The Martingale Approach to Operational and Financial Hedging

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October 2005.
Motivation

The Martingale Approach to Operational and Financial Hedging
Motivation

Problem:

Select an *operational strategy* and a *financial strategy* that optimize the value of the corporation.
Motivation

○ Operations and services models are now beginning to consider issues related to risk aversion and hedging

○ There are many good reasons for hedging
  – costs associated with financial distress
  – taxes
  – costs associated with raising capital
  – principal-agent issues
  – smoothing earnings for the Wall Street analysts!

○ And in practice corporations often do hedge
  – not inconsistent with Modigliani - Miller theory of corporate finance

○ Is there a modelling framework that might be useful for operational and financial decision-making?
Motivation

- A Methodology that effectively integrates Operational and Financial activities by
  - Maximizing the Economic Value of the corporation.
  - Modeling the risk preferences of decision makers.
  - Using financial markets for hedging purposes.
  - Recognizing the "incompleteness" of the problem.
  - Taking advantage of all available information.
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\max \{\text{Economic Value}\}
\]

subject to problem dynamics

and subject to risk management constraint(s).
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\max \{\text{Economic Value}\} \\
\text{subject to} \quad \text{problem dynamics} \\
\text{and subject to} \quad \text{risk management constraint(s)}. \]

- Operational risk in customers' taste, machine breakdowns, unreliable suppliers, etc...

- Financial risk in exchange rates, commodity prices, interest rates, etc...
Outline of Talk

- **Part I:** The Modelling Framework
  - Motivation for maximizing *economic* value of operating profits.
  - Financial market & informational structure

- **Part II:** Supply Contracts with Financial Hedging
  - The Wholesale Price Contract with Budget Constraint

- **Part III:** Models with Tail Constraints
  - A Procurement Example

- Conclusions & Further Research
Part I

Modelling Framework

Financial Market & Information
Motivating the Modelling Framework

- Consider two series cash-flows arising from two operating policies $I_1$ and $I_2$:

  \[ C^{(1)}(I_1) = (c_1^{(1)}(I_1), c_2^{(1)}(I_1), \ldots, c_n^{(1)}(I_1)) \]

  \[ C^{(2)}(I_2) = (c_1^{(2)}(I_2), c_2^{(2)}(I_2), \ldots, c_n^{(2)}(I_2)) \]

- If probability distribution of $C^{(1)}$ and $C^{(2)}$ identical then most operations models are indifferent between the two.

- However, if $C^{(1)}$ positively correlated with financial market for example, and $C^{(2)}$ negatively correlated with financial market then economic value of $C^{(2)}$ greater than economic value of $C^{(1)}$.

  - models should take this into account as they get ever more complex and include additional sources of uncertainty

- Do so by using an equivalent martingale measure, $Q$, or stochastic discount factor

  - consistent with only a hedging motivation for financial trading
The Modelling Framework

- $H(I)$ the time $T$ cumulative payoff from some operating policy, $I$
  - The operating policy may be static or dynamic.
The Modelling Framework

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- $G(\theta)$ the time $T$ gain from a self-financing trading strategy, $\theta$
The Modelling Framework

- \( H(I) \) the time \( T \) cumulative payoff from some operating policy, \( I \)
  - The operating policy may be static or dynamic.
- \( G(\theta) \) the time \( T \) gain from a self-financing trading strategy, \( \theta \)
- Operational and financial hedging problem is

\[
\min_{I,G(\theta)} \mathbb{E}^Q[H(I)]
\]

subject to  problem dynamics

and subject to  risk management constraint(s) \( \equiv \left\{ \begin{array}{c}
\mathcal{R}(H(I), G(\theta)) \leq 0 \quad a.s.
\mathbb{E}^P[\mathcal{R}(H(I), G(\theta))] \leq 0.
\end{array} \right. \)
The Modelling Framework

- $H(I)$ the time $T$ cumulative payoff from some operating policy, $I$
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$$\min_{I,G(\theta)} \mathbb{E}^Q[H(I)]$$

subject to

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and subject to

- risk management constraint(s) $\equiv \left\{ \begin{array}{lcl} \mathcal{R}(H(I), G(\theta)) \leq 0 & a.s. & \\ \mathbb{E}^P[\mathcal{R}(H(I), G(\theta))] \leq 0. & & \end{array} \right.$$

- consistent with only a hedging motivation for financial trading
  - $G(\theta)$ a martingale under $\mathbb{Q}$ so $\mathbb{E}^Q[G(\theta)] = 0$
  - assuming initial capital assigned to hedging strategy is zero
Financial Market Model

- Probability space \((\Omega, \mathcal{F}, \mathbb{P})\).
- \((W_{1t}, W_{2t})\) a standard Brownian motion.
- A single risky stock with price dynamics
  \[ dX_t = \mu X_t \, dt + \sigma X_t \, dW_{1t}. \]
- Cash account available with \(r \equiv 0\).
- \(\mathcal{F}_t\) is the filtration generated by \((W_{1t}, W_{2t})\).
- \(\mathcal{F}^X_t\) is the filtration generated by \(X_t\).
- Operating profits, \(H(I) \in \mathcal{F}_T\), a function of \((W_{1t}, W_{2t})\).
- The gain process, \(G(\theta)_t\) of a self-financing trading strategy \(\theta_t\) is
  \[ G(\theta)_t = \int_0^t \theta_s \, dX_s. \]
- An equivalent martingale measure, \(\mathbb{Q}\), under which \(X_t\) and \(G(\theta)_t\) are martingales.
Information

In general, operating profits are function of \((X_t, W_{2t})\), that is, \(H(I) \in \mathcal{F}_T\).
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- **Incomplete Information:**
  - Demand, product quality, customer tastes not always observable.
  - Only information related to \(X_t\) is available.

  \[ \Rightarrow \text{ Trading strategies must satisfy: } \theta_t \in \mathcal{F}_t^X, \quad t \in [0, T]. \]
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- **Complete Information:**
  - The evolution of \(X_t\) and \(B_{2t}\) are both available.

  \[\Rightarrow\] Trading strategies satisfy: \(\theta_t \in \mathcal{F}_t, \quad t \in [0, T].\)
Part II

Supply Contracts with Financial Hedging

The Wholesale Price Contract with Budget Constraint
The Wholesale Price Contract

Non-Cooperative Operation:

○ Solution Concept: Stackelberg game.

1. At time $t = 0$, Manufacturer offers a wholesale price $w$ to the Retailer.
2. At time $t = 0$, Retailer orders a quantity $q$.
3. At time $t = T$, a random clearance price $P(q)$ is realized: $P(q) = A - q$.

○ The random price $A$ is correlated to the financial market $X_t$.

○ Retailer operates under a budget constraint: $w q \leq B$. 
Types of Contracts

Simple Contract

Production takes place
$t = 0$
$t = T$

Clearance Price $P(q)$ is realized
Payoffs are determined

$(w,q)$ contract is decided
Types of Contracts

Simple Contract

Flexible Contract
Types of Contracts

Simple Contract

- $(w_t, q_t)$ contract is decided
- Production takes place at $t = 0$
- Payoffs are determined at $t = T$

Flexible Contract

- $(w_\tau, q_\tau)$ contract is decided
- Market Signal $X_\tau$ is observed at $t = \tau$
- Production takes place at $t = \tau$
- Payoffs are determined at $t = T$

Flexible Contract with Financial Hedging

- $(w_\tau, q_\tau)$ contract is decided
- Market Signal $X_\tau$ is observed at $t = \tau$
- Trading Gain $G_\tau$ is observed
- Financial hedging takes place at $t = 0$
- Production takes place at $t = \tau$
- Payoffs are determined at $t = T$
Types of Contracts

**Simple Contract**

- (w, q) contract is decided
- Production takes place
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- (w_τ, q_τ) contract is decided
- Market Signal $X_τ$ is observed
- Trading Gain $G_τ$ is observed
- Clearance Price P(q) is realized
- Payoffs are determined
- Financial hedging takes place
- Production takes place
- $t = 0$
- $t = T$
- $t = τ$

**Remark:** The time $τ$ can be a decision variable: a deterministic time or a $\mathcal{F}_τ^X$-stopping time.
Some Notation

*) $(\Omega, \mathcal{F}, \mathbb{Q})$ probability space.

*) $X_t$: Time $t$ value of a $\mathbb{Q}$-martingale tradable security ($t \in [0, T]$).

*) $\mathcal{F}_t = \sigma(X_s ; 0 \leq s \leq t)$ filtration generated by $X_t$ with $\mathcal{F}_T \subseteq \mathcal{F}$.

*) $\mathbb{E}_t^\mathbb{Q}[E] := \mathbb{E}[E|\mathcal{F}_t]$ for all $E \in \mathcal{F}$.

*) $P(q) = A - q$: retail price at time $T$ with $A \in \mathcal{F}$.

*) $\bar{A}_\tau = \mathbb{E}_{\tau}^\mathbb{Q}[A|\mathcal{F}_\tau]$ for any $\mathcal{F}_t$-stopping time $\tau \leq T$.

*) $c_\tau$: per unit manufacturing cost at time $\tau \in [0, T]$.

*) $w_\tau \in \mathcal{F}_\tau$: manufacturer’s wholesale price menu.

*) $q_\tau \in \mathcal{F}_\tau$: retailer’s ordering quantity menu.

*) $G_\tau \in \mathcal{F}_\tau$: retailer’s trading gains (or losses) with $\mathbb{E}_\tau^\mathbb{Q}[G_\tau] = 0$.

*) Budget constraint: $w_\tau q_\tau \leq B_\tau := B + G_\tau$.

Assumption. For all $\tau \in [0, T]$, $\bar{A}_\tau \geq c_\tau$. 

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Flexible Contract

Consider a fixed $\tau \in [0, T]$.

**Step 1:** At $t = 0$, the manufacturer offers a wholesale menu $w_\tau \in \mathcal{F}_\tau$.

**Step 2:** A retailer, with no access to the financial market, decides the ordering level $q_\tau \in \mathcal{F}_\tau$ solving

$$
\Pi^F_{\mathcal{R}}(w_\tau) = \mathbb{E}_0^Q \left[ \max_{q_\tau \geq 0} \left\{ \mathbb{E}_T^Q \left[ (A - q_\tau - w_\tau) q_\tau \right] \right\} \right]
$$

subject to $w_\tau q_\tau \leq B$, for all $\omega \in \Omega$.

The optimal solution (retailer’s reaction) is

$$
q(w_\tau) = \min \left\{ \left( \frac{\bar{A}_\tau - w_\tau}{2} \right)^+, \frac{B}{w_\tau} \right\}.
$$

**Step 3:** The manufacturer problem is (Stackelberg leader)

$$
\Pi^F_{\mathcal{M}} = \mathbb{E}_0^Q \left[ \max_{w_\tau \geq c_\tau} \left\{ (w_\tau - c_\tau) q_\tau(w_\tau) \right\} \right].
$$
Flexible Contract

**Proposition.** (Flexible Contract Solution)

*Under Assumption, the equilibrium solution for the flexible contract is*

\[ w^F_\tau = \frac{\bar{A}_\tau + \delta^F_\tau}{2} \quad \text{and} \quad q^F_\tau = \frac{\bar{A}_\tau - \delta^F_\tau}{4}, \]

*where*

\[ \delta^F_\tau := \max \left\{ c_\tau, \sqrt{(\bar{A}^2_\tau - 8 B)^+} \right\}. \]

*The equilibrium expected payoffs of the players are then given by*

\[ \Pi^F_{M|\tau} = \frac{(\bar{A}_\tau + \delta^F_\tau - 2 c_\tau) (\bar{A}_\tau - \delta^F_\tau)}{8} \quad \text{and} \quad \Pi^F_{R|\tau} = \frac{(\bar{A}_\tau - \delta^F_\tau)^2}{16}. \]

**Remarks:**

- \( \delta^F_\tau \) is a modified production cost \((\geq c_\tau)\) that is induced by a limited budget \( B \).

- \( w^F_\tau \) decreases in \( B \) \quad and \quad \( q^F_\tau, \Pi^F_{M|\tau} \) and \( \Pi^F_{R|\tau} \) increase in \( B \).
Flexible Contract vs. Simple Contract

Define

\[ w^F := \mathbb{E}_0^Q[w^F_\tau], \quad q^F := \mathbb{E}_0^Q[q^F_\tau], \quad \Pi^F_M := \mathbb{E}_0^Q[\Pi^F_M|\tau] \text{ and } \Pi^F_R := \mathbb{E}_0^Q[\Pi^F_R|\tau]. \]

**Proposition.** Suppose that \( B \leq \frac{\bar{A}^2_\tau - c^2_\tau}{8} \) almost surely. Then

\[ w^F \leq w^S, \quad q^F \geq q^S, \quad \Pi^F_M \leq \Pi^S_M \text{ and } \Pi^F_R \geq \Pi^S_R. \]

However, if \( B \geq \max \left\{ \frac{\bar{A}^2_\tau - c^2_\tau}{8}, \frac{\bar{A}^2 - c^2_0}{8} \right\} \) almost surely then

\[ w^F = w^S + \frac{c_\tau - c_0}{2} \quad \text{and} \quad q^F = q^S - \frac{c_\tau - c_0}{4} \]

and

\[ \Pi^F_M \geq \Pi^S_M \text{ and } \Pi^F_R \geq \Pi^S_R \quad \text{if and only if} \quad \mathbb{E}_0^Q[(\bar{A}_\tau - c_\tau)^2] \geq (\bar{A} - c_0)^2. \]
Flexible Contract vs. Simple Contract

\[ A_\tau \sim \text{Uniform}[1, 3], \quad c_0 = 0.3, \quad c_\tau = 0.35 \text{ (case 1)} \quad \text{and} \quad c_\tau = 0.7 \text{ (case 2)}. \]
Flexible Contract: Efficiency

The Martingale Approach to Operational and Financial Hedging

\[ \bar{A}_\tau \geq 3 \ c_\tau \]

\[ \bar{A}_\tau \leq 3 \ c_\tau \]

\[ \bar{A}_\tau = 2, \quad c_\tau = 0.6 \] (top) and \[ c_\tau = 1.2 \] (bottom).
Flexible Contract with Financial Hedging

- Contract is decided at $t = 0$
- Financial hedging takes place
- Market Signal $X_t$ is observed
- Trading Gain $G_t$ is observed
- Clearance Price $P(q)$ is realized
- Payoffs are determined
- Production takes place at $t = \tau$
- Time $t = T$
Flexible Contract with Financial Hedging

- **Step 1:** At $t = 0$, and for a fixed $\tau \leq T$, the manufacturer offers a price menu $w_\tau \in F^X_\tau$. 
Flexible Contract with Financial Hedging

- **Step 1:** At $t = 0$, and for a fixed $\tau \leq T$, the manufacturer offers a price menu $w_\tau \in \mathcal{F}_\tau^X$.

- **Step 2:** In response, at $t = 0$, the retailer selects an optimal ordering menu $q^*_\tau(w_\tau) \in \mathcal{F}_\tau^X$ solving

\[
\Pi^H_R(w_\tau) = \max_{q_{\tau} \geq 0, G_\tau} \mathbb{E}^Q[(A - q_{\tau}) q_{\tau} - w_{\tau} q_{\tau}]
\]

subject to

\[
w_{\tau} q_{\tau} \leq B + G_{\tau}, \quad \text{for all } \omega \in \Omega
\]

\[
\mathbb{E}^Q[G_{\tau}] = 0.
\]
Flexible Contract with Financial Hedging

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$$

subject to $w_{\tau} q_{\tau} \leq B + G_\tau$, for all $\omega \in \Omega$

$$
\mathbb{E}^Q[G_\tau] = 0.
$$

- **Step 3**: The manufacturer selects the optimal wholesale price menu $w^*_\tau$ solving

$$
\Pi^H_M(w_\tau) = \max_{w_{\tau}} \mathbb{E}^Q\left[w_{\tau} q^*_\tau(w_{\tau}) - c_{\tau} q^*_\tau(w_{\tau})\right].
$$
Flexible Contract with Financial Hedging

**Proposition.** (Retailer’s Optimal Strategy)

Let $Q$, $\mathcal{X}$ and $\mathcal{X}^c$ be defined as follows

$$Q_\tau \triangleq \left( \frac{A_\tau - w_\tau}{2} \right)^+, \quad \mathcal{X} \triangleq \{ \omega \in \Omega : B \geq Q_\tau w_\tau \}, \quad \text{and} \quad \mathcal{X}^c \triangleq \Omega - \mathcal{X}.$$
Flexible Contract with Financial Hedging

**Proposition.** (Retailer’s Optimal Strategy)

Let $Q_\tau$, $\mathcal{X}$ and $\mathcal{X}^c$ be defined as follows

$$Q_\tau \triangleq \left(\frac{\bar{A}_\tau - w_\tau}{2}\right)^+ , \quad \mathcal{X} \triangleq \{\omega \in \Omega : B \geq Q_\tau w_\tau\} , \quad \text{and} \quad \mathcal{X}^c \triangleq \Omega - \mathcal{X}.$$ 

**Case 1:** Suppose that $\mathbb{E}^Q\left[Q_\tau w_\tau\right] \leq B$. Then $q^*_\tau(w_\tau) = Q_\tau$ and there are infinitely many choices of the optimal claim, $G_\tau$. One natural choice is to take

$$G_\tau = [Q_\tau w_\tau - B] \cdot \left\{ \begin{array}{ll} \delta & \text{if } \omega \in \mathcal{X} \\ 1 & \text{if } \omega \in \mathcal{X}^c \end{array} \right.$$ 

$$\delta \triangleq \frac{\int_{\mathcal{X}^c} [Q_\tau w_\tau - B] \, dQ}{\int_{\mathcal{X}} [B - Q_\tau w_\tau] \, dQ}.$$
Flexible Contract with Financial Hedging

Proposition. (Retailer’s Optimal Strategy)

Let $Q_\tau$, $\mathcal{X}$ and $\mathcal{X}^c$ be defined as follows

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$$\delta \triangleq \frac{\int_{\mathcal{X}^c} [Q_\tau w_\tau - B] \, dQ}{\int_{\mathcal{X}} [B - Q_\tau w_\tau] \, dQ}.$$ 

Remark: In this case, it is possible to completely eliminate the budget constraint by trading in the financial market.
Flexible Contract with Financial Hedging

Proposition. (Continuation)

Case 2: Suppose that \( B < \mathbb{E}^Q [ Q_\tau w_\tau ] \). Then

\[
q_\tau(w_\tau) = \left( \frac{\bar{A}_\tau - w_\tau (1 + \lambda)}{2} \right)^+ \text{ where } \lambda \geq 0 \text{ solves } \mathbb{E}^Q \left[ w_\tau \left( \frac{\bar{A}_\tau - w_\tau (1 + \lambda)}{2} \right)^+ \right] = B.
\]
Flexible Contract with Financial Hedging

Proposition. (Continuation)

Case 2: Suppose that \( B < \mathbb{E}^Q [ Q_{\tau} w_{\tau} ] \). Then

\[
q_{\tau}(w_{\tau}) = \left( \frac{\bar{A}_{\tau} - w_{\tau} (1 + \lambda)}{2} \right)^+ \text{ where } \lambda \geq 0 \text{ solves } \mathbb{E}^Q \left[ w_{\tau} \left( \frac{\bar{A}_{\tau} - w_{\tau} (1 + \lambda)}{2} \right)^+ \right] = B.
\]

Proposition. (Producer’s Optimal Strategy and the Stackelberg Solution)

Let \( \phi^* \triangleq \inf \left\{ \phi \geq 1 \text{ such that } \mathbb{E}^Q \left[ \left( \frac{\bar{A}_{\tau}^2 - (\phi c_{\tau})^2}{8} \right)^+ \right] \leq B \right\} \).

Then, \( w^*_\tau = \frac{\bar{A}_{\tau} + \phi^* c_{\tau}}{2} \) and \( q^*_\tau = \left( \frac{\bar{A}_{\tau} - \phi^* c_{\tau}}{4} \right)^+ \) and the players’ expected payoffs satisfy

\[
\Pi_{H|\tau}^M = \left( \frac{\bar{A}_{\tau} + \phi^* c_{\tau} - 2 c_{\tau}}{8} \right) \left( \frac{\bar{A}_{\tau} - \phi^* c_{\tau}}{8} \right)^+ \quad \text{and} \quad \Pi_{H|\tau}^R = \left( \frac{\bar{A}_{\tau} - \phi^* c_{\tau}}{16} \right)^2.
\]
Flexible Contract with Financial Hedging

**Proposition.** (Continuation)

**Case 2:** Suppose that $B < \mathbb{E}^Q [Q \tau \, w_\tau]$. Then

$$q_\tau(w_\tau) = \left( \frac{\bar{A}_\tau - w_\tau (1 + \lambda)}{2} \right)^+$$

where $\lambda \geq 0$ solves $\mathbb{E}^Q \left[ w_\tau \left( \frac{\bar{A}_\tau - w_\tau (1 + \lambda)}{2} \right)^+ \right] = B$.

**Proposition.** (Producer’s Optimal Strategy and the Stackelberg Solution)

Let $\phi^* \triangleq \inf \left\{ \phi \geq 1 \text{ such that } \mathbb{E}^Q \left[ \left( \frac{\bar{A}_\tau^2 - (\phi \, c_\tau)^2}{8} \right)^+ \right] \leq B \right\}$.

Then, $w^*_\tau = \frac{\bar{A}_\tau + \phi^* \, c_\tau}{2}$ and $q^*_\tau = \left( \frac{\bar{A}_\tau - \phi^* \, c_\tau}{4} \right)^+$ and the players’ expected payoffs satisfy

$$\Pi^H_{M \mid \tau} = \frac{(\bar{A}_\tau + \phi^* \, c_\tau - 2c_\tau) (\bar{A}_\tau - \phi^* \, c_\tau)^+}{8} \quad \text{and} \quad \Pi^H_{R \mid \tau} = \left( \frac{(\bar{A}_\tau - \phi^* \, c_\tau)^+}{16} \right)^2.$$

**Remark:** When $q^*_\tau = 0$, the manufacturer decides to overcharge the retailer making the supply chain non-operative. This is never the case if the retailer does not have not access to the financial market.
Flexible Contract with Financial Hedging

**Proposition.** The manufacturer always prefers the H-contract to the F-Contract. On the other hand, the retailer’s preferences are
Proposition. The manufacturer always prefers the H-contract to the F-Contract. On the other hand, the retailer's preferences are:

- Small Budget: H-Contract preferred to F-Contract.
- Undetermined: H-Contract or F-Contract.
- Large Budget: H-Contract = F-Contract.

Graphs showing the preferences for Wholesale Price, Ordering Level, Producer’s Payoff, and Retailer’s Payoff.
Flexible Contract with Financial Hedging
Efficiency

- On path-by-path basis, the Centralized system is not necessarily more efficient than the Decentralized Supply Chain!

\[ \exists \omega \in \Omega \text{ such that } q_{\mathcal{C}|\tau}^H = 0 \text{ and } q_{\tau}^H > 0. \]

Remarks: This is never the case under a Flexible Contract without Hedging.
Flexible Contract with Financial Hedging Efficiency

- On path-by-path basis, the Centralized system is not necessarily more efficient than the Decentralized Supply Chain!

\[ \exists \omega \in \Omega \text{ such that } q_{c|\tau}^H = 0 \text{ and } q_{\tau}^H > 0. \]

**Remarks:** This is never the case under a Flexible Contract without Hedging.

- On average, the Centralized solution is more efficient than the Decentralized solution.

\[ \mathbb{E}_0^Q[q_{c|\tau}^H] \geq \mathbb{E}_0^Q[q_{\tau}^H]. \]
Summary

- Simple extension to the traditional wholesale contract.
Summary

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○ The proposed procurement contracts uses the Financial Market as:
  – A source of public information upon which contracts can be written.
  – A means for financial hedging to mitigate the impact of the budget constraint.
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○ Consistent with the notions of production postponement and demand forecast.
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○ Managerial Insights:
  – Manufacturer and Retailer incentives are not always aligned as a function of $B$.
  – Manufacturer prefers retailers that have access to the financial market.
  – With hedging, the supply chain might not operate in some states $\omega \in \Omega$.
  – In some cases, financial hedging **eliminates** the budget constraint.
  – Optimal time $\tau$ of the contract balances $\mathbb{V}ar(\bar{A}_\tau)$ and $c_\tau$. 
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  – Optimal time $\tau$ of the contract balances $\Var(\bar{A}_\tau)$ and $c_\tau$.

○ Extensions:
  – Other types of contracts: quantity discount, buy-back, etc.
  – Include other sources of uncertainty: exchange rates, interest rates, credit risk.