HW 4

1) Find the probability that a standard normal random variable is between
   A) 0 and 2
   B) −1.5 and 0
   C) 2 and 3
   D) −2 and 1
   E) −3 and −1.65.

2) Find the probability that a standard normal random variable is:
   A) Greater than zero
   B) Greater than −1.45
   C) Less than −.32.
   D) Equal to 1.

3) Find a value of a standard normal random variable Z (call it \( z_0 \)) such that
   A) \( \text{Prob}(Z < z_0) = .2090 \)
   B) \( \text{Prob}(Z > z_0) = .025 \)
   C) \( \text{Prob}(-z_0 < Z < z_0) = .8472 \)
   D) \( \text{Prob}(-z_0 < Z < 0) = .4798 \)
   E) \( \text{Prob}(-1 < Z < z_0) = .5328 \)

4) Suppose that X is normally distributed with mean 11 and standard deviation 2. Find
   A) \( \text{Prob}(10 < X < 12) \)
   B) \( \text{Prob}(6 < X < 10) \)
   C) \( \text{Prob}(X > 7.62) \)

5) A Pepsi machine in a Burger King store can be regulated so that it dispenses an
   average of \( \mu \) ounces per cup. If the amount dispensed is normally distributed with
   standard deviation 0.2 ounces, what should be the setting for \( \mu \) so that 8-ounce cups
   will overflow only 1% of the time?

6) Suppose that annual stock returns for a particular company are normally distributed
   with a mean of 16% and a standard deviation of 10%. You are going to invest in this
   stock for one year. (Note: In reality, annual returns tend to be more nearly normally
   distributed than daily returns. In the next two problems, you will study whether daily
   returns are normal.)
   A) Find that the probability that your one-year return will exceed 30%
   B) Find that probability that your one-year return will be negative.

7) A student of mine was asked this question in an interview. “Consider the following
   game: A cup is filled with 100 pennies. The cup is shaken, and the pennies are poured
   onto a table. If at least 60 of the pennies are Heads, you win $20. Otherwise, you lose
   $1. Is this a good game to play?” Here are some hints: Let \( X \) be the total number of
   Heads. What’s the distribution of \( X \)? Now, use the empirical rule, that is, pretend that
X is normally distributed, to get the probability of winning. Finally, use expected value to decide if it’s a good game to play.

8) **Are daily stock returns normally distributed?** Consider the excess market return. This is the variable MarketReturn in the file Market.MTP.

   A) Create the Graphical Summary for MarketReturn. The normal curve superimposed on the histogram has mean and standard deviation to match the values for the actual data, so in that sense it’s the normal distribution that best fits the data. But does it fit well? Why, or why not?

   B) Let the cursor rest on the most negative outlying value in the boxplot. Then identify the date and value for the most negative daily return in the data set.

   C) Compute the z-score for this most negative data point.

   D) What is the probability that a standard normal random variable would be less than the z-score from C)? We can’t use Table 6 to answer this question, since we are “off the chart”. Instead, use Minitab, Calc → Probability Distributions → Normal, Cumulative Probability, Input Constant. Then type in the z-score. As you will see, you are even off the chart for Minitab! Nevertheless, count how many zeros there are after the decimal point. How small must this probability be if after rounding to that many digits, all the digits are zero? (By the way, the actual probability is 6.9×10⁻⁷⁰.)

   E) Based on the actual probability from D), do you believe that stock returns are normally distributed? Why or why not?

9) **Are daily Stock returns normally distributed? (Continued).**

   A) Compute the proportion of the MarketReturn data that are more than 1,2,3,4 and 5 standard deviations from the mean. To do this, first use Calc→ Calculator to create Zscore, the z-scores corresponding to MarketReturn. Next, use Calc → Calculator to create Beyond1 (the proportion of returns more than one standard deviation from the mean) by typing the expression mean(abs(Zscore)>1). Similarly, create Beyond2,…,Beyond5 by changing the 1 in the above expression to 2,…,5.

   B) In a normal distribution, the probabilities of being more than 1,2,3,4 and 5 standard deviations from the mean are 0.32, 0.045, 0.0026, 0.000063 and 0.00000057, respectively. Do these numbers seem in line with the proportions you found in the actual data in Part A)? If not, do they suggest that actual daily returns have heavier tails or lighter tails than a normal distribution? (For a heavy-tailed distribution, the probability of being far from the mean is larger than in a normal distribution. Heavy-tailed distributions are also said to be leptokurtic, or to exhibit the “black swan” effect.)

   C) From a risk-management perspective, does it seem wise to act as if daily stock returns are normally distributed? Why or why not?

   D) Create a Normal Probability Plot for MarketReturn, using Stat → Basic Statistics → Normality Test. This gives a plot of the percentiles of the best-fitting normal distribution vs. the percentiles of the actual data. If the data come from a normal distribution, the plot should look like the straight line plotted in blue. For heavy-
tailed distributions, the plot should be S-shaped. Does the actual plot look straight or S-shaped?

E) If the data came from a normal distribution, the “p-value” provided in the plot would be less than .05 only 5% of the time, would be less than .01 only 1% of the time, etc. Is the actual p-value consistent with the hypothesis that the return data came from a normal distribution?

F) Finally, report the kurtosis from the graphical summary. High values indicate leptokurtosis. Values close to zero indicate a normal distribution. Based on the kurtosis, do you think that this data comes from a leptokurtic distribution?