Treasury Bond Futures

Concepts and Buzzwords

- Basic Futures Contract
- Futures vs. Forward
- Delivery Options
- Underlying asset, marking-to-market, convergence to cash, conversion factor, cheapest-to-deliver, wildcard option, timing option, end-of-month option, implied repo rate, net basis

Reading

- Veronesi, Chapters 6 and 11
- Tuckman, Chapter 14
Basic Futures Contract

- In a basic futures contract without delivery options, the buyer agrees to take delivery of an underlying asset from the seller at a specified expiration date $T$.

- Associated with the contract is the futures price, $G(t)$, which varies in equilibrium with time and market conditions.

- On the expiration date, the buyer pays the seller $G(T)$ for the underlying asset.

Marking to Market and Contract Value

- Each day prior to the expiration date, the long and short positions are marked to market:
  - The buyer gets $G(t) - G(t - 1 \text{ day})$.
  - The seller gets $-(G(t) - G(t - 1 \text{ day}))$.

- It costs nothing to get into or out of a futures contract, ignoring transaction costs.

- Therefore, in equilibrium, the futures price on any day is set to make the present value of all contract cash flows equal to zero.
Marking to Market...

- Consider buying the contract at any time \( t \) and selling it after just one day.

- It essentially costs nothing to buy and sell the contract, so the payoff from this strategy is just the profit or loss from the marking to market: \( G(t+1 \text{ day}) - G(t) \).

- \( G(t + 1 \text{ day}) \) is random.

- \( G(t) \) is set today to make the market value of the next day’s random payoff \( G(t+1 \text{ day}) - G(t) \) equal to zero.

Marking to Market...

- The market value of the random mark-to-market, \( G(t + 1 \text{ day}) - G(t) \), is the cost of replicating that payoff.

- We can represent that cost in the usual way as its discounted expected value under the risk-neutral probability distribution.

- To make this market value zero, today’s futures price must be the expected value of tomorrow’s futures price under the risk-neutral probability distribution:

\[
E_t \{ e^{r(t+1 \text{ day})} [G(t + 1 \text{ day}) - G(t)] \} = 0
\]

\[
=> G(t) = E_t \{ G(t + 1 \text{ day}) \}.
\]
Convergence to Cash

Consider entering the futures contract the instant before it expires.

The long position would instantly pay the futures price and receive the underlying asset.

The payoff would be $V(T) - G(T)$, where $V(T)$ is the spot price of the underlying on the expiration date.

In the absence of arbitrage, since it costs nothing to enter into either side of the contract, the (known) payoff must be zero:

$G(T) = V(T)$.

Determining the Futures Price Ignoring Delivery Options

Consider a “basic” futures contract on a bond.

To determine the current futures price, $G(0)$,

- we start at the expiration date of the futures, when the futures price is equal to the spot price of the underlying bond,

- then work backwards each mark-to-market date to determine the futures price that makes the next marking to market payoff worth zero.
**Example**

- Consider a futures on a 6%-coupon bond maturing at time 2.
- The futures expires at time 1.
- The futures contract is marked to market every 6 months.

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**Class Problem: Time 1 Price of the Underlying 6% Bond Maturing at Time 2**

<table>
<thead>
<tr>
<th>Time 0</th>
<th>Time 0.5</th>
<th>Time 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{uu}(1) = ?$</td>
<td>$V_{ud}(1) = ?$</td>
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Treasury Bond Futures
Class Problem: Futures Prices

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<td>$G(m)(1) =$ ?</td>
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Marking to Market: If you bought at time 0 and sold at time 1, what would be the cash flows you would receive along the path up-down?

Class Problem: SR $Duration of a Futures Contract

What is the SR $duration of this futures contract?
Class Problem:  
**Forward Price of 6% Bond Maturing at Time 2, for Settlement at Time 1**

For comparison, what would be the forward price negotiated at time 0 to pay at time 1 for the 6% bond maturing at time 2?

(Recall: the forward price is the spot price of the underlying, minus the pv of any payments prior to the settlement date, plus interest to the settlement date.)

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**Futures Price vs. Forward Price**

- When there are no further marks to market remaining before the expiration date of the contract, the forward price and futures price are the same.

- If interest rates are uncorrelated with the value of the underlying asset, then the forward price and futures price are the same. (May be reasonable to assume with stock index futures or commodity futures.)

- When the underlying asset is a bond, its value is negatively correlated with interest rates. This makes the futures price lower than the forward price. Why?
Futures Price < Forward Price

- The profit or loss from the forward contract is \( V(T) - F(0) = F(T) - F(0) \), which is received all at the end, at time \( T \), and \( \text{NPV}[F(T) - F(0)] = 0 \).

- The cumulative profit or loss from the futures contract is \( V(T) - G(0) = G(T) - G(0) \), but this is paid out intermittently through marks to market.

- Consider reinvesting all gains and losses from marking to market to the expiration date.

- Gains would be reinvested at low rates, losses at high rates, so to make the NPV equal to zero, the futures price must start out lower than the forward price.

- Example with expiration at \( T=1 \) and also marking-to-market at time 0.5:

\[
\text{Reinvested futures profit} = (G(0.5)-G(0)) (1 + 0.5r_1/2) + V(T) - G(0.5)
\]

\[
= V(T) - G(0) + (G(0.5)-G(0)) 0.5r_1/2.
\]

- \( \text{NPV}[G(0.5)-G(0) 0.5r_1/2] < 0 \) because of negative correlation between rates and marks-to-market.

- So \( \text{NPV}(V(T) - G(0)) > 0 \) to make total NPV = 0. So \( G(0) < F(0) \).

Exchange-Traded Interest Rate Futures Contracts

- Traded on the Chicago Board of Trade (CBOT) or the Chicago Mercantile Exchange (CME). For contract specifications see www.cmegroup.com/trading/interest-rates.

- Contracts expire in March, June, September, or December.

- Contracts on various assets include:
  - 5- and 10-year Treasury notes, 30-year Treasury bonds, and Ultra T-bonds, $100,000 par, CBOT
  - 2- and 3-year Treasury notes, $200,000 par, CBOT
  - 5-, 7-, 10-, and 30-year interest rate swaps, $100,000 Notional, CBOT
  - 13-week Treasury bills, $1,000,000 par, CME
  - Eurodollar futures (LIBOR), $1,000,000 par, CME
Exchange-Traded Futures Contracts...

* All buyers and sellers trade with a clearing corporation associated with each exchange so there is no counterparty risk. The marking-to-market provision limits the credit risk faced by the clearing corporation.

* Commissions on futures contracts are about $25 round-trip or less, and fully negotiable.

* Upon entering into a futures contract, the investor must post initial margin, which is interest-bearing. If the balance in the margin account falls below the maintenance margin, the investor must post variation margin to restore it to its initial level.

Delivery Options

* In practice, futures contracts give the seller various delivery options which make it very different than a forward contract.
  
  – Quality option: The seller can deliver any bond with maturity in a given range using a conversion factor.
  
  – Timing option: The seller can deliver any time during the expiration month
  
  – Wildcard option: The futures exchange closes early in the afternoon, but bonds keep trading. The seller can announce delivery any time until bond markets close.
  
  – End-of-month option: The futures stop trading 8 business days before the end of the month.

* The delivery options reduce the equilibrium futures price.
Treasury Bond Futures and the Quality Option

* The seller has the option to deliver any bond with at least 15 years to call or maturity.

* Each deliverable bond has a publicized conversion factor equal to the price of $1 par of the bond at a yield of 6%.

* If the seller delivers a given bond, he receives the futures price, times the conversion factor, plus accrued interest.

* The seller’s net cash flow from delivering is

\[ G \times CF(i) - \text{Price of Bond } i \]

Cheapest-to-Deliver with No Conversion Factors:

Suppose all bonds have a 6% coupon

* All bonds with a 6% coupon have conversion factor equal to 1.

* The seller’s payoff from delivering is \( G - \text{Price of bond } i \)

* The seller wants to deliver the cheapest bond. The cheapest-to-deliver will be:

  * a long-maturity bond when rates are higher than 6%,
  * a short-maturity bond when rates are lower than 6%.

* In equilibrium, the seller’s delivery payoff must be zero, since the contract can be sold before delivery at no cost.

* So the futures price at expiration will be the price of the cheapest-to-deliver: \( G(T) = \text{Price of cheapest-to-deliver} \)
Consider a futures contract expiring at time 1.

The seller can deliver either of two bonds.
- Bond 1: 6%-coupon bond maturing at time 1.5
- Bond 2: 6%-coupon bond maturing at time 2.

Cheapest-to-Deliver on Expiration Date (Time 1)

Zero rates: \( r_{1.5} = 6.915\% \), \( r_2 = 6.968\% \)
Price of Bond 1 = 99.5578, Price of Bond 2 = 99.0814
\[ G(1) = \min(99.5578, 99.0814) = 99.0814 \]
(bond 2 is cheapest-to-deliver)

Zero rates: \( r_{1.5} = 5.437\% \), \( r_2 = 5.479\% \)
Price of Bond 1 = 100.274, Price of Bond 2 = 100.501
\[ G(1) = \min(100.274, 100.501) = 100.274 \]
(bond 1 is cheapest-to-deliver)

Zero rates: \( r_{1.5} = 4.275\% \), \( r_2 = 4.308\% \)
Price of Bond 1 = 100.8443, Price of Bond 2 = 101.639
\[ G(1) = \min(100.8443, 101.639) = 100.8443 \]
(bond 1 is cheapest-to-deliver)
Futures Price with Quality Option

<table>
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<tr>
<td></td>
<td></td>
<td>99.0814</td>
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<td><img src="image1.png" alt="image" /></td>
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- The time 0 futures price with only bond 2 deliverable was 100.4306.
- When the seller has the option of delivering bond 1 instead, the futures price is lower.

Convexity of the Futures Price with the Quality Option

As rates fall, the futures price rises, but it starts tracking a lower duration bond. As rates rise, the futures price starts tracking a high duration bond. This switching of the underlying asset gives the futures price negative convexity.
Cheapest-to-Deliver with Conversion Factors: All bonds deliverable, not just 6% bonds

*If the yield curve were flat at 6% (and all bonds were noncallable) then the conversion factors would be “perfect” and the seller would be indifferent about which bond to deliver.

*Otherwise, the conversion factor is not “perfect”, and one bond becomes “cheapest to deliver.”

*In particular, the seller wants to maximize his profit at delivery:

\[-G*\text{Conversion Factor} - \text{Flat Price of Bond}\]

The Futures Price and the Cheapest-to-Deliver with All Bonds Deliverable (Conversion Factors)

*At expiration the seller will deliver the bond that maximizes the proceeds of delivery:

\[\max G*CF(bond \ i) - \text{Price(bond } i)\]

*In equilibrium, the proceeds from delivery must be zero, because it costs nothing to sell a contract.

*Therefore, \[\max G*CF(i) - \text{Price}(i) = 0.\]

*It turns out that this implies

\[G = \min \text{Price}(i)/CF(i)\]

at expiration.
Which bond is cheapest-to-deliver?

*A variety of factors determine which bond is cheapest-to-deliver, mainly yield level and bond duration effects.

*On the delivery date, the cheapest-to-deliver is the bond with the minimum Price/Conversion Factor ratio. Letting \( y \) denote a given bond’s yield, note that

\[
P/CF = \frac{P(y)}{P(6\%)} \sim 1 - \text{Duration}(6\%) \ast (y - 6\%)
\]

So if the yield curve is flat above 6% (say at 7%), then high duration bonds will be cheapest to deliver. Conversely, if the yield curve is flat below 6% (say at 5%) , then low duration bonds will be cheapest to deliver.

*When the yield curve is not flat, relatively higher yielding bonds look cheaper-to-deliver.

Example with two 5.5% bonds

*Consider a futures contract expiring at time 1.

*The seller can deliver either of two bonds.
  - Bond 1: 5.5%-coupon bond maturing at time 1.5
  - Bond 2: 5.5%-coupon bond maturing at time 2.

*The conversion factor for each bond equals the price of $1 par to yield 6%:
  - 0.9976 for bond 1
  - 0.9952 for bond 2.
Cheapest-to-Deliver on the Expiration Date (Time 1)

Zero rates: $r_{1.5} = 6.915\%$, $r_2 = 6.968\%$

Price of Bond 1 = 99.3162, Price of Bond 2 = 98.6063

$G(1) = \min(99.3162/0.9976, 98.6063/0.9952) = 98.6063/0.9952 = 99.0802$ (bond 2 is cheapest-to-deliver)

Zero rates: $r_{1.5} = 5.437\%$, $r_2 = 5.479\%$

Price of Bond 1 = 100.0306, Price of Bond 2 = 100.0207

$G(1) = \min(100.0306/0.9976, 100.0207/0.9952) = 100.0306/0.9976 = 100.274$ (bond 1 is cheapest-to-deliver)

Zero rates: $r_{1.5} = 4.275\%$, $r_2 = 4.308\%$

Price of Bond 1 = 100.5995, Price of Bond 2 = 101.1547

$G(1) = \min(100.5995/0.9976, 101.1547/0.9952) = 100.5995/0.9976 = 100.8443$ (bond 1 is cheapest-to-deliver)

Futures Price

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<tr>
<td>0.5(99.0802+100.2740) + 99.6771</td>
<td>99.0802</td>
<td>99.0802</td>
</tr>
<tr>
<td>= 100.1181</td>
<td>= 100.2740</td>
<td>= 100.2740</td>
</tr>
<tr>
<td>0.5(100.274 + 100.8443) + 100.5591</td>
<td>100.8443</td>
<td>100.8443</td>
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</table>
**Implied Repo Rate**

- Prior to the delivery date, some practitioners identify the cheapest-to-deliver bond as the one with the highest “implied repo rate.”
- The implied repo rate is the hypothetical rate of return earned from buying a deliverable bond, selling the futures, and then delivering the bond into the futures contract on an assumed date (ignoring marking to market, treating the futures like a forward).
- The implied repo rate is typically below the bond’s market repo rate because the seller of the futures can exploit other options (wildcard, end-of-month) as well.

**Net Basis**

- An alternative approach is to choose bond with the minimum “net basis.”
- This is the hypothetical loss incurred by buying the bond, financing the purchase in the repo market, selling the futures, and delivering the bond into the futures contract on an assumed delivery date. Again this ignores marking-to-market, treating the futures like a forward.
- This is typically negative, because the seller of the futures contract can exploit other delivery options (e.g., wildcard, end-of-month) as well.