Options

Concepts and Buzzwords

- Put-Call Parity
- Volatility Effects
- Call, put, European, American, underlying asset, strike price, expiration date

Readings

- Tuckman, Chapter 19
- Veronesi, Chapter 6
**Call Option**

- A *European* call option is a contract that gives the owner the *right* but not the *obligation* to buy
- an underlying asset
  - at a pre-specified *strike price*
  - on a pre-specified *expiration date*.
- An *American* call option gives the owner the right to buy the asset at the strike price any time on or before the expiration date.

**Put Option**

- A *European* put option is a contract that gives the owner the right but not the obligation to *sell*
- an underlying asset
  - at a pre-specified strike price
  - on a pre-specified expiration date.
- An *American* put option gives the owner the right to sell the asset at the strike price any time on or before the expiration date.
Call Payoff

Let $V_T$ represent the value of the underlying asset on the expiration date $T$.

Consider the payoff of a European call option with strike price $K$.

- If the underlying is worth more than $K$ at expiration, the option holder should exercise the option and buy the asset for $K$, for a net payoff of $V_T - K$.
- If the underlying is worth less than $K$, the option holder should leave the option unexercised, for a net payoff of 0.

To summarize, call payoff = max($V_T - K$, 0)

Put Payoff

Now consider the payoff of a European put option with strike price $K$.

- If the underlying is worth less than $K$ at expiration, the option holder should exercise the option and sell the asset for $K$, for a net payoff of $K - V_T$.
- If the underlying is worth more than $K$, the option holder should leave the option unexercised, for a net payoff of 0.

To summarize, put payoff = max($K - V_T$, 0)
Put-Call Parity: Payoffs

- Consider a European call and a European put on the same underlying asset with the same strike price and the same expiration date.
- Math identity: \( \text{Max}(V_T - K, 0) = \text{Max}(K - V_T, 0) + V_T - K \)
- I.e., Call payoff = Put payoff + \( V_T - K \)
- Verify:
  - If \( V_T > K \), then the call is in the money, with a payoff of \( V_T - K \), and the put is out of the money with a payoff of zero, so the payoff equation is satisfied:
    \[ V_T - K = 0 + V_T - K. \]
  - If \( V_T < K \) then the call is out of the money, the put is in the money, and the equation still holds:
    \[ 0 = K - V_T + V_T - K. \]

Put-Call Parity: Prices

- Suppose the underlying asset pays no cash flows before the option expiration date.
- Then the payoff of the call is the same as the payoff of a portfolio consisting of
  - the put
  - the underlying asset
  - a short position in the riskless zero with par value \( K \).
- Therefore, in the absence of arbitrage, the current price of the call must equal the current value of the portfolio:
  \[ C = P + V - d_T K. \]
+ **Example: Options on a Zero**

- Consider European call and put options on $100 par of a zero maturing at time 2.
- The options expire at time 1.
- The strike price of the options is $95.
- We’ll use the bond market model developed last lecture to price these options.
- At each node below, zero prices are listed in order of maturity.

<table>
<thead>
<tr>
<th>Time 0</th>
<th>Time 0.5</th>
<th>Time 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.973047</td>
<td>0.970857</td>
<td>0.966581</td>
</tr>
<tr>
<td>0.947649</td>
<td>0.941787</td>
<td>0.933802</td>
</tr>
<tr>
<td>0.922242</td>
<td>0.913180</td>
<td>0.973533</td>
</tr>
<tr>
<td>0.897166</td>
<td>0.976941</td>
<td>0.947382</td>
</tr>
<tr>
<td></td>
<td>0.976941</td>
<td>0.930855</td>
</tr>
<tr>
<td></td>
<td>0.953790</td>
<td>0.979071</td>
</tr>
<tr>
<td></td>
<td>0.993085</td>
<td>0.958270</td>
</tr>
</tbody>
</table>

+ **Class Problem: Call Valuation**

Fill in the tree of prices of the call option.

<table>
<thead>
<tr>
<th>Time 0</th>
<th>Time 0.5</th>
<th>Time 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = ?$</td>
<td>$C_u = ?$</td>
<td>$C_{uu} = ?$</td>
</tr>
<tr>
<td></td>
<td>$C_d = ?$</td>
<td>$C_{ud} = ?$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_{dd} = ?$</td>
</tr>
</tbody>
</table>
Class Problem: Put Valuation

Fill in the tree of prices of the put option.

<table>
<thead>
<tr>
<th>Time 0</th>
<th>Time 0.5</th>
<th>Time 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P = ?$</td>
<td>$P_u = ?$</td>
<td>$P_{uu} = ?$</td>
</tr>
<tr>
<td></td>
<td>$P_d = ?$</td>
<td>$P_{ud} = ?$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$P_{dd} = ?$</td>
</tr>
</tbody>
</table>

Class Problem: Put-Call Parity

1) Verify that put-call parity holds at time 0.

2) Verify put-call parity at time 0.5, up.

3) Verify put-call parity at time 0.5, down.
**Option Prices and Volatility**

- Consider changing the volatility of the bond market, holding the term structure constant.
- The higher the volatility of the underlying asset, the higher the value of both call and put options.
- Why? We can see more extreme bond prices will make the payoff of the (mostly out-of-the-money) call uniformly higher, so it’s risk-neutral expected value will be greater.
- By put-call parity, the put value must therefore be higher, too.

**Graphical Illustration of the Volatility Effect**

- Suppose that under distribution 1, the future asset price will be VL or VH with equal probability.
- Under the higher volatility distribution 2, the future asset price will be either VLL or VHH with equal probability.
- Then the expected option payoff will be higher under distribution 2.
Example: Increasing Volatility to $\sigma = 0.25$

- Suppose we increase the volatility parameter $\sigma$ from 0.17 to 0.25, and recalibrate the model to match the original term structure ($m_1 = -0.0955$, $m_2 = 0.0273$, and $m_3 = 0.003$).
- The resulting tree of prices of zeroes out to 2 years is below.

<table>
<thead>
<tr>
<th>Time 0</th>
<th>Time 0.5</th>
<th>Time 1</th>
<th>Time 1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.973047</td>
<td>0.969448</td>
<td>0.963274</td>
<td>0.9564</td>
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<tr>
<td>0.947649</td>
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<tr>
<td>0.922242</td>
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<td>0.973926</td>
<td>0.9780</td>
</tr>
<tr>
<td>0.897166</td>
<td>0.978350</td>
<td>0.981548</td>
<td>0.9780</td>
</tr>
<tr>
<td></td>
<td>0.956569</td>
<td>0.948128</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.934945</td>
<td>0.981548</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.963142</td>
<td></td>
</tr>
</tbody>
</table>

At each node, the top number is the call price and the bottom number is put price. Notice that both option prices are higher with higher volatility.

Call and Put Prices with $\sigma = 0.25$