Interpreting the Movements in Short-Term Interest Rates

Introduction

The so-called Fisher equation is probably the oldest and best known equilibrium asset pricing model. Much to the dismay of generations of economists, empirical tests of the model often fail to provide support for it. Modern asset pricing theory provides the additional determinants of nominal interest rates. In this article, we develop an empirical framework for evaluating the influence on interest rates of movements in expected inflation, the real rate of interest, and the risk premium.

The Consumption-based Capital Asset Pricing Model developed by Lucas (1978) and Breeden (1979) is used to derive a generalized ex ante Fisher model for interest rates. Our model differs from others (such as Benninga and Protopapadakis 1983; Ferson and Merrick 1987; and Shome, Smith, and Pinkerton 1988) by allowing

We are grateful for helpful comments from Stephen Cecchetti, Robert Cumby, Campbell Harvey, Joel Habrouck, Karen Lewis, James Lothian, Frederic Mishkin, and Stephen Smith. We have also benefited from the comments of seminar participants at the National Bureau of Economic Research, the City University Business School (London), the Western Finance Association (Santa Barbara), and the World Congress of the Econometric Society (Barcelona).

1. For empirical evidence on the simple Fisher model of short-term interest rates, see Summers (1983) and Barsky (1987). At the very best, the relationship is highly unstable, and the hypothesized coefficient of one for the full Fisher effect of inflation on interest rates is resoundingly rejected.

This article uses modern asset pricing theory to examine the behavior of short-term nominal interest rates over the past 25 years. The analysis investigates whether variation in the stochastic behavior of consumption growth and inflation can explain movements in the rate of interest. The model estimated allows for taste shocks to utility. Our results reveal that much of the month-to-month movement in nominal interest rates reflects changes in the real rate and the risk premia in addition to inflationary expectations.

Journal of Business, 1992, vol. 65, no. 3)
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0021-9398/92/6503-0005$01.50
for taste shocks to utility. The effects of multiplicative taste shocks have been studied in other contexts by Miron (1986) and Campbell (1986).

The proximate determinants of nominal interest rates are, in addition to the expected inflation rate and its variance, which determine the expected rate of depreciation of money, the conditional covariance of inflation and consumption growth and the conditional expectation and variance of consumption growth. Like other models, these additional variables identify the real rate and the risk premium. Our model provides new specifications for the real rate and the risk premium when taste shocks are present.

The conditional moments of consumption growth and inflation are not directly observable. In the second section, we describe the data and use modern time-series techniques to develop appropriate measures for each of the independent variables in the model. The specification chosen is an autoregressive model with time-varying parameters and autoregressive conditional heteroscedasticity (ARCH) residuals.

In the third section of the article, we investigate the ability of the model to explain monthly movements in the 1-month Treasury-bill rate and quarterly movements in the 3-month rate. Our results suggest that the parameters should vary through time. We are therefore led to examine empirical models that allow for time-varying parameters. These models provide an adequate explanation of movements in both 1-month and 3-month rates. Our results are robust with respect to different measures of inflation.

In the fourth section of the article, the estimated models are used to interpret the influence of movements in real rates and expected inflation on nominal interest rates. They provide us with interesting insights into the behavior of nominal interest rates. In particular, we are surprised to find little correlation between the monthly and quarterly changes in nominal interest rates and expected inflation (the so-called Fisher effect). Annual changes in interest rates are more strongly correlated with changes in expected inflation. Our model also allows us to interpret the historical movements in interest rates. We find that (i) the increase in interest rates in the 1970s was at first due to higher expected inflation, and in later years due to increases in the real rate of interest, (ii) the persistence of high nominal interest rates during the early 1980s can be attributed to persistently high real rates of interest, and (iii) the fall in nominal interest rates in the mid-1980s was at first due to declines in expected inflation and after 1986 was due to a fall in the real rate.

2. See Mishkin (1988) for a taxonomy of earlier results that relate nominal interest rates to the real rate and expected inflation.
A Generalization of the Fisher Equation

The generalization of the Fisher equation that forms the heart of our empirical investigation is derived from the Consumption-based Capital Asset Pricing Model (CCAPM) with taste shocks.

Determinants of Interest Rate

Consider a version of the CCAPM in which there is a single consumption good and where utility is isoelastic and time separable. The representative consumer maximizes expected utility over an infinite lifetime:

$$E_t \sum_{i=0}^{\infty} \Phi^i \Lambda_{t+i} \frac{1}{1-\gamma} C_{t+i}^{1-\gamma}, \quad 1 > \Phi > 0, \gamma > 0,$$

where $E_t$ denotes expectations conditional on information available in period $t$, $\Phi$ is the discount factor, and $\gamma$ is the coefficient of relative risk aversion. Taste shocks, $\Lambda_t$, follow an exogenous (serially correlated) stochastic process. Large realizations of $\Lambda_t$ make consumption in period $t$ particularly desirable.

Equilibrium asset returns are identified from the first-order conditions of the representative consumer’s maximization problem. Let $Q_t$ be the value of any asset at time $t$ expressed in terms of consumption goods. The first-order condition implies that

$$\Lambda_t C_t^{-\gamma} = \Phi E_t[\Lambda_{t+1} C_{t+1}^{-\gamma} (Q_{t+1}/Q_t)].$$

If the asset is a nominal bond with a known nominal interest rate of $I_t$, then the (ex post) real return on investing in nominal bonds between $t$ and $t+1$ is $(1 + I_t)P_t/P_{t+1} = Q_{t+1}/Q_t$, where $P_t$ is the nominal price of a good at $t$. The first-order condition for nominal bonds is therefore

$$\Lambda_t C_t^{-\gamma} = \Phi E_t[\Lambda_{t+1} C_{t+1}^{-\gamma} (1 + I_t)P_t/P_{t+1}].$$

In the absence of taste shocks (i.e., when $\Lambda_t$ is constant), (3) can form the basis for an estimable model following the approach popularized by Hansen and Singleton (1982). When taste shocks are present, we must make some further assumptions in order to estimate the model. To this end, we assume that the taste shock, prices, and consumption at $t + 1$ are (conditional on information available at time $t$) jointly log normally distributed. Applying log normality allows us to

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3. We study the single-good case for simplicity. See Breeden (1986) for a discussion of the differences between the single-good and multiple-good versions of the Fisher equation.
rewrite (3) as

\[ i_t = r_t + E_t \Delta p_{t+1} - \frac{1}{2} \text{var}_t(\Delta p_{t+1}) \]

\[ - \gamma \text{cov}_t(\Delta c_{t+1}; \Delta p_{t+1}) + \text{cov}_t(\Delta \lambda_{t+1}; \Delta p_{t+1}), \]  

where small letters are used for natural logarithms, \( \Delta \) is the first-difference operator, and \( \text{var}(\cdot) \) and \( \text{cov}(\cdot) \) denote variances and covariances conditional on information available at \( t \). The first term in (4), \( r_t \), is the log of the real rate of interest. The next two terms identify the expected rate of depreciation of money. Both the expected rate of inflation, \( E_t \Delta p_{t+1} \), and the variance of the price level, \( \text{var}(\Delta p_{t+1}) \), appear because the inflation rate is continuously compounded, and both determine the expected rate of depreciation of money, \( E(P_t/P_{t+1}) \).\(^4\)

The two covariance terms in (4) represent the risk premium on nominal bonds. The first term, \( \gamma \text{cov}(\Delta p_{t+1}; \Delta c_{t+1}) \), is familiar from the standard CCAPM. Risk-averse investors find nominal bonds more attractive in situations where unexpectedly high real returns coincide with high marginal utility. Since the real return on a nominal bond is inversely related to next period’s price level and marginal utility is decreasing in consumption, this means that, ceteris paribus, a rise in \( \text{cov}(\Delta p_{t+1}; \Delta c_{t+1}) \) should increase the demand for nominal bonds and lower the equilibrium nominal interest rate.

The second covariance term reflects the effects of taste shocks on marginal utility. When \( \text{cov}(\Delta \lambda_{t+1}; \Delta p_{t+1}) \) is positive, unexpectedly high marginal utility coincides with low realized real returns on nominal bonds. Ceteris paribus, this makes nominal bonds less attractive, making the equilibrium nominal interest rate higher.

The specification of \( r_t \) in (4) can be derived by considering the return on a real bond. If the known real rate of interest at time \( t \) is \( R_t \), then the purchase of a real bond in period \( t \) for \( Q_t \) consumption goods entitles the holder to \( Q_t(1 + R_t) = Q_{t+1} \) goods in \( t + 1 \). The first-order condition for the real bond is found by substituting in (2):

\[ \lambda_i c^{-\gamma(t+1)} = \Phi E_i[\lambda_{t+1} c^{-\gamma(t+1)}(1 + R_t)]. \]

Applying the assumption of log normality and rearranging defines the log of the real rate:

\[ r_t = \gamma E_t \Delta c_{t+1} - \frac{1}{2} \gamma^2 \text{var}_t(\Delta c_{t+1}) - \phi \]

\[ - E_t \Delta \lambda_{t+1} + \gamma \text{cov}_t(\Delta \lambda_{t+1}; \Delta c_{t+1}) - \frac{1}{2} \text{var}_t(\Delta \lambda_{t+1}), \]  

where \( \phi = \log \Phi \). The first three terms are familiar from the standard CCAPM, and the last three terms identify the effects of taste shocks.

\(^4\) Fama (1976), Amihud and Barnea (1977), and Shome, Smith, and Pinkerton (1988) discuss the importance of the expected depreciation of money in the Fisher equation. In our model, these terms are related by \( \exp[-E_t \Delta p_{t+1} + \frac{1}{2} \text{var}_t(\Delta p_{t+1})] = E_t(P_t/P_{t+1}) \).
Since expected marginal utility in \( t + 1 \) is raised by either an increase in \( E_t \Delta \lambda_{t+1} \), a fall in \( \text{cov}_t(\Delta \lambda_{t+1}, \Delta c_{t+1}) \), or a rise in \( \text{var}_t(\Delta \lambda_{t+1}) \), the equilibrium real interest rate will fall in each case.\(^5\)

An Estimable Model

Although equations (4) and (5) identify the determinants of equilibrium nominal and real interest rates, as they stand, the equations do not provide the basis for an estimable model. The reason is that some of the key elements, such as \( \text{cov}_t(\Delta \lambda_{t+1}, \Delta c_{t+1}) \) and \( \text{cov}_t(\Delta \lambda_{t+1}, \Delta p_{t+1}) \), are unidentified because taste shocks are unobserved.

One way of circumventing this difficulty is to utilize information about the structure of the economy in order to make identifying assumptions about the unobserved variables. For example, in an exchange economy with a nonstorable endowment, equilibrium consumption follows an exogenous process. In this case, there is no correlation between equilibrium consumption and taste shocks, so that \( \text{cov}_t(\Delta \lambda_{t+1}, \Delta c_{t+1}) = 0 \). However, such a simplification would be less plausible in a production economy (cf. Campbell 1986), and it would be inappropriate to develop an estimable model for interest rates with identifying assumptions that may hold only in certain endowment economies.

In a production economy, taste shocks can affect the equilibrium in many ways. A positive taste shock, for example, may lead to a rise in equilibrium consumption as production is reallocated toward current rather than future consumption. Taste shocks may also affect equilibrium prices through their impact on the demand for money. An induced increase in consumption may raise the demand for real transactions balances.\(^6\) Without an accommodating change in the money stock, this could lead to a fall in the equilibrium price level.

We will identify the unobservable covariance terms by exploiting the fact that, when equilibrium consumption and prices are correlated with taste shocks, observed consumption and prices will contain information about tastes. In particular, if \( \text{cov}_t(\Delta \lambda_{t+1}, \Delta c_{t+1}) \) and \( \text{cov}_t(\Delta \lambda_{t+1}, \Delta p_{t+1}) \) are nonzero, inferences about innovations in \( \lambda_t \) can be made from the observed innovations in consumption growth and/or inflation. Such inferences (see Sargent 1979) can be made from the simple re-

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5. To see this, note that expected marginal utility is increasing in the variance of log marginal utility and that \( \text{var}_t[\log(\lambda_{t+1}|C_{t+1})] = \text{var}_t(\Delta \lambda_{t+1}) + \gamma^2 \text{var}_t(\Delta c_{t+1}) - 2\gamma \text{cov}_t(\Delta \lambda_{t+1}, \Delta c_{t+1}) \).

6. Although money plays no role in our model, the first-order conditions in (2) and (3) are identical to those obtained in the cases where real balances and consumption separately enter utility. Such models (i.e., Grossman and Weiss 1982) can generate money demand functions that are increasing in consumption.
\[ \Delta \lambda_{t+1} - E_t \Delta \lambda_{t+1} = \pi^c [\Delta c_{t+1} - E_t \Delta c_{t+1}] + \pi^p [\Delta p_{t+1} - E_t \Delta p_{t+1}] + e_{t+1}. \]  

(6)

The coefficients \( \pi^c \) and \( \pi^p \) depend on the variances of tastes and other shocks and the equilibrium dependence of \( c_t \) and \( p_t \) on \( \lambda_t \). When the variance of taste shocks is small relative to other shocks, the signal-to-noise ratio will be low, making \( \pi^c \) and \( \pi^p \) close to zero.\(^7\) It is also possible that the signal-to-noise ratios change over the sample, making \( \pi^c \) and \( \pi^p \) time varying. We shall examine this possibility below.

Equation (6) allows us to derive an estimable model for nominal interest rates from (4) and (5). Given \( \pi^c \) and \( \pi^p \), we can write

\[
\text{cov}(\Delta \lambda_{t+1}, \Delta c_{t+1}) = \pi^c \text{var}(\Delta c_{t+1}) + \pi^p \text{cov}(\Delta p_{t+1}, \Delta c_{t+1}),
\]

\[
\text{cov}(\Delta \lambda_{t+1}, \Delta p_{t+1}) = \pi^p \text{var}(\Delta p_{t+1}) + \pi^c \text{cov}(\Delta p_{t+1}, \Delta c_{t+1}),
\]

and

\[
\text{var}(\Delta \lambda_{t+1}) = (\pi^p)^2 \text{var}(\Delta p_{t+1}) + (\pi^c)^2 \text{var}(\Delta c_{t+1}) + 2\pi^p\pi^c \text{cov}(\Delta p_{t+1}, \Delta c_{t+1}) + \text{var}(e_{t+1}).
\]

Substituting these expressions in (4) and (5) and simplifying yields

\[
i_t = E_t \Delta p_{t+1} - \frac{1}{2} \text{var}(\Delta p_{t+1}) + \gamma E_t \Delta c_{t+1} + \left[ \gamma \pi^c - \frac{1}{2} \gamma^2 - \frac{1}{2}(\pi^c)^2 \right] \text{var}(\Delta c_{t+1})
\]

\[
- \frac{1}{2}(\pi^p)^2 \text{var}(\Delta p_{t+1}) + (\gamma - \pi^c) \pi^p \text{cov}(\Delta c_{t+1}, \Delta p_{t+1}) + \delta
\]

\[
+ \pi^p \text{var}(\Delta p_{t+1}) + (\pi^c - \gamma) \text{cov}(\Delta c_{t+1}, \Delta p_{t+1}),
\]

(7)

where \( \delta = -\phi - \frac{1}{2} \text{var}(e_{t+1}) - E_t \Delta \lambda_{t+1} \).

The terms in the first line of (7) are the expected rate of depreciation of money, the next three lines define the real rate of interest, and the last line represents the risk premium. Generally, these expressions differ from those implied by the CCAPM without taste shocks. For example, the risk premium depends on the variance of inflation. This is because, in an equilibrium where prices are correlated with taste shocks, \( \pi^p \neq 0 \). Hence, variations in \( \text{var}(\Delta p_{t+1}) \) indicate changes in \( \text{cov}(\Delta \lambda_{t+1}, \Delta p_{t+1}) \), which affect the size of the premium.

Equation (7) is a generalization of the Fisher equation that combines the implications of the representative consumer’s first-order conditions with inferences about the covariance structure of taste shocks, con-

\(^7\) If \( \text{cov}(\Delta \lambda_{t+1}, \Delta c_{t+1}) > 0 \) and \( \text{cov}(\Delta \lambda_{t+1}, \Delta p_{t+1}) < 0 \), as suggested above, then \( \pi^c > 0 \) and \( \pi^p < 0 \). Here, a positive innovation in consumption will signal a positive taste shock, and a positive innovation in inflation will signal a negative taste shock.
sumption growth, and inflation. The parameters to be estimated are $\gamma$, $\delta$, $\pi^c$, and $\pi^d$.

Data and Time-Series Model

In this section, we first describe the data and then develop the time-series models that are used to derive estimates of the conditional moments of inflation and consumption growth. We will examine the performance of our generalized Fisher equation using monthly and quarterly data, so time-series models are estimated for both frequencies.

Data Definitions

Interest rates. Our measure of the short-run nominal interest rate in the monthly models is the 1-month Treasury-bill return from the Center for Research in Security Prices (CRSP) data tape (see Ibbotson and Sinquefield 1982) for the period 1964–87. For the quarterly models, we use the return on (zero-coupon) 3-month Treasury bills from 1954 to 1987 prepared by McCulloch (1990).

Inflation. We examine two alternative measures of the inflation rate, one based on the consumer price index (CPI) for all urban consumers, the other based on the implicit price deflator (IPD) for personal consumption expenditures on nondurables and services. Inflation rates are calculated as the change in the natural log of the price level. The CPI series—a consistent series with the rental equivalence measure for owner-occupied housing—is described in the Data Appendix (App. A). Appendix A also describes a correction that was introduced in the monthly CPI data to make sure that the estimates of the time-series structure were not influenced by Nixon-era price controls.

Consumption growth. The measure of real consumption that we use is the Commerce Department’s seasonally adjusted data on real per capita personal consumption expenditure on nondurables and services. Consumption growth is defined as the change in the log of consumption.

Recent work by Breeden, Gibbons, and Litzenberger (1989) points out two econometric problems peculiar to the use of consumption and inflation data in asset pricing models. First, the data include some interpolations since not all of the data are sampled every month. The measurement errors introduced by interpolation should result in serially correlated errors in regressions where a series without interpolations is regressed on a series with interpolations. We estimate models with both monthly and quarterly data to insure that our conclusions are not being unduly affected by such measurement errors.

Second, because the reported consumption data for the month is the integral of instantaneous consumption during the month, summation
biases will affect estimates of the variance of consumption growth and the covariance of consumption growth and inflation that we will construct below. Furthermore, summation bias also induces serial correlation in regression residuals. To insure that the presence of summation bias does not influence our main results, we will test for its effects in our interest rate models.

**Time-Series Model for Inflation and Output**

Several methods have been used to obtain the explanatory variables—the conditional first and second moments of inflation and consumption growth—necessary for the generalized Fisher equation. Shome, Smith, and Pinkerton (1988), for example, use the Livingston survey data to calculate proxies for the relevant moments. Alternatively, time-series methods can be used to generate estimates of the relevant moments. In particular, following a popular approach, we could model inflation and consumption growth as a bivariate ARCH or GARCH (generalized ARCH) process. Elements from the estimated conditional covariance matrix together with the predictions would provide us with all the necessary explanatory variables.

Unfortunately, traditional ARCH models suffer from a potentially serious drawback: they assume that the time-series process being modeled is stable over the whole sample. When the process is subject to structural shifts, the estimated conditional moments from simple ARCH models are biased. For example, estimates of \( \text{var}_t(\Delta c_{t+1}) \) and \( \text{var}_t(\Delta p_{t+1}) \) derived from a simple ARCH model will be biased below the true conditional variances because they ignore the contributions of unanticipated structural changes in either process.

Earlier research by Ferson and Merrick (1987) found significant structural changes in the quarterly consumption process over stages of the business cycle, and Evans (1991) found significant structural change in the monthly inflation process. We discovered similar structural instability in the consumption and inflation processes at both monthly and quarterly frequencies. So, to avoid the difficulties mentioned above, we explicitly allow for changes in the persistence of both inflation and consumption growth. Our models have the following general form:

\[
X_t = A_{0,t} + \sum_{i=1}^{q} A_{i,t} X_{t-i} + \epsilon_t, \quad E_{t-1} \epsilon_t \epsilon'_t = \Sigma_t. \tag{8}
\]

8. Recall that the theoretical model calls for the rate of change from the beginning to the end of the month in the instantaneous consumption rate.

9. These survey data are available only twice a year. Furthermore, the data do not appear to be rational (see Figlewski and Wachtel 1981) and are probably subject to large measurement errors.

10. The Technical Appendix shows how the bias arises. Evans (1991) contains a more complete discussion of the potential drawbacks in modeling inflation as an ARCH process.
where $X_t = [\Delta p_t^e, \Delta c_t^e]'$, $\epsilon_t$ is a $2 \times 1$ vector of serially uncorrelated disturbances with conditional covariance matrix $\Sigma_t$ that follows an ARCH process, $A_{q, i}$ is a $2 \times 1$ matrix, and the $A_{i, i}$ are $2 \times 2$ matrices. This specification allows for change over time in both the coefficient matrices and the conditional covariance matrix.

Structural variations may reflect the effects of changes in private-sector behavior, economic policy, and/or institutions and are represented by variations in the coefficient matrices. We have no specific model for these changes. On empirical grounds, we model each time-varying element as a random walk. This choice appears to be supported by the diagnostic tests reported below.

There are two theoretical justifications for the variation in the conditional covariance matrix $\Sigma_t$. First, shocks to inflation represent combinations of structural disturbances such as productivity, money supply, and price shocks. Particular structural disturbances may be more likely to occur during some periods than others. For example, the variance of monetary shocks may rise during high inflation episodes (see Ball 1990).

A second reason is provided by the Lucas critique (1976). Since the inflation process represents the result of a large number of price-setting decisions, shocks to the aggregate price level must in part depend on how individual price setters respond to structural disturbances. For example, the pricing strategies of individual firms will determine the aggregate effects of a given nominal demand shock. Such pricing rules are subject to change, altering the short-run susceptibility of inflation to structural disturbances and, hence, the conditional variance of inflation. Changing pricing policies may also affect the short-run susceptibility of real output to structural shocks, which, as a result, could lead to heteroscedasticity in equilibrium consumption.

We assume that the innovations $\epsilon_t$ in (8) are jointly normally distributed. The model is estimated by maximum likelihood in conjunction with a modified version of the Kalman Filter. The estimation process is described in the Technical Appendix (App. B).

Estimates of Time-Series Models

We estimated time-series models with four data sets, using the monthly and quarterly data with the IPD and CPI inflation data. Our chosen specifications for each model along with the residual autocorrelations are summarized in table 1. These statistics reveal that our models are successful in capturing all the serial correlation in both the first and

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11. The analysis in Ball, Mankiw, and Romer (1988), Evans (1989), and others suggests that the frequency of individual price changes should rise as the economy moves toward regimes of higher inflation, so that the aggregate price level will respond more quickly to nominal shocks. Under these circumstances, a prolonged increase in the rate of inflation will induce a rise in the conditional variance of inflation.
<table>
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<th>Equation</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
<th>( \rho_3 )</th>
<th>( \rho_4 )</th>
<th>( \rho_5 )</th>
<th>( \rho_6 )</th>
<th>Description</th>
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</tbody>
</table>

Note.—Autocorrelations are denoted by \( \rho_i \), \( i = 1, 2, \ldots, 6 \), for \( \epsilon_1 \sigma_1 \) (first line) and for \( (\epsilon_i)^2 \) \( \sigma_i^2 \) (second line), where \( \epsilon_i \) and \( \sigma_i^2 \) are the estimated residuals and associated conditional variance from each of the model's equations. The estimated models are shown in Appendix tables C1–C4.

12. The maximum likelihood estimates of the models, structural stability, and other specification test results are found in Appendix tables C1–C4.

13. Our estimated models support the idea that both time-varying pa...

second moments of each series. The maximum likelihood estimates of the models, structural stability, and other specification test results are found in Appendix tables C1–C4.

Our estimated models support the idea that both time-varying pa...
Parameters and ARCH effects are present in the processes for inflation and consumption growth. The intercepts \( A_{o,t} \) identify the trends in inflation and consumption growth. For inflation, Dickey-Fuller tests revealed that both the series contain a unit root, so that the intercepts in the inflation equations follow random walks. With the monthly CPI data, it reaches a peak of 7% in 1980; with the monthly deflator, it reaches peaks between 10% and 11% in 1973 and 1980; and with the quarterly data, the peaks are even larger. Trend consumption growth, by contrast, appears to be constant and is estimated to be about 2% at annual rates.

The estimates of the autoregressive coefficients (the diagonal elements of \( A_{1,t} \)) show the changes over time in the degree of persistence of inflation.\(^4\) These changes imply that the proportion of the variation in inflation that is forecastable, \( \text{var}(F_{t} \Delta p_{t+1})/\text{var}(\Delta p_{t+1}) \), varies through time. Since the strength of the ex post Fisher effect (the relationship between nominal interest rates and realized inflation) depends on this ratio (Barsky 1987), our inflation models imply that the strength of the ex post Fisher effect has varied over the past 25 years.

**The Generalized Fisher Equation**

In this section, we examine the generalized Fisher equation with the four data sets on expectations and the variance-covariance structure estimated in the previous section. We begin by considering simple models with and without taste shocks in which all the parameters are taken to be constant. While the results reveal that taste shocks have a significant effect on nominal interest rates, they also indicate that their influence has varied considerably over the past 25 years; our estimates of \( \delta \), \( \pi^e \), and \( \pi^c \) appear unstable over the sample.

Theoretically speaking, the coefficients \( \pi^e \) and \( \pi^c \) will be constant only when there is no change in the signal-to-noise ratios among the innovations in consumption, inflation, and tastes.\(^5\) Since both consumption and inflation are conditionally heteroscedastic, it is likely that \( \pi^c \) and \( \pi^e \) are time varying because they depend on the variances of tastes and other shocks and the equilibrium dependence of \( c \) and \( p \) on \( \lambda \).\(^6\) We examine this possibility by estimating an extended version of equation (7) where \( \delta \), \( \pi^e \), and \( \pi^c \) are time varying.

\(^4\) The monthly CPI and IPD data do not show exactly the same pattern of persistence. This is probably because the cross correlations between the two series are quite low, around 0.5 for lags 1–6. With the quarterly data, the persistence appears to be the same with both series.

\(^5\) In addition, \( \delta = \phi - \frac{1}{2} \text{var}(e_{t+1}) - E_{t} \Delta \lambda_{t+1} \) will be constant in the presence of taste shocks if \( \lambda \) follows a random walk and \( \text{var}(e_{t+1}) \) is constant. If, however, the taste shocks are serially correlated so that \( E_{t} \Delta \lambda_{t+1} \) varies through time, so too will \( \delta \).

\(^6\) It is also possible that the general equilibrium dependence of \( p_{t} \) and \( c_{t} \) on taste shocks has changed through time.
Constant Coefficient Estimates

The models are estimated by the Generalized Method of Moments (GMM) with an instrumental variables technique proposed by Pagan and Ullah (1988) that accounts for measurement error in our estimates of the moments of inflation and consumption growth. To begin, note that our theoretical model, equation (7), is an exact relationship between the nominal interest rate and the moments of inflation and consumption growth. To obtain an estimable equation, we first replace all the true moments that are multiplied by unknown coefficients in the theoretical model by realized values. That is, $E_t \Delta c_{t+1}$ is replaced by $\Delta c_{t+1}$, $\text{var}(\Delta c_{t+1})$ by $(e_{t+1}^c)^2$, $\text{var}(\Delta p_{t+1})$ by $(e_{t+1}^p)^2$, and $\text{cov}(\Delta c_{t+1}, \Delta p_{t+1})$ by $e_{t+1}^c e_{t+1}^p$, where the $e$'s are the estimated innovations to consumption growth and inflation from the time-series models.\(^{12}\) Next, the true rate of expected inflation is replaced by the time-series estimate of $E_t \Delta p_{t+1}$.\(^{18}\) After these replacements, we can rewrite (7) as

$$i_t = E_t \Delta p_{t+1} + \left[ \pi^p - \frac{1}{2}(\pi^p)^2 - \frac{1}{2} \right] (e_{t+1}^p)^2 + \gamma \Delta c_{t+1} + 8 + \left[ \gamma \pi^c - \frac{1}{2} \gamma^2 - \frac{1}{2} (\pi^c)^2 \right] (e_{t+1}^c)^2 \quad (9)$$

$$+ \left[ \pi^c - \gamma \right] t(1 - \pi^p)(e_{t+1}^c e_{t+1}^p) + \omega_{t+1}.$$  

The error term $\omega_{t+1}$ is a linear combination of the differences between the true moments and their (estimated) realizations, and the difference between expected inflation and our time-series estimate. By construction, $\omega_{t+1}$ is orthogonal to variables known at time $t$.\(^{19}\) Thus, the parameters $\gamma$, $\delta$, $\pi^c$, and $\pi^p$ can be estimated by GMM using instruments that are known at $t$ and highly correlated with $\Delta c_{t+1}$, $(e_{t+1}^c)^2$, $(e_{t+1}^p)^2$, and $(e_{t+1}^c e_{t+1}^p)$. We use the second, third, and fourth lagged values of $E_t \Delta c_{t+1}$, $\text{var}(\Delta c_{t+1})$, $\text{var}(\Delta p_{t+1})$, and $\text{cov}(\Delta c_{t+1}, \Delta p_{t+1})$ from the time-series model and the ex post real rate, $i_t - \Delta p_{t+1}$, as instruments.\(^{20}\)

The constant coefficient estimates of the generalized Fisher equations are shown in table 2 for each of the four data sets. The first equation in each panel eliminates the taste shocks in utility, and the next three allow for taste shocks that are correlated with consumption growth and/or inflation.

---

17. Theoretically speaking, we should use the true rather than the estimated innovations. However, this substitution need not affect the limiting properties of the resulting parameter estimates. See proposition 3 in Pagan and Ullah (1988).

18. We also ran experiments where the expected inflation is replaced with ex post inflation. The results were similar to those we report.

19. To be precise, $\omega_{t+1}$ is orthogonal to all the $t$-dated variables that are used to estimate the right-hand-side variables in (9).

20. The first lagged values are omitted to reduce the problems caused by summation bias. The use of "generated instruments" also causes some potential problems which are discussed in the Technical Appendix. On the basis of numerous experiments, we believe that our results are robust to the exact choice of instruments.
TABLE 2  Estimates of Generalized Fisher Models

<table>
<thead>
<tr>
<th>Monthly data, inflation-CPI:</th>
<th>Monthly data, inflation-deflator:</th>
<th>Quarterly data, inflation-CPI:</th>
<th>Quarterly data, inflation-deflator:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8(10^{-2})$ $\gamma$ $\pi^c$ $\pi^p$ SEE($10^{-2}$) $\rho_1$ $\rho_2$ $\rho_3$ $\chi^2_{(100)}$ $\chi^2_{(100)}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.047* (.021) .483* .766* .352 .105 .213 .177 36.22 19.62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.171* (.032) .515* -7.673* .458 .078 .161 .070 28.56 15.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.068 (.128) (2.419)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.215* (.080) .431* -27.545* .449 .229 .071 .127 27.81 15.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7.570)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.257* (.105) .428* -4.68 .292 .280* .486 .235 .053 .109 24.34 12.53</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.285) (8.465)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.030 (.019) .362* .282 .376 .441 .376 62.07 32.73</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.058 (.019) .297* .655 .4595 .295 .347 .408 .322 44.57 22.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(.055) (.116) (2.890)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.173* (.061) .284* .5171* .344 .240 .325 .207 33.71 21.61</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6.215)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.463* (.145) .601 11.921 .29988* .563 .223 .148 .024 28.95 8.68</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.105) (8.906)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note.—All models are estimated by GMM using three lags of $E_{t-1}S_{t-1}$, var($E_{t-1}S_{t-1}$), cov($E_{t-1}S_{t-1}, E_{t-1}S_{t-1}$) and the expected real rate, $r_{t} = \Delta P_{t}$. In all cases, the first lag is excluded. All standard errors (reported below the parameter estimates) allow for heteroscedasticity and correct for MA(2) serially correlated errors using the Newey-West procedure. $\rho_1$, $\rho_2$, and $\rho_3$ are the first three autocorrelations of the residuals. $\chi^2_{(100)}$ denotes Hansen’s test of the overidentifying restrictions. $\chi^2_{(100)}$ is a joint stability test on $\gamma$, $\pi^c$, and $\pi^p$ (if present) over the period 1979:3–1982:4. Both tests allow for heteroscedasticity and serial correlation.

* Significantly different from zero at the 5% level

The estimates of the coefficient of relative risk aversion, $\gamma$, are always less than one and almost always significantly different from zero at the 5% level.\textsuperscript{21} The estimates of $\pi^p$ and $\pi^c$ suggest that in some cases taste shocks do affect interest rates. In particular, the significant

\textsuperscript{21} The standard errors reported in Table 2 allow for conditional heteroscedasticity and serial correlation (up to an MA(2) process) using the White-Newey-West procedure.
estimates for \( \pi^p \) imply that the variance of inflation contributes significantly to the risk premium in the monthly models.

The right-hand columns of the table indicate how the models perform statistically. The autocorrelations, \( p_c \), indicate the presence of a small amount of serial correlation in the residuals for the monthly models and a large amount for the quarterly models. The serial correlation does not decline when we allow for the effects of taste shocks. Similar serial correlation patterns have been reported in other studies (i.e., Singleton 1990).\(^{21}\) Tests of the overidentifying restrictions (Hansen and Singleton 1982) are also reported in the table. The statistics, \( \chi^2_{(\text{ideal})} \), are highly significant in every case.

To test for instability in \( \delta, \pi^c, \) and \( \pi^p \), we reestimated all the models in table 2 allowing these three coefficients to take different values during a subperiod that coincides with the Federal Reserve’s experiment with monetary aggregates targeting. Studies by Clarida and Friedman (1984), Huizinga and Mishkin (1986), and others indicate that the behavior of interest rates changed during this period. The test statistics, \( \chi^2_{(\text{tab})} \), in the last column of table 2 indicated significant parameter instability (at the 5% level) in all of the models.\(^{23}\)

The results in table 2 suggest that the dynamics of taste shocks differ from those assumed in our constant coefficient estimates. Specifically, the coefficient instability and residual serial correlation indicate that we should examine models with time-varying estimates of the parameters \( \delta, \pi^c, \) and \( \pi^p \).

**Time-varying Coefficient Estimates**

In order to examine models where the dynamics are left unrestricted, we reformulate our theoretical equation (7) to allow the coefficients \( \delta, \pi^c, \) and \( \pi^p \) to vary through time:

\[
i_t = E_t \Delta \pi_{t+1} + \left[ \pi^p_t - \frac{1}{2} (\pi^p_t)^2 - \frac{1}{2} \psi \right] \text{var}_t(\Delta \pi_{t+1}) \\
+ \gamma E_t \Delta c_{t+1} + \delta_t + \left[ \gamma \pi^c_t - \frac{1}{2} \gamma^2 - \frac{1}{2} (\pi^c_t)^2 \right] \text{var}_t(\Delta c_{t+1}) (7') \\
+ \left[ \pi^p_t - \gamma \right] [1 - \pi^p_t] \text{cov}_t(\Delta c_{t+1}, \Delta \pi_{t+1}).
\]

Here, changes in \( \delta_t \) reflect variations in \( E_t \Delta \lambda_{t+1} \) and \( \text{var}_t(\epsilon_{t+1}) \). Changes in the conditional regression coefficients of tastes shocks on inflation and consumption are reflected by variations in \( \pi^c_t \) and \( \pi^p_t \).

---

22. Some of the serial correlation appears related to the fact that \( i_t - E_t \Delta \rho_{t+1} + \frac{1}{2} \text{var}_t(\Delta \rho_{t+1}) \) is a highly persistent process, while the largest autocorrelations in \( E_t \Delta \rho_{t+1}, \) \( \text{var}_t(\Delta \rho_{t+1}), \) and \( \text{cov}_t(\Delta \rho_{t+1}, \Delta \rho_{t+1}) \)—the variables determining the real rate and the risk premium—are approximately \( \pm 0.2 \).

23. The test is a Wald test for the equality of the parameter estimates across subperiods. The test statistic allows for heteroscedasticity and an MA(2) process in the residuals. Although we only report joint tests in table 2, we also reject stability of \( \sigma^p \) and \( \delta \) individually in the monthly models.
Equation (7') is estimated with a Kalman Filter using the moments of consumption and inflation from the time-series model. This procedure assumes that the estimated moments from the time-series model contain no measurement errors. While this contrasts with our earlier assumptions, the diagnostic tests described below do not lead us to believe that the estimation method suffers from a serious errors-in-variables problem. Nonetheless, it is important to account for the generated data when conducting inference. The standard errors reported in the tables are therefore calculated using estimates from the complete model: a model that combines the inflation, consumption, and interest rate equations and imposes the full set of cross-equation restrictions.

Our chosen specifications for $\delta$, $\pi_r$, and $\pi_t$, together with the maximum likelihood estimates, are shown in tables 3–6. In the monthly models, the estimates of $\gamma$ are small and significant. There is also significant evidence of variations in $\delta$, $\pi_r$, and $\pi_t$. The specifications for $\delta$, $\pi_r$, and $\pi_t$ were chosen to minimize the serial correlation in the innovations to interest rates. Autocorrelations and tests for errors in variables are at the bottom of each panel. To test for errors-in-variables bias, we conducted the “omitted variables” form of the Hausman test as described in Godfrey (1988); see the Technical Appendix for details. All the test statistics are too small to indicate a significant errors-in-variables bias.

The results from the quarterly models are quite different. When $\gamma$ was estimated freely, we repeatedly obtained negative estimates, which are outside the concave region of the parameter space. Therefore, tables 3–6 report the maximum likelihood estimates where $\gamma$ is constrained above zero. The likelihood functions find a maximum with $\gamma$ extremely close to zero. Unlike the monthly models, we were unable to find any evidence that $\pi_r$ and $\pi_t$ are different from zero. By contrast, $\delta$ is highly variable.

In the Technical Appendix, we show that the Kalman Filter can be viewed as a rolling regression technique. This means that the estimates of $\pi_r$ and $\pi_t$ can be interpreted as coefficients from a (nonlinear) regression of $i_t$ on the moments $\text{var}(\Delta p_{t+1})$, $\text{var}(\Delta c_{t+1})$, $\text{cov}(\Delta p_{t+1}, \Delta c_{t+1})$, given other variables, using data up to $t = T$. We can therefore interpret the results in tables 3–6 in terms of the multiple correlations in the data up to $t = T$ between $i_t$ and these moments.

24. The errors-in-variables problem arises in this context if the information set used by investors to forecast inflation and consumption growth differs markedly from that spanned by past inflation and consumption growth used in our time-series models. It may also arise if our estimates of the consumption and inflation moments are subject to significant summation bias.

25. Dunn and Singleton (1986), among others, found similar results when estimating CCAPM models for the term structure.
TABLE 3  Varying Coefficient Estimates of the Generalized Fisher Equation (Monthly Data, Inflation-CPI)

\[ i_t = E_t \Delta p_{t+1} + \gamma (\pi_t^{r})^2 - \frac{1}{2} \text{var}(\Delta p_{t+1}) \]
\[ + \gamma E_t \Delta c_{t+1} + \delta + [\gamma \pi_t^{r} - \frac{1}{2} \gamma^2 - \frac{1}{2} \gamma (\pi_t^{r})^2] \text{cov}(\Delta c_{t+1}, \Delta p_{t+1}) \]
\[ + [\pi_t^{r} - \gamma] [1 - \pi_t^{r}] \text{cov}(\Delta c_{t+1}, \Delta p_{t+1}) \]

A. Model Estimates

<table>
<thead>
<tr>
<th>( \gamma = .132 )</th>
<th>( .024 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_t = \delta_{t-1} + u_t^\delta )</td>
<td>( \text{var}(u_t^\delta) = .440^2 )</td>
</tr>
<tr>
<td>( \pi_t^\delta = .350 \pi_{t-1}^\delta + u_t^\delta + .512 u_{t-1}^\delta )</td>
<td>( \text{var}(u_t^\delta) = 5.396^2 + 9.122^2 (u_t^\delta)^2 )</td>
</tr>
<tr>
<td>( \pi_t^\gamma = \pi_{t-1}^\gamma + u_t^\gamma - .100 u_{t-1}^\gamma )</td>
<td>( \text{var}(u_t^\gamma) = 4.247^2 + .854^2 (u_t^\gamma)^2 )</td>
</tr>
</tbody>
</table>

B. Test Statistics

<table>
<thead>
<tr>
<th>( \eta / h_t )</th>
<th>( \eta / \eta_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_1 )</td>
<td>( -.059 )</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>( -.012 )</td>
</tr>
<tr>
<td>( \rho_3 )</td>
<td>( -.021 )</td>
</tr>
<tr>
<td>( \rho_4 )</td>
<td>( .017 )</td>
</tr>
<tr>
<td>( \rho_5 )</td>
<td>( -.046 )</td>
</tr>
<tr>
<td>( \rho_6 )</td>
<td>( .116 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hausman tests</th>
<th>( .033 )</th>
<th>( 1.941 )</th>
<th>( 2.765 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( .855 )</td>
<td>( .164 )</td>
<td>( .906 )</td>
<td></td>
</tr>
</tbody>
</table>

Notes.—The standard errors, reported in parentheses, are corrected for the presence of “generated regressors” (i.e., \( E_t \Delta p_{t+1} \), \( E_t \Delta c_{t+1} \), \( \text{var}(\Delta p_{t+1}) \), \( \text{cov}(\Delta c_{t+1}, \Delta p_{t+1}) \), and \( \text{var}(\Delta c_{t+1}) \)) using the procedure described in the Technical Appendix. Autocorrelations are denoted by \( \rho_i \), \( i = 1, 2, \ldots, 6 \), for \( \eta / h_t \) (first line) and for \( \eta / \eta_t \) (second line), where \( \eta_t \) and \( h_t \) are the estimated innovations to interest rates and associated conditional variance. Column 1 = the Hausman test for the effects of measurement errors in \( \gamma E_t \Delta c_{t+1} \) = \( \gamma \text{var}(\Delta c_{t+1}) \) = \( \gamma \text{cov}(\Delta p_{t+1}, \Delta c_{t+1}) \); col. 2 = the Hausman test for the effects of measurement errors in \( \text{var}(\Delta p_{t+1}) + \text{cov}(\Delta p_{t+1}, \Delta c_{t+1}) \); col. 3 = the Hausman test for the effects of measurement errors in \( \gamma \text{var}(\Delta c_{t+1}) + \text{cov}(\Delta p_{t+1}, \Delta c_{t+1}) \). Significance levels are reported below the test statistics.

We can attribute the differences between the monthly and quarterly estimates of \( \pi^r \) and \( \pi^\gamma \) to the differences in these multiple correlations. The correlations vary significantly in the monthly models so that the estimates of \( \pi^r \) and \( \pi^\gamma \) change through time. In the quarterly models, the correlations are always very close to zero. This is consistent with the constant coefficient estimates in table 2.

Overall, the estimates in tables 3–6 show that monthly and quarterly interest rates are driven by different factors. Because our estimate of \( \gamma \) is positive in the monthly models, interest rates respond to consumption growth and the risk premium. In the quarterly models, we find that \( \gamma \) is very close to zero, so that neither factor is significant. This
TABLE 4  Varying Coefficient Estimates of the Generalized Fisher Equation  
(Monthly Data, Inflation-Deflator)  

\[ i_t = E\Delta p_{t+1} + [\bar{\pi}_t - \bar{\pi}(\pi_t^2) - \frac{1}{2}] \text{var}(\Delta p_{t+1}) \]
\[ + \gamma E\Delta c_{t+1} + \delta_t + [\gamma \bar{\pi}_t - 1/2 \gamma^2 - 1/2(\pi_t^2)]\text{var}(\Delta c_{t+1}) \]
\[ + \{\pi_t^2 - \gamma(1 - \pi_t)\text{cov}(\Delta c_{t+1}, \Delta p_{t+1})\} \]

A. Model Estimates  
\[ \gamma = .098 \]
\[ (.028) \]
\[ \delta_t = \delta_{t-1} + \omega_t^\delta - .341 \omega_{t-1}^\delta \]
\[ (.106) \]
\[ \text{var}(\omega^\delta_{t+1}) = .290^2 + .781^2(\omega^\delta_t)^2 \]
\[ (.091) \]
\[ (.096) \]
\[ \pi_t^\delta = -.644 \pi_{t-1} + \omega_t^\delta \]
\[ (.089) \]
\[ \text{var}(\omega^\delta_{t+1}) = 1.032^2 \]
\[ (.798) \]
\[ \pi_t^\gamma = .820 \pi_{t-1} + \omega_t^\gamma \]
\[ (.145) \]
\[ \text{var}(\omega^\gamma_{t+1}) = 4.274^2 \]
\[ (.823) \]

B. Test Statistics  

<table>
<thead>
<tr>
<th>Autocorrelations</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
<th>( \rho_3 )</th>
<th>( \rho_4 )</th>
<th>( \rho_5 )</th>
<th>( \rho_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_{1/1} )</td>
<td>.044</td>
<td>-.066</td>
<td>-.050</td>
<td>-.094</td>
<td>-.057</td>
<td>.030</td>
</tr>
<tr>
<td>( \bar{\eta}_{1/2} )</td>
<td>.165</td>
<td>-.039</td>
<td>-.067</td>
<td>-.078</td>
<td>-.047</td>
<td>.084</td>
</tr>
<tr>
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<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hausman tests  
\[ .375 \]
\[ (.540) \]
\[ .088 \]
\[ (.766) \]
\[ .781 \]
\[ (.377) \]

Note — The standard errors, reported in parentheses, are corrected for the presence of "generated regressors" (i.e., \( E\Delta p_{t+1}, E\Delta c_{t+1}, \text{var}(\Delta p_{t+1}), \text{cov}(\Delta c_{t+1}, \Delta p_{t+1}) \), and \( \text{var}(\Delta c_{t+1}) \) using the procedure described in the Technical Appendix. Autocorrelations are denoted by \( \rho_i \), \( i = 1, 2, \ldots, 6 \), for \( \eta_{1/1} \) (first line) and for \( \eta_{1/2}, \bar{\eta}_{1/2} \) (second line), where \( \eta_t \) and \( \bar{\eta}_t \) are the estimated innovations to interest rates and associated conditional variance. Column 1 = the Hausman test for the effects of measurement errors in \( \gamma E\Delta c_{t+1} \), \( \bar{\eta}_{1/2} \text{var}(\Delta c_{t+1}) \), \( \gamma \text{cov}(\Delta c_{t+1}, \Delta p_{t+1}) \); col. 2 = the Hausman test for the effects of measurement errors in \( \bar{\eta}_{1/2} \text{var}(\Delta c_{t+1}) \), \( \gamma \text{cov}(\Delta c_{t+1}, \Delta p_{t+1}) \); col. 3 = the Hausman test for the effects of measurement errors in \( \gamma \text{cov}(\Delta c_{t+1}, \Delta p_{t+1}) \). Significance levels are reported below the test statistics.  

The difference has important consequences for interpreting the behavior of interest rates at monthly and quarterly frequencies.

Interpreting Interest Rate Movements  
In this section, we will use the model estimates in tables 3–6 to examine some of the stylized facts about interest rates and to decompose movements in the nominal interest rate into its components. Our model indicates that the nominal interest rate can be decomposed into terms that relate to the expected price change, the risk premium and the real
TABLE 5  Varying Coefficient Estimates of the Generalized Fisher Equation (Quarterly Data, Inflation-CPI)

\[
i_t = E_t \Delta p_{t+1} + \left[ \pi_t^e - \frac{1}{2}(\pi_t^e)^2 - \frac{1}{2} \right] \text{var}(\Delta p_{t+1})
+ \gamma E_t \Delta c_{t+1} + \delta_t + \left[ \gamma \pi_t^e - \frac{1}{2} \gamma^2 - \frac{1}{2}(\pi_t^e)^2 \right] \text{var}(\Delta c_{t+1})
+ \left( \pi_t^e - \delta \right)(1 - \pi_t^e) \text{cov}(\Delta c_{t+1}, \Delta p_{t+1})
\]

A. Model Estimates

\[
\begin{align*}
\gamma &= 0, \quad \pi_t^e = 0, \quad \pi_t^e = 0 \\
\delta_t &= 1.501 + .930 \delta_t + .599 u_t^b, \\
& (0.603) (0.977) (0.996) \\
\text{var}(u_t^b) &= .929 \sigma_t^2 + .717 \sigma_t^2 \\
& (0.030) (0.152)
\end{align*}
\]

B. Test Statistics

<table>
<thead>
<tr>
<th></th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\rho_3$</th>
<th>$\rho_4$</th>
<th>$\rho_5$</th>
<th>$\rho_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_t^e/\delta_t$</td>
<td>.105</td>
<td>-.113</td>
<td>.048</td>
<td>.029</td>
<td>-.027</td>
<td>.038</td>
</tr>
<tr>
<td>$\sigma_t^2/\delta_t$</td>
<td>.263</td>
<td>-.055</td>
<td>.020</td>
<td>-.031</td>
<td>.039</td>
<td>.134</td>
</tr>
</tbody>
</table>

Note.—The standard errors, reported in parentheses, are corrected for the presence of "generated regressors" (i.e., $E_t \Delta p_{t+1}, E_t \Delta c_{t+1}, \text{var}(\Delta p_{t+1}), \text{cov}(\Delta c_{t+1}, \Delta p_{t+1}),$ and $\text{var}(\Delta c_{t+1})$) using the procedure described in the Technical Appendix. Autocorrelations are denoted by $\rho_i$, $i = 1, 2, \ldots, 6$, for $\pi_t^e$ (first line) and for $\sigma_t^2/\delta_t$ (second line), where $\pi_t^e$ and $\delta_t$ are the estimated innovations to interest rates and associated conditional variance.

rate of interest. In terms of the variables and parameters of the model structure, we can write:

\[
\begin{align*}
\text{expected inflation} &= E_t \Delta p_{t+1}, \\
\text{expected rate of depreciation of money} &= -E_t \Delta p_{t+1} + \frac{1}{2} \text{var}(\Delta p_{t+1}), \\
\text{risk premium} &= \pi_t^e \text{var}(\Delta p_{t+1}) \\
& + \left[ \pi_t^e - \gamma \right] \text{cov}(\Delta p_{t+1}, \Delta c_{t+1}), \text{ and}
\end{align*}
\]

\[
\begin{align*}
\text{real rate} &= \delta_t + \gamma E_t \Delta c_{t+1} \\
& + \left[ \gamma \pi_t^e - \frac{1}{2} \gamma^2 - \frac{1}{2}(\pi_t^e)^2 \right] \text{var}(\Delta c_{t+1}) \\
& + \left( \gamma - \pi_t^e \right) \pi_t^e \text{cov}(\Delta p_{t+1}, \Delta c_{t+1}) \\
& - \frac{1}{2}(\pi_t^e)^2 \text{var}(\Delta p_{t+1}).
\end{align*}
\]

The asset pricing model indicates that the nominal rate should depend on the expected rate of depreciation of money, while most popu-
TABLE 6  Varying Coefficient Estimates of the Generalized Fisher Equation (Quarterly Data, Inflation Deflator)

\[ i_t = E\Delta \rho_{t+1} + [u_t^2 - \frac{1}{2} \sigma_t^2 - \frac{1}{2} \text{var}(\Delta \rho_{t+1})] \]
\[ + \gamma E\Delta c_{t+1} + \delta_t + [\gamma \pi_t^c - \frac{1}{2} \gamma^2 - \frac{1}{2} \text{var}(\Delta c_{t+1})] \]
\[ + [\pi_t^c - \gamma] \text{var}(\Delta c_{t+1}, \Delta \rho_{t+1}) \]

A. Model Estimates

\[ \gamma = 0, \quad \pi_t^c = 0, \quad \pi_t^\rho = 0 \]
\[ \delta_t = .744 + .897 \delta_{t-1} + u_t^\delta - .025 u_{t-1}^\delta \]
\[ (.083) \quad (.080) \quad (.109) \]
\[ \text{var}(u_{t+1}^\delta) = .925^2 + .712(u_t^\delta)^2 \]
\[ (.944) \quad (.042) \]

B. Test Statistics

<table>
<thead>
<tr>
<th>Autocorrelations</th>
<th>(\hat{\rho}_1)</th>
<th>(\hat{\rho}_2)</th>
<th>(\hat{\rho}_3)</th>
<th>(\hat{\rho}_4)</th>
<th>(\hat{\rho}_5)</th>
<th>(\hat{\rho}_6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelations</td>
<td>-.106</td>
<td>-.190</td>
<td>.057</td>
<td>.003</td>
<td>-.043</td>
<td>.134</td>
</tr>
<tr>
<td></td>
<td>.387</td>
<td>.049</td>
<td>-.009</td>
<td>-.003</td>
<td>-.018</td>
<td>-.011</td>
</tr>
</tbody>
</table>

Note.—The standard errors, reported in parentheses, are corrected for the presence of "generated regressors" (i.e., \(E\Delta \rho_{t+1}, E\Delta c_{t+1}, \text{var}(\Delta \rho_{t+1}), \text{cov}(\Delta c_{t+1}, \Delta \rho_{t+1})\), and \(\text{var}(\Delta c_{t+1})\)) using the procedure described in the Technical Appendix. Autocorrelations are denoted by \(\hat{\rho}_i, i = 1, 2, \ldots, 6\), for \(\eta_i/\hat{h}_i^2\) (first line) and \((\eta_i/\hat{h}_i^2\) (second line), where \(\eta_i\) and \(\hat{h}_i^2\) are the estimated innovations to interest rates and associated variance.

lar specifications of Fisher relationships relate interest rates to the expected rate of inflation. However, the differences between the two series are always very small in our data; they can be used interchangeably.

The definitions above make clear that the estimated variability of \(\pi_t^c\) and \(\pi_t^\rho\) in the monthly models contribute to the movements in the real rate and the risk premium. To calculate the contributions of \(\pi_t^c\) and \(\pi_t^\rho\), we regressed the estimates of the change in the real rate and the risk premium obtained from the monthly models on a constant and the estimates of \(\Delta \pi_t^c\) and \(\Delta \pi_t^\rho\). The \(R^2\)'s from these regressions measure how much of the variation in the real rate and risk premium can be attributed to movements in \(\pi_t^c\) and \(\pi_t^\rho\) independent of other factors. In the model using the CPI inflation data, we find that \(\pi_t^c\) and \(\pi_t^\rho\) contribute 84% to the variability in the real rate and 83% to the variability of the risk premium, and the results with the deflator are similar.26

Figure 1 shows that in the monthly data expected depreciation of

26. For comparison, we also ran regressions using the first differences of \(\text{var}(\Delta \rho_{t+1})\), \(\text{cov}(\Delta \rho_{t+1}, \Delta c_{t+1})\), and \(\text{var}(\Delta c_{t+1})\) as explanatory variables. The \(R^2\)'s from these regressions are 0.05 and 0.13.
money and the nominal interest rate generally move together. However, there are numerous short-term fluctuations in the expected depreciation that are not matched by changes in interest rates. Figure 1 also illustrates a recent puzzle in the behavior of nominal interest rates: their failure to fall during the disinflation of the early 1980s. Discussions of this puzzle often suggest that the change in the Federal Reserve's operating procedure in late 1979 led to an increase in the risk premium in interest rates (see, e.g., Bodie, Kane, and McDonald 1984; and Mankiw 1986). However, some of the largest gaps between nominal interest rates and expected depreciation appear in 1983–85, when the Fed's flirtation with the targeting of monetary aggregates was at an end.

Figure 2 shows the 1-month nominal and real interest rates calculated from the CPI inflation model. During the late 1960s, the real rate was stable; in the 1970s, it became negative and variable; and in the 1980s, it was high and variable (see table 7). The results are similar with the deflator.27

Our interpretation of the movements in nominal interest rates is summarized in table 8, which reports the changes in annual averages

27. In the quarterly model for the 1-month rate (using the deflator), the mean real rate was $-0.10\%$ between 1955 and 1970, $-1.35\%$ between 1970 and 1980, and $2.56\%$ after 1980. The associated standard deviations were $0.89\%$, $1.42\%$, and $2.52\%$. 
of the nominal rate and its components over various periods. The decompositions are shown with the monthly model that uses the CPI data and the quarterly model that uses the deflator. There are differences between the two panels because they reflect changes in the term structure of interest rates as well as overall changes in the level of interest rates. Generally speaking, there are large movements in both the real rate and expected inflation components of nominal interest rates and smaller changes in the risk premium. The risk premium in the quarterly model does not change; it is always zero because the estimates of $\gamma$, $\pi^e$, and $\pi^c$ are zero.

The table shows that the run-up in 1- and 3-month nominal interest

28. The differences between these results and the decompositions with the other inflation data of the same frequency are inconsequential.
<table>
<thead>
<tr>
<th>TABLE 8</th>
<th>Decomposition of Interest Rate Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nominal Rate</td>
</tr>
<tr>
<td>1-month rate with CPI:</td>
<td></td>
</tr>
<tr>
<td>January 1965 to 1969</td>
<td>2.48</td>
</tr>
<tr>
<td>1969-72</td>
<td>-2.46</td>
</tr>
<tr>
<td>1972-78</td>
<td>-2.81</td>
</tr>
<tr>
<td>1978-81</td>
<td>-7.01</td>
</tr>
<tr>
<td>1981-86</td>
<td>-7.86</td>
</tr>
<tr>
<td>1986-87</td>
<td>-1.66</td>
</tr>
<tr>
<td>3-month rate (quarterly) with deflator:</td>
<td></td>
</tr>
<tr>
<td>1955:3-1969</td>
<td>5.45</td>
</tr>
<tr>
<td>1969-72</td>
<td>-1.98</td>
</tr>
<tr>
<td>1972-78</td>
<td>.69</td>
</tr>
<tr>
<td>1978-81</td>
<td>6.22</td>
</tr>
<tr>
<td>1981-86</td>
<td>-4.13</td>
</tr>
<tr>
<td>1986-87</td>
<td>-1.92</td>
</tr>
</tbody>
</table>

**Note.**—Entries are the changes between the annual averages for the years indicated of the nominal interest rate and its components calculated from the estimates in tables 3 and 8. See the text for the definition of each component. 1955:3 = the third quarter of 1955.

Rates from the start of each data set to 1969 was largely due to increases in the expected inflation rates. The ensuing fall in rates, which brought the interest rate in 1972 back to its 1965 level, was due to a large decline in the real rate. The expansion of the mid-1970s (1972-78) was marked by a large increase in the expected inflation rate. The large increase in the nominal interest rates between 1978 and 1981 was due to a very large increase in the 1-month real rate and, for the 3-month rate, large increases in both the real rate and the expected inflation rate.

Nominal interest rates declined during the disinflation of the 1980s. For the 1-month rate, the decline was due at first to a fall in the expected inflation rate and then to a decline in the real rate of interest. For the 3-month rate, the real rate component increased from 1981 to 1986. It was only in 1987 that the real rate returned to a level that might be viewed as closer to historical norms.

It is also informative to look at the correlations between nominal rates, real rates, expected inflation, and the risk premium. These are shown in table 9 with the monthly CPI data set and the quarterly data set with the deflator. The left-hand column shows that changes in the real rate have a negative correlation with changes in expected inflation. The negative correlation is consistent with the earlier results of Hui-zinga and Mishkin (1984). The correlations of expected inflation and the nominal interest rates are a simple and informative way of looking at the strength of the Fisher effect. The second column in table 9 shows that the correlation of the changes is very weak with high frequency
TABLE 9  Correlations of Changes

<table>
<thead>
<tr>
<th></th>
<th>Real Rate and Expected Inflation</th>
<th>Nominal Rate and Expected Inflation</th>
<th>Real Rate</th>
<th>Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-month rates with CPI:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>January 1965 to December 1987</td>
<td>-.765</td>
<td>-.047</td>
<td>.679</td>
<td>.571</td>
</tr>
<tr>
<td>January 1965 to September 1979</td>
<td>-.831</td>
<td>.048</td>
<td>.516</td>
<td>.350</td>
</tr>
<tr>
<td>October 1979 to October 1982</td>
<td>-.725</td>
<td>-.192</td>
<td>.815</td>
<td>.761</td>
</tr>
<tr>
<td>November 1982 to December 1987</td>
<td>-.775</td>
<td>.058</td>
<td>.586</td>
<td>.449</td>
</tr>
<tr>
<td>Annual averages, 1965–87</td>
<td>-.511</td>
<td>.638</td>
<td>.336</td>
<td>.024</td>
</tr>
</tbody>
</table>

|                      |                                  |                                    |           |              |
| 3-month rates (quarterly) with deflator: |                                |                                    |           |              |
| 1955:3–1967:4        | -.477                           | .079                              | .837      | .00          |
| 1955:3–1979:3        | -.711                           | .063                              | .654      | .00          |
| 1979:4–1982:3        | -.381                           | -.103                             | .959      | .00          |
| 1983:4–1987:4        | -.201                           | .322                              | .863      | .00          |
| Annual averages, 1956–87 | -.476                           | .514                              | .506      | .00          |

Note: Entries are sample correlations of the changes in the components of nominal interest calculated from the estimates in tables 3 and 6. See the text for the definition of each component. 1955:3 = the third quarter of 1955.

(monthly or quarterly) data. A different picture emerges when we look at changes between annual averages; in the long run, there is support for the existence of the Fisher effect. Table 9 also reveals one further stylized fact: short-run movements in nominal interest rates are more closely related to real rate variation than to variation in expected inflation. Short-run movements in 1-month nominal rates are also highly correlated with the risk premium.

These results have at least one fairly striking policy implication. High frequency movements in nominal interest rates are more likely to be a signal of change in the real rate (or risk premium) than in the expected inflation rate. Thus, policymakers who interpret a monthly increase in the nominal rate as an indication of rising inflationary expectations and advocate a policy tightening could well be mistaken. A monthly rise in the nominal interest rate is probably an indication of a rise in the real rate, suggesting the need for more liquidity if a recession is to be avoided. Of course, sustained changes in nominal rates are likely to be indicative of higher inflationary expectations.

Conclusions

Modern asset pricing theory shows that the simple Fisher relationship between interest rates and expected inflation is only one part of the equilibrium relationship that provides a structural model for interest rates. That simple model omits an appropriate specification for the real rate, the risk premium, and the expected rate of depreciation of money.
We estimate a generalized model that allows for taste shocks to utility and provides us with new insights into the behavior of nominal interest rates. Our estimates link the structural modeling of interest rate equilibrium to an understanding of the observed movements in interest rates in recent years. Our results indicate that there is little correlation between changes in nominal interest rates and expected inflation with quarterly or monthly data. That is, the high frequency data provide little support for the so-called Fisher effect.

Appendix A
Data Appendix

Since 1983, the CPI utilizes a rental equivalence calculation for the owner-occupied housing component that eliminates the undue weighting of mortgage interest costs, which was a source of severe criticism of the CPI in the 1970s. However, the Bureau of Labor Statistics (BLS) does not revise the historical data when revisions to the CPI are introduced. We use a consistent series for the whole period that uses the rental equivalence calculation. For the period 1967–83, the CPI based on the rental equivalence measure (called CPIX) is available from the BLS. The Congressional Budget Office has worked that series back, and their data for 1964–67 were obtained from Frederic Mishkin.

For estimation of the monthly time-series model, the CPI inflation data were corrected for the influence of the Nixon price controls. The correction is made so that the period of controlled prices would not influence the estimates of the time-series structure of inflation.

The inflation series was regressed (without a constant term) on a dummy variable that varies from zero to one based on the proportion of the CPI that was covered by price controls. The proportions were estimated by Blinder (1979, p. 125) and were nonzero for 33 months from 1971 to 1974. The residuals from this regression are inflation purged of the influence of the controls. For the estimation of the interest rate equations, the effects of the price controls were added back in to derive the appropriate series for expected inflation. This was done by adding the controls effect on $\Delta p_t$ (i.e., the predicted values from the dummy variable regression for $\Delta p_t$) to the predicted value of $\Delta p_t$ from the time-series model.

For the quarterly models, the CPI inflation rate is derived from the level of the series in the last month of the quarter, and no correction is made for the influence of the price controls.

The use of dummy variables does not appear to affect our main results. The correlation of our monthly expected inflation rate with estimates from a time-series model without the dummies is 0.96. Furthermore, when we estimated the time-varying parameter model using data without dummies and calculated the interest rate decompositions, we obtained very similar results to those in tables 8 and 9.
Appendix B

Technical Appendix

This appendix describes the algorithm used to estimate both the consumption-inflation model and the time-varying version of our generalized Fisher equation. The filtering results presented below are discussed in Chow (1983, ch. 10). The appendix also discusses the use of "generated instruments" to produce the results in table 2 and describes the Hausman tests used to test for errors-in-variables bias in tables 3–6.

Time-varying Parameter Models

The time-varying parameter models (both the models for consumption growth and inflation and the interest rate models) can generally be written as

\[ y_t = x_t f(B_t) + \epsilon_t, \quad E_{t-1} [\epsilon_t \epsilon_t'] = R_t, \]

and

\[ B_t = \Gamma B_{t-1} + C U_t, \quad E_{t-1} [U_t U_t'] = Q_t, \]

where \( \Gamma \) is \( k \times k \), \( C \) is \( k \times \ell_t \), \( Q \) is \( \ell_t \times \ell_t \), and \( B_t \) is the \( k \times 1 \) vector of time-varying parameters with covariance matrix \( C_t Q_t C_t' \).

The time-series models for consumption growth and inflation are linear in the time-varying parameters, so \( f(B_t) \) simplifies to \( B_t \). The model shown in Appendix tables C1 and C2 is given by

\[ y_t = [\Delta \rho_t, \Delta c_t]', \]

\[ B_t = [\beta_1, \beta_2, \beta_3, \beta_4]', \]

and

\[ x_t = \begin{bmatrix} 1, & \Delta \rho_{t-1}, & 0, & 0, & \Delta \varsigma_{t-1} \end{bmatrix}. \]

Furthermore, \( \Gamma = C = I_k \), and \( Q_t \) is a time-invariant matrix with \( Q_{33} = 0 \). The models in Appendix tables C3 and C4 can be similarly represented.

The time-varying coefficient interest rate models in tables 3–6 can be written in the form of (B1) if we set \( R_t = 0 \):

\[ y_t = [\Delta \rho_{t-1}, \Delta \varsigma_{t-1}, 0, 0, \Delta \varsigma_{t-1}'], \]

\[ x_t = \begin{bmatrix} \text{var}(\Delta \rho_{t-1}), \text{var}(\Delta \varsigma_{t-1}), \text{cov}(\Delta \rho_{t-1}, \Delta \varsigma_{t-1}), 1, \end{bmatrix}, \]

\[ B_t = [\pi_t', \delta_t, u_t^\rho, u_t^\varsigma, u_t^\delta]', \]

\[ U_t = [u_t^\rho, u_t^\varsigma, u_t^\delta]', \]

and

\[ f(B_t) = [\pi_t', \sqrt{\lambda(\pi_t')^2}, \gamma \pi_t' - \sqrt{n(\pi_t')^2}, \pi_t'(1 - \pi_t') + \gamma \pi_t', \delta_t']]. \]
In the case where \( \delta_t, \pi^t, \) and \( \pi^t' \) follow ARMA(1,1) processes, which is the most general specification in tables 3–6, we specify that

\[
\Gamma = \begin{bmatrix}
\Gamma_1 & 0 & 0 & \Gamma_4 & 0 & 0 \\
0 & \Gamma_2 & 0 & 0 & \Gamma_5 & 0 \\
0 & 0 & \Gamma_3 & 0 & 0 & \Gamma_6 \\
0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\
0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\
\end{bmatrix},
\]

and

\[
C = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}.
\]

In the models where \( \pi^t = \pi^t' = 0 \), the vectors \( x_t \) and \( B_t \) and the matrices \( \Gamma \) and \( C \) simplify in obvious ways.

**The Extended Kalman Filter**

The filtering equations for the model in (B1) are

\[
y_t = x_t f(B_{a(t-1)}) + \eta_t, \quad \text{(B2)}
\]

\[
H_t = x_t \nabla f(B_{a(t-1)}) \nabla f_{x_t} x_t' + R_t, \quad \text{(B3)}
\]

\[
B_{t+1|t} = A(B_{a(t-1)} + K_{a(t-1)} \eta_t), \quad \text{(B4)}
\]

\[
P_{t+1|t} = A [U_k - K_{a(t-1)} \nabla f_{x_t} P_{a(t-1)} A'] + C Q_c C',
\]

and

\[
K_{a(t-1)} = P_{a(t-1)} \nabla f_{x_t} x_t' H_t^{-1}, \quad \text{(B6)}
\]

where \( B_{a(t-1)} \) is the estimate of \( B_t \) given information at \( t - 1 \), \( \nabla f_{x_t} \) is the gradient matrix of \( f(\cdot) \) evaluated at \( B_{a(t-1)} \), and \( P_{a(t-1)} \) is the covariance matrix of \( B_t \) given information available at \( t - 1 \).

Several points are worth noting about the filtering equations:

1. They simplify considerably for the models for consumption growth and inflation because \( f(B_{a(t-1)}) = B_{a(t-1)} \) and \( \nabla f_{x_t} = I_k \).
2. When \( f(\cdot) \) is nonlinear, the filter effectively replaces \( f(B_t) \) with the linear approximation \( f(B_{a(t-1)}) + \nabla f_{x_t}(B_t - B_{a(t-1)}) \).
3. Equation (B3) shows how uncertainty about \( B_t \) contributes with \( R_t \) to \( H_t \), the covariance matrix of innovations to \( y_t \), defined as \( \eta_t \) in (B2). When there is no uncertainty about \( B_t \) (i.e., \( P_{a(t-1)} \) is equal to the null matrix), \( R_t = H_t \), so that the model encompasses a simple ARCH specification as a special case. Under other circumstances, the diagonal elements in \( R_t \) will be biased below the true conditional variances in \( H_t \), because \( \nabla f_{x_t} P_{a(t-1)} \nabla f_{x_t} \) will be
positive definite. Notice also that the covariance terms in $R_t$ will not represent the true covariances in $H_t$.

**Estimating the Consumption/Inflation Models**

Some of these models take $R_t$ to follow an ARCH process. Since $B_{it} = B_{it-1} + K_{ii-1}^{-1} \cdot \eta_{it-1}$, in linear models the necessary estimates of the past forecast errors can be obtained from

$$
\hat{e}_t = y_t - x_t B_{it} = [I - x_t K_{ii-1}] \cdot \eta_t.
$$

(B7)

Estimates of $Q$, and the ARCH parameters in $R_t$, are obtained by maximizing the likelihood

$$
\sum_{t=1}^{T} \log(2\pi) - \log|H_t| = \frac{1}{2}(\eta_t'H_t^{-1}\eta_t),
$$

(B8)

where $\eta_t$ and $H_t$ are derived from (B2)–(B7).

**Estimating the Interest Rate Models**

Computational considerations make it necessary to estimate the model in (7') by limited rather than full information maximum likelihood. Specifically, we treat the estimates of expected consumption growth and inflation together with their conditional covariance matrix as data when estimating our generalized version of the Fisher model.

Since there is no residual in (7'), $R_t = 0$. Conditional heteroscedasticity is introduced by allowing the covariance matrix $Q_t$ to follow a multivariate ARCH process. The past errors are obtained as elements from $CU_t$:

$$
CU_t = B_{it} - B_{it-1} = K_{ii-1}^{-1} \cdot \eta_t.
$$

(B9)

Using these definitions, the likelihood (B8) can be formed using (B2)–(B6) and maximized in the usual way.

The standard errors reported in tables 3–6 are derived from the score vector for the complete model (i.e., a model that combines the consumption, inflation, and interest rate equations and imposes the full set of cross-equation restrictions) evaluated at the parameter estimates reported in tables C1–C4 and 3–6. This procedure avoids the inference problems caused by the presence of "generated regressors" in the interest rate equation.

**The Kalman Filter as a Rolling Regression Technique**

The model in (B1) can also be written in the form of a regression. The time-varying coefficient vectors $B_{1t}, \ldots, B_{k-1}$ can be expressed as functions of $B_t$ as long as $\Gamma^{-1}$ exists. This follows because we can write

$$
B_{t} = \Gamma B_{t-1} + CU_t
= \Gamma^2 B_{t-2} + CU_t + \Gamma CU_{t-1}
= \Gamma^{t-1} B_1 + \Gamma^{t-2} CU_2 + \ldots + \Gamma CU_{t-1} + CU_t.
$$
The regression observations can be written as

\[
\begin{bmatrix}
\tilde{y}_1 \\
\tilde{y}_2 \\
\vdots \\
\tilde{y}_{t-1} \\
\tilde{y}_t
\end{bmatrix} = 
\begin{bmatrix}
\hat{x}_{t-1} \Gamma^{-1} \\
\hat{x}_{t-2} \Gamma^{-2} \\
\vdots \\
\hat{x}_1 \\
\hat{x}_t
\end{bmatrix} B_0 + 
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\vdots \\
\epsilon_{t-1} \\
\epsilon_t
\end{bmatrix} - 
\begin{bmatrix}
\check{x}_{t-1} \Gamma^{-1} \\
\check{x}_{t-2} \Gamma^{-2} \\
\vdots \\
\check{x}_1 \\
\check{x}_t
\end{bmatrix} C U_2 - 
\begin{bmatrix}
\check{x}_{t-1} \Gamma^{-1} \\
\check{x}_{t-2} \Gamma^{-2} \\
\vdots \\
\check{x}_1 \\
\check{x}_t
\end{bmatrix} C U_{t-1}
\]

or more compactly as

\[
\tilde{y}_t = z_t \beta + \epsilon_t - M U_t, \tag{B10}
\]

where, for linear models, \( \tilde{y}_t = y_t \) and \( \hat{x}_t = x_t \) and, for nonlinear models, \( \tilde{y}_t = y_t - x_t f(B_{\psi \theta_t \lambda_{\psi \theta_t}}) + \nabla f B_{\psi \theta_t \lambda_{\psi \theta_t}} \) and \( \hat{x}_t = x_t \nabla f \), for \( i \leq t \). The filtered estimates of \( B \) from the Kalman Filter are equivalent to the generalized least squares (GLS) estimate of \( B \) in the regression model (B10). Thus, the Kalman Filter can be viewed as a rolling GLS technique that expands the data set each period. Additionally, in the nonlinear case, \( \tilde{y}_t \) and \( \hat{x}_t \) for \( i \leq t \) are calculated from the estimate of \( B_{\psi \theta_t \lambda_{\psi \theta_t}} \).

Equation (B10) can be used to interpret the results in tables 3–6. Regression theory implies that \( B = \text{var}(z_t' z_t)^{-1} \text{cov}(y_t', z_t) \), where \( \text{var}(\cdot) \) and \( \text{cov}(\cdot) \) are the sample variance and covariance matrices. In the monthly models, the elements of \( \text{cov}(y_t' z_t) \) are nonzero, and \( \text{var}(z_t' z_t)^{-1} \text{cov}(y_t' z_t) \) varies significantly from period to period. Given the definitions of \( y_t \) and \( z_t \), this means that the changes in \( \pi_t^c \) and \( \pi_t^f \) represent variations in the (multiple) sample correlations between \( \text{var}(\Delta p_{\rho_t \lambda_{\rho_t}}) \), \( \text{var}(\Delta c_{\rho_t \lambda_{\rho_t}}) \), \( \text{cov}(\Delta p_{\rho_t \lambda_{\rho_t}} \Delta c_{\rho_t \lambda_{\rho_t}}) \), and \( \epsilon_t \). Thus, \( \pi_t^c \) and \( \pi_t^f \) are not free to vary with the level of interest rates.

**Generated Instruments**

The results in table 2 use the time-series estimates of \( E_t \Delta c_{t+1} \), \( \text{var}(\Delta c_{t+1}) \), \( \text{var}(\Delta p_{\rho_t \lambda_{\rho_t}}) \), and \( \text{cov}(\Delta p_{\rho_t \lambda_{\rho_t}} \Delta c_{t+1}) \) as instruments. Strictly speaking, these estimated moments would not be valid instruments if they were generated from time-series models with constant coefficients because the coefficients used to calculate the instruments depend on the full data sample. Equation (B10) shows that estimates based on the time-varying parameter models differ from these because the coefficient estimates do not depend directly on future data. Of course, if some of the parameters in the model used to estimate \( E_t \Delta p_{\rho_t \lambda_{\rho_t}} \) and \( E_t \Delta c_{t+1} \) are fixed (as they are in the quarterly models), then \( E_t \Delta p_{\rho_t \lambda_{\rho_t}} \) and \( E_t \Delta c_{t+1} \) cease to be strictly valid instruments. For this reason, we also estimated the model using lags of the ex post real rate, inflation, and consumption growth. The results were similar to those we report.

**Hausman Tests**

Consider the regression

\[
y_t = x_t \beta + \epsilon_t. \tag{B11}
\]
We wish to test whether measurement errors in the scalar $$x_i$$ affect the estimate of $$\beta$$. Let $$w_t$$ be a set of valid instruments. Godfrey shows that the Hausman test is equivalent to the Lagrange multiplier (LM) test of $$\psi = 0$$ in

$$y_i = x_i\beta + x_i(w)\psi + e_i,$$  \hspace{1cm} (B12)

where $$x_i(w)$$, are the predicted values from the regression of $$x_i$$ on the instruments $$w_t$$.

Tables 3 and 4 report three Hausman tests for the monthly models. Lagged values of actual consumption growth and the ex post real rate are used as the instruments in each case. We report results for $$w_t$$ equal to $$\gamma E_{t}\Delta c_{t+1} - \psi \gamma^2 \text{var}_{t}(\Delta c_{t+1}) - \gamma \text{cov}(\Delta p_{t+1}, \Delta c_{t+1})$$, $$\text{var}_{t}(\Delta p_{t+1}) + \gamma \text{cov}(\Delta p_{t+1}, \Delta c_{t+1})$$, and $$\gamma \text{var}_{t}(\Delta c_{t+1}) + \gamma \text{cov}(\Delta p_{t+1}, \Delta c_{t+1})$$. Other choices for $$w_t$$ gave similar results.
Appendix C

Table Appendix

<table>
<thead>
<tr>
<th>TABLE C1</th>
<th>Time-Series Model for Consumption Growth and Inflation (Based on the CPI), Monthly Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Model Estimates</td>
<td></td>
</tr>
<tr>
<td>( \Delta p_t = \beta_1 + \beta_2 \Delta p_{t-1} + \varepsilon_t ) &amp; ( \text{var}(\varepsilon_t) = 5.775 + .168(\varepsilon_t)^2 )</td>
<td></td>
</tr>
<tr>
<td>( \Delta c_t = .187 + \beta_2 \Delta c_{t-1} + \varepsilon_t ) &amp; ( \text{var}(\varepsilon_t) = 41.408 + .226(\varepsilon_t)^2 )</td>
<td></td>
</tr>
<tr>
<td>&amp; ( \text{cov}(\varepsilon_t, \varepsilon_{t-1}) = 3.530 - .033(\varepsilon_t \varepsilon_{t-1}) )</td>
<td></td>
</tr>
<tr>
<td>( \beta_{ux1} + u_u ) &amp; ( \text{var}(u_u) = .455^2 )</td>
<td></td>
</tr>
<tr>
<td>( \beta_{ux2} + u_x ) &amp; ( \text{var}(u_x) = .026^2 )</td>
<td></td>
</tr>
<tr>
<td>( \beta_{ux3} = \beta_1 + u_x ) &amp; ( \text{var}(u_x) = .020^2 )</td>
<td></td>
</tr>
<tr>
<td>&amp; ( \text{cov}(\varepsilon_t, \varepsilon_{t-1}) = 3.530 - .033(\varepsilon_t \varepsilon_{t-1}) )</td>
<td></td>
</tr>
</tbody>
</table>

B. Test Statistics

<table>
<thead>
<tr>
<th>Stabilty tests*</th>
<th>Inflation</th>
<th>Consumption Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi^2 )</td>
<td>7.52 () .01 )</td>
<td>4.08 () .04 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exclusion tests†</th>
<th>Real Rate</th>
<th>Consumption Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi^2 )</td>
<td>3.33 () .19 )</td>
<td>5.14 () .08 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Random walk test‡</th>
<th>Inflation and Consumption Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi^2 )</td>
<td>6.81 () .08 )</td>
</tr>
</tbody>
</table>

Note.—Asymptotic \( t \)-statistics are reported in parentheses below the parameter estimates. Significance levels are reported in parentheses below the test statistics. CPI = consumer price index.

* The largest \( \chi^2 \) statistic recorded from a sequence of Chow stability tests on a constant coefficient model for inflation and consumption growth. The test statistic allows for conditional heteroscedasticity.

† LM statistics testing the exclusion of the lagged ex post real rate or lagged consumption growth from both the inflation and consumption equations.

‡ LM test for misspecification in the random walk model for the coefficient vector \( \beta \). The alternative process for \( B_t \) is \( B_{t+1} = B_t + B_t[B_t - B_0] + V_{t+1} \), where \( V_t \sim N(0, Q) \), \( B_0 \) is a \( k \times k \) matrix, and \( B_0 \) is the long-run level of \( B_t \). The models above restrict \( B_t \) to be equal to the identity matrix. That is the restriction tested.
TABLE C2  Time-Series Model for Consumption Growth and Inflation
(Based on the Deflator), Monthly Data
A. Model Estimates

$$\Delta \rho_t = \beta_{1t} + \beta_{2t} \Delta \rho_{t-1} + \epsilon_{t}$$  \hspace{1cm} \text{var}(\epsilon_{t+1}^2) = 5.241 + .063(\epsilon_t^2)^2$$

(5.041) \hspace{1cm} (8.281) \hspace{1cm} (6.89)

$$\Delta c_t = 2.093 + \beta_{1t} \Delta c_{t-1} + \epsilon_{c_t}$$  \hspace{1cm} \text{var}(\epsilon_{c_t+1}^2) = 39.978 + .263(\epsilon_c_t^2)^2$$

(5.041) \hspace{1cm} (8.263) \hspace{1cm} (2.535)

$$\text{cov}(\epsilon_{t+1}^2, \epsilon_{c_t+1}) = .673 + .155(\epsilon_t^2)(\epsilon_c_t^2)$$

(6.49) \hspace{1cm} (2.101)

$$\beta_{1t+1} = \beta_{1t} + \nu_{1t}, \hspace{1cm} \text{var}(\nu_{1t}) = .551^2$$

(4.707)

$$\beta_{2t+1} = \beta_{2t} + \nu_{2t}, \hspace{1cm} \text{var}(\nu_{2t}) = .022^2$$

(1.483)

$$\theta_{3t+1} = \theta_{3t} + \nu_{3t}, \hspace{1cm} \text{var}(\nu_{3t}) = .022^2$$

(1.964)

B. Test Statistics

<table>
<thead>
<tr>
<th>Stability tests$^\star$</th>
<th>Inflation</th>
<th>Consumption Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8.27</td>
<td>4.08</td>
</tr>
<tr>
<td></td>
<td>(.01)</td>
<td>(.04)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Real Rate</th>
<th>Consumption Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.62</td>
<td>.60</td>
</tr>
<tr>
<td>(.44)</td>
<td>(.74)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Random walk test$^\dagger$</th>
<th>Inflation and Consumption Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.89</td>
</tr>
<tr>
<td></td>
<td>(.27)</td>
</tr>
</tbody>
</table>

Note.—Asymptotic t-statistics are reported in parentheses below the parameter estimates. Significance levels are reported in parentheses below the test statistics.

$^\star$ The largest $\chi^2$ statistic recorded from a sequence of Chow stability tests on a constant coefficient model for inflation and consumption growth. The test statistic allows for conditional heteroscedasticity.

$^\dagger$ LM statistics testing the exclusion of the lagged ex post real rate or lagged consumption from both the inflation and consumption equations.

$^\ddagger$ LM test for misspecification in the random walk model for the coefficient vector $B$. The alternative process for $B_t$ is $B_{t+1} = B_0 + B_1[B_t - B_0] + V_{t+1}$, where $V_t \sim N(0, Q)$. $B_0$ is a $k \times k$ matrix, and $B_0$ is the long-run level of $B_t$. The models above restrict $B_0$ to be equal to the identity matrix. This is the restriction tested.
TABLE C3  Time-Series Model for Consumption Growth and Inflation (Based on the CPI, Quarterly Data)

A. Model Estimates

\[
\Delta p_t = \beta_{1t} - .403 \Delta p_{t-1} - .286 \Delta p_{t-2} + \epsilon_t^1 \\
\quad \text{(2.857)} \\
\Delta c_t = 1.551 + \beta_{2t} \Delta c_{t-1} + \epsilon_t^2 \\
\quad \text{(6.736)} \\
\text{cov} (\epsilon_t^1, \epsilon_t^2) = -1.008 \\
\quad \text{(3.098)} \\
\beta_{1t+1} = \beta_{1t} + \nu_{1t} \\
\quad \text{var}(\nu_{1t}) = 1.393^2 \\
(5.062) \\
\beta_{2t+1} = \beta_{2t} + \nu_{2t} \\
\quad \text{var}(\nu_{2t}) = 0.23^2 \\
(1.713)
\]

B. Test Statistics

<table>
<thead>
<tr>
<th>Stability test*</th>
<th>Inflation</th>
<th>3.26</th>
<th>(.07)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exclusion test†</td>
<td>Real Rate</td>
<td>.31</td>
<td>(.86)</td>
</tr>
<tr>
<td></td>
<td>Consumption Growth</td>
<td>.14</td>
<td>(.93)</td>
</tr>
<tr>
<td>Random walk test‡</td>
<td>Inflation and Consumption Growth</td>
<td>1.30</td>
<td>(.52)</td>
</tr>
</tbody>
</table>

Note — Asymptotic t-statistics are reported in parentheses below the parameter estimates. Significance levels are reported in parentheses below the test statistics. CPI = consumer price index.

* The largest \( \chi^2 \) statistic recorded from a sequence of Chow stability tests on a constant coefficient model for inflation. The test statistic allows for conditional heteroskedasticity.

† LM statistic testing the exclusion of the lagged ex post real rate or lagged consumption from both the inflation and consumption equations.

‡ LM test for misspecification in the random walk model for the coefficient vector \( B_t \). The alternative process for \( B_{t+1} \) is \( B_{t+1} = B_t + B_t (B_t - B_t) + V_{t+1} \), where \( V_t \sim N(0, \sigma^2) \), \( B_t \) is a \( k \times k \) matrix, and \( B_{t+1} \) is the lagged level of \( B_t \). The models above restrict \( B_t \) to be equal to the identity matrix. This is the restriction tested.
TABLE C4  Time-Series Models for Consumption Growth and Inflation  
(Based on the Deflator), Quarterly Data

A. Model Estimates

\[ \Delta p_t = \beta_{1t} - .045 \Delta p_{t-1} + \epsilon_{1t}^p \quad \text{var} (\epsilon_{1t}^p) = .490 + 1.269 (\epsilon_{1t}^p)^2 \]
\[ \Delta C_t = 1.661 + \beta_{2t} \Delta C_{t-1} + \epsilon_{2t} \quad \text{var} (\epsilon_{2t}) = 3.859 + .068 (\epsilon_{2t})^2 \]
\[ \text{cov} (\epsilon_{1t}^p, \epsilon_{2t}) = .139 - .014 (\epsilon_{1t}^p \epsilon_{2t}) \]

\[ \beta_{1t+1} = \beta_{1t} + \nu_{1t} \quad \text{var} (\nu_{1t}) = .812 \]
\[ \beta_{2t+1} = \beta_{2t} + \nu_{2t} \quad \text{var} (\nu_{2t}) = .022 \]

B. Test Statistics

<table>
<thead>
<tr>
<th>Stability test†</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.26</td>
</tr>
<tr>
<td></td>
<td>(.07)</td>
</tr>
</tbody>
</table>

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<tr>
<th>Exclusion tests†</th>
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<th>Consumption Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.35</td>
<td>.21</td>
</tr>
<tr>
<td></td>
<td>(.84)</td>
<td>(.90)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Random walk test‡</th>
<th>Inflation and Consumption Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.18</td>
</tr>
<tr>
<td></td>
<td>(.20)</td>
</tr>
</tbody>
</table>

Note.—Asymptotic t-statistics are reported in parentheses below the parameter estimates. Significance levels are reported in parentheses below the test statistics.

† The largest χ² statistic recorded from a sequence of Chow stability tests on a constant coefficient model for inflation. The test statistic allows for conditional heteroscedasticity.

‡ LM statistic testing the exclusion of the lagged ex post real rate or lagged consumption from both the inflation and consumption equations.

§ LM test for misspecification in the random walk model for the coefficient vector \(B_t\). The alternative process for \(B_t\) is \(B_{t+1} = B_0 + R_t \phi - B_{l-1} + V_{t+1}\), where \(V_t \sim \mathcal{N}(0, \Omega_t)\), \(B_0\) is a \(k \times k\) matrix, and \(B_0\) is the long-run level of \(B_t\). The models above restrict \(B_0\) to be equal to the identity matrix. This is the restriction tested.
References


