Order price impact and manipulation

- “Manipulation”
  - There are no universally accepted definitions in economics or law.
- Most definitions suggest something like
  - Trading to deliberately move the price, to establish an artificial price, a price that does not reflect true supply and demand.
  - Most manipulations involve deception.
These definitions aren’t precise

- It is almost impossible to trade without moving the price.
- Many accepted strategies attempt to obscure the trader’s true information, intentions and plans.

The following material illustrates certain possible manipulations based on order price impact.

The key question is, “For a given impact function can an uninformed trader execute a series of profitable buys and sells based on the price movements that his orders generate?”

Note: many of the schemes discussed here are illegal. They are presented to facilitate discussion of what features make a market prone to manipulation, so that, to the greatest extent possible, these features may be avoided in actual securities markets.

Recall the impact function used to analyze order splitting.

- \( p_t = p_{t-1} + \alpha + \lambda S_t + u_t \)
  - \( S_t \) is the net number of shares actively purchased in interval \( t \).
  - \( \lambda > 0 \) is the impact coefficient.
- We will look at strategies that start “flat” (with no position) and end flat.
  - Example: buy 10 shares using 10 orders of one share, then sell using 10 orders of one share, OR two orders of five shares, OR ...
- We’ll assume that \( \alpha = 0 \).
  - If \( \alpha > 0 \), just buy. Sell after the price has gone up.
  - If \( \alpha < 0 \), ...
- We’ll also ignore risk (\( u_t = 0 \))
- \( p_t = p_{t-1} + \lambda S_t \)
Attempted manipulation 1

- Starting at \( p_0 \), buy 5 shares slowly, one at a time.
  - \( p_1 = p_0 + \lambda \times 1 = p_0 + \lambda \)
  - \( p_2 = (p_0 + \lambda) + \lambda \times 1 = p_0 + 2\lambda \)
  - ...
  - \( p_5 = p_0 + 5\lambda \)
  - Average purchase price is \( \frac{p_1 + p_2 + \cdots + p_5}{5} = p_0 + 3\lambda \)
- Now sell the shares, one at a time
  - \( p_6 = p_5 - \lambda \times 1 = p_0 + 5\lambda - \lambda = p_0 + 4\lambda \)
  - \( p_7 = p_6 - \lambda \times 1 = p_0 + 4\lambda - \lambda = p_0 + 3\lambda \)
  - ...
  - \( p_{10} = p_9 - \lambda \times 1 = p_0 + \lambda - \lambda = p_0 \)
  - Average sale price is \( \frac{p_6 + p_7 + \cdots + p_{10}}{5} = p_0 + 2\lambda \)
- The average profit per share is \( -(p_0 + 3\lambda) + (p_0 + 2\lambda) = -\lambda \) (a loss)
- The receipts don’t cover the expenditure. The manipulation doesn’t work.

Attempted manipulation 2

- Buy the five shares slowly (as before)
  - Average price is \( p_0 + 3\lambda \)
- Sell the shares all at once:
  - \( p_6 = p_5 - \lambda \times 5 = p_0 + 5\lambda - 5\lambda = p_0 \)
- Manipulation profits are \( -(p_0 + 3\lambda) - p_0 = -3\lambda \)
- This, too, leads to a loss.
“Theorem”

- If the price impact function is linear and constant over time, profitable manipulation isn’t possible.

- Are there non-linear or time-varying price impact functions that allow for manipulation?
- Yes

Time variation in impact

- Suppose that initially $\lambda = 1$, and we know that it will drop to $\lambda = 0.1$.
- Then we can buy two units
  - $p_1 = p_0 + \lambda \times 1 = p_0 + 1$
  - $p_2 = p_1 + \lambda \times 1 = p_0 + 2$
  - Average share price is $p_0 + 1.5$
- ... and sell them when $\lambda = 0.1$
  - $p_3 = p_2 - \lambda \times 1 = p_2 - 0.1 = p_0 + 1.9$
  - $p_4 = p_3 - \lambda \times 1 = p_0 + 1.8$
  - Average share price is $p_0 + 1.85$
- Manipulation profits are $- (p_0 + 1.5) + p_0 + 1.85 = 0.35 > 0$
Asymmetry in the impact function

- Suppose that $\lambda$ for buys is $\lambda_{Buy} = 0.1$ and $\lambda$ for sells is $\lambda_{Sell} = 1$.
- We (short) sell two shares
  - $p_1 = p_0 - \lambda_{Sell} \times 1 = p_0 - 1$
  - $p_2 = p_1 - \lambda_{Sell} \times 1 = p_0 - 2$
  - Average price is $p_0 - 1.5$
- Now we cover our short sales
  - $p_3 = p_2 + \lambda_{Buy} \times 1 = p_2 - 2 + .1 = p_0 - 1.9$
  - $p_4 = p_3 + \lambda_{Buy} \times 1 = p_0 - 1.8$
  - Average price is $p_0 - 1.85$
- Manipulation profits are $(p_0 - 1.5) - (p_0 - 1.85) = 0.35 > 0$

General structure of manipulations

- To establish the position, first trade to maximize the price impact.
  - This doesn’t necessarily mean “buy”; sometimes the initial position is short.
- To unwind the position, trade to minimize the price impact.
- Time variation
- Asymmetry
Nonlinearities in the impact function ...

The concave case, for purchases

Suppose that 
\[ p_t = \begin{cases} 
  p_{t-1} + \lambda \sqrt{S_t} & \text{for buy orders, } S_t > 0 \\
  p_{t-1} - \lambda \sqrt{-S_t} & \text{for sell orders, } S_t < 0 
\end{cases} \]

The average price for a purchase of \( q^* \) shares is \( p^*/q^* \).

This is lower for large traders.
A series of small trades vs. one large trade

Small trades: high impact

Large trade: lower impact

Concave example

- Suppose that \( p_t = \begin{cases} p_{t-1} + \lambda \sqrt{S_t} & \text{for buy orders, } S_t > 0 \\ p_{t-1} - \lambda \sqrt{-S_t} & \text{for sell orders, } S_t < 0 \end{cases} \)

- Buy 8 units with eight 1-unit trades
- Sell 8 units with two 4-unit trades
Purchases
- \( p_1 = p_0 + \lambda \times \sqrt{1} = p_0 + \lambda \)
- ...
- \( p_8 = p_7 + \lambda \times \sqrt{1} = p_0 + 8 \lambda \)
- Average purchase price is \( p_0 + 4.5 \lambda \)

Sales
- \( p_9 = p_8 - \lambda \times \sqrt{4} = p_0 + 6\lambda \)
- \( p_{10} = p_9 - \lambda \times \sqrt{4} = p_0 + 4\lambda \)
- Average sale price is \( p_0 + 5 \lambda \)

Profits per share are \((p_0 + 5\lambda) - (p_0 + 4.5 \lambda) = 0.5 \lambda > 0\)

The convex case, for purchases

Will the same buy-small, sell-large manipulation work?
Suppose that $p_t = \begin{cases} p_{t-1} + \lambda S_t^2 & \text{for buy orders, } S_t > 0 \\ p_{t-1} - \lambda S_t^2 & \text{for sell orders, } S_t < 0 \end{cases}$

- We’ll buy eight shares with two trades of 4 shares.
- Sell eight shares with eight sales of 1 share.

Purchases
- $p_1 = p_0 + \lambda \times 4^2 = p_0 + 16\lambda$
- $p_2 = p_1 + \lambda \times 4^2 = p_0 + 32\lambda$
- Average purchase price is $p_0 + 24\lambda$

Sales
- $p_3 = p_2 - \lambda \times 1 = p_0 + 31\lambda$
- $p_4 = p_3 - \lambda \times 1 = p_0 + 30\lambda$
- ...
- $p_{10} = p_9 - \lambda \times 1 = p_0 + 24\lambda$
- Average sale price is $p_0 + 27.5\lambda$

Manipulation profits are $3.5\lambda$ per share
Is manipulation really possible when the price impact function is non-linear, buy-sell asymmetric, or time varying?

- Other costs (like bid-ask spread, commissions) might reduce profits.
- There are risks:
  - We don’t know for sure what the price impact function looks like.
  - Prices change for reasons other than incoming orders.
- What is the empirical evidence?

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The square-root “law”

- Many practitioners and academics believe that price impact goes up with the square root of order size.
  - Example: \( p_t = p_{t-1} + \lambda \sqrt{S_t} \)
- This seems to fit many samples of financial data. Typically:
  - A broker looks at the price impact of all its customer orders.
  - A hedge fund looks at the price impact of all of its own orders.
Manipulations

- Manipulations based on the order impact model involve only the direct effects of our orders.
- Other traders may have strategies that are indirectly affected by our trades.
- Momentum traders buy when they think an upward price trend has just started.
  - ... and sell when they think the trend has stopped.
  - Can a manipulator start a trend?

“Pump and Dump”

- The manipulator starts a trend, the trend draws in momentum traders, the trend accelerates, the manipulator reverses his trades.
- The trend can be started by
  - A series of large trades in one direction.
  - Prearranged wash sales. I buy 100 shares from my partner at $10.00; she buys them from me at $10.10; I buy them from her at $10.20 ... Prearranged trades are generally illegal.
  - The trend can be “validated” by spreading rumors in blogs and social media.
- Momentum traders buy when the price goes up and sell when the price drops.
  - But so do traders who are trying to hedge a short position in a call option.
Spoofing and layering

- Spoofing: entering a bid or offer that is not intended for execution.
- Layering: entering *large* bids/offers not intended for execution *priced away* from the market.
- Priced away: a buy limit order priced below the bid or a sell limit order priced above the offer.
- Why?

BATS book in PBR (Petrobras) on April 27, 2015

- Large quantities at the best bid and offer and away from the best bid an offer.
- Conveys the sense of a liquid market.
Suppose we suddenly place an order to sell 20,000 shares at 85.35.

What inferences would the market draw?

What action might ensue?

Layering case brought by the Commodities Futures Trading Commission (civil suit) and the Department of Justice (criminal prosecution), filed April 2015.

*Read pp. 1-5*

The complaint alleges ...

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**US v. Nav Sarao Futures Limited PLC and Navinder Singh Sarao**

Layering case brought by the Commodities Futures Trading Commission (civil suit) and the Department of Justice (criminal prosecution), filed April 2015.

*Read pp. 1-5*

The complaint alleges ...
Defendants engaged in a massive effort to manipulate the Chicago Mercantile Exchange's (CME's) E-mini S&P 500 futures contract ... by utilizing a variety of exceptionally large, aggressive, and persistent spoofing tactics.

Beginning in June 2009, Defendants schemed to design and utilize an automated system to manipulate the E-mini S&P price to their benefit. ... Navinder Sarao (Sarao) and/or his company Nav Sarao Futures Limited PLC (Sarao Futures), ..., used this automated system, as well as a variety of manual techniques, to place, modify, and cancel hundreds of thousands of orders with no intention of executing such orders so as to affect the E-mini S&P price such that Defendants could profit from their other trading.

U.S. CFTC v. IGOR B. OYSTACHER and 3 RED TRADING LLC

Read p. 1-4; filed Oct 19, 2015, the complaint alleges that

From December 2011 through at least January 2014, Igor B. Oystacher and the company he owns and controls, 3 Red Trading LLC, intentionally and repeatedly engaged in a manipulative and deceptive spoofing scheme while placing orders for and trading futures contracts.
- Oystacher and 3 Red engaged in this scheme by manually placing large ... passive order(s) on one side of the market at or near the best bid or offer price, which were intended to be canceled before execution.

- Defendants placed these order(s) (the "spoof orders") ... to create the false impression of market depth and book pressure in a certain direction (to either buy or sell) and induce other market participants into placing orders on the same side of the market and at similar price levels as the spoof orders.

- The Defendants would then cancel ... all of the spoof order(s) before they were executed and virtually simultaneously "flip" their position from buy to sell (or vice versa) by placing at least one aggressive order on the other side of the market at the same or better price to trade with market participants that had been induced to enter the market by the spoof orders they just canceled.

- This strategy allowed Defendants to buy or sell futures contracts in quantities and/or at price levels that would not have otherwise been available to them in the market, absent the spoofing conduct.