Forward Rate Agreements

Concepts
- Forward Rate Agreement (FRA)
- Forward Contract
- Valuation
- FRAs and Swaps

Reading
- Veronesi, pp. 162-167
A forward rate agreement (FRA) is a contract between two counterparties to exchange a fixed interest payment for a floating interest payment on a single date.

- Large, liquid, over-the counter market.
- $47 trillion notional amount outstanding in 2009.
- Most contracts are linked to LIBOR or Eurobor.
- Contracts can be customized.

To take a simple example, consider a contract on a 0.5-year rate. The fixed receiver pays interest at some maturity date $t$ at the floating rate $r_t$ in exchange for interest at fixed rate $f$, on an agreed notional amount $N$.

There would be a single cash flow at time $t$. The fixed payer would pay the fixed receiver $N \times (f - t^{-0.5}r_t)/2$

Actually, in practice, the payoff is settled when it becomes known at time $t-0.5$ at $\left[ N \times (f - t^{-0.5}r_t)/2 \right]/(1 + t^{-0.5}r_t/2)$ but this does not change the value.
**FRA as Forward Contract on a Zero**

- The fixed receiver’s payoff \((f - \frac{t-0.5}{2}r_t)/2\) at time \(t\) is the same as \((1+f/2) - (1+\frac{t-0.5}{2}r_t)/2\) at time \(t\).

- This can be generated by:
  - buying \((1+f/2)\) par of a zero maturing at time \(t\),
  - selling 1 par of a zero maturing at time \(t-0.5\), and
  - selling a 0.5-year par bond at time \(t-0.5\):
    
    \[
    \begin{array}{c|c|c}
    0 & t-0.5 & t \\
    \hline
    +1 & -1 & -(1+\frac{t-0.5}{2}r_t)/2 \\
    \end{array}
    \]

- The trade at time \(t-0.5\) is self-financing, so the fixed receiver’s side is equivalent to:
  - long \((1+f/2)\) par of a zero maturing at time \(t\),
  - short 1 par of a zero maturing at time \(t-0.5\).

- Thus, the value of the FRA is \((1+f/2) d_t - d_{t-0.5}\)

- At inception, the fixed rate \(f\) is set to make the FRA worth zero, so
  
  \[
  (1+f/2) d_t - d_{t-0.5} = 0 \Rightarrow 1/(1+f/2) = d_t/d_{t-0.5} \\
  \Rightarrow f = f_{t-0.5}^t
  \]

- i.e., the forward rate is the fixed rate that makes the FRA worth zero.

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**FRAs and Forward Rates**

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  \]

- i.e., the forward rate is the fixed rate that makes the FRA worth zero.
FRAs and Swaps

- Clearly, a swap can be viewed as a portfolio of FRAs.
- Each swap cash flow is the cash flow of an FRA with fixed rate equal to the swap rate \( k \).
- Thus, there is a no-arbitrage relation between the LIBOR swap term structure and the term structure of forward rates in LIBOR-based FRAs.

Swap Rates in Terms of Forward Rates

- The FRA valuation shows that replacing each floating rate in the swap with the corresponding fixed forward rate leaves the swap value unchanged.
- Thus the swap rate \( k_T \) that makes the \( T \)-year swap worth zero must satisfy
  \[
  (k_T - f_{0.5})d_{0.5} + (k_T - f_{0.5})d_1 + \ldots + (k_T - f_{T-0.5})d_T = 0
  \]
  or
  \[
  k_T = (f_{0.5}d_{0.5} + f_{0.5}d_1 + \ldots f_{T-0.5}d_T)/(d_{0.5} + \ldots + d_T)
  \]
  i.e., the swap rate is the average forward rate weighted by the zero prices.
- This is an alternative par rate formula, equivalent to
  \[
  k_T = 2(1-d_t)/(d_{0.5} + \ldots + d_T)).
  \]