Optimal Operating Control and Dividend Distribution Policies

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**Extended Abstract**

1 Overview

In 1958 Modigliani and Miller demonstrated that firms that operate in perfect and complete capital markets need not to worry about simultaneously coordinating operational and financial decisions. Consistently, most of the OM research has treated both functions as if they were indeed independent. However, the real world is far from the Platonic Modigliani-Miller’s world. Recently, there has been a rapidly growing body of literature investigating the interactions between finance and operations in firms that have limited working capital and that operate in imperfect and incomplete capital markets. This new stream of research has provided interesting new insights that could help budget-constrained firms reduce the risk of bankruptcy increasing shareholders’ value.

In this paper, we consider a firm whose net earnings evolve according to a Brownian motion which is influenced by the firm’s operating strategy. The firm has to decide on the optimal operating policy (by controlling the drift and the volatility of the earnings process) as well as the leverage ratio and the distribution of dividends. The manager’s objective is to maximize the expected discounted stream of payments to the shareholders. However, an overly aggressive dividend policy can increase the firm’s risk of bankruptcy, which we model as the first time the net earnings process hits a predetermined (lower) threshold. This is precisely the trade-off that we investigate in this paper by characterizing optimal operating policies as well as capital structure decisions in terms of long and short term debts.

2 Related Research

In a closely related paper, Li et al. [7] propose a multi-period inventory management framework in which a firm decides every period on how much to borrow (only short-term loans), how much to produce and how much dividend to issue before knowing the period’s demand. The firms seeks to maximize the present value of dividends net of capital subscriptions. Two types of bankruptcy are envisioned: reorganization bankruptcy where the default causes a costly restructuring but operations continue, and wipeout bankruptcy which causes a permanent halt of operations. They find that the integration of the production/inventory problem with the cash flow and financial problems of a firm may be relevant and worthwhile specially for firms operating with thin budgets in volatile markets. Hu and Sobel [6] further expand the latter model by adding a capital structure (long-term debt) and find that optimal coordinated decisions, in comparison to decentralized decisions, yield lower inventories, require less working capital, make larger short-term loans, require less long-term debt, have a lower
default risk, and yield higher expected dividends net of capital subscriptions. Buzacott and Zhang [2] incorporate asset-based financing into production decisions by modeling the available cash in each period as a function of the firm’s assets and liabilities. Other recent papers that analyze models of financially constrained firms include Babich and Sobel [1], Xu and Birge [10] and Dada and Hu [4].

Our formulation is based on Radner and Shepp [9] in which the revenue stream of the firm is modeled by means of a Brownian motion. Different operating strategies yield different combinations of expected return and volatility, the Brownian motion’s parameters (drift/volatility pairs) can be chosen from a finite set which emulates the operational strategies that are accessible to the firm. The firm must also decide on how much (if any) dividends to distribute to its shareholders and its objective is to maximize the expected present value of this stream of dividend payments. Other authors like Hjgaard and Taksar [5] and Cadenillas, Chouilli, Taksar and Zhang [3] have explored similar models where there is a single drift/volatility pair available, but the firm can affect it proportionally. This emulates an insurance company that can control the reinsurance rate.

In this paper, we develop a model similar to the one studied by Radner and Shepp [9] where the firm must also make capital structure decisions selecting long and short term debts. We address this issue by taking both a static and a dynamic approach. In the static model debt levels can be chosen only once at the beginning of the planning horizon (thus emulating long-term debt). In the dynamic model the leverage level may be modified at any point in time. Both approaches will help deepen our understanding of the interactions between finance and operations for different types of firms. For the sake of exposition, in what follows we describe our model and main results for the case in which the firm uses only long-term debt.

3 Model Description and Main Results

Consider a firm whose net operating income are uncertain, and can be modeled by a diffusion process. The firm’s operational policy controls the parameters (drift and volatility) of this diffusion. At time 0, the firm can acquire a long-term loan which will remain constant throughout its lifetime. At any time the firm can pay out dividends to its shareholders. The firm’s available cash evolves as the differences between the operating income and the dividend it pays over time. If this cash reservoir falls to zero the firm is considered to be bankrupt and it ceases to exist. Finally, the value of the firm is defined as the expected present value of all future dividend payments until it reaches bankruptcy (an event which might never occur).

We assume that the firm has access to a finite number of operational strategies (pairs drift/volatility). The assets that the firm owns and uses to operate (machinery, warehouses, etc) are considered completely illiquid. Also, we do not consider the possibility of reinvestment of revenues on additional assets that could make available new, and better, operational strategies. We also leave out of the model the addition of extra capital (e.g., raising new equity) that could save the firm from bankruptcy. Since dividends must be nonnegative and debt cannot be modified the only way the firm’s cash reserve can grow is through revenues.

Our first result characterizes the subset of operating strategies that are used in an optimal policy.

Proposition 1 Consider the positive orthant where the X-axis measures volatility and the Y-axis measures drift. In this space, the firm’s operating strategies are represented by a finite set of points. Let us add the origin to this set. Then, in an optimal policy the firm only uses those strategies that belong to the increasing portion of the upper envelop of this set.

Intuitively, this result is consistent with the notion of an efficient frontier in a mean-variance setting. Our second result establishes that the firm will pay dividend only if the cash reserve is sufficiently high, otherwise operating incomes are fully retained.
Proposition 2  The firm’s optimal dividend policy is of a bang-bang type, that is, the company pays dividends only when the cash reserve hits a fixed upper threshold. This threshold acts as a reflecting barrier in the evolution of the firm’s cash reserve. As result, under an optimal policy, the firm reaches bankruptcy with probability one.

This result is somehow counterintuitive as it reveals that in order to maximize shareholder value, managers must eventually lead their companies to bankruptcy.

Based on the previous results, we are able to provide a complete characterization of an optimal solution together with a simple recursive algorithm that computes the firm’s optimal operating strategy and long-term debt decisions. In order to model the terms of this debt, we use a competitive financial market assumption under which the interest rate on the debt is computed assuming risk neutrality. Hence, this interest rate is endogenously determined solving a fixed-point condition; the firm’s optimal strategy depends on the interest rate of the loan, which on its turn depends on the firm’s operating strategy. The following result summarizes some of the key features of an optimal policy.

Proposition 3

1. Let Z be the upper threshold identified in Proposition 2. The interval [0, Z] (which represents the domain of the firm’s cash reserves) is partitioned into a set of disjoint intervals such that there is a one-to-one mapping between these intervals and the set of optimal operating strategies identified in Proposition 1. The mapping is monotone in the sense that the firm increases the risk (volatility) of its operating policy as its cash reserve increases, and viceversa.

2. The relative value of leverage (debt) and the leverage ratio decrease with the firm’s initial level of cash.

3. The ratio of the firm’s time to default with and without leverage is U-shaped as a function of its initial level of cash.

Our theoretical results are complemented by a set of numerical experiments that we use to perform sensitivity analysis on the optimal amount of leverage, the value added by leverage, the risk premium of the optimal debt and the expected time-to-default. Our results show that for firms with limited capital, leverage can both add value and help stay in business for a longer time if the risk free rate is low enough. Firms with greater amounts of initial equity may also increase their value by acquiring a small amount of debt, although this will actually shorten their expected lifetime.

References


