Equity Instruments & Markets: Part I
Discounted Cash Flow Valuation
B40.3331

Aswath Damodaran
Discounted Cashflow Valuation: Basis for Approach

\[
\text{Value} = \sum_{t=1}^{t=n} \frac{\text{CF}_t}{(1+r)^t}
\]

where \(\text{CF}_t\) is the cash flow in period \(t\), and \(r\) is the discount rate appropriate given the riskiness of the cash flow and \(t\) is the life of the asset.

**Proposition 1:** For an asset to have value, the expected cash flows have to be positive some time over the life of the asset.

**Proposition 2:** Assets that generate cash flows early in their life will be worth more than assets that generate cash flows later; the latter may however have greater growth and higher cash flows to compensate.
Equity Valuation versus Firm Valuation

- Value just the equity stake in the business
- Value the entire business, which includes, besides equity, the other claimholders in the firm
I. Equity Valuation

- The value of equity is obtained by discounting expected cashflows to equity, i.e., the residual cashflows after meeting all expenses, tax obligations and interest and principal payments, at the cost of equity, i.e., the rate of return required by equity investors in the firm.

\[
\text{Value of Equity} = \sum_{t=1}^{t=n} \frac{\text{CF to Equity}_t}{(1 + k_e)^t}
\]

where,

- \(\text{CF to Equity}_t = \text{Expected Cashflow to Equity in period } t\)
- \(k_e = \text{Cost of Equity}\)

- The dividend discount model is a specialized case of equity valuation, and the value of a stock is the present value of expected future dividends.
II. Firm Valuation

The value of the firm is obtained by discounting expected cashflows to the firm, i.e., the residual cashflows after meeting all operating expenses and taxes, but prior to debt payments, at the weighted average cost of capital, which is the cost of the different components of financing used by the firm, weighted by their market value proportions.

\[
\text{Value of Firm} = \sum_{t=1}^{n} \frac{\text{CF to Firm}_t}{(1+WACC)^t}
\]

where,

\(\text{CF to Firm}_t = \text{Expected Cashflow to Firm in period } t\)

\(WACC = \text{Weighted Average Cost of Capital}\)
Firm Value and Equity Value

To get from firm value to equity value, which of the following would you need to do?
- Subtract out the value of long term debt
- Subtract out the value of all debt
- Subtract the value of all non-equity claims in the firm, that are included in the cost of capital calculation
- Subtract out the value of all non-equity claims in the firm

Doing so, will give you a value for the equity which is
- greater than the value you would have got in an equity valuation
- lesser than the value you would have got in an equity valuation
- equal to the value you would have got in an equity valuation
Cash Flows and Discount Rates

- Assume that you are analyzing a company with the following cashflows for the next five years.

<table>
<thead>
<tr>
<th>Year</th>
<th>CF to Equity</th>
<th>Int Exp (1-t)</th>
<th>CF to Firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$50</td>
<td>$40</td>
<td>$90</td>
</tr>
<tr>
<td>2</td>
<td>$60</td>
<td>$40</td>
<td>$100</td>
</tr>
<tr>
<td>3</td>
<td>$68</td>
<td>$40</td>
<td>$108</td>
</tr>
<tr>
<td>4</td>
<td>$76.2</td>
<td>$40</td>
<td>$116.2</td>
</tr>
<tr>
<td>5</td>
<td>$83.49</td>
<td>$40</td>
<td>$123.49</td>
</tr>
<tr>
<td></td>
<td>Terminal Value</td>
<td></td>
<td>$1603.008</td>
</tr>
</tbody>
</table>

- Assume also that the cost of equity is 13.625% and the firm can borrow long term at 10%. (The tax rate for the firm is 50%).

- The current market value of equity is $1,073 and the value of debt outstanding is $800.
Method 1: Discount CF to Equity at Cost of Equity to get value of equity
- Cost of Equity = 13.625%
- PV of Equity = $1073

Method 2: Discount CF to Firm at Cost of Capital to get value of firm
- Cost of Debt = Pre-tax rate (1- tax rate) = 10% (1-.5) = 5%
- WACC = 13.625% (1073/1873) + 5% (800/1873) = 9.94%
- PV of Firm = $1873
- PV of Equity = PV of Firm - Market Value of Debt
  = $1873 - $800 = $1073
First Principle of Valuation

- Never mix and match cash flows and discount rates.
- The key error to avoid is mismatching cashflows and discount rates, since discounting cashflows to equity at the weighted average cost of capital will lead to an upwardly biased estimate of the value of equity, while discounting cashflows to the firm at the cost of equity will yield a downward biased estimate of the value of the firm.
The Effects of Mismatching Cash Flows and Discount Rates

**Error 1: Discount CF to Equity at Cost of Capital to get equity value**
- PV of Equity = \( \frac{50}{1.0994} + \frac{60}{1.0994^2} + \frac{68}{1.0994^3} + \frac{76.2}{1.0994^4} + \frac{(83.49 + 1603)}{1.0994^5} \) = $1248
- Value of equity is overstated by $175.

**Error 2: Discount CF to Firm at Cost of Equity to get firm value**
- PV of Firm = \( \frac{90}{1.13625} + \frac{100}{1.13625^2} + \frac{108}{1.13625^3} + \frac{116.2}{1.13625^4} + \frac{(123.49 + 2363)}{1.13625^5} \) = $1613
- PV of Equity = $1612.86 - $800 = $813
- Value of Equity is understated by $260.

**Error 3: Discount CF to Firm at Cost of Equity, forget to subtract out debt, and get too high a value for equity**
- Value of Equity = $1613
- Value of Equity is overstated by $540
Discounted Cash Flow Valuation: The Steps

- Estimate the **discount rate** or rates to use in the valuation
  - Discount rate can be either a cost of equity (if doing equity valuation) or a cost of capital (if valuing the firm)
  - Discount rate can be in nominal terms or real terms, depending upon whether the cash flows are nominal or real
  - Discount rate can vary across time.

- Estimate the **current earnings and cash flows** on the asset, to either equity investors (CF to Equity) or to all claimholders (CF to Firm)

- Estimate the **future earnings and cash flows** on the asset being valued, generally by estimating an expected growth rate in earnings.

- Estimate **when** the firm will reach “**stable growth**” and what characteristics (risk & cash flow) it will have when it does.

- Choose the **right DCF model** for this asset and value it.
Generic DCF Valuation Model

DISCOUNTED CASHFLOW VALUATION

**Cash flows**
- Firm: Pre-debt cash flow
- Equity: After debt cash flows

**Expected Growth**
- Firm: Growth in Operating Earnings
- Equity: Growth in Net Income/EPS

**Value**
- Firm: Value of Firm
- Equity: Value of Equity

**Discount Rate**
- Firm: Cost of Capital
- Equity: Cost of Equity

**Length of Period of High Growth**

**Terminal Value**
- Firm is in stable growth: Grows at constant rate forever

**Forever**

EQUITY VALUATION WITH DIVIDENDS

Dividends
Net Income
* Payout Ratio
= Dividends

Expected Growth
Retention Ratio * Return on Equity
Firm is in stable growth: Grows at constant rate forever

Terminal Value = \( \frac{\text{Dividend}_{n+1}}{k_e - g_n} \)

Discount at Cost of Equity

Value of Equity

Dividend 1 Dividend 2 Dividend 3 Dividend 4 Dividend 5 Dividend \( n \) 

Forever

Discount at Cost of Equity

Cost of Equity

Riskfree Rate:
- No default risk
- No reinvestment risk
- In same currency and in same terms (real or nominal as cash flows

Beta
- Measures market risk

Risk Premium
- Premium for average risk investment

Type of Business
Operating Leverage
Financial Leverage
Base Equity Premium
Country Risk Premium
EQUITY VALUATION WITH FCFE

Cashflow to Equity
Net Income
- (Cap Ex - Depr) (1- DR)
- Change in WC (!-DR)
= FCFE

Expected Growth
Retention Ratio *
Return on Equity

Firm is in stable growth:
Grows at constant rate forever

Terminal Value = \( \frac{FCFE_{n+1}}{k_e - g_n} \)

Value of Equity

Discount at Cost of Equity

Cost of Equity

Riskfree Rate:
- No default risk
- No reinvestment risk
- In same currency and in same terms (real or nominal as cash flows)

Risk Premium
- Premium for average risk investment

β
- Measures market risk

Type of Business
Operating Leverage
Financial Leverage
Base Equity Premium
Country Risk Premium
VALUING A FIRM

Cashflow to Firm
EBIT (1-t)
- (Cap Ex - Depr)
- Change in WC
= FCFF

Expected Growth
Reinvestment Rate
* Return on Capital

Firm is in stable growth:
Grows at constant rate
forever

Terminal Value = FCFF_{n+1}/(r-g_n)

Value of Operating Assets
+ Cash & Non-op Assets
= Value of Firm
- Value of Debt
= Value of Equity

Cost of Equity
Cost of Debt
(Riskfree Rate + Default Spread) (1-t)

Discount at
WACC = Cost of Equity (Equity/(Debt + Equity)) + Cost of Debt (Debt/(Debt + Equity))

Weights
Based on Market Value

Riskfree Rate:
- No default risk
- No reinvestment risk
- In same currency and
in same terms (real or nominal as cash flows

Beta
- Measures market risk

Risk Premium
- Premium for average risk investment

Type of Business
Operating Leverage
Financial Leverage
Base Equity Premium
Country Risk Premium
Discounted Cash Flow Valuation: The Inputs

Aswath Damodaran
I. Estimating Discount Rates

DCF Valuation
Critical ingredient in discounted cashflow valuation. Errors in estimating the discount rate or mismatching cashflows and discount rates can lead to serious errors in valuation.

At an intuitive level, the discount rate used should be consistent with both the riskiness and the type of cashflow being discounted.

- **Equity versus Firm:** If the cash flows being discounted are cash flows to equity, the appropriate discount rate is a cost of equity. If the cash flows are cash flows to the firm, the appropriate discount rate is the cost of capital.
- **Currency:** The currency in which the cash flows are estimated should also be the currency in which the discount rate is estimated.
- **Nominal versus Real:** If the cash flows being discounted are nominal cash flows (i.e., reflect expected inflation), the discount rate should be nominal.
Cost of Equity

- The cost of equity should be higher for riskier investments and lower for safer investments.
- While risk is usually defined in terms of the variance of actual returns around an expected return, risk and return models in finance assume that the risk that should be rewarded (and thus built into the discount rate) in valuation should be the risk perceived by the marginal investor in the investment.
- Most risk and return models in finance also assume that the marginal investor is well diversified, and that the only risk that he or she perceives in an investment is risk that cannot be diversified away (i.e., market or non-diversifiable risk).
## The Cost of Equity: Competing Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Expected Return</th>
<th>Inputs Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPM</td>
<td>$E(R) = R_f + \beta (R_m - R_f)$</td>
<td>Riskfree Rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Beta relative to market portfolio</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Market Risk Premium</td>
</tr>
<tr>
<td>APM</td>
<td>$E(R) = R_f + \sum_{j=1}^{# \text{Factors}} \beta_j (R_j - R_f)$</td>
<td>Riskfree Rate; # of Factors; Betas relative to each factor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Factor risk premiums</td>
</tr>
<tr>
<td>Multi factor</td>
<td>$E(R) = R_f + \sum_{j=1,\ldots,N} \beta_j (R_j - R_f)$</td>
<td>Riskfree Rate; Macro factors Betas relative to macro factors</td>
</tr>
<tr>
<td>Proxy</td>
<td>$E(R) = a + \sum_{j=1,\ldots,N} b_j Y_j$</td>
<td>Proxies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regression coefficients</td>
</tr>
</tbody>
</table>

*Aswath Damodaran*
The CAPM: Cost of Equity

Consider the standard approach to estimating cost of equity:
Cost of Equity = \( R_f + \text{Equity Beta} \times (E(R_m) - R_f) \)

where,
\( R_f \) = Riskfree rate
\( E(R_m) \) = Expected Return on the Market Index (Diversified Portfolio)

In practice,
- Short term government security rates are used as risk free rates
- Historical risk premiums are used for the risk premium
- Betas are estimated by regressing stock returns against market returns
Short term Governments are not riskfree

- On a riskfree asset, the actual return is equal to the expected return. Therefore, there is no variance around the expected return.
- For an investment to be riskfree, then, it has to have
  - No default risk
  - No reinvestment risk
- Thus, the riskfree rates in valuation will depend upon when the cash flow is expected to occur and will vary across time
- A simpler approach is to match the duration of the analysis (generally long term) to the duration of the riskfree rate (also long term)
- In emerging markets, there are two problems:
  - The government might not be viewed as riskfree (Brazil, Indonesia)
  - There might be no market-based long term government rate (China)
Estimating a Riskfree Rate

- Estimate a range for the riskfree rate in local terms:
  - *Upper limit:* Obtain the rate at which the largest, safest firms in the country borrow at and use as the riskfree rate.
  - *Lower limit:* Use a local bank deposit rate as the riskfree rate.

- Do the analysis in real terms (rather than nominal terms) using a real riskfree rate, which can be obtained in one of two ways –
  - from an inflation-indexed government bond, if one exists
  - set equal, approximately, to the long term real growth rate of the economy in which the valuation is being done.

- Do the analysis in another more stable currency, say US dollars.
A Simple Test

- You are valuing Brahma, a Brazilian company, in U.S. dollars and are attempting to estimate a riskfree rate to use in the analysis. The riskfree rate that you should use is
  - The interest rate on a Brazilian Real denominated long term Government bond
  - The interest rate on a US $ denominated Brazilian long term bond (called a C-Bond)
  - The interest rate on a US $ denominated Brazilian Brady bond (which is partially backed by the US Government)
  - The interest rate on a US treasury bond
Everyone uses historical premiums, but..

- The historical premium is the premium that stocks have historically earned over riskless securities.
- Practitioners never seem to agree on the premium; it is sensitive to
  - How far back you go in history…
  - Whether you use T.bill rates or T.Bond rates
  - Whether you use geometric or arithmetic averages.
- For instance, looking at the US:

<table>
<thead>
<tr>
<th>Historical period</th>
<th>Stocks - T.Bills</th>
<th>Stocks - T.Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Arith Geom</td>
<td>Arith Geom</td>
</tr>
<tr>
<td>1926-1999</td>
<td>9.41% 8.14%</td>
<td>7.64% 6.60%</td>
</tr>
<tr>
<td>1962-1999</td>
<td>7.07% 6.46%</td>
<td>5.96% 5.74%</td>
</tr>
<tr>
<td>1990-1999</td>
<td>13.24% 11.62%</td>
<td>16.08% 14.07%</td>
</tr>
</tbody>
</table>
If you choose to use historical premiums....

- Go back as far as you can. A risk premium comes with a standard error. Given the annual standard deviation in stock prices is about 25%, the standard error in a historical premium estimated over 25 years is roughly:

  Standard Error in Premium = \( \frac{25\%}{\sqrt{25}} = \frac{25\%}{5} = 5\% \)

- Be consistent in your use of the riskfree rate. Since we argued for long term bond rates, the premium should be the one over T.Bonds

- Use the geometric risk premium. It is closer to how investors think about risk premiums over long periods.

- Never use historical risk premiums estimated over short periods.

- For emerging markets, start with the base historical premium in the US and add a country spread, based upon the country rating and the relative equity market volatility.
### Assessing Country Risk Using Currency Ratings: Latin America

<table>
<thead>
<tr>
<th>Country</th>
<th>Rating</th>
<th>Default Spread over US T.Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>Ba3</td>
<td>525</td>
</tr>
<tr>
<td>Bolivia</td>
<td>B1</td>
<td>600</td>
</tr>
<tr>
<td>Brazil</td>
<td>B2</td>
<td>750</td>
</tr>
<tr>
<td>Chile</td>
<td>Baa1</td>
<td>150</td>
</tr>
<tr>
<td>Colombia</td>
<td>Baa3</td>
<td>200</td>
</tr>
<tr>
<td>Ecuador</td>
<td>B3</td>
<td>850</td>
</tr>
<tr>
<td>Paraguay</td>
<td>B2</td>
<td>750</td>
</tr>
<tr>
<td>Peru</td>
<td>Ba3</td>
<td>525</td>
</tr>
<tr>
<td>Uruguay</td>
<td>Baa3</td>
<td>200</td>
</tr>
<tr>
<td>Venezuela</td>
<td>B2</td>
<td>750</td>
</tr>
</tbody>
</table>
Country ratings measure default risk. While default risk premiums and equity risk premiums are highly correlated, one would expect equity spreads to be higher than debt spreads.

- One way to adjust the country spread upwards is to use information from the US market. In the US, the equity risk premium has been roughly twice the default spread on junk bonds.
- Another is to multiply the bond spread by the relative volatility of stock and bond prices in that market. For example,
  - Standard Deviation in Merval (Equity) = 42.87%
  - Standard Deviation in Argentine Long Bond = 21.37%
  - Adjusted Equity Spread = 5.25% \( \frac{42.87}{21.37} \) = 10.53%
### Assessing Country Risk Using Currency Ratings: Western Europe

<table>
<thead>
<tr>
<th>Country</th>
<th>Rating</th>
<th>Default Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>Aa1</td>
<td>75</td>
</tr>
<tr>
<td>Denmark</td>
<td>Aaa</td>
<td>0</td>
</tr>
<tr>
<td>France</td>
<td>Aaa</td>
<td>0</td>
</tr>
<tr>
<td>Germany</td>
<td>Aaa</td>
<td>0</td>
</tr>
<tr>
<td>Greece</td>
<td>A2</td>
<td>120</td>
</tr>
<tr>
<td>Ireland</td>
<td>Aaa</td>
<td>0</td>
</tr>
<tr>
<td>Italy</td>
<td>Aa3</td>
<td>90</td>
</tr>
<tr>
<td>Netherlands</td>
<td>Aaa</td>
<td>0</td>
</tr>
<tr>
<td>Norway</td>
<td>Aaa</td>
<td>0</td>
</tr>
<tr>
<td>Portugal</td>
<td>Aa2</td>
<td>85</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>Aaa</td>
<td>0</td>
</tr>
</tbody>
</table>
Country ratings measure default risk. While default risk premiums and equity risk premiums are highly correlated, one would expect equity spreads to be higher than debt spreads.

- One way to adjust the country spread upwards is to use information from the US market. In the US, the equity risk premium has been roughly twice the default spread on junk bonds.
- Another is to multiply the bond spread by the relative volatility of stock and bond prices in that market. For example,
  - Standard Deviation in MIB30 (Equity) = 15.64%
  - Standard Deviation in Italian long bond = 9.2%
  - Adjusted Equity Spread = 0.90% \times (15.64/9.2) = 1.53\%
From Country Spreads to Risk premiums

- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,

\[ E(\text{Return}) = \text{Riskfree Rate} + \text{Country Spread} + \beta \text{(US premium)} \]

Implicitly, this is what you are assuming when you use the local Government’s dollar borrowing rate as your riskfree rate.

- Approach 2: Assume that a company’s exposure to country risk is similar to its exposure to other market risk.

\[ E(\text{Return}) = \text{Riskfree Rate} + \beta \text{(US premium + Country Spread)} \]

- Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)

\[ E(\text{Return}) = \text{Riskfree Rate} + \beta \text{(US premium)} + \lambda \text{(Country Spread)} \]
Estimating Exposure to Country Risk

Different companies should be exposed to different degrees to country risk. For instance, an Italian firm that generates the bulk of its revenues in Western Europe should be less exposed to country risk in Italy than one that generates all its business within Italy.

The factor \( \lambda \) measures the relative exposure of a firm to country risk. One simplistic solution would be to do the following:

\[
\lambda = \frac{\% \text{ of revenues domestically}_{\text{firm}}}{\% \text{ of revenues domestically}_{\text{avg firm}}}
\]

For instance, if a firm gets 35% of its revenues domestically while the average firm in that market gets 70% of its revenues domestically

\[
\lambda = \frac{35\%}{70\%} = 0.5
\]

There are two implications

- A company’s risk exposure is determined by where it does business and not by where it is located
- Firms might be able to actively manage their country risk exposures
Estimating E(Return) for Siderar: An Argentine Steel Company

- Assume that the beta for Siderar is 0.71, and that the riskfree rate used is 6.00%. (US Long Term Bond rate)

- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,
  
  \[
  E(\text{Return}) = 6.00\% + 10.53\% + 0.71 \times 5.5\% = 20.44\%
  \]

- Approach 2: Assume that a company’s exposure to country risk is similar to its exposure to other market risk.
  
  \[
  E(\text{Return}) = 6.00\% + 0.71 \times (5.5\% + 10.53\%) = 17.38\%
  \]

- Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)
  
  \[
  E(\text{Return}) = 6.00\% + 0.71 \times 5.5\% + 1.10 \times 10.53\% = 21.49\%
  \]

In 1998, Siderar got 76.3% of its revenues from Argentina. The average across all Argentinan firms is closer to 70%.
Implied Equity Premiums

- If we use a basic discounted cash flow model, we can estimate the implied risk premium from the current level of stock prices.
- For instance, if stock prices are determined by a variation of the simple Gordon Growth Model:
  - Value = Expected Dividends next year/ (Required Returns on Stocks - Expected Growth Rate)
  - Dividends can be extended to included expected stock buybacks.
  - Plugging in the current level of the index, the dividends on the index and expected growth rate will yield a “implied” expected return on stocks. Subtracting out the riskfree rate will yield the implied premium.
- This model can be extended to allow for two stages of growth - an initial period where the entire market will have earnings growth greater than that of the economy, and then a stable growth period.
Estimating Implied Premium for U.S. Market:
Jan 1, 2000

- Level of the index = 1469
- Treasury bond rate = 6.50%
- Expected Growth rate in earnings (next 5 years) = 10% (Consensus estimate for S&P 500)
- Expected growth rate after year 5 = set equal to T.Bond rate
- Expected dividends + stock buybacks = 1.68% of index

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Dividends</td>
<td>$27.23</td>
<td>$29.95</td>
<td>$32.94</td>
<td>$36.24</td>
</tr>
</tbody>
</table>

+ Stock Buybacks

Expected dividends + buybacks in year 6 = 39.86 (1.065) = $42.45

1469 = 27.23/(1+r) + 29.95/(1+r)^2 + 32.94/(1+r)^3 + 36.24/(1+r)^4 + (39.86+(42.45/(r-.065)))/(1+r)^5

Solving for r, r = 8.60%. (Only way to do this is trial and error)

Implied risk premium = 8.60% - 6.50% = 2.10%
Level of the Index (Merval) = 430.06
Dividends on the Index = 3.45% of 430.06 (Used weighted yield)

Other parameters
- Riskfree Rate = 6%
- Expected Growth (in nominal dollar terms)
  - Next 5 years = 12% (Used expected growth rate in Earnings from ADRs)
  - After year 5 = 6%

Solving for the expected return:
- Expected return on Equity = 10.81%
- Implied Equity premium = 10.81% - 6.00% = 4.81%
Implied Premium for Italian Market: June 1, 1999

- Level of the Index = 35152
- Dividends on the Index = 2.15% of 35152 (Used weighted yield)
- Other parameters
  - Riskfree Rate = 4.24%
  - Expected Growth (in nominal dollar terms)
    - Next 5 years = 10% (Used expected growth rate in Earnings)
    - After year 5 = 5%
- Solving for the expected return:
  - Expected return on Equity = 7.82%
  - Implied Equity premium = 3.58%
The standard procedure for estimating betas is to regress stock returns ($R_j$) against market returns ($R_m$) -

$$R_j = a + b R_m$$

- where $a$ is the intercept and $b$ is the slope of the regression.

The slope of the regression corresponds to the beta of the stock, and measures the riskiness of the stock.

This beta has three problems:
- It has high standard error
- It reflects the firm’s business mix over the period of the regression, not the current mix
- It reflects the firm’s average financial leverage over the period rather than the current leverage.
Beta Estimation: The Noise Problem
Beta Estimation: The Index Effect
Determinants of Betas

- **Product or Service**: The beta value for a firm depends upon the sensitivity of the demand for its products and services and of its costs to macroeconomic factors that affect the overall market.
  - Cyclical companies have higher betas than non-cyclical firms
  - Firms which sell more discretionary products will have higher betas than firms that sell less discretionary products

- **Operating Leverage**: The greater the proportion of fixed costs in the cost structure of a business, the higher the beta will be of that business. This is because higher fixed costs increase your exposure to all risk, including market risk.

- **Financial Leverage**: The more debt a firm takes on, the higher the beta will be of the equity in that business. Debt creates a fixed cost, interest expenses, that increases exposure to market risk.
Equity Betas and Leverage

- The beta of equity alone can be written as a function of the unlevered beta and the debt-equity ratio

\[ \beta_L = \beta_u \left( 1 + \left(1-t\right)\frac{D}{E} \right) \]

where

- \( \beta_L \) = Levered or Equity Beta
- \( \beta_u \) = Unlevered Beta
- \( t \) = Corporate marginal tax rate
- \( D \) = Market Value of Debt
- \( E \) = Market Value of Equity

- While this beta is estimated on the assumption that debt carries no market risk (and has a beta of zero), you can have a modified version:

\[ \beta_L = \beta_u \left( 1 + \left(1-t\right)\frac{D}{E} \right) - \beta_{\text{debt}} \left(1-t\right)\frac{D}{D+E} \]
Solutions to the Regression Beta Problem

- Modify the regression beta by
  - changing the index used to estimate the beta
  - adjusting the regression beta estimate, by bringing in information about the fundamentals of the company

- Estimate the beta for the firm using
  - the standard deviation in stock prices instead of a regression against an index.
  - accounting earnings or revenues, which are less noisy than market prices.

- Estimate the beta for the firm from the bottom up without employing the regression technique. This will require
  - understanding the business mix of the firm
  - estimating the financial leverage of the firm

- Use an alternative measure of market risk that does not need a regression.
Bottom-up Betas

■ The bottom up beta can be estimated by:
  • Taking a weighted (by sales or operating income) average of the unlevered betas of the different businesses a firm is in.
    \[
    \sum_{j=1}^{j=k} \beta_j \left( \frac{\text{Operating Income}_j}{\text{Operating Income}_{\text{firm}}} \right)
    \]
  (The unlevered beta of a business can be estimated by looking at other firms in the same business)
  • Lever up using the firm’s debt/equity ratio
    \[\beta_{\text{levered}} = \beta_{\text{unlevered}} \left[ 1 + (1 - \text{tax rate}) \left( \frac{\text{Current Debt}}{\text{Equity Ratio}} \right) \right] \]

■ The bottom up beta will give you a better estimate of the true beta when
  • It has lower standard error \( \text{SE}_{\text{average}} = \text{SE}_{\text{firm}} / \sqrt{n} \) (\( n \) = number of firms)
  • It reflects the firm’s current business mix and financial leverage
  • It can be estimated for divisions and private firms.
### Bottom-up Beta: Firm in Multiple Businesses

**Boeing in 1998**

<table>
<thead>
<tr>
<th>Segment</th>
<th>Estimated Value</th>
<th>Unlevered Beta</th>
<th>Segment Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commercial Aircraft</td>
<td>30,160.48</td>
<td>0.91</td>
<td>70.39%</td>
</tr>
<tr>
<td>Defense</td>
<td>12,687.50</td>
<td>0.80</td>
<td>29.61%</td>
</tr>
</tbody>
</table>

Unlevered Beta of firm = 0.91 (.7039) + 0.80 (.2961) = 0.88

#### Levered Beta Calculation

- Market Value of Equity = $33,401
- Market Value of Debt = $8,143
- Market Debt/Equity Ratio = 24.38%
- Tax Rate = 35%

Levered Beta for Boeing = 0.88 (1 + (1 - .35) (.2438)) = 1.02
Telecom Italia’s Bottom-up Beta

<table>
<thead>
<tr>
<th>Business</th>
<th>Unlevered D/E Ratio</th>
<th>Levered Beta</th>
<th>Riskfree Rate</th>
<th>Risk Premium</th>
<th>Cost of Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telecom</td>
<td>0.79</td>
<td>18.8%</td>
<td>0.87</td>
<td>4.24%</td>
<td>7.03%</td>
</tr>
</tbody>
</table>

Proportion of operating income from telecom = 100%

Unlevered Beta for Telecom Italia = 0.79

Assume now that Telecom Italia decides to go into the internet business, and that the unlevered beta for that business is 1.75. Assuming that 25% of Telecom Italia’s business looking forward will come from this business, what will the firm’s beta be?
Siderar’s Bottom-up Beta

<table>
<thead>
<tr>
<th>Business</th>
<th>Unlevered D/E Ratio</th>
<th>Levered Beta</th>
<th>Riskfree Rate</th>
<th>Risk Premium</th>
<th>Cost of Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>0.68</td>
<td>5.97%</td>
<td>0.71</td>
<td>6.00%</td>
<td>16.03%</td>
</tr>
</tbody>
</table>

Proportion of operating income from steel = 100%

Levered Beta for Siderar = 0.71
The Cost of Equity: A Recap

Cost of Equity = Riskfree Rate + Beta \times (\text{Risk Premium})

Preferably, a bottom-up beta, based upon other firms in the business, and firm’s own financial leverage

Has to be in the same currency as cash flows, and defined in same terms (real or nominal) as the cash flows

**Historical Premium**
1. Mature Equity Market Premium: Average premium earned by stocks over T.Bonds in U.S.
2. Country risk premium = Country Default Spread* \left( \frac{\sigma_{\text{Equity}}}{\sigma_{\text{Country bond}}} \right)

**Implied Premium**
Based on how equity market is priced today and a simple valuation model
## Valuing a Firm from Different Risk Perspectives

*Firm is assumed to have a cash flow of 100 each year forever.*

<table>
<thead>
<tr>
<th>Investor Type</th>
<th>Project Risk</th>
<th>Cares about</th>
<th>Risk Measure</th>
<th>Cost of Equity</th>
<th>Firm Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Business: Owner has all his wealth invested in the business</td>
<td></td>
<td>Total Risk</td>
<td>Standard Deviation</td>
<td>40%</td>
<td>100/0.4=250</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sector Risk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Int’l Risk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Market Risk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Venture Capitalist: Has wealth invested in a number of companies in one sector</td>
<td></td>
<td>Risk added to sector portfolio</td>
<td>Beta relative to sector</td>
<td>25%</td>
<td>100/0.25=400</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sector Risk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Int’l Risk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Market Risk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Publicly traded company with investors who are diversified domestically or IPO to investors who are domestically diversified</td>
<td></td>
<td>Risk added to domestic portfolio</td>
<td>Beta relative to local index</td>
<td>15%</td>
<td>100/0.15=667</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Int’l Risk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Market Risk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Publicly traded company with investors who are diversified globally or IPO to global investors</td>
<td></td>
<td>Risk added to global portfolio</td>
<td>Beta relative to global index</td>
<td>10%</td>
<td>100/0.10=1000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Market Risk</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Estimating the Cost of Debt

The cost of debt is the rate at which you can borrow at currently. It will reflect not only your default risk but also the level of interest rates in the market.

The two most widely used approaches to estimating cost of debt are:

- Looking up the yield to maturity on a straight bond outstanding from the firm. The limitation of this approach is that very few firms have long term straight bonds that are liquid and widely traded.
- Looking up the rating for the firm and estimating a default spread based upon the rating. While this approach is more robust, different bonds from the same firm can have different ratings. You have to use a median rating for the firm.
- When in trouble (either because you have no ratings or multiple ratings for a firm), estimate a synthetic rating for your firm and the cost of debt based upon that rating.
Estimating Synthetic Ratings

- The rating for a firm can be estimated using the financial characteristics of the firm. In its simplest form, the rating can be estimated from the interest coverage ratio.

  \[
  \text{Interest Coverage Ratio} = \frac{\text{EBIT}}{\text{Interest Expenses}}
  \]

- For Siderar, for instance

  \[
  \text{Interest Coverage Ratio} = \frac{161}{48} = 3.33
  \]

  - Based upon the relationship between interest coverage ratios and ratings, we would estimate a rating of A- for Siderar. With a default spread of 1.25\% (given the rating of A-)

- For Telecom Italia, for instance

  \[
  \text{Interest Coverage Ratio} = \frac{4313}{306} = 14.09
  \]

  - Based upon the relationship between interest coverage ratios and ratings, we would estimate a rating of AAA for Telecom Italia.
## Interest Coverage Ratios, Ratings and Default Spreads

<table>
<thead>
<tr>
<th>If Interest Coverage Ratio is</th>
<th>Estimated Bond Rating</th>
<th>Default Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 8.50</td>
<td>AAA</td>
<td>0.20%</td>
</tr>
<tr>
<td>6.50 - 8.50</td>
<td>AA</td>
<td>0.50%</td>
</tr>
<tr>
<td>5.50 - 6.50</td>
<td>A+</td>
<td>0.80%</td>
</tr>
<tr>
<td>4.25 - 5.50</td>
<td>A</td>
<td>1.00%</td>
</tr>
<tr>
<td>3.00 - 4.25</td>
<td>A–</td>
<td>1.25%</td>
</tr>
<tr>
<td>2.50 - 3.00</td>
<td>BBB</td>
<td>1.50%</td>
</tr>
<tr>
<td>2.00 - 2.50</td>
<td>BB</td>
<td>2.00%</td>
</tr>
<tr>
<td>1.75 - 2.00</td>
<td>B+</td>
<td>2.50%</td>
</tr>
<tr>
<td>1.50 - 1.75</td>
<td>B</td>
<td>3.25%</td>
</tr>
<tr>
<td>1.25 - 1.50</td>
<td>B –</td>
<td>4.25%</td>
</tr>
<tr>
<td>0.80 - 1.25</td>
<td>CCC</td>
<td>5.00%</td>
</tr>
<tr>
<td>0.65 - 0.80</td>
<td>CC</td>
<td>6.00%</td>
</tr>
<tr>
<td>0.20 - 0.65</td>
<td>C</td>
<td>7.50%</td>
</tr>
<tr>
<td>&lt; 0.20</td>
<td>D</td>
<td>10.00%</td>
</tr>
</tbody>
</table>
Cost of Debt computations

- Companies in countries with low bond ratings and high default risk might bear the burden of country default risk
  - For Siderar, the rating estimated of A- yields a cost of debt as follows:
    Pre-tax Cost of Debt
    \[= \text{US T.Bond rate} + \text{Country default spread} + \text{Company Default Spread}\]
    \[= 6\% + 5.25\% + 1.25\% = 12.50\%\]

- For Telecom Italia, the rating of AAA yields a cost of debt in Euros as follows:
  Pre-tax Cost of Debt = Riskfree Rate + Default Spread
  \[= 4.24\% + 0.20\% = 4.44\%\]
Synthetic Ratings: Some Caveats

- The relationship between interest coverage ratios and ratings, developed using US companies, tends to travel well, as long as we are analyzing large manufacturing firms in markets with interest rates close to the US interest rate.
- They are more problematic when looking at smaller companies in markets with higher interest rates than the US.
Weights for the Cost of Capital Computation

- The weights used to compute the cost of capital should be the market value weights for debt and equity.
- There is an element of circularity that is introduced into every valuation by doing this, since the values that we attach to the firm and equity at the end of the analysis are different from the values we gave them at the beginning.
- As a general rule, the debt that you should subtract from firm value to arrive at the value of equity should be the same debt that you used to compute the cost of capital.
It is also often argued that book values are more reliable than market values since they are not as volatile. Do you agree?

- Yes
- No

It is often argued that using book value weights is more conservative than using market value weights. Do you agree?

- Yes
- No
Estimating Cost of Capital: Telecom Italia

- **Equity**
  - Cost of Equity = 4.24% + 0.87 (7.03%) = 10.36%
  - Market Value of Equity = 9.92* 5255.13 = 52,110 Mil (84.16%)

- **Debt**
  - Cost of debt = 4.24% + 0.2% (default spread) = 4.44%
  - Market Value of Debt = 9,809 Mil (15.84%)

- **Cost of Capital**
  Cost of Capital = 10.36 % (.8416) + 4.44% (1- .4908) (.1584)) = 9.07%
Telecom Italia: Book Value Weights


Is this more conservative?
Estimating Cost of Capital: Siderar

- **Equity**
  - Cost of Equity = 6.00% + 0.71 (16.03%) = 17.38%
  - Market Value of Equity = 3.20* 310.89 = 995 million (94.37%)

- **Debt**
  - Cost of debt = 6.00% + 5.25% (Country default) +1.25% (Company default) = 12.5%
  - Market Value of Debt = 59 Mil (5.63%)

- **Cost of Capital**

  Cost of Capital = 17.38 % (.9437) + 12.50% (1-.3345) (.0563))
  = 17.38 % (.9437) + 8.32% (.0563)) = 16.87%
Dealing with Hybrids and Preferred Stock

- When dealing with hybrids (convertible bonds, for instance), break the security down into debt and equity and allocate the amounts accordingly. Thus, if a firm has $125 million in convertible debt outstanding, break the $125 million into straight debt and conversion option components. The conversion option is equity.

- When dealing with preferred stock, it is better to keep it as a separate component. The cost of preferred stock is the preferred dividend yield. (As a rule of thumb, if the preferred stock is less than 5% of the outstanding market value of the firm, lumping it in with debt will make no significant impact on your valuation).
Recapping the Cost of Capital

Cost of Capital = Cost of Equity \( \frac{\text{Equity}}{\text{Debt} + \text{Equity}} \) + Cost of Borrowing \( (1-t) \frac{\text{Debt}}{\text{Debt} + \text{Equity}} \)

Cost of borrowing should be based upon
1. synthetic or actual bond rating
2. default spread
Cost of Borrowing = Riskfree rate + Default spread

Marginal tax rate, reflecting tax benefits of debt

Weights should be market value weights

Cost of equity based upon bottom-up beta
II. Estimating Cash Flows

DCF Valuation
Steps in Cash Flow Estimation

- Estimate the current earnings of the firm
  - If looking at cash flows to equity, look at earnings after interest expenses - i.e. net income
  - If looking at cash flows to the firm, look at operating earnings after taxes

- Consider how much the firm invested to create future growth
  - If the investment is not expensed, it will be categorized as capital expenditures. To the extent that depreciation provides a cash flow, it will cover some of these expenditures.
  - Increasing working capital needs are also investments for future growth

- If looking at cash flows to equity, consider the cash flows from net debt issues (debt issued - debt repaid)
Measuring Cash Flows

Cash flows can be measured to

**All claimholders in the firm**

1. EBIT (1 - tax rate)
2. \( - (\text{Capital Expenditures} - \text{Depreciation}) \)
3. \( - \text{Change in non-cash working capital} \)
4. \( = \text{Free Cash Flow to Firm (FCFF)} \)

**Just Equity Investors**

1. Net Income
2. \( - (\text{Capital Expenditures} - \text{Depreciation}) \)
3. \( - \text{Change in non-cash Working Capital} \)
4. \( - (\text{Principal Repaid} - \text{New Debt Issues}) \)
5. - Preferred Dividend
6. **Dividends**
7. + Stock Buybacks
Measuring Cash Flow to the Firm

\[
\text{EBIT} \times (1 - \text{tax rate}) - (\text{Capital Expenditures} - \text{Depreciation}) - \text{Change in Working Capital} = \text{Cash flow to the firm}
\]

Where are the tax savings from interest payments in this cash flow?
Earnings Checks

When estimating cash flows, we invariably start with accounting earnings. To the extent that we start with accounting earnings in a base year, it is worth considering the following questions:

- Are there any one-time charges that might be depressing income in the base year?
- Are the earnings negative, and if so, why?
- Are there any financial or capital expenses intermingled with the operating expenses, and if so, how do we correct for them?
One-Time Charges

Assume that you are valuing a firm that is reporting a loss of $500 million, due to a one-time charge of $1 billion. What is the earnings you would use in your valuation?

- A loss of $500 million
- A profit of $500 million

Would your answer be any different if the firm had reported one-time losses like these once every five years?

- Yes
- No
To get earnings right...

- We need to normalize earnings, if the base year earnings are negative or abnormally low.
- We need to adjust earnings to reflect the effects of the accounting treatment of
  - Some financing expenses as operating expenses
  - Some capital expenses as operating expenses
Negative Earnings: Why they are a problem

- When earnings are negative, you cannot start with that number in the base year and expect to grow yourself out of the problem.
- The key to valuation, when earnings are negative, is to some how work with the numbers until the earnings become positive. Exactly how this is done will depend upon why the earnings are negative in the first place.
- In fact, this applies even if your earnings are positive but lower than normal.
A Framework for Dealing with Negative Earnings

Why are the earnings negative or abnormally low?

Temporary Problems

Cyclicality: Eg. Auto firm in recession

Structural Problems: Eg. Cable co. with high infrastructure investments.

Leverage Problems: Eg. An otherwise healthy firm with too much debt.

Long-term Operating Problems: Eg. A firm with significant production or cost problems.

Normalize Earnings

If firm’s size has not changed significantly over time

Average Dollar Earnings (Net Income if Equity and EBIT if Firm made by the firm over time)

If firm’s size has changed over time

Use firm’s average ROE (if valuing equity) or average ROC (if valuing firm) on current BV of equity (if ROE) or current BV of capital (if ROC)

Value the firm by doing detailed cash flow forecasts starting with revenues and reduce or eliminate the problem over time:

(a) If problem is structural: Target for operating margins of stable firms in the sector.

(b) If problem is leverage: Target for a debt ratio that the firm will be comfortable with by end of period, which could be its own optimal or the industry average.

(c) If problem is operating: Target for an industry-average operating margin.
Correcting Accounting Earnings

- The Operating Lease Adjustment: While accounting convention treats operating leases as operating expenses, they are really financial expenses and need to be reclassified as such. This has no effect on equity earnings but does change the operating earnings.

- The R & D Adjustment: Since R&D is a capital expenditure (rather than an operating expense), the operating income has to be adjusted to reflect its treatment.
The Magnitude of Operating Leases

Operating Lease expenses as % of Operating Income

- Market
- Apparel Stores
- Furniture Stores
- Restaurants
Dealing with Operating Lease Expenses

- Operating Lease Expenses are treated as operating expenses in computing operating income. In reality, operating lease expenses should be treated as financing expenses, with the following adjustments to earnings and capital:
  - Debt Value of Operating Leases = PV of Operating Lease Expenses at the pre-tax cost of debt
  - Adjusted Operating Earnings = Operating Earnings + Pre-tax cost of Debt * PV of Operating Leases.
Operating Leases at The Home Depot in 1998

- The pre-tax cost of debt at the Home Depot is 6.25%

<table>
<thead>
<tr>
<th>Yr</th>
<th>Operating Lease Expense</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$294</td>
<td>$277</td>
</tr>
<tr>
<td>2</td>
<td>$291</td>
<td>$258</td>
</tr>
<tr>
<td>3</td>
<td>$264</td>
<td>$220</td>
</tr>
<tr>
<td>4</td>
<td>$245</td>
<td>$192</td>
</tr>
<tr>
<td>5</td>
<td>$236</td>
<td>$174</td>
</tr>
<tr>
<td>6-15</td>
<td>$270</td>
<td>$1,450 (PV of 10-yr annuity)</td>
</tr>
</tbody>
</table>

Present Value of Operating Leases = $2,571

- Debt outstanding at the Home Depot = $1,205 + $2,571 = $3,776 mil
  (The Home Depot has other debt outstanding of $1,205 million)

- Adjusted Operating Income = $2,016 + 2,571 (.0625) = $2,177 mil
The Effects of Capitalizing Operating Leases

- Debt: will increase, leading to an increase in debt ratios used in the cost of capital and levered beta calculation
- Operating income: will increase, since operating leases will now be before the imputed interest on the operating lease expense
- Net income: will be unaffected since it is after both operating and financial expenses anyway
- Return on Capital will generally decrease since the increase in operating income will be proportionately lower than the increase in book capital invested
The Magnitude of R&D Expenses

R&D as % of Operating Income

- Market
- Petroleum
- Computers

Aswath Damodaran
R&D Expenses: Operating or Capital Expenses

- Accounting standards require us to consider R&D as an operating expense even though it is designed to generate future growth. It is more logical to treat it as capital expenditures.

- To capitalize R&D,
  - Specify an amortizable life for R&D (2 - 10 years)
  - Collect past R&D expenses for as long as the amortizable life
  - Sum up the unamortized R&D over the period. (Thus, if the amortizable life is 5 years, the research asset can be obtained by adding up 1/5th of the R&D expense from five years ago, 2/5th of the R&D expense from four years ago...:}
Capitalizing R&D Expenses: Bristol Myers

R & D was assumed to have a 10-year life.

<table>
<thead>
<tr>
<th>Year</th>
<th>R&amp;D Expense</th>
<th>Unamortized portion</th>
<th>Amortization this year</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1385.00</td>
<td>1.00</td>
<td>1385.00</td>
</tr>
<tr>
<td>-2</td>
<td>1276.00</td>
<td>0.90</td>
<td>1148.40</td>
</tr>
<tr>
<td>-3</td>
<td>1199.00</td>
<td>0.80</td>
<td>959.20</td>
</tr>
<tr>
<td>-4</td>
<td>1108.00</td>
<td>0.70</td>
<td>775.60</td>
</tr>
<tr>
<td>-5</td>
<td>1128.00</td>
<td>0.60</td>
<td>676.80</td>
</tr>
<tr>
<td>-6</td>
<td>1083.00</td>
<td>0.50</td>
<td>541.50</td>
</tr>
<tr>
<td>-7</td>
<td>983.00</td>
<td>0.40</td>
<td>393.20</td>
</tr>
<tr>
<td>-8</td>
<td>881.00</td>
<td>0.30</td>
<td>264.30</td>
</tr>
<tr>
<td>-9</td>
<td>789.00</td>
<td>0.20</td>
<td>157.80</td>
</tr>
<tr>
<td>-10</td>
<td>688.00</td>
<td>0.10</td>
<td>68.80</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td>$6,370.60</td>
</tr>
</tbody>
</table>

Value of research asset = $ 6,371 million
Amortization of research asset in 1998 = $ 1,052 million
Adjustment to Operating Income = $ 1,385 million - $ 1,052 million = $ 333 million
The Effect of Capitalizing R&D

- Operating Income will generally increase, though it depends upon whether R&D is growing or not. If it is flat, there will be no effect since the amortization will offset the R&D added back. The faster R&D is growing the more operating income will be affected.
- Net income will increase proportionately, depending again upon how fast R&D is growing.
- Book value of equity (and capital) will increase by the capitalized Research asset.
- Capital expenditures will increase by the amount of R&D; Depreciation will increase by the amortization of the research asset; For all firms, the net cap ex will increase by the same amount as the after-tax operating income.
What tax rate?

- The tax rate that you should use in computing the after-tax operating income should be
  - The effective tax rate in the financial statements (taxes paid/Taxable income)
  - The tax rate based upon taxes paid and EBIT (taxes paid/EBIT)
  - The marginal tax rate
  - None of the above
  - Any of the above, as long as you compute your after-tax cost of debt using the same tax rate
The Right Tax Rate to Use

- The choice really is between the effective and the marginal tax rate. In doing projections, it is far safer to use the marginal tax rate since the effective tax rate is really a reflection of the difference between the accounting and the tax books.

- By using the marginal tax rate, we tend to understate the after-tax operating income in the earlier years, but the after-tax tax operating income is more accurate in later years.

- If you choose to use the effective tax rate, adjust the tax rate towards the marginal tax rate over time.

- The tax rate used to compute the after-tax cost of debt has to be the marginal tax rate, and can be different from the tax rate used to compute after-tax operating income.
Assume that you are trying to estimate the after-tax operating income for a firm with $1 billion in net operating losses carried forward. This firm is expected to have operating income of $500 million each year for the next 3 years, and the marginal tax rate on income for all firms that make money is 40%. Estimate the after-tax operating income each year for the next 3 years.

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT</td>
<td>$500</td>
<td>$500</td>
</tr>
<tr>
<td>Taxes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EBIT (1-t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax rate</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Net Capital Expenditures

- Net capital expenditures represent the difference between capital expenditures and depreciation. Depreciation is a cash inflow that pays for some or a lot (or sometimes all of) the capital expenditures.
- In general, the net capital expenditures will be a function of how fast a firm is growing or expecting to grow. High growth firms will have much higher net capital expenditures than low growth firms.
- Assumptions about net capital expenditures can therefore never be made independently of assumptions about growth in the future.
Capital expenditures should include

- Research and development expenses, once they have been re-categorized as capital expenses. The adjusted cap ex will be
  
  \[
  \text{Adjusted Net Capital Expenditures} = \text{Net Capital Expenditures} + \text{Current year’s R&D expenses} - \text{Amortization of Research Asset}
  \]

- Acquisitions of other firms, since these are like capital expenditures. The adjusted cap ex will be
  
  \[
  \text{Adjusted Net Cap Ex} = \text{Net Capital Expenditures} + \text{Acquisitions of other firms} - \text{Amortization of such acquisitions}
  \]

Two caveats:

1. Most firms do not do acquisitions every year. Hence, a normalized measure of acquisitions (looking at an average over time) should be used

2. The best place to find acquisitions is in the statement of cash flows, usually categorized under other investment activities
Working Capital Investments

- In accounting terms, the working capital is the difference between current assets (inventory, cash and accounts receivable) and current liabilities (accounts payables, short term debt and debt due within the next year).

- A cleaner definition of working capital from a cash flow perspective is the difference between non-cash current assets (inventory and accounts receivable) and non-debt current liabilities (accounts payable).

- Any investment in this measure of working capital ties up cash. Therefore, any increases (decreases) in working capital will reduce (increase) cash flows in that period.

- When forecasting future growth, it is important to forecast the effects of such growth on working capital needs, and building these effects into the cash flows.
Changes in non-cash working capital from year to year tend to be volatile. A far better estimate of non-cash working capital needs, looking forward, can be estimated by looking at non-cash working capital as a proportion of revenues.

Some firms have negative non-cash working capital. Assuming that this will continue into the future will generate positive cash flows for the firm. While this is indeed feasible for a period of time, it is not forever. Thus, it is better that non-cash working capital needs be set to zero, when it is negative.
In the strictest sense, the only cash flow that an investor will receive from an equity investment in a publicly traded firm is the dividend that will be paid on the stock.

Actual dividends, however, are set by the managers of the firm and may be much lower than the potential dividends (that could have been paid out)
- managers are conservative and try to smooth out dividends
- managers like to hold on to cash to meet unforeseen future contingencies and investment opportunities

When actual dividends are less than potential dividends, using a model that focuses only on dividends will understate the true value of the equity in a firm.
Some analysts assume that the earnings of a firm represent its potential dividends. This cannot be true for several reasons:

- Earnings are not cash flows, since there are both non-cash revenues and expenses in the earnings calculation.
- Even if earnings were cash flows, a firm that paid its earnings out as dividends would not be investing in new assets and thus could not grow.
- Valuation models, where earnings are discounted back to the present, will over estimate the value of the equity in the firm.

The potential dividends of a firm are the cash flows left over after the firm has made any “investments” it needs to make to create future growth and net debt repayments (debt repayments - new debt issues):

- The common categorization of capital expenditures into discretionary and non-discretionary loses its basis when there is future growth built into the valuation.
Estimating Cash Flows: FCFE

- Cash flows to Equity for a Levered Firm
  
  Net Income
  - (Capital Expenditures - Depreciation)
  - Changes in non-cash Working Capital
  - (Principal Repayments - New Debt Issues)
  = Free Cash flow to Equity

- I have ignored preferred dividends. If preferred stock exist, preferred dividends will also need to be netted out.
Estimating FCFE when Leverage is Stable

Net Income
- (1 - δ) (Capital Expenditures - Depreciation)
- (1 - δ) Working Capital Needs
= Free Cash flow to Equity

δ = Debt/Capital Ratio

For this firm,

- Proceeds from new debt issues = Principal Repayments + d (Capital Expenditures - Depreciation + Working Capital Needs)

In computing FCFE, the book value debt to capital ratio should be used when looking back in time but can be replaced with the market value debt to capital ratio, looking forward.
Estimating FCFE: Disney

- Net Income = $1,533 Million
- Capital spending = $1,746 Million
- Depreciation per Share = $1,134 Million
- Non-cash Working capital Change = $477 Million
- Debt to Capital Ratio = 23.83%


Net Income $1,533 Mil
- (Cap. Exp - Depr)*(1-DR) $465.90
Chg. Working Capital*(1-DR) $363.33
= Free CF to Equity $704 Million

Dividends Paid $345 Million
FCFE and Leverage: Is this a free lunch?

[Bar chart showing the relationship between Debt Ratio and FCFE for Disney, with the y-axis labeled 'FCFE' and the x-axis labeled 'Debt Ratio'.]
FCFE and Leverage: The Other Shoe Drops
In a discounted cash flow model, increasing the debt/equity ratio will generally increase the expected free cash flows to equity investors over future time periods and also the cost of equity applied in discounting these cash flows. Which of the following statements relating leverage to value would you subscribe to?

- Increasing leverage will increase value because the cash flow effects will dominate the discount rate effects
- Increasing leverage will decrease value because the risk effect will be greater than the cash flow effects
- Increasing leverage will not affect value because the risk effect will exactly offset the cash flow effect
- Any of the above, depending upon what company you are looking at and where it is in terms of current leverage
Estimating FCFE: Brahma

- Net Income (1996) = 325 Million BR
- Capital spending (1996) = 396 Million
- Depreciation (1996) = 183 Million BR
- Non-cash Working capital Change (1996) = 12 Million BR
- Debt Ratio = 43.48%


- Earnings per Share 325.00 Million BR
- (Cap Ex-Depr) (1-DR) = (396-183)(1-.4348) = 120.39 Million BR
- Change in Non-cash WC (1-DR) = 12 (1-.4348) = 6.78 Million BR

Free Cashflow to Equity 197.83 Million BR

- Dividends Paid 232.00 Million BR
III. Estimating Growth

DCF Valuation
Ways of Estimating Growth in Earnings

- Look at the past
  - The historical growth in earnings per share is usually a good starting point for growth estimation

- Look at what others are estimating
  - Analysts estimate growth in earnings per share for many firms. It is useful to know what their estimates are.

- Look at fundamentals
  - Ultimately, all growth in earnings can be traced to two fundamentals - how much the firm is investing in new projects, and what returns these projects are making for the firm.
I. Historical Growth in EPS

- Historical growth rates can be estimated in a number of different ways
  - Arithmetic versus Geometric Averages
  - Simple versus Regression Models
- Historical growth rates can be sensitive to
  - the period used in the estimation
- In using historical growth rates, the following factors have to be considered
  - how to deal with negative earnings
  - the effect of changing size
## Disney: Arithmetic versus Geometric Growth Rates

<table>
<thead>
<tr>
<th>Year</th>
<th>EPS</th>
<th>Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td>1991</td>
<td>1.20</td>
<td>-20.00%</td>
</tr>
<tr>
<td>1992</td>
<td>1.52</td>
<td>26.67%</td>
</tr>
<tr>
<td>1993</td>
<td>1.63</td>
<td>7.24%</td>
</tr>
<tr>
<td>1994</td>
<td>2.04</td>
<td>25.15%</td>
</tr>
<tr>
<td>1995</td>
<td>2.53</td>
<td>24.02%</td>
</tr>
<tr>
<td>1996</td>
<td>2.23</td>
<td>-11.86%</td>
</tr>
</tbody>
</table>

**Arithmetic Average = 8.54%**

**Geometric Average = \((2.23/1.50) (1/6) – 1 = 6.83\%\) (6 years of growth)**

- The arithmetic average will be higher than the geometric average rate
- The difference will increase with the standard deviation in earnings
Disney: The Effects of Altering Estimation Periods

<table>
<thead>
<tr>
<th>Year</th>
<th>EPS</th>
<th>Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>1.20</td>
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</tr>
<tr>
<td>1996</td>
<td>2.23</td>
<td>-11.86%</td>
</tr>
</tbody>
</table>

Taking out 1990 from our sample, changes the growth rates materially:
Arithmetic Average from 1991 to 1996 = 14.24%
Geometric Average = \((2.23/1.20)^{1/5}\) = 13.19% (5 years of growth)
Disney: Linear and Log-Linear Models for Growth

<table>
<thead>
<tr>
<th>Year</th>
<th>Year Number</th>
<th>EPS</th>
<th>ln(EPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>1</td>
<td>$1.50</td>
<td>0.4055</td>
</tr>
<tr>
<td>1991</td>
<td>2</td>
<td>$1.20</td>
<td>0.1823</td>
</tr>
<tr>
<td>1992</td>
<td>3</td>
<td>$1.52</td>
<td>0.4187</td>
</tr>
<tr>
<td>1993</td>
<td>4</td>
<td>$1.63</td>
<td>0.4886</td>
</tr>
<tr>
<td>1994</td>
<td>5</td>
<td>$2.04</td>
<td>0.7129</td>
</tr>
<tr>
<td>1995</td>
<td>6</td>
<td>$2.53</td>
<td>0.9282</td>
</tr>
<tr>
<td>1996</td>
<td>7</td>
<td>$2.23</td>
<td>0.8020</td>
</tr>
</tbody>
</table>

- EPS = 1.04 + 0.19 (t): EPS grows by $0.19 a year
  
  Growth Rate = $0.19/$1.81 = 10.5% ($1.81: Average EPS from 90-96)

- ln(EPS) = 0.1375 + 0.1063 (t): Growth rate approximately 10.63%
You are trying to estimate the growth rate in earnings per share at Time Warner from 1996 to 1997. In 1996, the earnings per share was a deficit of $0.05. In 1997, the expected earnings per share is $0.25. What is the growth rate?

- 600%
- +600%
- +120%
- Cannot be estimated
Dealing with Negative Earnings

- When the earnings in the starting period are negative, the growth rate cannot be estimated. \((0.30/-0.05 = -600\%)\)

- There are three solutions:
  - Use the higher of the two numbers as the denominator \((0.30/0.25 = 120\%)\)
  - Use the absolute value of earnings in the starting period as the denominator \((0.30/0.05=600\%)\)
  - Use a linear regression model and divide the coefficient by the average earnings.

- When earnings are negative, the growth rate is meaningless. Thus, while the growth rate can be estimated, it does not tell you much about the future.
The Effect of Size on Growth: Callaway Golf

<table>
<thead>
<tr>
<th>Year</th>
<th>Net Profit</th>
<th>Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>1.80</td>
<td></td>
</tr>
<tr>
<td>1991</td>
<td>6.40</td>
<td>255.56%</td>
</tr>
<tr>
<td>1992</td>
<td>19.30</td>
<td>201.56%</td>
</tr>
<tr>
<td>1993</td>
<td>41.20</td>
<td>113.47%</td>
</tr>
<tr>
<td>1994</td>
<td>78.00</td>
<td>89.32%</td>
</tr>
<tr>
<td>1995</td>
<td>97.70</td>
<td>25.26%</td>
</tr>
<tr>
<td>1996</td>
<td>122.30</td>
<td>25.18%</td>
</tr>
</tbody>
</table>

Geometric Average Growth Rate = 102%
## Extrapolation and its Dangers

<table>
<thead>
<tr>
<th>Year</th>
<th>Net Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>$122.30</td>
</tr>
<tr>
<td>1997</td>
<td>$247.05</td>
</tr>
<tr>
<td>1998</td>
<td>$499.03</td>
</tr>
<tr>
<td>1999</td>
<td>$1,008.05</td>
</tr>
<tr>
<td>2000</td>
<td>$2,036.25</td>
</tr>
<tr>
<td>2001</td>
<td>$4,113.23</td>
</tr>
</tbody>
</table>

If net profit continues to grow at the same rate as it has in the past 6 years, the expected net income in 5 years will be $4.113 billion.
II. Analyst Forecasts of Growth

While the job of an analyst is to find under and over valued stocks in the sectors that they follow, a significant proportion of an analyst’s time (outside of selling) is spent forecasting earnings per share.

- Most of this time, in turn, is spent forecasting earnings per share in the next earnings report
- While many analysts forecast expected growth in earnings per share over the next 5 years, the analysis and information (generally) that goes into this estimate is far more limited.

Analyst forecasts of earnings per share and expected growth are widely disseminated by services such as Zacks and IBES, at least for U.S companies.
How good are analysts at forecasting growth?

Analysts forecasts of EPS tend to be closer to the actual EPS than simple time series models, but the differences tend to be small.

<table>
<thead>
<tr>
<th>Study</th>
<th>Time Period</th>
<th>Analyst Forecast Error</th>
<th>Time Series Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collins &amp; Hopwood</td>
<td>Value Line Forecasts 1970-74</td>
<td>31.7%</td>
<td>34.1%</td>
</tr>
<tr>
<td>Brown &amp; Rozeff</td>
<td>Value Line Forecasts 1972-75</td>
<td>28.4%</td>
<td>32.2%</td>
</tr>
<tr>
<td>Fried &amp; Givoly</td>
<td>Earnings Forecaster 1969-79</td>
<td>16.4%</td>
<td>19.8%</td>
</tr>
</tbody>
</table>

The advantage that analysts have over time series models:
- tends to decrease with the forecast period (next quarter versus 5 years)
- tends to be greater for larger firms than for smaller firms
- tends to be greater at the industry level than at the company level

Forecasts of growth (and revisions thereof) tend to be highly correlated across analysts.
Are some analysts more equal than others?

A study of All-America Analysts (chosen by Institutional Investor) found that

- There is no evidence that analysts who are chosen for the All-America Analyst team were chosen because they were better forecasters of earnings. (Their median forecast error in the quarter prior to being chosen was 30%; the median forecast error of other analysts was 28%)

- However, in the calendar year following being chosen as All-America analysts, these analysts become slightly better forecasters than their less fortunate brethren. (The median forecast error for All-America analysts is 2% lower than the median forecast error for other analysts)

- Earnings revisions made by All-America analysts tend to have a much greater impact on the stock price than revisions from other analysts

- The recommendations made by the All America analysts have a greater impact on stock prices (3% on buys; 4.7% on sells). For these recommendations the price changes are sustained, and they continue to rise in the following period (2.4% for buys; 13.8% for the sells).
The Five Deadly Sins of an Analyst

- **Tunnel Vision**: Becoming so focused on the sector and valuations within the sector that they lose sight of the bigger picture.

- **Lemmingitis**: Strong urge felt by analysts to change recommendations & revise earnings estimates when other analysts do the same.

- **Stockholm Syndrome** (shortly to be renamed the Bre-X syndrome): Refers to analysts who start identifying with the managers of the firms that they are supposed to follow.

- **Factophobia** (generally is coupled with delusions of being a famous story teller): Tendency to base a recommendation on a “story” coupled with a refusal to face the facts.

- **Dr. Jekyll/Mr. Hyde**: Analyst who thinks his primary job is to bring in investment banking business to the firm.
Propositions about Analyst Growth Rates

- **Proposition 1**: There is far less private information and far more public information in most analyst forecasts than is generally claimed.

- **Proposition 2**: The biggest source of private information for analysts remains the company itself which might explain
  - why there are more buy recommendations than sell recommendations (information bias and the need to preserve sources)
  - why there is such a high correlation across analysts forecasts and revisions
  - why All-America analysts become better forecasters than other analysts after they are chosen to be part of the team.

- **Proposition 3**: There is value to knowing what analysts are forecasting as earnings growth for a firm. There is, however, danger when they agree too much (lemmingitis) and when they agree to little (in which case the information that they have is so noisy as to be useless).
III. Fundamental Growth Rates

- Current Earnings = \( \text{Investment in Existing Projects} \times \text{Current Return on Investment on Projects} \)
  - \( = \$1000 \times 12\% \) = \$120

- Next Period's Earnings = \( \text{Investment in Existing Projects} \times \text{Next Period's Return on Investment} + \text{Investment in New Projects} \times \text{Return on Investment on New Projects} \)
  - \( = \$1000 \times 12\% + \$100 \times 12\% \) = \$132

- Change in Earnings = \( \text{Investment in Existing Projects} \times \text{Change in ROI from current to next period} + \text{Investment in New Projects} \times \text{Return on Investment on New Projects} \)
  - \( = \$1000 \times 0\% + \$100 \times 12\% \) = \$12
Growth Rate Derivations

In the special case where ROI on existing projects remains unchanged and is equal to the ROI on new projects:

\[
\text{Investment in New Projects} \times \text{Return on Investment} = \frac{\text{Change in Earnings}}{\text{Current Earnings}}
\]

\[
\frac{100}{120} \times 12\% = \frac{12}{120}
\]

Reinvestment Rate:

\[
\frac{83.33\%}{12\%} = 10\%
\]

In the more general case where ROI can change from period to period, this can be expanded as follows:

\[
\text{Investment in Existing Projects} \times (\text{Change in ROI}) + \text{New Projects (ROI)} = \frac{\text{Change in Earnings}}{\text{Current Earnings}}
\]

\[
\text{Investment in Existing Projects} \times \text{Current ROI}
\]

For instance, if the ROI increases from 12% to 13%, the expected growth rate can be written as follows:

\[
\frac{1,000 \times (.13 - .12) + 100 \times (.13)}{100 \times .12} = \frac{23}{120} = 19.17\%
\]
Expected Long Term Growth in EPS

- When looking at growth in earnings per share, these inputs can be cast as follows:
  - Reinvestment Rate = Retained Earnings/ Current Earnings = Retention Ratio
  - Return on Investment = ROE = Net Income/Book Value of Equity
- In the special case where the current ROE is expected to remain unchanged
  - $g_{EPS} = \frac{\text{Retained Earnings}_{t-1}}{\text{NI}_{t-1}} \times \text{ROE}$
  - $= \text{Retention Ratio} \times \text{ROE}$
  - $= b \times \text{ROE}$
- Proposition 1: The expected growth rate in earnings for a company cannot exceed its return on equity in the long term.
Estimating Expected Growth in EPS: ABN Amro

- Current Return on Equity = 15.79%
- Current Retention Ratio = 1 - DPS/EPS = 1 - 1.13/2.45 = 53.88%
- If ABN Amro can maintain its current ROE and retention ratio, its expected growth in EPS will be:

  Expected Growth Rate = 0.5388 (15.79%) = 8.51%
Expected ROE changes and Growth

Assume now that ABN Amro’s ROE next year is expected to increase to 17%, while its retention ratio remains at 53.88%. What is the new expected long term growth rate in earnings per share?

Will the expected growth rate in earnings per share next year be greater than, less than or equal to this estimate?

- greater than
- less than
- equal to
Changes in ROE and Expected Growth

- When the ROE is expected to change,
  \[ g_{\text{EPS}} = b \times \text{ROE}_{t+1} + (\text{ROE}_{t+1} - \text{ROE}_t) \times \text{ROE}_t \]
- Proposition 2: Small changes in ROE translate into large changes in the expected growth rate.
  - The lower the current ROE, the greater the effect on growth of changes in the ROE.
- Proposition 3: No firm can, in the long term, sustain growth in earnings per share from improvement in ROE.
  - Corollary: The higher the existing ROE of the company (relative to the business in which it operates) and the more competitive the business in which it operates, the smaller the scope for improvement in ROE.
Changes in ROE: ABN Amro

Assume now that ABN’s expansion into Asia will push up the ROE to 17%, while the retention ratio will remain 53.88%. The expected growth rate in that year will be:

\[
g_{\text{EPS}} = b \times \text{ROE}_{t+1} + (\text{ROE}_{t+1} - \text{ROE}_t) (\text{BV of Equity}_t) / \text{ROE}_t (\text{BV of Equity}_t)
\]

\[= (0.5388)(0.17) + (0.17 - 0.1579) / (0.1579)\]

\[= 16.83\%
\]

Note that 1.21% improvement in ROE translates into almost a doubling of the growth rate from 8.51% to 16.83%.
ROE and Leverage

- ROE = ROC + D/E (ROC - i (1-t))
- where,
  ROC = (Net Income + Interest (1 - tax rate)) / BV of Capital
  = EBIT (1- t) / BV of Capital
  D/E = BV of Debt/ BV of Equity
  i = Interest Expense on Debt / BV of Debt
  t = Tax rate on ordinary income
- Note that BV of capital = BV of Debt + BV of Equity.
Decomposing ROE: Brahma

- Real Return on Capital = 687 (1-.32) / (1326+542+478) = 19.91%
  - This is assumed to be real because both the book value and income are inflation adjusted.
- Debt/Equity Ratio = (542+478)/1326 = 0.77
- After-tax Cost of Debt = 8.25% (1-.32) = 5.61% (Real BR)
- Return on Equity = ROC + D/E (ROC - i(1-t))
  \[ 19.91% + 0.77 (19.91% - 5.61%) = 30.92\% \]
Decomposing ROE: Titan Watches

- Return on Capital = \( \frac{713 \times (1-.25)}{1925+2378+1303} = 9.54\% \)
- Debt/Equity Ratio = \( \frac{2378 + 1303}{1925} = 1.91 \)
- After-tax Cost of Debt = 13.5\% \times (1-.25) = 10.125\%
- Return on Equity = ROC + D/E \times (ROC - i(1-t))
  \[ 9.54\% + 1.91 \times (9.54\% - 10.125\%) = 8.42\% \]
Expected Growth in EBIT And Fundamentals

- When looking at growth in operating income, the definitions are:
  
  Reinvestment Rate = (Net Capital Expenditures + Change in WC)/EBIT(1-t)
  
  Return on Investment = ROC = EBIT(1-t)/(BV of Debt + BV of Equity)

- Reinvestment Rate and Return on Capital

  \[ g_{EBIT} = \frac{(Net \ Capital \ Expenditures + \ Change \ in \ WC)}{EBIT(1-t)} \times ROC \]

  = Reinvestment Rate \times ROC

- Proposition: No firm can expect its operating income to grow over time without reinvesting some of the operating income in net capital expenditures and/or working capital.

- Proposition: The net capital expenditure needs of a firm, for a given growth rate, should be inversely proportional to the quality of its investments.
No Net Capital Expenditures and Long Term Growth

- You are looking at a valuation, where the terminal value is based upon the assumption that operating income will grow 3% a year forever, but there are no net cap ex or working capital investments being made after the terminal year. When you confront the analyst, he contends that this is still feasible because the company is becoming more efficient with its existing assets and can be expected to increase its return on capital over time. Is this a reasonable explanation?

- Yes
- No
- Explain.
Estimating Growth in EBIT: Disney

- Reinvestment Rate = 50%
- Return on Capital = 18.69%
- Expected Growth in EBIT = 0.5(18.69%) = 9.35%
Estimating Growth in EBIT: Hansol Paper

- Net Capital Expenditures = (150,000-45000) = 105,000 Million WN
  (I normalized capital expenditures to account for lumpy investments)
- Change in Working Capital = 1000 Million WN
- Reinvestment Rate = (105,000+1,000)/(109,569*.7) = 138.20%
- Return on Capital = 6.76%
- Expected Growth in EBIT = 6.76% (1.382) = 9.35%
A Profit Margin View of Growth

- The relationship between growth and return on investment can also be framed in terms of profit margins:
- In the case of growth in EPS
  
  \[
  \text{Growth in EPS} = \text{Retention Ratio} \times \text{ROE}
  \]
  
  \[
  = \text{Retention Ratio} \times \frac{\text{Net Income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{BV of Equity}}
  \]
  
  \[
  = \text{Retention Ratio} \times \text{Net Margin} \times \text{Equity Turnover Ratio}
  \]

- Growth in EBIT = Reinvestment Rate \times \text{ROC}
  
  \[
  = \text{Reinvestment Rate} \times \frac{\text{EBIT}(1-t)}{\text{BV of Capital}}
  \]
  
  \[
  = \text{Reinvestment Rate} \times \text{AT Operating Margin} \times \text{Capital Turnover Ratio}
  \]
IV. Growth Patterns

Discounted Cashflow Valuation
Stable Growth and Terminal Value

- When a firm’s cash flows grow at a “constant” rate forever, the present value of those cash flows can be written as:
  \[ \text{Value} = \frac{\text{Expected Cash Flow Next Period}}{(r - g)} \]
  where,
  \[ r = \text{Discount rate (Cost of Equity or Cost of Capital)} \]
  \[ g = \text{Expected growth rate} \]

- This “constant” growth rate is called a **stable growth rate** and cannot be higher than the growth rate of the economy in which the firm operates.

- While companies can maintain high growth rates for extended periods, they will all approach “stable growth” at some point in time.

- When they do approach stable growth, the valuation formula above can be used to estimate the “terminal value” of all cash flows beyond.
A key assumption in all discounted cash flow models is the period of high growth, and the pattern of growth during that period. In general, we can make one of three assumptions:

- there is no high growth, in which case the firm is already in stable growth
- there will be high growth for a period, at the end of which the growth rate will drop to the stable growth rate (2-stage)
- there will be high growth for a period, at the end of which the growth rate will decline gradually to a stable growth rate (3-stage)
Determinants of Growth Patterns

- Size of the firm
  - Success usually makes a firm larger. As firms become larger, it becomes much more difficult for them to maintain high growth rates.

- Current growth rate
  - While past growth is not always a reliable indicator of future growth, there is a correlation between current growth and future growth. Thus, a firm growing at 30% currently probably has higher growth and a longer expected growth period than one growing 10% a year now.

- Barriers to entry and differential advantages
  - Ultimately, high growth comes from high project returns, which, in turn, comes from barriers to entry and differential advantages.
  - The question of how long growth will last and how high it will be can therefore be framed as a question about what the barriers to entry are, how long they will stay up and how strong they will remain.
Stable Growth and Fundamentals

The growth rate of a firm is driven by its fundamentals - how much it reinvests and how high project returns are. As growth rates approach “stability”, the firm should be given the characteristics of a stable growth firm.

<table>
<thead>
<tr>
<th>Model</th>
<th>High Growth Firms usually</th>
<th>Stable growth firms usually</th>
</tr>
</thead>
<tbody>
<tr>
<td>DDM</td>
<td>1. Pay no or low dividends</td>
<td>1. Pay high dividends</td>
</tr>
<tr>
<td></td>
<td>2. Have high risk</td>
<td>2. Have average risk</td>
</tr>
<tr>
<td></td>
<td>3. Earn high ROC</td>
<td>3. Earn ROC closer to WACC</td>
</tr>
<tr>
<td>FCFE/</td>
<td>1. Have high net cap ex</td>
<td>1. Have lower net cap ex</td>
</tr>
<tr>
<td>FCFF</td>
<td>2. Have high risk</td>
<td>2. Have average risk</td>
</tr>
<tr>
<td></td>
<td>3. Earn high ROC</td>
<td>3. Earn ROC closer to WACC</td>
</tr>
<tr>
<td></td>
<td>4. Have low leverage</td>
<td>4. Have leverage closer to</td>
</tr>
<tr>
<td></td>
<td></td>
<td>industry average</td>
</tr>
</tbody>
</table>
Consider the example of ABN Amro. Based upon its current return on equity of 15.79% and its retention ratio of 53.88%, we estimated a growth in earnings per share of 8.51%.

Let us assume that ABN Amro will be in stable growth in 5 years. At that point, let us assume that its return on equity will be closer to the average for European banks of 15%, and that it will grow at a nominal rate of 5% (Real Growth + Inflation Rate in NV).

The expected payout ratio in stable growth can then be estimated as follows:

\[
\text{Stable Growth Payout Ratio} = 1 - \frac{g}{\text{ROE}} = 1 - \frac{0.05}{0.15} = 66.67\%
\]

\[g = b \times \text{ROE}\]

\[b = \frac{g}{\text{ROE}}\]

\[\text{Payout} = 1 - b\]
To estimate the net capital expenditures in stable growth, consider the growth in operating income that we assumed for Disney. The reinvestment rate was assumed to be 50%, and the return on capital was assumed to be 18.69%, giving us an expected growth rate of 9.35%.

In stable growth (which will occur 10 years from now), assume that Disney will have a return on capital of 16%, and that its operating income is expected to grow 5% a year forever.

Reinvestment Rate = Growth in Operating Income/ROC = 5/16

This reinvestment rate includes both net cap ex and working capital.

Estimated EBIT (1-t) in year 11 = $ 9,098 Million

Reinvestment = $9,098(5/16) = $2,843 Million

Net Capital Expenditures = Reinvestment - Change in Working Capital

= $ 2,843m -105m = 2,738m
V. Beyond Inputs: Choosing and Using the Right Model

Discounted Cashflow Valuation
Summarizing the Inputs

In summary, at this stage in the process, we should have an estimate of the
- the current cash flows on the investment, either to equity investors (dividends or free cash flows to equity) or to the firm (cash flow to the firm)
- the current cost of equity and/or capital on the investment
- the expected growth rate in earnings, based upon historical growth, analysts forecasts and/or fundamentals

The next step in the process is deciding
- which cash flow to discount, which should indicate
- which discount rate needs to be estimated and
- what pattern we will assume growth to follow
Which cash flow should I discount?

- **Use Equity Valuation**
  
  (a) for firms which have stable leverage, whether high or not, and  
  (b) if equity (stock) is being valued

- **Use Firm Valuation**

  (a) for firms which have leverage which is too high or too low, and expect to change the leverage over time, because debt payments and issues do not have to be factored in the cash flows and the discount rate (cost of capital) does not change dramatically over time.  
  
  (b) for firms for which you have partial information on leverage (eg: interest expenses are missing..)  
  
  (c) in all other cases, where you are more interested in valuing the firm than the equity. (Value Consulting?)
Given cash flows to equity, should I discount dividends or FCFE?

- **Use the Dividend Discount Model**
  - (a) For firms which pay dividends (and repurchase stock) which are close to the Free Cash Flow to Equity (over a extended period)
  - (b) For firms where FCFE are difficult to estimate (Example: Banks and Financial Service companies)

- **Use the FCFE Model**
  - (a) For firms which pay dividends which are significantly higher or lower than the Free Cash Flow to Equity. (What is significant? ... As a rule of thumb, if dividends are less than 80% of FCFE or dividends are greater than 110% of FCFE over a 5-year period, use the FCFE model)
  - (b) For firms where dividends are not available (Example: Private Companies, IPOs)
What discount rate should I use?

- **Cost of Equity versus Cost of Capital**
  - If discounting cash flows to equity -> Cost of Equity
  - If discounting cash flows to the firm -> Cost of Capital

- **What currency should the discount rate (risk free rate) be in?**
  - Match the currency in which you estimate the risk free rate to the currency of your cash flows

- **Should I use real or nominal cash flows?**
  - If discounting real cash flows -> real cost of capital
  - If nominal cash flows -> nominal cost of capital
  - If inflation is low (<10%), stick with nominal cash flows since taxes are based upon nominal income
  - If inflation is high (>10%) switch to real cash flows
Which Growth Pattern Should I use?

- If your firm is
  - large and already growing at a rate close to or lower than the overall growth rate of the economy, or
  - constrained by regulation from growing at rate faster than the economy
  - has the characteristics of a stable firm (average risk & reinvestment rates)
    Use a Stable Growth Model

- If your firm
  - is large & growing at a moderate rate ($\leq$ Overall growth rate + 10%) or
  - has a single product & barriers to entry with a finite life (e.g. patents)
    Use a 2-Stage Growth Model

- If your firm
  - is small and growing at a very high rate ($>\,$ Overall growth rate + 10%) or
  - has significant barriers to entry into the business
  - has firm characteristics that are very different from the norm
    Use a 3-Stage Model
The Building Blocks of Valuation

<table>
<thead>
<tr>
<th>Choose a Cash Flow</th>
<th>Dividends</th>
<th>Cashflows to Equity</th>
<th>Cashflows to Firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Dividends to Stockholders</td>
<td>Net Income</td>
<td>- (1 - δ) (Capital Exp. - Deprec’n)</td>
<td>EBIT (1 - tax rate)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- (1 - δ) Change in Work. Capital</td>
<td>- (Capital Exp. - Deprec’n)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= Free Cash flow to Equity (FCFE)</td>
<td>- Change in Work. Capital</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[δ = Debt Ratio]</td>
<td>= Free Cash flow to Firm (FCFF)</td>
</tr>
</tbody>
</table>

& A Discount Rate

**Cost of Equity**
- **Basis:** The riskier the investment, the greater is the cost of equity.
- **Models:**
  - CAPM: Riskfree Rate + Beta (Risk Premium)
  - APM: Riskfree Rate + Σ Beta (Risk Premium): \(n\) factors

**Cost of Capital**
WACC = \(k_e\) (E/ (D+E)) + \(k_d\) (D/(D+E))
\(k_d\) = Current Borrowing Rate (1-t)
E,D: Mkt Val of Equity and Debt

& a growth pattern

<table>
<thead>
<tr>
<th>Stable Growth</th>
<th>Two-Stage Growth</th>
<th>Three-Stage Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>g</td>
<td>g</td>
</tr>
<tr>
<td>t</td>
<td>High Growth</td>
<td>High Growth</td>
</tr>
<tr>
<td>Stable</td>
<td>Stable</td>
<td>Transition</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stable</td>
</tr>
</tbody>
</table>
6. Tying up Loose Ends
Dealing with Cash

- The simplest and most direct way of dealing with cash and marketable securities is to keep it out of the valuation - the cash flows should be before interest income from cash and securities, and the discount rate should not be contaminated by the inclusion of cash. (Use betas of the operating assets alone to estimate the cost of equity).

- Once the firm has been valued, add back the value of cash and marketable securities.
  - If you have a particularly incompetent management, with a history of overpaying on acquisitions, markets may discount the value of this cash.
How much cash is too much cash?

Cash as a Percentage of Value of Firm: US Firms in December 1997

- Cash % of Value
- Number of Firms

- < 2.5%
- 2.5 - 5%
- 5 - 7.5%
- 7.5 - 10%
- 10 - 15%
- 15 - 20%
- 20 - 25%
- 25 - 30%
- > 30%
Implicitly, we are assuming here that the market will value cash at face value. Assume now that you are buying a firm whose only asset is marketable securities worth $100 million. Can you ever consider a scenario where you would not be willing to pay $100 million for this firm?

- Yes
- No

What is or are the scenario(s)?
The Case of Closed End Funds

- Closed end funds are mutual funds, with a fixed number of shares. Unlike regular mutual funds, where the shares have to trade at net asset value (which is the value of the securities in the fund), closed end funds shares can and often do trade at prices which are different from the net asset value.

- The average closed end fund has always traded at a discount on net asset value (of between 10 and 20%) in the United States.
Closed End Funds: Price and NAV

Closed End Equity Funds: December 31, 1997

Number of Funds

Discount: 15-20%
Discount: 10-15%
Discount: 5-10%
Discount: 0-5%
Discount: <0%
Premium: 0-5%
Premium: 5-10%
Premium: 10-15%
Premium: 15-20%
Premium: 20-25%
Premium: >25%
A Simple Explanation for the Closed End Discount

- Assume that you have a closed-end fund that invests in ‘average risk” stocks. Assume also that you expect the market (average risk investments) to make 11.5% annually over the long term. If the closed end fund underperforms the market by 0.50%, estimate the discount on the fund.
Some closed end funds trade at a premium on net asset value. For instance, the Thai closed end funds were trading at a premium of roughly 40% on net asset value and the Indonesian fund at a premium of 80%+ on NAV on December 31, 1997. Why might an investor be willing to pay a premium over the value of the marketable securities in the fund?
Berkshire Hathaway

![Graph showing the value per share, market value/share, book value/share, and premium over book value for Berkshire Hathaway from 1987 to 1997. The graph indicates a significant increase in value per share and premium over book value over the years.]
Dealing with Other Non-Operating Assets

- The more difficult assets to value are minority holdings in subsidiaries. The right way to value these holdings is to value the subsidiaries themselves, and take the firm’s ownership portion of this value. Unfortunately, accounting standards do not allow for much transparency, especially when the subsidiaries are not
Assume that you have done an equity valuation of Microsoft. The total value for equity is estimated to be $400 billion and there are 5 billion shares outstanding. What is the value per share?
An added fact

In 1999, Microsoft had 500 million options outstanding, granted to employees over time. These options had an average exercise price of $20 (the current stock price is $80). Estimate the value per share.
Equity Value and Per Share Value

- The conventional way of getting from equity value to per share value is to divide the equity value by the number of shares outstanding. This approach assumes, however, that common stock is the only equity claim on the firm.

- In many firms, there are other equity claims as well including:
  - warrants, that are publicly traded
  - management and employee options, that have been granted, but do not trade
  - conversion options in convertible bonds
  - contingent value rights, that are also publicly traded.

- The value of these non-stock equity claims has to be subtracted from the value of equity before dividing by the number of shares outstanding.
Option Grants at US Firms

Snowballing
Annual stock option grants as a percentage of shares outstanding at 200 big companies.
Warrants

- A warrant is a security issued by a company that provides the holder with the right to buy a share of stock in the company at a fixed price during the life of the warrant.
- A warrant is therefore a long term call option on the equity of the firm and can be valued using option pricing models.
- Warrants and other equity options issued by the firm are claims on the equity of the firm and have to be treated as equity, which has relevance for:
  - estimating debt and equity for the leverage calculation
  - estimating per share value from total equity value
Why firms use warrants and options

- Warrants are priced based upon the implied volatility assigned to the underlying stock; the greater the volatility, the greater the value. To the degree that the market overestimates the firm’s volatility, the firm may gain by using warrants and option-like securities.

- Warrants, by themselves, create no cash obligations at the time of the issue. Consequently, issuing warrants is a good way for a high growth firm to raise funds, especially when current cash flows are low or nonexistent.

- For financial officers who are sensitive to the dilution created by issuing common stock, warrants seem to provide the best of both worlds — they do not create any new additional shares currently, while they raise equity investment funds for current use.
A convertible bond is a bond that can be converted into a predetermined number of shares, at the option of the bond holder. While it generally does not pay to convert at the time of the bond issue, conversion becomes a more attractive option as stock prices increase.

A convertible bond can be considered to be made up of two securities - a straight bond and a conversion option.

Firms generally add conversions options to bonds to lower the interest rate paid on the bonds.
Embedded in every convertible bond is a straight bond component. The easiest way to value the straight bond component is to act as if the conversion option does not exist and value the bond. This can be accomplished as follows:

- Step 1: Obtain the coupon rate on the convertible bond (which will generally be low because of the conversion option)
- Step 2: Estimate the interest rate that the company would have had to pay if it had issued a straight bond. This can be obtained either from other bonds that the company has outstanding or from its bond rating.
- Step 3: Using the maturity of the convertible bond, the coupon rate and the market interest rate, estimate the value of the bond as:

  \[ \text{Value of Bond} = \text{PV of coupons at market interest rate} + \text{PV of face value of bond at market interest rate} \]

The straight bond component is clearly debt.
Factors in Using Option Pricing Models to Value Convertibles and Warrants

- Option pricing models can be used to value the conversion option with three caveats –
  - conversion options are long term, making the assumptions about constant variance and constant dividend yields much shakier,
  - conversion options result in stock dilution, and
  - conversion options are often exercised before expiration, making it dangerous to use European option pricing models.
- These problems can be partially alleviated by using a binomial option pricing model, allowing for shifts in variance and early exercise, and factoring in the dilution effect.
Steps in Getting to Value Per Share

- Step 1: Value the firm, using discounted cash flow or other valuation models.
- Step 2: Subtract out the value of the outstanding debt to arrive at the value of equity. Alternatively, skip step 1 and estimate the value of equity directly.
- Step 3: Subtract out the market value (or estimated market value) of other equity claims:
  - Value of Warrants = Market Price per Warrant * Number of Warrants
  - Alternatively estimate the value using OPM
  - Value of Conversion Option = Market Value of Convertible Bonds - Value of Straight Debt Portion of Convertible Bonds
- Step 4: Divide the remaining value of equity by the number of shares outstanding to get value per share.
An Example: Valuing Sterling Software

- The equity in Sterling Software was valued at $2,036 million, based upon projected cash flows.
- The firm has two equity options outstanding:
  - The firm has 100,000 bonds outstanding, each of which can be converted into 20 shares of stock. The market price of each convertible bond is $1,750 and the face value is $1,000; coupon rate of 5.75%; expires in 8 years; Bond Rating is A-; Interest rate on comparable debt = 7.50%;
  - The firm has 1.8 million warrants outstanding, with a strike price of $55 per share; these are trading at $30 per share.
Estimating the Value of Options

- Convertible Debt has market value of $175 million; face value of $115 million; coupon rate of 5.75%; expires in 8 years;
  - Bond Rating is A-; Interest rate on comparable debt = 7.50%;
  - Coupon on Convertible Debt = 0.0575 * 115 million = $6.6125 million
  - Value of Straight Debt Portion of Convertible Debt = $6.6125 (PV of Annuity, 7.5%, 8 years) + $115 million / 1.075^8 = $103.21 million
  - Value of Conversion Option in Debt = Market Value of Convertible Debt - Straight Debt Portion = $175 - $103 = $72 million : Equity

- Value of Warrants = Number of warrants * Warrant Price = 1.8 million warrants * $30 = $54 million
## Value Per Share: Sterling Software

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of Equity</td>
<td>$2,036 million</td>
</tr>
<tr>
<td>- Value of Equity in Convertible Debt</td>
<td>$72 million</td>
</tr>
<tr>
<td>- Value of Equity in Warrants</td>
<td>$54 million</td>
</tr>
<tr>
<td>Value of Equity in Common Stock</td>
<td>$1,910 million</td>
</tr>
<tr>
<td>/ Number of Shares outstanding</td>
<td>25.50 million</td>
</tr>
<tr>
<td>Value per Share</td>
<td>$74.90</td>
</tr>
</tbody>
</table>
A Comparison to Other Approaches

- **The Conservative Approach**: Estimate the total number of shares outstanding, including those in the options.

  Value of Equity per share = \( \frac{\text{Value of Equity}}{\text{Fully diluted # of shares}} \)
  
  \[
  = \frac{2,036}{(25.50 + 2 + 1.8)} = \$ 69.49
  \]

- **The Treasury Stock Approach**: Add the expected proceeds from exercise to the numerator before dividing by the number of shares outstanding:

  Value of Equity per share = \( \frac{\text{Value of Equity} + \text{Proceeds from Exercise}}{\text{Fully diluted number of shares}} \)
  
  \[
  = \frac{2036 + 100 + 1.8*55}{25.5 + 2 + 1.8} = \$ 76.28
  \]