Eurodollar Futures

Concepts
- Eurodollar Futures (EDF)
- Futures rate
- Convexity adjustment

Reading
- Veronesi, Chapter 6
- Tuckman, Chapter 17
- Sundaresan, Chapter 15
Eurodollar Futures (EDF)

- Eurodollar futures are cash-settled futures contracts with final futures price based on three-month LIBOR at the expiration date: 
  \[ G(T) = 100(1 - 7^{T+.25}) \]
- For example, if 3-month LIBOR is 1% on the futures expiration date, the EDF price is 99.00.
- Contracts are based on $1,000,000 par, but marked to market based on the change in the unannualized rate. I.e., each one basis point change in the EDF price induces a mark-to-market of $25 = 1,000,000 \times 0.0001/4.

The Eurodollar Futures Market

- EDFs are traded on the Chicago Mercantile Exchange.
- For quotes and contract specifications, see http://www.cmegroup.com/trading/interest-rates/stir/eurodollar_quotes_globex.html
- Contracts with expiration dates every month for nearest 6 months, and then every quarter March, June, September, December, out ten years.
- Contracts with expiration up to three years are very liquid. These futures prices form the basis for calibrating the short end of the LIBOR term-structure for LIBOR-based derivative pricing models. LIBOR swap rates are used for the long end of the LIBOR term structure.
Example

- Let’s consider a stylized example of an EDF based on the 0.5-year riskless rate \( r_{T+0.5} \) in our model.
- Suppose the contract expires at time 0.5.

\[
\begin{array}{c|c}
\text{Time 0} & \text{Time 0.5} \\
94.6375 & 93.996 = 100-6.004 \\
95.279 = 100-4.721 & \\
=0.5(93.996+95.279) \\
\end{array}
\]

The Futures Rate

- Define the futures rate as
  \[
g = \frac{(100-\text{EDF price})}{100}.
\]
- The time 0 futures rate for this contract is
  \[
g_{0.5} = \frac{(100-94.6375)}{100} = 5.3625\%
\]

- Class Problem: What is the forward rate \( f_{0.5} \)?
The Convexity Adjustment (I)

- The futures rate is higher than the corresponding forward rate. Thus, to extract forward rates from EDF rates, it is necessary to make an adjustment commonly called the “convexity adjustment.”
- The difference arises for two reasons. Here is one:
  - The futures rate is the risk-neutral expected future rate:
    \[ G_T^{T+0.25} = E\{100(1-r_{T+0.25})\} \Rightarrow g_T^{T+0.25} = E\{r_{T+0.25}\} \]
  - Similarly, in our stylized example, \[ g_T^{T+0.5} = E\{r_{T+0.5}\} \].
- But for \( T = 0.5 \), which is one period out in our model,
  \[ 1/(1+f_{0.5}/2) = F_{0.5} < d_1/d_{0.5} = E\{0.5d_1\} = E\{1/(1+0.5r_1/2)\} \]
  \[ > 1/(1+E\{0.5r_1/2\}) \] because \( 1/(1+0.5r_1/2) \) is convex in \( 0.5r_1 \)
  \[ \Rightarrow f_{0.5} < E\{0.5r_1\} = g_{0.5} \].

The Convexity Adjustment (II)

- For expiration dates farther out, there is the additional effect of marking to market.
- For example, in a FRA, all the “marking-to-market” \( f-r \) comes at the end.
- In the EDF, the sum of marks-to-market are
  \[ g(0)-g(1) + g(1)-g(2) + \ldots + g(T-1)-r= g(0)-r \]
  but negative marks are reinvested at higher rates, while positive marks are reinvested at lower rates, so the futures rate \( g \) must be higher than the forward rate \( f \) to compensate for this adverse effect.
- This is the same as the way that the marking to market in a bond futures contract makes the futures price lower than the forward price of the underlying bond.
Class Problem

- Consider again a stylized example of a EDF based on the 0.5-year riskless rate $r_{1.5}$ in our model.
- Suppose the contract expires at time 1 and the contract is marked to market every 0.5 years.
- Fill in the tree of EDF prices below:

<table>
<thead>
<tr>
<th>Time 0</th>
<th>Time 0.5</th>
<th>Time 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Class Problems…

- What is the time 0 futures rate $g_{1.5}$?

- What is the time 0 forward rate $f_{1.5}$?