \quad \forall j \in J, \ \forall t \in T
\]

(44)

35