HW 3

1) For the example on logic analyzers from Handout 6, verify the following two claims made there: If you schedule 17 machines, then the probability that all will work is 0.596; if you schedule 18 machines, then \( \text{Prob}\{\text{At least 17 work}\}=0.900 \). Please show your calculations for the binomial coefficients (defined in Handout 6).

2) If \( X \) is a binomial random variable with \( n=100, \ p=0.7 \), use the formula in Handout 6 to find the mean and standard deviation of \( X \).

3) Use the first table in Handout 6 to find the probability of getting at least 6 correct forecasts out of 10 for the direction of the Dow, assuming that the Dow is a random walk.

4) A multiple-choice quiz has 10 questions. Each question has five possible answers, of which only one is correct.
   A) What is the probability that sheer guesswork will yield at least 10 correct answers? (Use the first table in Handout 6.)
   B) What is the expected number of correct answers by sheer guesswork?
   C) Suppose 10 points are awarded for a correctly answered question. How many points should be deducted for an incorrectly answered question, so that for a student guessing randomly, the expected score on a question is zero? (Most standardized tests use this method to set penalties for guessing).
   D) If a student is able to correctly eliminate one option as a possible correct answer but is still guessing randomly, what happens to his/her expected score for that question? Use your answer to C) as the number of points being deducted for an incorrect answer.

5) The No-Tell Motel has 10 bedrooms. From past experience, the manager knows that 20% of the people who make room reservations don’t show up. The manager accepts 15 reservations. If a customer with a reservation shows up and the motel has run out of rooms, it is the motel’s policy to pay $100 as compensation to the customer. What is the expected value of the compensation that the motel must pay?

6) Gamblers talk about a “Law of Averages”, essentially a feeling that their winnings will balance out in the long run. The Oxford American Dictionary defines the law of averages as “the proposition that the occurrence of one extreme will be matched by that of the other extreme so as to maintain the normal average”. Most gamblers interpret this to mean that if you’ve been losing badly your luck will have to improve if you keep playing. This is not true!
   (But most people believe it. Maybe you believe it too. Ask yourself the following questions, just as a warm-up for this problem. In Roulette, if you’ve seen a long streak of red, does it make sense to start betting on black? In craps, if the shooter throws a crap on the come-out, does this improve his/her chance of getting a seven on
the next roll? Craps players call this an “apology”. If the market has been moving up recently, does this increase the chances of a sudden drop? Financial analysts call this a “correction”.)

Meanwhile, there is something called the Law of Large Numbers (LLN), which says that if you repeat the same experiment independently, then the proportion of the time that a given event occurs will approach the probability of that event. The LLN has been proven to be true, and it helps to justify the relative frequency approach to probability. In this problem, we will explore the differences between the Law of Averages and the LLN.

A) If a fair coin is tossed independently 11 times, and $A=\{\text{First 10 tosses are Heads}\}$, $B=\{11^{\text{th}} \text{ toss is Heads}\}$ then what is the conditional probability $P(B|A)$? Is this any different from $P(B)$? What does this suggest about the Law of Averages?

B) Let’s use Minitab to simulate a gambler’s winnings in playing a fair game. Use Calc $\rightarrow$ Random Data $\rightarrow$ Bernoulli, Number of rows of data to generate: 1000, store in column: HeadsOrTails, Event Probability: 0.5 $\rightarrow$ OK. (This simulates the results of 1000 coin tosses, with, let’s say, 1 representing heads and zero representing tails.) Next, let’s imagine a fair game where you win $1 for Heads, and lose $1 for Tails. Compute the winnings for the current round (either $1 or $-1) using Calc $\rightarrow$ Calculator $\rightarrow$ Store Result in Variable: ThisRoundsWinnings, Expression: $2*\text{HeadsOrTails}-1$ $\rightarrow$ OK. Finally, calculate your current profit, using Calc $\rightarrow$ Calculator $\rightarrow$ Store Result in Variable: Profit $\rightarrow$ pars(ThisRoundsWinnings) $\rightarrow$ OK. This uses the pars function in Minitab, which is a cumulative sum. Thus, for example, Profit at time 10 represents your winnings minus losses after 10 rounds of the game. Make a time series plot of Profit, without the dots. (Use Data View in the time series plot command and uncheck the box for Symbols.) Do you see any winning streaks or losing streaks? Clearly, these streaks are real in the historical sense, but are they real in the predictive sense? (Remember your answer to Part A). Does the Profit seem to approach zero in the long run? Note for future reference: The Profit series you created here is an example of a random walk. Random walks are used as models for efficient markets.

C) Now, let’s create the cumulative frequency of heads. First create a time variable going from 1 to 1000 using Calc $\rightarrow$ Make Patterned Data $\rightarrow$ Simple Set of Numbers $\rightarrow$ Store Patterned Data in: NumTosses, From First Value: 1, To Last Value: 1000 $\rightarrow$ OK. Now, create the cumulative frequency of heads using Calc $\rightarrow$ Calculator $\rightarrow$ Store Result in Variable: ProportionOfHeads $\rightarrow$ pars(HeadsOrTails)/NumTosses. Make a time series plot of ProportionOfHeads. Does the proportion of heads seem to approach the true probability of heads, 0.5? In what ways does this plot appear different from that of Profit, above?

D) Let’s try to reconcile the Law of Large Numbers with the Law of Averages (also known as the Gambler’s Fallacy.) If you repeatedly toss a fair coin (independently), why doesn’t the profit need to go to zero in order for the proportion of heads to go to $1/2$? (Hint: the proportion has a denominator that increases. The profit doesn’t.)