Trading to hedge: Static hedging

Hedging: risk reduction

- We have some risk exposure that can’t be directly mitigated (reduced).
- Example: A bank portfolio of loans might be exposed to risk from unexpected interest rate changes.
  - The bank can’t simply sell the loans because
    - The loans are earning returns that the bank can’t get elsewhere.
    - There might be no market for the loans.
Example: An airline is exposed to risk arising from changes in the price of fuel.
- It might enter into long-term fixed-price contracts, but if the airline’s projected fuel needs change, it will be difficult to modify the contracts.

Example: A pension fund with a large portfolio of stocks has a negative market outlook in the short run (weeks or months).
- Selling the stocks and repurchasing them will lead to substantial trading costs.

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We won’t try to eliminate *all* risks.

- Hedging is expensive.
  - Most hedges will incur trading costs.
- The securities that we need may not exist.
- There are some risk exposures that we (or our investors) might want us to keep.
  - A bond fund with expertise in credit scoring might want to hedge interest rate risk, but not credit risk.
  - Investors in gold mining stocks usually want some exposure to the price of gold. They don’t want the firm to eliminate this exposure.
- We want to be thoughtful and selective about the risks we hedge and the risks we keep.
The basic hedging principle

- Reduce risk by establishing a position in a security that is negatively correlated with the risk exposure.
- Negative correlation: the *value of the hedge* moves against or opposite to the risk exposure.
  - The ideal hedging security is cheap to buy, easy to trade, and very highly correlated with the risk exposure.
  - If we can go long or short the hedging security, it doesn’t matter if the correlation is positive or negative.

Static hedging

- When we buy/sell the hedging security, we need to trade quickly.
  - Until the hedging position is established we have risk.
  - But if we trade too quickly we’ll incur high trading costs.
- The trade-off is risk vs. cost
- If the hedge just needs to be set up initially, and doesn’t have to be modified, it is a *static hedge*.
  - The *trading* aspect of a static hedge is usually easy. At the outset buy or sell what you need.
  - It can be complicated to figure out what you need.
Dynamic hedging

- In some situations the hedge position must be adjusted after the initial set-up. This is a *dynamic hedge*.
- The need for dynamic hedging typically arises in
  - Stock portfolios that have put and call options.
  - Bond portfolios that try to match the duration of some liability.
- The RIT H3 case involves a dynamic hedge.

Static Hedge Situation 1: Removing the market *return* in CAT

- CAT is the ticker symbol for Caterpillar (a manufacturer of heavy equipment)
- Portfolio manager Beth has $10 Million to invest.
- If she thinks that Caterpillar is undervalued, she simply buys CAT.
- Suppose that Beth thinks that Caterpillar is undervalued *relative to the market*.
  - She’s analyzed the heavy equipment industry, but has no opinion on interest rates, commodity prices, consumer spending or any of the many other things that drive the market.
  - She wants to invest in the difference between the return on CAT and the return on the market.
Betting on the return difference, $r_{CAT} - r_M$

- If the return on the market is $r_M = 5\%$ and $r_{CAT} = 7\%$, she wants a return of 2%.
  - If $r_M = -11\%$ and $r_{CAT} = -8\%$, she wants a return of 3%
- She wants to be long CAT and short the market.
- She’ll use the Standard and Poors Composite Index to approximate “the market”.
- To mirror the market “$M$,” there are two candidate hedge securities.
  - She can go long or short the SPDR (ticker symbol “SPY”)
  - She can go long or short the S&P Composite E-mini futures contract.

The index, the ETF, and the futures contract

- The S&P composite index is a weighted average of the prices of 500 stocks. It is computed every fifteen seconds.
  - Many market data systems use “SPX” to denote the index.
    - But since it is not a traded security “SPX” is not a real ticker symbol.
  - As of November, 2014, $SPX \approx 2,000$.
- Ticker symbol SPY refers to the exchange-traded-fund (ETF) based on the index.
  - It actually is traded. SPY is a real ticker symbol.
  - It is constructed to have a value of one-tenth the index.
    - As of November, 2014 its price is $SPY \approx 200$.
  - The SPY tracks the SPX closely, but not perfectly.
    - Discrepancies arise due to dividends, management fees, and so on.
The E-mini S&P futures contract

- Ticker symbols for futures contracts have a two-character product code ("SP") followed by a month/year code that denotes the maturity of the contract.
  - We’ll use “SP” to denote the nearest maturity.
- The SP price quotes are reported in index points.
- The size of the contract is $50 \times SPX$.
- The contract is cash settled.
  - Suppose I go long the contract today (time 0) at a price of $SP_0 = 2,000$.
  - Suppose at maturity (time $T$) the index is at $SPX_T = 2,100$.
  - I receive (from the short side)
    $$(SPX_T - SP_0) \times 50 = (2,100 - 2,000) \times 50 = 5,000$$
- Note: this discussion is somewhat simplified. It ignores margin and daily resettlement.

Method I: Buying CAT and shorting the SPY

- Suppose that CAT is about $100 per share, and that the $SPX \approx 2,000$
- Buy $10,000,000 / 100 = 100,000$ sh of CAT
- The SPY represents one-tenth of the S&P index. $SPY \approx 200$
  - Beth goes short $10,000,000 / 200 = 50,000$ sh of SPY.
    - She borrows 50,000 sh of SPY and sells them.
- She’s long 100,000 sh of CAT and short 50,000 sh of SPY
Suppose that \( r_{CAT} = 7\% \) and \( r_M = 5\% \)

- CAT stock goes from $100 to $107.
  - Beth's 100,000 shares are now worth $10,700,000.
- The SPY is initially at $200.
  - A 5\% return corresponds to a price of $210.
  - The value of Beth's short position is \( 50,000 \times $210 = $10,500,000 \).
- The net value of Beth's overall position (CAT + SPY) has gone up by $200,000
- This is a 2\% return on the $10 Million initial investment.

Suppose that \( r_{CAT} = -8\% \) and \( r_M = -11\% \)

- CAT stock goes from $100 to $92.
  - Beth's 100,000 shares are now worth $9,200,000.
- The SPY is initially at $200.
  - \( r_{SPY} = -11\% \) return corresponds to a price of $178.
  - The value of Beth's short position is \( 50,000 \times $178 = $8,900,000 \).
- The net value of the overall position (CAT + SPY) has gone up by $300,000
- This is a 3\% return on the $10 Million initial investment.
Problem: suppose that $r_{CAT} = -10\%$ and $r_M = -6\%$. Work out the return on Beth’s $10$ Million investment.

- Answer:
  - The price of CAT goes from $100$ to $90$.
    - Beth’s shares are worth $9,000,000$
  - The SPY goes from $200$ to $188$.
  - The value of Beth’s short position is $50,000 \times $188 = $9,400,000$.
  - The net change is $-400,000$, a $-4\%$ return.

Method II: Buying CAT and shorting the futures contract

- As in method I, Beth buys $100,000$ sh of CAT
- As of November, 2014 (time “0”), the level of the S&P index is about $SPX_0 = 2,000$.
- An E-Mini S&P index futures contract has a notional value of $50 \times SPX = 50 \times 2,000 = $100,000/contract$.
- She goes short $\frac{10,000,000}{$100,000} = 100$ contracts at $2,000$
Suppose that $r_{CAT} = 7\%$ and $r_M = 5\%$

- CAT stock goes from $100$ to $107$.
  - Beth’s $100,000$ shares are now worth $10,700,000$.
- “$r_M = 5\%$”: The SPX goes from $2,000$ to $2,100$
  - To settle her $100$ short contracts, Beth pays
    \[(2,100 - 2,000) \times 50 \times 100 = 500,000\]
  - The net gain is $200,000$ (a $2\%$ return on the $10$ Million initial investment).

Suppose that $r_{CAT} = -8\%$ and $r_M = -11\%$

- CAT stock goes from $100$ to $92$.
  - Beth’s $100,000$ shares are now worth $9,200,000$.
- “$r_M = -11\%$”: The SPX goes from $2,000$ to $1,780$
  - To settle her $100$ short contracts, Beth pays
    \[(1,780 - 2,000) \times 50 \times 100 = -1,100,000\]
  - Beth receives $1,100,000
- Her positions are now worth $10,300,000$: (a $3\%$ return on the $10$ Million initial investment).
Problem: suppose that $r_{\text{CAT}} = -10\%$ and $r_{\text{M}} = -6\%$. Work through the numbers for method II. (How much to settle the futures contracts? What is the net percentage return?)

- Answer:
  - Beth’s shares are worth $9,000,000
  - “$r_{\text{M}} = -6\%$”: The SPX goes from 2,000 to 1,880
    - To settle her 100 short contracts, Beth pays $(1,880 - 2,000) \times 50 \times 100 = -600,000$
    - Beth receives $600,000
  - Her net position is now worth $9,600,000.
  - This is a loss of $400,000, a $-4\%$ return.

Static hedging situation 2: Removing the market risk from CAT

- Beth owns $10$ Million worth of CAT
- She likes CAT, but would like to eliminate the market risk in CAT.
  - Market risk: randomness in CAT’s return that is driven by the market.
- We need a model of the joint randomness in CAT and the market.
  - We’ll use a simple linear regression.
  - Regress the returns on CAT vs the returns on $M$. 
Alternative ways to do simple linear regression in Excel

1. Use in-cell formulas, SLOPE and INTERCEPT
2. Use array formula, LINEST. Can also be used for multiple regression.
3. Using Excel’s charting menus: plot data on an XY scatterplot; add an estimated trend line; display the equation of the trend line.
4. Use Excel’s data analysis menu to run the regression and display the output. Can also be used for multiple regression.
   - This method computes more diagnostic statistics.
   - But the results do not automatically update if the data change. You need to rerun the regression.

Approach

- Download prices for CAT stock and the SPY (or the S&P index)
- We’ll use month-end prices from 2009-2013.
- Construct monthly returns for CAT and the SPY.
- Plot them and find the best fit linear regression line.
  - A linear regression takes two variables “x and y” and relates them as a straight line plus an error.
  - For data point \( i \), \( y_i = \alpha + \beta \times x_i + e_i \)
- The data and details are in workbook H1.xlsx, worksheet CATSPY, posted to the web.
- You’ll be doing similar calculations for the stocks in the RIT hedging case.
Going from prices to returns

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Method 1: Use the SLOPE and INTERCEPT functions

Beta and intercept from SLOPE and INTERCEPT functions:

\[
\begin{align*}
\text{Beta} &= \text{SLOPE}(F4:F62,G4:G62) \\
\text{Intercept} &= \text{INTERCEPT}(F4:F62,G4:G62)
\end{align*}
\]

\[
\begin{align*}
\text{Beta} &= 1.8589 \\
\text{Intercept} &= -0.0037
\end{align*}
\]

Beta with zero intercept

\[
\begin{align*}
\text{Beta} &= \frac{\text{SUMPRODUCT}(F4:F62,G4:G62)}{\text{SUMPRODUCT}(G4:G62,G4:G62)}
\end{align*}
\]

\[
\begin{align*}
\text{Beta} &= 1.8336
\end{align*}
\]
Method 2: Make a scatterplot with a trendline

\[ y = 1.8589x - 0.0037 \]
\[ R^2 = 0.6147 \]

Method 3: Use the LINEST array function

\[
\begin{array}{cc}
1.8589 & -0.0037 \\
0.1949 & 0.0090 \\
0.6147 & 0.0654 \\
90.9345 & 57 \\
0.3886 & 0.2436 \\
\end{array}
\]
Method 4: Use the DATA > Analysis > Regression menu

If you don’t see “Data Analysis” on the DATA menu, you may need to enable the Analysis-ToolPak add-in. You can reach this menu from FILE⇒Options⇒Add-Ins

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Interpretation of one observation

- $r_{CAT,t} = \alpha_{CAT} + \beta_{CAT} \times r_{SPY,t} + e_{SPY,t}$
- In $t = \text{June, 2009}$, $r_{CAT,t} = 0.334$ (33.4%) and $r_{SPY,t} = 0.075$ (7.5%)
- Statistical interpretation:
  - $0.334 = -0.004 + 1.859 \times 0.075 + 0.199$  
    Predicted value of $r_{CAT,t}$
  - Regression error $= 0.135$ (13.5%)
- Economic interpretation:
  - “In June, 2009, factors in the broader market caused CAT to go up by 13.5%. An additional return of 19.9% came from factors unrelated to the market.”
  - These unrelated factors would be due to industry- and company-specific effects.
Decomposition of CAT’s risk

- \( r_{\text{CAT},t} = \alpha_{\text{CAT}} + \beta_{\text{CAT}} \times r_{\text{SPY},t} + e_{\text{CAT},t} \)
- \( \text{Var}(r_{\text{CAT},t}) = \sigma_{\text{CAT}}^2 = \beta_{\text{CAT}}^2 \times \sigma_{\text{SPY}}^2 + \sigma_{e,\text{CAT}}^2 \)
- Note: \( \alpha_{\text{CAT}} \) is constant and doesn’t contribute any risk.
- Interpretation: \( \frac{\sigma_{\text{CAT}}^2}{\text{Total risk of CAT}} = \frac{\beta_{\text{CAT}}^2 \times \sigma_{\text{SPY}}^2}{\text{CAT’s market risk}} + \frac{\sigma_{e,\text{CAT}}^2}{\text{CAT’s firm-specific risk}} \)

Implications for hedging

- \( r_{\text{CAT},t} = \alpha_{\text{CAT}} + \beta_{\text{CAT}} \times r_{\text{SPY},t} + e_{\text{CAT},t} \)
- \( \beta_{\text{CAT}} \approx 1.86 \) is a multiplier
  - If the market is up 1%, then all else equal, we expect CAT to be up 1.86%
  - If we are long $1 in CAT, we should be short \( \beta_{\text{CAT}} \times $1 \approx $1.86 \) of the SPY.
- To eliminate the market risk in $10 Million worth of CAT we can
  - Short $18.6 Million worth of SPY
    - \( \frac{$18.6\text{Million}}{$200} \approx 93,000 \text{ shares of SPY} \)
    - Or, short $18.6 Million notional of the index futures contract
    - \( \frac{$18.6\text{Million}}{2,000 \times $50} \approx 186 \text{ Contracts} \)
Example

- If \( r_{SPY} = 0.01(= 1\%) \), then we expect (all else equal, ignoring \( \alpha_{CAT} \)) that \( r_{CAT} = 0.0186 \).
- Our $10 Million position in \( CAT \) goes up by $186,000.
- A 1% gain on SPY corresponds to the S&P going from 2,000 to 2,020.
  - We settle our 186 futures contracts by paying \( 186 \times (2,020 - 2,000) \times 50 = $186,000 \)
  - This is a total offset.

The static hedging case

- See the details on the course web site under *Announcements*
- The market index is the RTX. The current value of the RTX is 1,050.
  - The RTX futures contract has a notional value of \( RTX \times 250 \). At present, this is \( 1,050 \times 250 = $262,500 \).
  - The contract is cash settled. At maturity the long side receives \( (RTX_{maturity} - 1,050) \times 250 \).
  - Example. If the RTX in one month is 1,045, then the long side receives \( (1,045 - 1,050) \times 250 = -$1,250 \)
  - Since the RTX has declined, the long side pays the short side $1,250.
Materials (workbook H1.xlsx, posted to web)

- Worksheet CATSPY (already used earlier)
- Worksheet Portfolio contains the composition of the portfolio.
- Worksheet Securities has the price history for the portfolio’s ten securities.

Worksheet Portfolio

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Two ways to estimate the portfolio beta, $\beta_P$

- Compute the portfolio weights $w_1, w_2, ..., w_{10}$.
- Using the returns on the individual stocks, estimate the individual beta's: $\beta_1, \beta_2, ..., \beta_{10}$.
- Compute the portfolio beta as the weighted average of the individual betas:
  $$\beta_P = w_1\beta_1 + w_2\beta_2 + \cdots + w_{10}\beta_{10}$$

- Compute the portfolio weights $w_1, w_2, ..., w_{10}$.
- For each month $t$, compute the portfolio return as the wtd avg
  $$r_{Pt} = w_1r_{1t} + w_2r_{2t} + \cdots + w_{10}r_{10t}$$
- Using the portfolio returns, estimate $\beta_P$