Private Information I

Private information and the bid-ask spread

Readings (links active from NYU IP addresses)

- STPP Chapter 10
  - Stable URL: http://www.jstor.org/stable/4470784
  - https://getit.library.nyu.edu/go/9369829?umlaut.institution=NYU

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Information

- Public information is known to everyone
- Private information known to at least one trader but not everyone.
- Some useful distinctions
  - Private information vs. private values.
  - Symmetric and asymmetric private information.

Common and private value

- The dividend discount model of stock valuation (DDM) asserts that the intrinsic value of a stock is
  \[ V = \text{Present Value of expected dividends} \]
  \[ = \frac{ED_1}{(1 + k)} + \frac{ED_2}{(1 + k)^2} + \frac{ED_3}{(1 + k)^3} + \ldots \]
- where
  - \( ED_1 \) is (today’s) expectation of the dividend in one year, \( ED_2 \) is ... in two years, and so on.
  - \( k \) is some interest rate that reflects the riskiness of the dividends.
    - Often we get \( k \) from the Security Market Line (SML) of the Capital Asset Pricing Model (CAPM).
Some inputs to the DDM are investor-specific

- **Tax rates**
  - An investor who is subject to personal taxes would set up this model so that both the expected dividends *and* the discount rate included some tax adjustment.

- **Risk**
  - If the investor is an employee of the company, the company’s stock is especially risky.
    - A large part of her income (her salary) is positively correlated with the company’s return.
    - She might require a higher $k$ than someone who has no exposure to the company.

Investor-specific risk and tax effects are private values

- **Private values**: they don’t affect anyone else’s valuations.
  - If the investor’s personal tax situation or her job changes, her intrinsic valuate calculation is affected, but not anyone else’s.

- In the art market, a private value would come from an individual’s tastes and preferences.

- Private information about private values is “safe” for markets.
  - As a first approximation, it won’t move market prices.
Common values ...

- ... arise from economic and financial developments that affect everyone's intrinsic value.
  - Changes in corporate dividends, investment policy, governance, etc.
  - Changes in national tax policy.
- In the art market, collective shifts in tastes give rise to changes in common values.
  - “Soon after, another Monet — ‘Poirier en Fleurs,’ from 1885 — went up to $8.56 million, again more than the high estimate. The picture ... belongs in the same vein of conventional Impressionism that had gone out of favor but seems to be making a comeback.” (NYT comment on a Sotheby's auction, May, 2013.)
- Private information about common values can introduce risk and “unfairness” in a market.

Private information: symmetric vs. asymmetric

- Symmetric information might be different across traders, but each piece of information is of the same quality.
  - Example: a firm has three divisions. Each of three securities analysts knows one division very well, but not the others.
- Symmetric private information poses challenges for markets, but doesn't generally lead to excessive risk and unfairness.
  - A well-functioning securities market should aggregate symmetric private information.
    - “Aggregate”: collect and average so that the full information is reflected in the security price.
Asymmetric private information (about common values)

- With asymmetric information, some people have better information.
  - Information obtained by better analysis.
  - Illegal insider information
- Asymmetric information is very problematic.
  - The markets need some asymmetric information (to induce people to do research).
  - But too much of or the “wrong kind” of asymmetric information can lead to market failure and perceived unfairness.

Summary

- Private information about private values does not usually hurt a market.
- Private information about common values
  - Generally okay if it is symmetric
  - Problematic if it is asymmetric
Informed trading in the F1 game

- As a player in the F1 game, you were informed:
  - You knew important information before the market completely reacted.
- Most people playing the game made large profits.
- Who in the game “lost”?

Effects of superior private information

- Increase the bid-ask spread.
  - Informed traders impose costs on dealers (and everyone else)
- Buy and sell orders have long-term impacts on security prices (order-price impact)
  - Buy and sell orders are signals because they might have originated from informed traders.
- Market failure: in extreme cases, the market will shut down
Private information and trading: the first pass

- Suppose the stock of ABC is offered at $51.
- Ivan knows that ABC will be the target of a takeover bid at $60.
  - He will lift ABC’s offer, walking through the ask side of the book until the offer reaches $60.
  - Note: this is illegal in the US
- Other traders will only see that ABC is offered at $60.
  - They won't know why, until the takeover bid is announced.
- Once the information is made public, Ivan can sell his stock, at about $60, realizing a cash profit.

Necessary conditions

- Trading
  - The market must be open and liquid.
  - Ivan must be able to trade on the information.
- Revelation
  - The private information must be made public before Ivan can close out his position at a profit.
- A buy and hold investor with favorable private information can patiently sit back and collect higher dividends.
- A (buy and sell) trader needs a market, needs to trade, and needs the information to be revealed.
Problems with the simple story

- Prior to the announcement of the takeover, there is no new fundamental public information.
  - Once Ivan has lifted the limit orders at $51, why don’t sellers simply replenish them, at $51?
  - Why doesn’t Ivan wait until the offer comes back to $51?
  - If someone is willing to pay above the $51 ("public information") value, why don’t short sellers step in?
  - How can the offer ever reach $60 before the revelation of the takeover bid?

One analysis of the problem due to "Walter Bagehot"
- A 19th c. British journalist.
- Pen name for Jack Treynor (Treynor Capital Management)

Treynor’s/Bagehot’s logic

- Many investors lose money in the market because they can’t overcome trading costs.
- A significant portion of the trading cost comes from the bid-ask spread.
- The bid-ask spread is set by the market-maker (dealer or specialist)

The dealer faces two clienteles of incoming traders

- *Liquidity traders (uninformed traders)* are motivated by their own idiosyncratic portfolio needs that are completely unrelated to the stock’s value.
  - Buyers and sellers arrive at rates that are roughly equal.
  - The dealer earns the half-spread on each arrival.
- *Informed traders* possess knowledge about impending developments, announcements.
  - At any given time, informed order flow is one-sided. If the information is positive, they will buy; if negative, sell.
  - The dealer always loses to these traders.
Informed trading and the bid-ask spread

- The dealer’s expected losses to informed traders depend on
  - How many there are, how much they trade, their rate of arrival.
  - The strength of their information.
- To stay in business, the dealer’s profits from the liquidity traders must exceed the losses to informed traders.
- To maintain a given level of profitability, when informed traders grow more numerous or obtain better information, the dealer must offset the increased losses by increasing profits from the liquidity traders.
  - The dealer raises the bid-ask spread.

A simple asymmetric information model

- The end-of-day value of a share of stock, $V$, is random.
- After the end of trading, the will be an announcement: either $V = High = 150$ or $V = Low = 100$, each with equal probability.
- Before trading starts, the expected value is
  - $EV = \frac{1}{2} \times 100 + \frac{1}{2} \times 150 = 125$
  - This is the unconditional expected value.
- There are two types of active traders
  - Informed traders know $V$.
  - Uninformed (“retail”) traders don’t know $V$.
  - The dealer doesn’t know $V$
Traders’ behavior

- Uninformed traders are equally likely to buy or sell.
- Informed traders
  - If they know that $V = High$, they always buy.
  - If they know that $V = Low$, they always sell.
- After the dealer sets the bid and offer quote, a trader arrives. The trader’s type is random.
  - Suppose $P(informed) = 0.2$.
  - That is, 20% of the incoming traders are informed.
  - $P(uninformed) = 0.8$.

The first quotes and the first trade

- The dealer sets bid and ask quotes.
- A random trader arrives and buys or sells.
  - The dealer doesn’t know if she is informed or not.
- How should the dealer set their bids and asks?
- Next slides show the event tree, the sequencing of possible events
  - and conditional probabilities.
  - or joint/total probabilities.
Conditional probabilities are in italics

“Given that we're here the probability of going there next is …”

Total probabilities in bold

Starting with the value draw, the probability that we'll end up here is …
After the trade, what does the dealer know?

- The dealer knows whether the trader has bought or sold.
- The total probability of a buy is 0.5
  - The total probability of sell is also 0.5
- The dealer doesn’t know for sure what path ended up in “buy” or “sell”.
- Suppose the incoming trader sells...
Conditional probabilities

- For two events, $A$ and $B$, we write the probability of $A$ conditional on $B$ as $P(A|B)$
  - Think of "|" as "conditional on" or "given"
- Bayes’ Rule:
  $$P(A|B) = \frac{P(A, B)}{P(B)} = \frac{P(A \text{ and } B \text{ together})}{P(B)}$$

Examples

- Consider a draw of one card at random from a deck of fifty-two playing cards.
  $$P(\text{Heart}|\text{Ace}) = \frac{P(\text{Heart and Ace})}{P(\text{Ace})} = \frac{1/52}{1/13} = \frac{1}{4}$$
  $$P(\text{Ace}|\text{Heart}) = \frac{P(\text{Heart and Ace})}{P(\text{Heart})} = \frac{1/52}{1/4} = \frac{1}{13}$$
- Heart and Ace are independent:
  $$P(\text{Heart and Ace}) = P(\text{Heart}) \times P(\text{Ace})$$
Conditional probability

- Suppose that the bid is hit (the trader has sold).
- Before the trade, the dealer thought there was a 50% chance that $V = Low$. What the dealer think now?
- The conditional probability is
  
  \[
  P(V = Low | Sell) = \frac{P(V = Low, Sell)}{P(Sell)} = \frac{0.1 + 0.2}{0.5} = 0.6
  \]
- Note: in this particular case $P(V = Low | Sell) = P(Sell | V =\

Conditional expectation

- The expected security value given that there’s been a sell is
  
  \[
  E[V | Sell] = P(V = Low | Sell) \times V_{Low} + P(V = High | Sell) \times V_{High}
  \]
  
  \[
  = 0.6 \times 100 + 0.4 \times 150 = 120
  \]
- Embedded problem: show $E[V | Buy] = 130$
  
  (Answer is in the online version of this handout.)
On the ask side. Given that there’s a buy ...

- The conditional probability is

\[
P(V = \text{Low}|\text{Buy}) = \frac{P(V=\text{Low}, \text{Buy})}{P(\text{Buy})} = \frac{0.2}{0.5} = 0.4
\]

- The expected security value is
  
  \[
  E[V|\text{Buy}] = P(V = \text{Low}|\text{Buy}) \times V_{\text{Low}} + P(V = \text{High}|\text{Buy}) \times V_{\text{High}}
  \]

  \[
  = 0.4 \times 100 + 0.6 \times 150 = 130
  \]

How should the dealer set his bid?

- If he bids 121 and he’s hit, he’ll have an (expected) loss.
- If he bids 119 and he’s hit, he’ll have an expected profit.
  - But someone else will compete, bidding, say, 119.10.
- The forces of competition will drive the bid to the point where \( Bid = E[V|\text{sell}] = 120 \)
- What should be the ask?
Suppose someone hits the dealer’s bid (at $120)

- If seller is uninformed,  
  \[ \text{dealer's profit} = \frac{125 - 120}{EV} = 5 \]

- If seller is informed,  
  \[ \text{dealer's profit} = \frac{100 - 120}{EV|\text{informed}} = -20 \]

\[ \frac{0.8}{P(\text{uninformed})} \times 5 + \frac{0.2}{P(\text{informed})} \times (-20) = 0 \]

- The dealer’s expected profits from uninformed just cover the expected losses to the informed.

Embedded problems

- Show that if the probability of an informed trader is 30% (rather than 20%) the market will be $117.50 bid, offered at $132.50.

- Show that if the probability of an informed trader is 40% the market will be $115.00 bid, offered at $135.00.

- Answers in the online version of this handout.
Answer (with probability of informed trader=30%)

<table>
<thead>
<tr>
<th>Value Realization</th>
<th>Trader arrival</th>
<th>Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low, 0.5</td>
<td>Informed, 0.15</td>
<td>Buy, 0</td>
</tr>
<tr>
<td></td>
<td>Uninformed, 0.35</td>
<td>Sell, 0.15</td>
</tr>
<tr>
<td>High, 0.5</td>
<td>Informed, 0.15</td>
<td>Buy, 0.175</td>
</tr>
<tr>
<td></td>
<td>Uninformed, 0.35</td>
<td>Sell, 0.175</td>
</tr>
</tbody>
</table>

- The tree now looks like this:
- \[ P(V = \text{Low}|\text{Sell}) = \frac{0.15 + 0.175}{0.5} = 0.65 \]
- \[ E(V|\text{Sell}) = 0.65 \times 100 + 0.35 \times 150 = 117.5 \]
- Similarly, \( E(V|\text{Buy}) = 132.5 \)

Answer (with probability of informed trader=40%)

<table>
<thead>
<tr>
<th>Value Realization</th>
<th>Trader arrival</th>
<th>Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low, 0.5</td>
<td>Informed, 0.2</td>
<td>Buy, 0</td>
</tr>
<tr>
<td></td>
<td>Uninformed, 0.3</td>
<td>Sell, 0.2</td>
</tr>
<tr>
<td>High, 0.5</td>
<td>Informed, 0.2</td>
<td>Buy, 0.15</td>
</tr>
<tr>
<td></td>
<td>Uninformed, 0.3</td>
<td>Sell, 0.15</td>
</tr>
</tbody>
</table>

- The tree now looks like this:
- \[ P(V = \text{Low}|\text{Sell}) = \frac{0.35}{0.5} = 0.7 \]
- \[ E(V|\text{Sell}) = 0.7 \times 100 + 0.3 \times 150 = 115 \]
- Similarly, \( E(V|\text{Buy}) = 135 \)
Questions

- If an important news announcement is scheduled for 8:30, why might the spread widen before 8:30?
- A dealer might prefer to quote different spreads to different clienteles (retail, institutional, proprietary, etc.) Can a dealer pay brokers to send her the retail customer orders?
- A NASDAQ dealer can be selective about which orders they take, an Exchange can’t discriminate.
- Suppose that the order flow is half retail, half institutional. 0% of retail orders and 90% of the institutional traders are “informed,” will the NBBO spread be driven by the average (that is, that 45% of all orders are informed)?

Private information II

Private information and order impact (on prices)
The first trader has just hit the dealer’s bid. What’s next?

- **Recap**
  - $V = Low = $100$ or $V = High = $150$
  - $P(V = Low) = P(V = High) = 0.5$
  - For any incoming trader, $P(\text{Informed}) = 0.2$
  - The dealer sets $\text{bid} = $120 and $\text{offer} = $130

- Suppose that the incoming trader hits the bid (sells to the dealer).
- How should the dealer revise his quotes in anticipation of the second trade?

- From the analysis leading up to the first trade, the dealer knows that
  - $P(V = Low|Sell_1) = 0.6$ and $P(V = High|Sell_1) = 0.4$
  - where we’ve added a subscript: $Sell_1$ indicates that the first trade was a sell.

- Without knowing whether the first trade is a buy or a sell, the dealer thinks $P(V = Low) = 0.5$.

- The event tree leading up to the second trade has the same form as the first tree, but with this one probability changed.
Analysis

- Conditional probabilities

\[
P(V = \text{Low}|Sell_2) = \frac{P(V = \text{Low}, Sell_2)}{P(Sell)} = \frac{0.12 + 0.24}{0.12 + 0.24 + 0.16} = 0.692
\]

\[
P(V = \text{High}|Sell_2) = 0.308
\]

- Conditional expectations

\[
E(V|Sell_2) = P(V = \text{Low}|Sell_2) \times 100 + P(V = \text{High}|Sell_2) \times 150 = 115.385
\]

- Buy the same logic as for the first trade, this should be the dealer’s bid.

- Embedded problem: Show \( P(V = \text{Low}|Buy_2) = 0.5 \)

and \( E(V|Buy_2) = 125 \) (= Ask)

Answer to embedded problem

- Conditional probabilities

\[
P(V = \text{Low}|Buy_2) = \frac{P(V = \text{Low}, Buy_2)}{P(Buy_2)} = \frac{0.24}{0.08 + 0.24 + 0.16} = 0.5
\]

\[
P(V = \text{High}|Buy_2) = 0.5
\]

- A buy followed by a sell leaves the dealer with the same beliefs as he held initially.

- When the order flow is balanced we learn nothing.

- Conditional expectations

\[
E(V|Buy_2) = P(V = \text{Low}|Buy_2) \times 100 + P(V = \text{High}|Buy_2) \times 150 = 125
\]
Summary

- Prior to any trade, $EV = 125$
- Prior to the first trade:
  \[ Bid = 120 = E[V|sell_1] \] and \[ Ask = 130 = E[V|buy_1] \]
- Suppose the first trade is a sell. Everyone now thinks that the stock is worth 120.
- The new quotes prior to the second trade are:
  \[ Bid = E[V|sell_1, sell_2] = 115.385 \] and
  \[ Ask = E[V|sell_1, buy_2] = 125 \]
- Order price impact: “The first sell drives down the price.”

Trade sequences

- For any given hypothetical sequence buys and sells, we can construct the path of the bid and offer
- Generally, as trading advances:
  - The bid and offer move in the direction of the orders.
  - A buy drives the quotes up; a sell drives the quotes down.
- Since the informed tend to trade in one direction, the order flow tends to become increasingly one-sided,
  - The price converges toward the true value.
  - The spread drops (because there is less uncertainty).
- Next: some example plots
### Trade sequence: $s, s, b, s, s, b, s, s, s, s, s, s$

<table>
<thead>
<tr>
<th>Price</th>
<th>Trade sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100$</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12</td>
</tr>
<tr>
<td>$110$</td>
<td></td>
</tr>
<tr>
<td>$120$</td>
<td></td>
</tr>
<tr>
<td>$130$</td>
<td></td>
</tr>
<tr>
<td>$140$</td>
<td></td>
</tr>
</tbody>
</table>

### Trade sequence: $s, b, b, s, b, b, s, b, b, b, b, b$

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<td></td>
</tr>
<tr>
<td>$130$</td>
<td></td>
</tr>
<tr>
<td>$140$</td>
<td></td>
</tr>
<tr>
<td>$150$</td>
<td></td>
</tr>
</tbody>
</table>
Order price impacts

- When a trade occurs, there is new public information: the trade itself (and, usually, its direction).
- The market interprets the order direction as a signal of private information, and adjusts the price accordingly.
- The market does not know which traders are informed.
  - Analysis of the trades, even in retrospect, won’t tell us which traders (if any) were informed.
  - The market reaction tells us what the market thought, not the reality.

Uninformed orders move the market just as much as informed orders

- The price movements in response to uninformed orders aren’t warranted.
  - But they won’t be corrected until there’s authoritative information.
  - Or the sustained absence of new information.
Trading and creation of *new* information asymmetries.

- Suppose I’m an uninformed trader, and I lift the offer.
- The bid and offer rise. The market (and market maker, if there is one) think, “Maybe he had some positive information.”
- I know that I *don’t* have any information. My knowledge of my own ignorance gives me an advantage over the market.
- I know (and only I know) that bids and offers are higher than they should be.
- Can an uninformed trader move the market so as to give the appearance of information?

Open questions

- How long do informational asymmetries persist?
- How should we trade if we have a monopoly on better information?
- If there are multiple informed traders, will they quickly compete away their advantage?
- In the stock market, private information might come from a few people who have advance knowledge of an earnings announcement or a planned acquisition.
  - But government bond and FX markets also seem to exhibit order price impact.
  - What is “private information” in the FX or government bond market?