Modern macroeconomic theory describes the economy of a country (any country) as a collection of firms and households, which have well defined objectives (maximize their owners’ wealth in one case and maximize their life-time utility or satisfaction in the other) and pursue them by exchanging goods and services with other firms and households.

To start with, we will make many simplifying assumptions. The most obvious are that we will abstract from the existence of the rest of the world (i.e. we will study an economy in isolation) and that we will not consider the role of the government. Finally, you will notice that we won’t even mention money and monetary policy.

This is not to say that we believe the interaction with the rest of the world and the policies of government and central bank to be unimportant for macroeconomic outcomes. On the contrary. The idea is to start simple and then introduce those pieces of the puzzle later on.

A further simplification is that in this note we will abstract from uncertainty. We will re-introduce it later in the course.

The basic postulates of modern macroeconomic theory are that people are smart and react to incentives. People are smart in the sense that, either consciously or unconsciously, they understand the consequences of major events (e.g. changes in government policy, changes in oil prices, improvements in technology) on current and future levels of wages and interest rates (among these rates are returns on investment and borrowing rates such as mortgage rates). What do we mean when we say that people react to incentives? Well, we simply mean that households, given current and future values of wages and interest rates, are able to select those choices (how much to consume? how much to save? how much to work?) that maximize their level of satisfaction.

Here is a stylized view of the economy of a country in some year $t$. Firms use inputs such as capital ($k_t$) and labor ($l_t$) to produce goods and services. Let’s call firms’ output $y_t$. (From now on, capital letters will be only used to identify aggregate quantities)

The capital input (or capital stock) $k_t$ is the total value of plant and equipment used in production. Ideally, this measure is constructed by evaluating the single assets at their current market price and then summing the values so obtained. Equivalently,
we can value single assets at their purchase price and then depreciate their value over time to account for the decline in their productive efficiency. (A sort of economic amortization).

The ideal measure of labor input is the total number of hours worked. Sometimes, it is expressed as the number of people on the payroll. Since workers differ in their productivity, it may make sense to weigh hours by some measure of skill. At this point, this would unnecessarily complicate the picture. We leave it for later.

At the beginning of year $t$, the capital is given. This is because we assume it takes one year to adjust its level. In order to produce, firms will also have to hire labor. Then, capital and labor will imply a level of output $y_t$. Our first observation is that:

- firms express demand for labor services
- firms supply goods and services

What about households? At the beginning of year $t$, every households will have financial assets, which we call $a_t$, yielding a return $r_t$. We think of $a_t$ as the value of the household’s portfolio of securities: it can be as simple as the money invested in a checking account and as exotic as the value of a hedge fund or private equity fund holding. If an household’s assets are negative, it means that it is borrowing at the rate $r_t$. The total resources available to an household will be the sum of financial wealth, $a_t(1 + r_t)$, and labor income $w_t l_t$, where $w_t$ is the wage and $l_t$ is the amount of hours worked (supplied to the market). The decisions of the household will be: how much to consume of the goods and services available on the market (we call consumption expenditure $c_t$), how much to work ($l_t$), and how much to re-invest in financial assets, assets that will yield a return $r_{t+1}$. That is:

- households express demand for goods and services (demand for consumption)
- households supply labor services
- households supply financial capital

When a household has positive financial wealth, it is providing resources either to other households (in form of direct or intermediated loans) or to firms (in forms of equity or debt).

At year $t$, every firm will determine its desired level of capital for the following year, $k_{t+1}$. Summing the desired level of capital across firms gives the demand for capital for the next year, $K_{t+1}$. Aggregate investment will be $I_t = K_{t+1} - (1-\delta)K_t$. Therefore we can say that
• firms express a demand for capital
• firms express a demand for investment.

Notice that, while gross investment cannot be negative, net investment will be negative whenever gross investment falls short of depreciation.

The bottom line is that in pursuing their objectives, firms and households demand and supply several goods and services. These demands and supply meet in three markets: the market for labor, the market for capital, and the market for goods and services. In the labor market, firms and households exchange labor services. There will be equilibrium in that market when the demand for labor expressed by firms equals the supply expressed by households. Similarly, the capital market will be in equilibrium when the supply of capital from households (their total financial wealth) equals the demand for capital. In the goods market, the supply is given by the total value of the goods produced by firms. The market is in equilibrium when such supply equals the demand for investment (expressed by the firms themselves) and for consumption (expressed by households).

In this note, we will investigate in some detail the decision processes the lead firms to express demands for labor and capital goods, and those that lead households to determine labor supply, consumption demand, and savings.

Before doing any of that, however, we need to pause and introduce the useful conceptual tool known as production function. Differently from the aggregate production function introduced earlier in the course, we will be concerned with the individual production function of a single firm. Under certain conditions, we will be able to recover the former simply by aggregating the latter.

The production function

Economic organizations transform inputs (factories, office buildings, machines, labor with a variety of skills, intermediate inputs, and so on) into outputs. Boeing, for example, owns factories, hires workers, buys electricity and avionics, and uses them to produce aircraft. American Express’s credit card business uses computers, buildings, labor, and small amounts of plastic to produce payment services. Pfizer hires scientists, MBAs, and others to develop, produce, and market drugs. McKinsey takes labor and information technology to produce consulting services.

A production function is a mathematical relation between inputs and output that makes this idea concrete:

\[ y = A \cdot F(k, l), \]
The variable $A$ is known as Total Factor Productivity – TFP. It is a measure of how efficiently capital and labor are combined in the production process.

We now illustrate a couple of conditions that we impose on the function $F$.

- More input leads to more output. Or, in other words, the marginal products of capital and labor are positive. This assumption seems rather uncontroversial to us. In mathematical terms, the function $F$ increases in both $k$ and $l$:

$$\frac{\partial F}{\partial k} > 0, \quad \frac{\partial F}{\partial l} > 0.$$  

Consult Chapter 1 of the book if it is not clear to you why.

- Diminishing marginal products of capital and labor. An increase in labor for given capital leads to increases in output, but it does so at a decreasing rate: the more labor we add, the less additional output we get. This property is illustrated in Figure 1: for a given capital stock $\bar{k}$, increasing labor by $\Delta$ starting from $l_1$ has a larger effect on output than increasing labor by the same amount starting from $l_2$. That is: $AF(\bar{k}, l_1 + \Delta) - AF(\bar{k}, l_1) > AF(\bar{k}, l_2 + \Delta) - AF(\bar{k}, l_2)$. We believe that you’ll be rather comfortable with this assumption: increasing the number of workers in a factory without expanding it will probably increase output, but most probably at a decreasing rate. We assume that the same obtains when we increase capital for given labor. These conditions translate into properties of the second derivatives of the production function:

$$\frac{\partial^2 F}{\partial k^2} < 0, \quad \frac{\partial^2 F}{\partial l^2} < 0.$$  

TFP is related to other concepts you may be familiar with, but it is distinct from them. The most common measure of productivity is the ratio of output to labor input, i.e. the average product of labor. It differs from the marginal product of labor for the same reason that average cost differs from marginal cost: the average product is output produced in average by the units of labor in place, while the marginal product is the increment in output that derives from the addition of one unit to the existing ones. TFP measures the overall efficiency in transforming inputs into outputs. Mathematically, the three definitions are

- Average Product of Labor $= \frac{y}{l}$
- Marginal Product of Labor $= \frac{\partial y}{\partial l}$
- Total Factor Productivity $= \frac{y}{F(k, l)}$.

When $F(k, l) = k^\alpha l^{1-\alpha}$ (we call this the Cobb-Douglas production function), they are

- Average Product of Labor $= A(k/l)^\alpha$
- Marginal Product of Labor $= (1 - \alpha)A(k/l)^\alpha$
- Total Factor Productivity $= A$. 
Holding $A$ constant, the first two increase when we increase the ratio of capital to labor. The idea? You can be more productive if you have (say) more equipment to work with. TFP is an attempt to measure productivity independently of the amount of capital each worker has.

**Demand for labor, capital, and investment**

The purpose of this section is to characterize the main determinants of the aggregate (economy-wide) demand for labor, capital, and investment. In order to accomplish this task, we will study the decision problem common to all firms, i.e. the choice of the capital and labor input, given values for the interest rate and the wage rate. To determine the aggregate desired level of capital and labor, we just need to add up the single decisions. The desired level of capital next period, given the capital currently in place, will determine the demand for investment.

We are going to characterize the firm problem using a very simple framework. We will assume that firms produce output using the Cobb-Douglas production function

$$y_t = A_t F(k_t, l_t) = A_t k_t^\alpha l_t^{1-\alpha}.$$  

The wage rate is denoted by $w_t$. With $r_t$ we denote the opportunity cost of capital, i.e. the return its shareholders would earn if they invested in other financial assets rather than in the company.

If, for simplicity, the price of the product the company produces equals 1, then the value that it produces for the shareholders is given by

$$A_t F(k_t, l_t) - r_t k_t - w_t l_t.$$
Here is the first question: for given capital stock $k_t$, what is the optimal level of labor input for the firm? That is, how many workers will it hire? We know the answer: the firm will keep on hiring as long as the gain in revenues from increasing the labor factor is larger than the cost of doing so. The gain in revenues of adding $\Delta$ labor is simply

$$A_t F(k_t, l_t + \Delta) - A_t F(k_t, l_t) = \frac{A_t F(k_t, l_t + \Delta) - A_t F(k_t, l_t)}{\Delta}.$$ 

When $\Delta$ is small, we know that the gain in revenues is approximately

$$\frac{\partial A_t F(k_t, l_t)}{\partial l_t} \Delta.$$ 

What is the cost to the firm of increasing the labor input by $\Delta$? Simply $w_t \Delta$. Figure 2 illustrates the point: the desired level of labor is the one that maximizes profits. The bottom line is that the firm will keep on increasing labor as long as $\frac{\partial A_t F(k_t, l_t)}{\partial l_t} \Delta > w_t \Delta$ or $\frac{\partial A_t F(k_t, l_t)}{\partial l_t} > w_t$. It will stop once $\frac{\partial A_t F(k_t, l_t)}{\partial l_t} = w_t$.

![Figure 2: Determining labor demand.](image)

The latter condition tells us exactly how much labor the firm will want to hire, given a wage $w_t$. In the case of Cobb-Douglas production function, the condition is

$$(1 - \alpha)A_t k_t^{\alpha} l_t^{1-\alpha} = w_t.$$ 

Solving this equation for $l_t$ yields the firm’s optimal level of labor, which is also labor demand:

$$l_t = k_t \left[ \frac{A_t(1 - \alpha)}{w_t} \right]^{\frac{1}{\alpha}}.$$
The aggregate demand for labor services is the sum of the demands across all firms. Notice that, as common sense dictates, the aggregate demand for labor is decreasing in the wage level and increasing in the TFP level, $A_t$.

Now we can ask the next question: for given employed labor $l_{t+1}$, what is the desired amount of investment? Notice that, given the current level of capital $k_t$, this amounts to figuring out $k_{t+1}$, i.e. the desired level of capital in year $t+1$. We also know the answer to this one: the firm will keep on accumulating capital as long as the gain in revenues from increasing it is larger than the cost of doing so. The gain in revenues of adding $\Delta$ capital is simply

$$A_tF(k_{t+1} + \Delta, l_{t+1}) - A_tF(k_{t+1}, l_{t+1}) = \frac{A_tF(k_{t+1} + \Delta, l_{t+1}) - A_tF(k_{t+1}, l_{t+1})}{\Delta}. $$

When $\Delta$ is small, we know that the gain in revenues is approximated by

$$\frac{\partial A_tF(k_{t+1}, l_{t+1})}{\partial k_{t+1}} \Delta.$$

What is the cost to the shareholders of increasing the capital input by $\Delta$? Simply $r_{t+1}\Delta$. In fact $r_{t+1}$ is the opportunity cost for the shareholders.

The bottom line is that the firm will keep on increasing capital as long as $\frac{\partial F(k_{t+1}, l_{t+1})}{\partial k_{t+1}} \Delta > r_{t+1}$ or $\frac{\partial F(k_{t+1}, l_{t+1})}{\partial k_{t+1}} = r_{t+1}$. It will cease once $\frac{\partial F(k_{t+1}, l_{t+1})}{\partial k_{t+1}} = r_{t+1}$. The latter condition tells us exactly how much capital the firm will want to employ, given an interest rate $r_{t+1}$. In the case of Cobb-Douglas production function, the condition is

$$\alpha A_{t+1} k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} = r_{t+1}. $$

Solving this equation for $k_{t+1}$ yields the firm’s optimal capital stock, i.e. its desired level:

$$k_{t+1} = l_{t+1} \left[ \frac{A_{t+1}^{\alpha}}{r_{t+1}} \right]^{\frac{1}{\alpha-1}}. $$

The desired level of the capital stock at the aggregate level is the sum of the optimal capital stocks across all firms. Notice that, as common sense dictates, the desired level of capital and the demand for investment are decreasing in the interest rate and increasing in $A_{t+1}$.

**Summary**

- The aggregate demand for labor is higher the higher TFP and the lower the wage rate
- The aggregate desired level of capital is higher the higher TFP and the lower the interest rate
- The aggregate demand for investment is higher the higher TFP and the lower the interest rate
Choosing how much to consume and how much to work

In this section we will study how households choose their consumption and work effort (i.e. how many hours to work). For simplicity, we will disregard the future. That is, we will assume that households live for only 1 year. We will re-introduce the time dimension below.

Again for simplicity, we will assume that there is only one consumption good available. Let $c_t$ denote the level of consumption. In order to describe households’ preferences, we will use a handy construct known as utility function. Given levels of consumption and work effort are going to imply a level of utility, or satisfaction. The more we consume, the better off we are. This means that the utility function must be increasing in $c_t$. It turns out we also have preferences on how much time we devote to work. Everything else equal, the less we work, the better off we are. Therefore our utility is decreasing in work effort. We define leisure time as the fraction of non-sleeping time (18 hours a day?) that we do not dedicate to work. If we denote the time worked as $l_t$ and we normalize the amount of non-sleeping hours to 1, leisure turns out to be $1 - l_t$.

The trade-off that is induced by the choice problem at hand is very simple. We would love to stay home from work all the time. In that case, however, we would not be able to consume anything. This point is illustrated by the budget constraint:

$$c_t \leq w_t l_t + m_t.$$ 

The amount of consumption expenditure will be at most equal to $m_t$ (any non-wage income) plus the wage compensation $w_t l_t$.

Optional We denote our utility function as $u(c_t, 1 - l_t)$. Given that the more we consume the better off we are, the budget constraint will always hold with equality. That is, we will spend everything (remember that there is no future). Therefore the decision problem can be cast in the following way:

$$\max_{c_t, l_t} u(c_t, 1 - l_t)$$

s.t. $c_t = w_t l_t + m_t$

This means that we pick consumption and work effort so to maximize utility, subject to the budget constraint. It is very easy to draw the budget constraint on the cartesian plane $(c_t, 1 - l_t)$. We do this in Figure 5 below.

Figure 3 hints that the representation of the utility function is not as easy, but with a little thought we can do a good job at it. We use a device known as indifference curve. Most probably you have already seen it in “Firms and Markets” and in “Foundations of Finance”, when talking about portfolio choice. An indifference curve is simply a slice of the utility function. Along an indifference curve, the level of utility is constant. As we travel along one of them from the top-left corner of the graph towards the bottom-right, the level of consumption decreases, inducing lower utility. However, the level of work effort decreases as well, maintaining the level of utility constant. Why are the indifference curves convex? Because when we work very hard, we are willing to give up a
lot of consumption for a small increase in leisure. When we do not work as hard, to compensate for the same decrease in work effort, we are willing to give up less consumption. Obviously, the level of utility is not the same across indifference curves. The farther away an indifference curve is from the origin, the higher the level of utility. The bottom line is that an optimizing individual chooses the consumption and leisure pair that lies on the budget constraint in correspondence of the indifference curve characterized by the highest utility. It is easy to see that the chosen pair lies at the point of tangency between the budget constraint and the selected indifference curve.

You can verify that the graphic solution coincides with the analytical solution. That is, the optimal choice of consumption and work \((c^*_t, l^*_t)\) satisfies the following condition:

\[
\frac{u_1(c_t, 1 - l_t)}{u_2(c_t, 1 - l_t)} = w_t
\]

or

\[
\frac{u_2(c_t, 1 - l_t)}{u_1(c_t, 1 - l_t)} = w_t.
\]

Here \(u_1(\cdot, \cdot)\) denotes the derivative with respect to consumption and \(u_2(\cdot, \cdot)\) the derivative with respect to leisure. The optimality condition will not surprise you. It just says that in correspondence of \((c^*_t, l^*_t)\) the slope of the budget constraint \((-w_t)\) equals the slope of the indifference curve \((-\frac{u_2(c_t, 1 - l_t)}{u_1(c_t, 1 - l_t)})\).
Here we are interested in understanding how the households’ optimal choices change when it becomes wealthier. The idea is to think of leisure as a good that we can purchase. The price of leisure is the wage income that we forego by laying on the couch (or training for a marathon) instead of going to work. Then it is reasonable to think that as we become wealthier, we would like to spend more money both in
consumption and in leisure. This implies that more wealth should lead to the choices of increasing consumption and decreasing the number of hours worked. Are these conclusions supported by the data?

It is not surprising that the wealth effect on consumption is positive. Around the world, consumption per person has grown with the rise in per capita income. The negative wealth effect on work effort is harder to verify. Here are a couple of facts that seem to attest that the wealth effect on work effort is indeed negative. In the United States, average hours worked per week for workers in manufacturing declined from about 60 in 1890 to 42 in 1996. Other developed countries show similar patterns. If we look across countries at a point in time, we see that average hours worked are negatively correlated with income per capita.

Optional An increase in wealth can be thought of as an increase in non-wage income from $m_t$ to $\hat{m}_t$. Figure 6 illustrates the mechanism. The increase in wealth implies a higher budget constraint. The new optimal pair implies more consumption and more leisure (less work). In economists’ jargon, the wealth effect is positive for consumption (i.e. more wealth implies more consumption) and negative for work effort (i.e. more wealth implies less work).

![Figure 6: An increase in wealth.](image)

**Effects of an increase in wages**

Here we are interested in understanding the impact of changes in the wage level on the optimal choices of consumption and leisure. In a sense, when the wage increases, leisure becomes more expensive. By not working, you forego a larger amount of consumption. For this reason, you would like to work more (and consume more). This is what economists call the substitution effect: when a good becomes relatively cheaper, you would like to substitute others in its favor. This is the case both with
apples Vs. bananas and with consumption Vs. leisure. On the other hand, a higher wage makes you wealthier. The wealth effect kicks in. From the above discussion, being wealthier makes you eager to buy more leisure (i.e. work less).

The bottom line is that with respect to the consumption choice, substitution and wealth effect reinforce each other: a higher wage surely implies more consumption. Instead, with respect to the work choice, they act in opposite ways. In reality, whether households decide to work more or less when their wage increases is an empirical question. The data seem to suggest that the answer depends on the level of development. In the United States, for example, in the last forty years there has been no strong trend in average hours worked per week in manufacturing for the United States. A likely conclusion is that with respect to the work choice wealth and substitution effects roughly cancel out. There was, however, a major decline in average hours worked at earlier stages of economic development.

Optional Figure 7 provides an example in which the substitution effect prevails. The increase in wage induces an upward rotation of the budget constraint. The new optimal pair lies on a higher indifference curve, in correspondence of a higher consumption and lower leisure (more work).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{example.png}
\caption{An increase in wage.}
\end{figure}

\textbf{Example 1} Assume that the utility function is given by \( u(c,1-l) = c^\gamma(1-l)^{1-\gamma}, \gamma \in (0,1) \). Determine the optimal choices of consumption and work effort. How do such choices change when the wage increases? We have that \( u_1(c,1-l) = \gamma c^{\gamma-1}(1-l)^{1-\gamma} \) and \( u_2(c,1-l) = (1-\gamma)c^\gamma(1-l)^{-\gamma} \). Therefore the optimality condition is given by

\[ \frac{1-\gamma}{\gamma} \frac{c}{1-l} = w. \]
This equation, along with the budget constraint, determines the optimal choices of work and consumption:

\[
c = \gamma (w + m),
\]
\[
l = \gamma - (1 - \gamma) \frac{m}{w}.
\]

It turns out that when the wage increases, both consumption and work effort increase.

Summary

- Households respond to greater wealth by consuming more and by working less
- Households react to higher wages by consuming more.
- The response of work effort to higher wages depends on the relative magnitude of the wealth and substitution effect. On the one hand, a higher wage makes leisure costlier (by not working, the household gives up more). This is the substitution effect, and works in the direction of making the household work more. On the other hand, a higher wage implies that the household is wealthier. The wealth effect works in the direction of making the household work less.
- The empirical evidence suggests that the wealth effect prevails at low levels of development. For higher levels of development, the two effects roughly cancel out, so that hours worked do not respond very much to increases in wage.

Choosing how much to consume and how much to save

In this section we will study how households take their consumption and saving decisions over time. We will still assume that there is only one consumption good, but we will introduce the time dimension. For now, we will abstract from the choice of work effort. We will reintroduce it later.

For the time being, the household’s income at year \( t \) is exogenously given, and denoted by \( m_t \). Households can borrow or save at some interest rate \( r_{t+1} \). Borrowing can consist of credit card debt or a mortgage, for example. Saving means investing in assets that can be as simple as a saving account or as complicated as a portfolio of stock. In the simple world we are describing, all of these assets yield the same return. The budget constraint at year \( t \) will look as follows:

\[
c_t + a_{t+1} = m_t + a_t (1 + r_t),
\]

where \( a_{t+1} \) represents the wealth accumulated as of time \( t \). If we are currently in year \( t = 1 \), then the budget constraint is

\[
c_1 + a_2 = m_1 + a_1 (1 + r_1). \tag{1}
\]
The value $a_1$ represents the household’s initial financial assets. The interest proceeds on the asset are $a_1 r_1$. Notice that $a_1$ is something that the household inherits from the past and $m_1$ and $r_1$ are quantities that it takes as given. What the household can choose is how much to consume ($c_1$) and how much to save (invest in financial assets) ($a_2$). If it decides to consume a lot, it may have to borrow. In such case, we will obtain that that $a_2 < 0$.

Next year (year 2), the budget constraint will be

$$c_2 + a_3 = m_2 + a_2(1 + r_2).$$  \hspace{1cm} (2)

This implies that the decisions taken now (year 1) will have an effect on year 2 as well. In fact the value of $a_2$, which is decided now (year 1) will constitute the net wealth the household will start with next year.

We can rewrite equation (2) as

$$a_2 = \frac{c_2 + a_3 - m_2}{1 + r_2}.$$

Then we can substitute the latter expression in (1) to obtain

$$c_1 + \frac{c_2}{1 + r_2} = m_1 + \frac{m_2}{1 + r_2} + a_1(1 + r_1) - \frac{a_3}{1 + r_2}.$$

You will recognize the left-hand side as the present value of consumption and the right-hand side as the total wealth, i.e. the year-1 value of all cash-flows.

A first, important consideration, is that the possibility of borrowing and lending eliminates the link between current cash inflows and current cash outflow (consumption expenditures). This means that the problem households really face is to figure out the present value of wealth (the right-hand side) and then decide how to spend it across years 1 and 2. In the simple scenario under consideration (two years only), the household needs to figure out its incomes in periods 1 and 2 ($m_1$ and $m_2$, respectively), the initial level of financial assets $a_1$, and the end of period-2 assets $a_3$. These quantities and the interest rates tell how much its wealth is. Let’s call such wealth $x_1$. Then the problem is to pick $c_1$ and $c_2$ in order to maximize utility. In this case, utility is defined over $c_1$ and $c_2$.

Optional The mathematical formulation of the problem is

$$\max_{c_1, c_2} u(c_1, c_2)$$

subject to

$$c_1 + \frac{c_2}{1 + r_2} = x_1.$$  

The solution is illustrated in Figure 8. Combinations of high year-1 consumption and low year-2 consumption will be achieved by running down financial assets (i.e. setting $a_2$ low and eventually negative). On the other hand, combinations of low year-1 consumption and high year-2 consumption will be achieved by increasing financial assets (i.e. by saving a lot, so to increase $a_2$).
Solving a simple example confirms our intuition. Let’s consider the case in which

\[ u(c_1, c_2) = \log(c_1) + \log(c_2). \]

Substituting the budget constraint into the maximization problem yields

\[ \max_{c_2} \log \left( x_1 - \frac{c_2}{1 + r_2} \right) + \log(c_2) \]

The first-order condition for the problem is

\[ - \frac{1}{c_1(1 + r_2)} + \frac{1}{c_2}, \]

which implies \( c_2/c_1 = (1 + r_2) \). This equation can be used along with the budget constrained to solve explicitly for the levels of consumption. We obtain that \( c_1 = x_1/2 \) and \( c_2 = x_1(1 + r_2)/2 \).

**Effects of an increase in wealth**

Here we ask what is the impact of an increase in wealth \( x_1 \) on \( c_1 \) and \( c_2 \). The wealth can increase for a variety of reasons. For example, current income \( m_1 \) may increase. This will allow the household to spend more on consumption not only in period 1, but also in period 2. In fact it can decide to save part of the increase in \( m_1 \), thereby increasing disposable income in year 2. What if the increase is in \( m_2 \), rather than in \( m_1 \)? This raises the spending capacity not only in year 2, but also in year 1. In fact the household can borrow against that future income. The important lesson is that it does not matter which of the cash inflows increase. What is important is that the year-1 value of wealth grows. The increase in wealth will impact consumption in all years. This is what we refer to as *consumption smoothing*. 
Optional We can use the usual graphical analysis to figure out the answer. Figure 9 says that consumption in both periods increases, following an increase in wealth from $x_1$ to $\hat{x}_1$. Consider also the simple example with logarithmic utility. The expressions for $c_1$ and $c_2$ reveal that consumption in both periods rises with the level of wealth.

![Graph](image)

Figure 9: An increase in wealth.

**Effects of an increase in the interest rate**

Here the idea is that a rise in the interest rate $r_2$ lowers the cost of year-2 consumption relative to that of year 1. This is the case because a person can obtain more units of consumption next period (period 2) for each unit of current consumption forgone today (period 1). This change in relative costs motivates people to substitute future goods ($c_2$) for current ones ($c_1$). You may recognize this as yet another form of substitution effect: when apples get cheaper with respect to bananas, we tend to buy more apples. When year-2 consumption becomes more convenient, we tend to buy more of it (i.e. save more in year 1).

Optional Consider once again the simple example with logarithmic utility. The expressions for $c_1$ and $c_2$ reveal that for given wealth $x_1$, the household chooses to increase consumption in period 2. This is the substitution effect at work. This analysis is not exhaustive though. As it was the case when we considered the effects of a wage increase, an increase in the interest rate induces a change in the wealth $x_1$ as well. The impact on wealth of a higher interest rate will be negative if $m_2 - a_3 > 0$ and will be positive otherwise. In general, higher interest rate increase the life-time wealth of creditors (positive $a_3$) and decrease the life-time wealth of debtors (negative $a_3$).

Optional In the case depicted in Figure 10, it is assumed that when the interest rate $r_2$ increases, the wealth $x_1$ decreases in such a way that the pair initially chosen lies on the new budget constraint and therefore is still available for choice. In such a case, it is clear that $c_1$ decreases and $c_2$ increases.
The individual reacts by saving more. Again, this is the substitution effect at work. What happens in general? A higher interest rate will always tilt the consumer choice towards consuming more in the future. That is: an higher interest rate always implies more savings. However, if an higher rate implies higher wealth, both $c_1$ and $c_2$ might increase.

Reintroducing the choice of work effort

It is now time to re-introduce the choice of how much to work. We want to understand how households choose work effort over time. What we need to do is to specify household’s income as the sum of its components: $q_1 = m_1 + w_1l_1$ and $q_2 = m_2 + w_2l_2$. That is, income in year $t$ is given by the sum of non-wage income $m_t$ and wage income $w_tl_t$. Now the household has to choose the work efforts $l_1$ and $l_2$.

We know what happens if either $m_1$ or $m_2$ or both increase. Since the wealth effect on the effort choice is negative, the households will decide to work less in both periods. That is, both $l_1$ and $l_2$ will decrease.

What is the impact of an increase in the interest rate $r_2$ on the labor supply decisions? When $r_2$ rises, year-2 leisure becomes cheaper relative to year-1 leisure. Therefore, an increase in the interest rate motivates people to substitute toward year-2 leisure and away from year-1’s. In other words, year-1 work effort rises relative to year-2’s. Also, recall that the substitution effect induces a decrease in $c_1$ that has the effect of increasing savings. The increase in $l_1$, by raising current income, contributes to further increase savings.

Optimal decisions over the household’s planning horizon
So far in this section we have used a simple framework with two years only: 1 and 2. Households, however, make plans for much longer horizons. How long? As long as people do not care about the well-being of their descendants, it seems sensible to talk about life-time horizon. If however, as it seems to be the case, people care about their offsprings, the offsprings of their offsprings, and so on, then we are led to conclude that the planning horizon actually extends to infinity. The good news is that for our analysis the length of the planning horizon does not matter very much. Sure, if we were to perform a quantitative analysis (i.e. put numbers instead of letters), there would be differences. But here we are only interested in a qualitative analysis. That is: we are interested in understanding whether current work effort increases or decreases because of a change in the interest rate, but we are not interested in knowing by how much it changes.

The qualitative analysis of households’ choices is the same, whether the planning horizon is \( T = 2 \), as above, or \( T = 60 \), or \( T = \infty \). For a planning horizon \( T \), the budget constraint will read

\[
c_1 + \frac{c_2}{1 + r_2} + \frac{c_3}{(1 + r_2)(1 + r_3)} + \ldots + \frac{c_T}{\prod_{t=2}^{T}(1 + r_t)} =
q_1 + \frac{q_2}{1 + r_2} + \frac{q_3}{(1 + r_2)(1 + r_3)} + \ldots + \frac{q_T}{\prod_{t=2}^{T}(1 + r_t)} + a_1(1 + r_1) - \frac{a_{T+1}}{\prod_{t=2}^{T}(1 + r_t)}.
\]

Given the year-1 value of life-time wealth, an individual can still substitute between \( c_1 \) and \( c_2 \). Just as before, for each unit of \( c_1 \) forgone, a household can obtain \( 1 + r_2 \) additional units of \( c_2 \). But there is nothing special about years 1 and 2. People can substitute in a similar manner between \( c_2 \) and \( c_3 \), \( c_3 \) and \( c_4 \), and so on.

The above implies that an increase in \( r_t+1 \) always lowers \( c_t \) relative to \( c_{t+1} \). Also, an increase in \( r_t+1 \) will imply that \( l_t \) rises relative to \( l_{t+1} \).

**A permanent increase in income**

Recall that for every year \( t \), \( q_t = m_t + w_t l_t \). What happens if the non-wage income \( m_t \) increases in all years? (We call this a permanent increase in income). The answer is a simple generalization of the answer we have given in the case in which \( T = 2 \). Consumption in all years will increase, and work effort in all years will decrease. This is the wealth effect at work. Suppose that \( m_t \) increases by one dollar in every period. By how much will the value of consumption increase? It turns out that it will approximately increase by one dollar. That is, the propensity to consume will be about 1, or the propensity to save will be about 0. Again this is the result of consumption smoothing. People like smooth (flat) consumption patterns over their lifetime. Therefore, a 1 dollar permanent increase in income is likely to trigger a 1 dollar permanent increase in consumption. There are plenty of examples of permanent increases in income. Here’s one: if you have a government job, any increase in salary is guaranteed until retirement and will be reflected in your Social Security check.
A temporary increase in income

Now suppose that the increase in income is limited to the current period. That is, suppose only $m_1$ increases (Economists refer to such an event as a temporary increase in income). Households would like to spread the extra income over consumption in all periods. In order to raise future consumption, they have to raise current saving. Therefore, the propensity to consume will be very small (close to zero if the planning horizon is long enough) and the propensity to save will be very close to 1. There are also very good examples of temporary increases in income: a one-time tax rebate, an inheritance, a bonus...

An increase in the wage rate

Once again, the effects of an increase in the wage rate will depend on whether the change is temporary or permanent.

A permanent increase in the wage rate (i.e. an increase in $w_t$) in all years $t$ will imply a higher work effort at all times and an higher income. In turn, this will imply an increase in consumption. The propensity to consume will be about 1. Hence, there will be no effects on savings.

A temporary increase in the wage rate is an increase in compensation limited to one or few years. For example, consider an increase in $w_1$. The work effort will increase in the current period, but consumption will increase very little. The reason is that households like to spread the windfall over their lifetime. The propensity to consume will be close to zero. Savings instead will increase: the propensity to save will be close to 1.

Summary

- The possibility of borrowing and lending eliminates the link between current cash flows and current cash outflows (consumption expenditures). What matters for consumption decisions is the present value of lifetime wealth.
- Households like to smooth consumption over their lifetime.
- A permanent increase in income implies a propensity to consume of about one (households increase consumption by an amount roughly equal to the increase in income) and therefore a propensity to save about equal to zero. The wealth effect implies that work effort drops.
- A temporary increase in income implies a propensity to consume close to zero and a propensity to save close to one. This is the case because people like to smooth consumption. The effect on work effort is negative but negligible.
- An increase in the interest rate increases savings, lowers current consumption, and increases current work effort.