Optimal Control and Hedging of Operations in the Presence of Financial Markets

René Caldentey
Stern School of Business, New York University, 44 West Fourth Street, New York, New York 10012, rcaldent@stern.nyu.edu

Martin Haugh
Department of Industrial Engineering and Operations Research, Columbia University, 331 S.W. Mudd Building, 500 West 120th Street, New York, New York 10027-6902, martin.haugh@columbia.edu

We consider the problem of dynamically hedging the profits of a corporation when these profits are correlated with returns in the financial markets. In particular, we consider the general problem of simultaneously optimizing over both the operating policy and the hedging strategy of the corporation. We discuss how different informational assumptions give rise to different types of hedging and solution techniques. Finally, we solve some problems commonly encountered in operations management to demonstrate the methodology.

Key words: operations management; portfolio optimization; stochastic control; incomplete markets
MSC2000 subject classification: Primary: 90B05, 91B28; secondary: 90B30, 91B70
OR/MS subject classification: Primary: inventory/production: policies; secondary: finance: portfolio
History: Received May 13, 2003; revised: May 13, 2004, February 16, 2005, and August 14, 2005.

1. Introduction. In this paper, we propose a framework for modelling the operations of a nonfinancial corporation that also trades in the financial markets. The corporation must simultaneously choose an optimal operating policy and an optimal trading strategy in the financial markets. In practice, it has long been observed that nonfinancial corporations do, in fact, hedge using financial markets. The goal of this paper then is to describe a method by which operations and hedging might be conducted.

One immediate difficulty that arises when modelling this problem is that most of the operations literature assumes that corporations are risk neutral. Indeed, this is supported by the famous work of Modigliani and Miller [24] who argue that in a frictionless world there is no need for corporations to hedge as shareholders can do so themselves. While this argument has some merit, we do, of course, live in a world with many frictions. These frictions include the costs of financial distress, taxes, and agency costs, as well as frictions in the capital markets. As a result, it is often the case that corporations should and do hedge. Once this is recognized, it is no longer plausible to assume that corporations are always risk neutral.

In this paper, we therefore consider the problem of dynamically hedging the profits of a risk-averse corporation when these profits are correlated with returns in the financial markets. The central modelling insight is to view the operations and facilities of the corporation as an asset in the corporation’s portfolio. This view enables us to pose the problem as one of financial hedging in incomplete markets, a problem that has been studied extensively in the recent literature in mathematical finance, e.g., Schweizer [31]. Though we pose the problem as one of hedging in incomplete markets, we also have the added complexity of simultaneously seeking to choose an optimal operating policy. As a result, we also have some control over the type of asset to be hedged. This is a distinguishing feature of this paper that is generally not found in the mathematical finance literature. We also discuss how different informational assumptions give rise to different types of hedging and solution techniques. In particular, the class of feasible hedging strategies that are available to the corporation will depend on whether or not the corporation can observe the evolution of all relevant state variables.

To maintain tractability, we will assume that the corporation has a mean-variance objective function. While this of course is somewhat restrictive, it is often used in practice and can serve as a useful first approximation. The techniques that we use are based on the mean-variance analysis of Schweizer [30], and the martingale approach of Cox and Huang [8] and Karatzas et al. [19]. While we are aware of the very recent progress that has been made towards solving hedging problems for more general utility functions and general price processes (e.g., Bertsimas et al. [2], Delbaen et al. [10], Gouriéroux et al. [16], Laurent and Pham [22], Lim [23], Pham et al. [28], Schweizer [31]), we have not attempted to apply this work here. Indeed, much of this literature is concerned with issues regarding existence and uniqueness of solutions and does not lend itself easily to the computation of such solutions. Moreover, the purpose of this paper is simply to highlight the modelling framework and demonstrate that it can be used to solve some interesting problems in operations management.