Provision of Incentives for Information Acquisition: Forecast-based Contracts versus Menus of Linear Contracts

Abstract

In the producer-seller relationship, the seller, besides his role of selling, is often in an ideal position to gather useful market information for the producer's operations planning. Incentive alignment is critical to motivate both information-acquisition and sales efforts. Two popular contract forms are investigated. One is the forecast-based contract that requires the seller to submit a demand forecast, and he obtains commissions from the realized sales but is also obliged to pay a penalty for any deviation of the sales from the forecast. The other is the menu of linear contracts, from which the seller can choose one that specifies a unique commission rate and a fixed payment. The conventional understanding suggests that the menu of linear contracts is superior, but it is often assumed that information is exogenously endowed. In contrast, we find that with an endogenous information-acquisition effort, the menu of linear contracts may suffer from the conflicted moral hazard effect that creates the friction between motivating the two efforts. The forecast-based contract can however decouple these two tasks and thus dominate the menu of the linear contracts. We further find that when ensuring interim participation is necessary (e.g., renegotiation cannot be prevented after information acquisition), the performance of the forecast-based contract might be affected by the adverse selection effect as it is unable to effectively separate different types, in which the menu of linear contracts excels. We show that when the demand and supply mismatch cost is substantial, the conflicted moral hazard effect dominates the adverse selection effect and the forecast-based contract is more efficient, and it is the converse otherwise. These findings can enrich the understanding of these two contract forms and are useful for sales and operations planning.

1 Introduction

A prevalent way to organize supply chain activities is to have production and selling done by different entities. Manufacturers may engage third-parties to market and sell their products on their behalf. For example, a maker of fashion apparel may engage a department store to sell its seasonal product lines; a high-end bicycle manufacturer may contract with a dealer to sell its new designs; a home appliance producer may sell its products through telemarketers. A commonly recognized issue in such producer-seller relationships is the incentive conflict between the producer's desire for more sales and the seller's aversion to exerting selling effort. Sales commissions are generally used to incentivize the sellers. However, in an environment with separate production and selling entities, the problem of incentive alignment often goes beyond simply motivating sales. The seller has close contacts with the end customers and thus is in an ideal position to gather market information, and fresh information from the field helps the producer to plan the right production quantity. The importance of aligned sales and operations planning (S&OP) is widely recognized. But, here lies another incentive problem that is often overlooked: the fresh information from the field, while highly valuable to the producer, is costly for the seller to gather. Therefore, a more complete characterization of the incentive problem in such producer-seller relationships shall take the above two aspects both into account; that is, how can the seller be motivated to exert costly effort to gather market information about the producer's product, share that information with the producer, and work hard to promote sales of the product?

Ideally, the producer would compensate the seller for the amounts of efforts he has devoted to the tasks, both for information acquisition and for selling. However, in many practical settings, such efforts are often not observable, and the producer has to contract the seller based on the sales. The question hence is how an incentive contract should be designed so that it maximizes the producer's profit. To study this problem, we employ a principal-agent model in which a producer (she) engages a seller (he) to sell a product over a single sales season. The demand for the product in the season is influenced by the market condition as well as the seller's sales effort. A key feature of our model is that the seller has an opportunity to collect a signal about the market condition before the sales season, and he can improve the quality of the signal by expending an information-acquisition effort. The producer is interested in improving her knowledge about the market condition for better production planning.

In general, many contract forms can be designed to maximize the producer's profit. However, not any contract can be easily implemented in a practical environment. In this study, we focus on two contract forms for their popularity and simplicity: the forecast-based contract (FC) and the menu of linear contracts (MLC). These contracts have also been widely studied in the literature. In particular, under the FC contract form, the seller can exert an effort to improve the market information he obtains and he is asked to submit a demand forecast before the sales season, based on which the producer can fine tune her production decision. The seller is penalized if the final sales volume deviates from his forecast, and thus he has an incentive to invest in collecting better market information, which improves the accuracy of his demand forecast. In addition to the penalty term, the contract also contains a linear term that is increasing in the total sales, rewarding the seller for increasing sales and thus inducing his selling effort. This forecast-based contract is a formalization of the Gonik (1978) scheme, which was originally designed to extract information and motivate selling effort from a salesforce and was implemented by IBM's Brazilian unit many years ago. Interestingly, the Gonik scheme is alive and well. For instance, Turner et al. (2007) describe that such forecasted-based incentive schemes are being widely used in the pharmaceutical industry in Europe.

Under the MLC contract form, the producer provides the seller with a list of contracts, and each contract is a distinct linear, nondecreasing function that maps the total sales to the seller's compensation. The seller has an opportunity to collect market information before choosing a contract from the menu. Because better information enables the seller to make a better contract choice, the seller is motivated to exert an information-acquisition effort. The information of the market condition is conveyed to the producer through the seller's contract choice, and the producer can prepare her production accordingly. After the sales season has begun, the seller also has an incentive to exert a selling effort because the more he sells the higher his compensation.

Despite the popularity of the forecast-based contract in practice, the academic literature seems to suggest that the menu of linear contracts is superior. In particular, the menu of linear contracts has been shown to be optimal under various settings where a risk-neutral agent holds private information, see, e.g., Laffont and Tirole (1986) and Rao (1990). In a context related to ours with operations planning, Chen (2005) shows that the menu of linear contracts is more efficient than the forecast-based contract even when the agent is risk averse. However, a common assumption made in this literature is that the agent is exogenously endowed with private information. In contrast, our paper studies a context that has an extra incentive problem: the seller can exert a costly effort to improve the quality of the information. While this addition does not change the fundamental incentive conflicts between the producer and the seller, it alters the producer's objective. Interestingly, we find that the forecast-based contract can now substantially outperform the menu of linear contracts, and it can even achieve the *optimality* under our setting. The intuition is intriguing and useful for understanding the performance comparison between these two contract forms.

In our context, the producer wants to motivate the seller to exert both the sales effort and the information-acquisition effort. Even though the menu of linear contracts can be effective to elicit information and motivate the sales effort alone, it fails to achieve the efficiency if the producer also wants to improve the information quality. These two objectives are in conflict under the menu of linear contracts. In particular, motivating the information-acquisition effort calls for a *broader* menu of contracts with more distinct commission rates so that it is important for the seller to improve the information quality to make more accurate contract choice. Whereas, to induce the optimal sales effort requires a *narrower* menu of contracts with similar commission rates. This conflict causes the loss of efficiency, which we call the *conflicted moral hazard effect* of the menu of linear contracts. The forecast-based contract, however, can achieve a separation of these two objectives. A unique feature of this contract is that the seller is requested to submit a forecast and a penalty is imposed on any deviation of the realized sales from the forecast. This penalty term acts to motivate the seller to improve the accuracy of his information and convey it to the producer. On the other hand, the linear commission plan in the contract can separately motivate the seller to exert the sales effort. The advantage of the forecast-based contract is the largest, when the cost of demand and supply mismatch is substantial and the effort to improve the information quality is intermediate so that the conflicted moral hazard effect is the most significant.

To have a deeper understanding, we further examine an extended case where ensuring interim participation is necessary. That is, after the seller obtains the signal, the producer's original contract needs to ensure his expected profit is no less than his reservation profit for any signal value. This extra participation constraint surfaces the adverse selection problem, which the menu of linear contracts can effectively deal with. The forecast-based contract that contains a single commission plan, however, suffers from the information rents it has to yield to the seller. We call this effect the *adverse selection effect*. We find that when the demand and supply mismatch cost is large, the producer wants to induce a large information-acquisition effort, which leads to the dominance of the conflicted moral hazard effect. The forecast-based contract is thus preferred to the menu of linear contracts. The comparison is the converse when the demand and supply mismatch cost is small and the adverse selection effect dominates the conflicted moral hazard effect. The information-acquisition cost also plays an important role. The forecast-based contract has the best chance of outperforming the menu of linear contract when the information-acquisition cost is intermediate. When the information-acquisition cost is either costless or extremely costly, the conflicted moral hazard effect will varnish, which renders the dominance of the menu of linear contracts. These observations significantly enrich the understanding of these two popular contract forms. They demonstrate that in determining an effective incentive alignment for sales and operations planning, one needs to carefully examine the value of the information for operations planning, the cost of information acquisition, as well as the seller's participation condition.

The rest of the paper is organized as follows. In Section 2, we review the relevant literature. In Section 3, we describe the model and present some preliminary results. Sections 4 to 6 contain, respectively, the analysis of the first-best solution, the menu of linear contracts, and the forecast-based contract. Section 7 discusses the implications of the interim participation constraint and concludes.

2 Literature

In a supply chain context, we study how the producer can use incentive contract to elicit information and motivate the seller to improve information acquisition. Related to our work, Shin and Tunca (2010) investigate how a producer can use a market-based pricing scheme to effectively induce multiple competing retailers to acquire demand information. Focusing on channel coordination, Fu and Zhu (2010) examine the performance of several commonly used contracts (e.g., buy-back, revenue sharing) under costly retailer information acquisition. These studies assume that the information once acquired by the retailer is observable to the producer. Differently, Guo (2009) explores a setting where the information acquired by the retailer is unobservable to the producer unless it is strategically disclosed. He investigates the effects of information acquisition and disclosure on the supply chain parties' profits under the wholesale price contract. Based on a similar setting but allowing nonlinear pricing, Li et al. (2014) show that disclosing just the information status (whether or not gained information) instead of the content can be beneficial for the retailer. Different from the above studies, our work considers both information-acquisition and sales efforts and focuses on the incentive contracts to motivate such efforts.

Notice that the seller's private information in our study is conveyed to the producer by his contract choice or his response to the contract terms. More strategically, information dissemination in supply chains might also be achieved through voluntary sharing. For instance, Li (2002) and Zhang (2002) show that when there is downstream competition, sellers might be willing to share their private demand information with their common supplier. Information sharing can also arise in competing supply chains while the sellers' incentives to do so are influenced by the contract choice, the accuracy of information as well as the production efficiency in the supply chains (Ha and Tong 2008, Ha et al. 2011). The aforementioned studies assume information is shared through installed facilities (e.g., shared database, joint marketing campaign) which is thus verifiable. Some recent research studies find that sellers might be able to truthfully share their demand forecast with their supply chain parties simply by communication when there are conflicting tradeoffs, e.g., announcing a large demand forecast may induce large capacity investment but may also trigger price increase or downstream competition (Chu et al. 2013, Shamir and Shin 2013). This information-sharing literature generally does not investigate the sellers' information-acquisition effort or sales effort.

Our study is naturally related to the literature that explores incentive contracts to motivate an agent to exert effort with private information. In a seminal work, Laffont and Tirole (1986) show that the menu of linear contracts is the optimal contract form to motivate the risk-neutral agent to truthfully report his private cost information and exert cost reduction effort in the procurement context. This is affirmed in a separate study by Picard (1987). In an alternative salesforce context, Rao (1990) also derives that the menu of linear contracts achieves optimum for motivating a risk-neutral sales agent to exert sales effort with private demand information. Under a setting similar to ours with operations planning, Chen (2005) compares the performance of the menu of the linear contracts with that of the forecast-based contract to elicit information from the agent and motivate him to exert sales effort. He shows that the menu of linear contracts is still superior to the forecast-based contract even if the agent is risk averse. The menu of linear contracts is also applied in Khanjari et al. (2014) as an efficient incentive scheme to elicit private information from a sales agent. They discuss whether the manufacturer or the retailer in the supply chain should hire the sales agent, considering the effect of information asymmetry among the supply chain parties. A common assumption made in these studies is that the agent is exogenously endowed with private information. One exception is Lewis and Sappington (1997) who relax the setting in Laffont and Tirole (1986) by assuming that the agent can decide whether or not to learn the production condition before choosing a contract from the menu offered by the principal. They show that this agent's choice of whether to learn the market condition can lead to significant modification of the classical menu of linear contracts.

Different from the above studies, the seller in our sales and operations planning context can exert an information-acquisition effort to improve the quality of the demand information he will receive, and our focus is on the comparison between the performances of the menu of linear contracts and the forecast-based contract. This reasonable addition to the incentive problem in our study results in a useful finding that the menu of linear contracts can be inferior to the forecast-based contract, especially in an environment where the information of the market condition is important for the producer's operations planning. It is worth noting that the forecast-based contract is also related to the so-called bottom-up approach used by many firms in practice to gain information from their salesforce. Under this approach, the firm requires its salesforce to submit forecast information and ties this information to their sales quotas and performance goals (see, e.g., Mantrala and Raman 1990, Mishra and Prasad 2004). Therefore, the forecast-based contract can be viewed as a specific incarnation of the bottom-up approach.

3 The Model

We consider a risk-neutral producer who produces a single product and employs a risk-neutral seller to market and sell the product. The demand x is jointly determined by a market condition θ, the seller's sales effort a, and a random noise ε via the following additive form: $x = \theta + a + \varepsilon$. Here, the seller's effort can represent the activities of reaching out to potential customers and persuading them to purchase. Assume that θ and ε are two independent normally distributed random variables with $\theta \sim N(\mu_\theta, \sigma_\theta^2)$ and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. The seller incurs a cost of $V(a)$ for exerting sales effort a. For simplicity, let $V(a) = a^2/2$.[1]

Before determining the level of sales effort, the seller has an opportunity to acquire a private signal s about the market condition. Assume that $s = \theta + \eta$, where η is independent of θ and ε with $\eta \sim N(0, \sigma_\eta^2)$. Let $\sigma^2 \equiv \sigma_\eta^2/(\sigma_\eta^2 + \sigma_\theta^2)$. From the conjugate property of the normal distribution, the posterior distribution of θ for a given signal s is

$$\theta | s \sim N(\mu_{ps}, \sigma_{ps}^2), \tag{1}$$

where

$$(\mu_{ps}, \sigma_{ps}^2) \equiv (\sigma^2 \mu_\theta + (1 - \sigma^2)s, \sigma_\theta^2 \sigma^2). \tag{2}$$

[1]These assumptions are often made in agency models to evaluate the performance of specific contracts, see, e.g., Lal and Srinivasan (1993).

Note that σ_η is a measure of the signal's precision in predicting the market condition. The larger the value of σ_η, the less information the signal has about the market condition. The seller can select a value of σ_η, or equivalently σ, by exerting information-acquisition effort. This effort represents the investment that the seller can make to improve the quality of the information (e.g., develop forecasting tools, hire marketing professionals, conduct customer survey). If $\sigma = 0$, the signal reveals the exact value of the market condition. On the other hand, as $\sigma \to 1$, the posterior distribution of the market condition is identical to its prior distribution, i.e., the signal contains no useful information. Let $\Gamma(\sigma)$ be the cost the seller incurs for collecting a signal of precision σ, $\sigma \in (0, 1]$, with $\Gamma(1) = 0$. We assume that $\Gamma(\sigma)$ is strictly decreasing and convex, i.e., $\Gamma'(\sigma) < 0$ and $\Gamma''(\sigma) > 0$ for $\sigma \in (0, 1]$.[2]

The producer must decide how much to produce before the sales season. This is often true where the production lead time is long relative to the length of the sales season, rendering a make-to-order system impractical. Denote the unit production cost by c. The sales price is $1 + c$ (hence the profit margin is normalized to 1). If the total demand exceeds the initial production quantity, the excess demand is satisfied by emergency production with a unit cost c'; otherwise, the leftover inventory is salvaged for v per unit. Reasonably, $v < c < c' < 1 + c$. Thus, if the demand is x and the initial production quantity is q, then the producer's profit (excluding the seller's compensation) is $x - L(q - x)$, where $L(\cdot)$ represents the total cost of demand and supply mismatch, with $L(z) = (c - v)\max\{0, z\} + (c' - c)\max\{0, -z\}$. Clearly, the more accurate market information the producer has, the less the mismatch cost she will incur.

However, as in standard agency theory, the seller's information-acquisition effort and sales effort are both unobservable to the producer. Moreover, the market signal the seller collects is also inaccessible to the producer. As such, the only measure of the seller's efforts in our model is the realized sales, upon which the producer can contract the seller's service. We consider two types of contracts for their popularity and simplicity: the forecast-based contract and the menu of linear contracts. These contracts have been widely studied in the literature. Let $w(x, y)$ denote the compensation the seller receives under the producer's contract, which is a function of the realized sales x and a parameter y chosen by the seller according to the contract form (it is either a forecast submitted by the seller or the index of a contract selected from the menu). By observing the seller's response y, the producer may gain some information about the market condition. Let I be the producer's information at the time of making the production decision. The producer's objective

[2]Similar setting for information improvement is used in the literature, see, e.g., Winkler (1981).

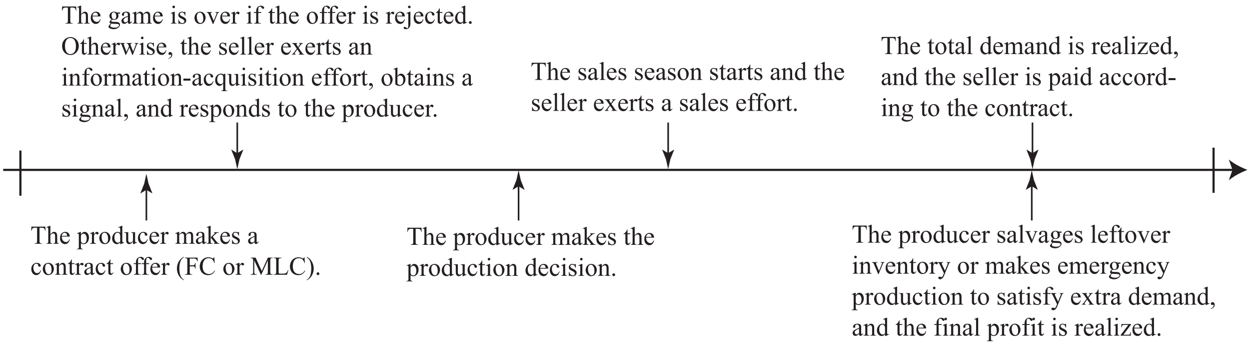

Figure 1: The timeline of the model.

is to maximize her expected profit, $E[x - w(x, y)] - E[E[L(q - x)]|I]$. The seller's objective is to maximize his expected net income, which is equal to the compensation received (w) less the effort costs ($V(a)$ and $\Gamma(\sigma)$). Without loss of generality, the seller's minimum requirement for his expected net income is normalized to 0. Thus, the producer's contract offer must generate for the seller a nonnegative expected net income in order for him to accept it.

Figure 1 details the timeline of the model. First, the producer makes a contract offer to the seller (either a forecast-based contract or a menu of linear contracts). If the offer is rejected, the game is over. Otherwise, the seller decides how much effort to expend to improve the quality of the information he will receive, and then, he obtains a signal. The seller responds to the producer according to the specification in the contract offer (either to submit a demand forecast under the forecast-based contract, or to select one compensation plan under the menu of linear contracts). Based on this response, the producer makes her initial production decision. Then, the sales season starts and the seller decides his sales effort. After the total demand is realized, the producer salvages the leftover inventory if there is any or makes emergency production to meet the extra demand. The seller is paid according to the contract, and the producer's final profit is realized.

4 The First-Best Benchmark

We present the first-best benchmark in this section by assuming that the seller's efforts are contractible and the signal the seller obtains is observable to the producer. As such, the producer can implement any effort level and simply compensate the seller for his cost.

Suppose that the producer implements a contingent optimal sales effort $a(s)$ after the signal s of the market condition is obtained. The distribution of the market condition θ given s is normal with mean μ_{ps} and variance $\sigma_\theta^2\sigma^2$ (see (1) and (2)). Since $x = \theta + a(s) + \varepsilon$ and ε is a normal random variable independent of s, the distribution of the demand x given s is also normal with mean $\mu_{ps}+a(s)$ and variance $\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2$. Note that in our model, the production decision facing the producer is a standard newsvendor problem. Hence, the producer's optimal production quantity can be expressed as: $q(s)=\mu_{ps}+a(s)+\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\Phi^{-1}((c'-c)/(c'-v))$, under which the demand and supply mismatch cost is: $E[L(q(s)-x)|s]=\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}$, where $\rho=(c'-v)\phi(\Phi^{-1}((c'-c)/(c'-v)))$ with $\phi(\cdot)$ being the standard normal density function and $\Phi^{-1}(\cdot)$ the inverse of the standard normal distribution function. Notice that ρ can sufficiently capture the producer's operations cost structure (i.e., c, c' and v), which will thus be used throughout the analysis.

We can write the producer's optimization problem with respect to the effort levels as:

$$\text{(P1).}\quad \max_{\sigma\in(0,1],a(\cdot)\geq 0}\ \mu_\theta+E_s[a(s)]-E_s[V(a(s))]-\Gamma(\sigma)-\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}.$$

Proposition 1. *The optimal solution $\{\sigma_o,a_o(\cdot)\}$ to (P1) is:*

$$\sigma_o = \arg\min_{\sigma\in(0,1]}\left\{\Gamma(\sigma)+\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\right\} \quad (3)$$

$$a_o(\cdot) = a_o=\arg\max_{a\geq 0}\{a-V(a)\}=1, \quad (4)$$

based on which the producer's maximum expected profit is

$$\Pi_o=\mu_\theta+1/2-\Gamma(\sigma_o)-\rho\sqrt{\sigma_\theta^2\sigma_o^2+\sigma_\varepsilon^2}. \quad (5)$$

The benefit of having a more accurate signal is a smaller posterior variance of the market condition and thus a lower expected demand and supply mismatch cost. Hence the optimal level of the signal precision is obtained by minimizing the total cost (see (3)). Due to the additive sales response, the optimal sales effort remains constant for any value of the realized signal. The optimal level of sales effort is derived by balancing the gross profit due to sales effort (profit margin is 1) and the cost of the effort. This explains (4). The producer's profit Π_o is simply her gross profit ($\mu_\theta+a_o$) less the effort costs compensated to the seller and the expected demand and supply mismatch cost, as given in (5).

5 The Menu of Linear Contracts

In this section, we characterize the optimal menu of linear contracts for the environment where neither of the seller's two efforts is contractible nor his received information is observable. Clearly, offered with a menu of contracts, the seller will select one that yields him the highest expected compensation, upon receiving the market signal. Hence, to induce the seller to convey his information, the contracts in the menu should have sufficient difference so that the seller makes distinct choices for different signal values. Further, to motivate the seller to exert sales effort, each contract should reward him for more realized sales; on the other hand, to induce the information-acquisition effort, the overall menu should be designed in a way such that it is beneficial for the seller to make more accurate contract choices. In the following, we formulate the seller's and the producer's problems under the menu of linear contracts.

Let $w(x, \mu_{ps}) = \alpha(\mu_{ps})x + \beta(\mu_{ps})$ denote the seller's compensation which is a linear function of the realized sales x, with the commission rate $\alpha(\mu_{ps}) \geq 0$ and the fixed transfer $\beta(\mu_{ps})$ each being a function of μ_{ps}, the posterior mean of the market condition reported by the seller. We denote the menu by $\{\alpha(\cdot), \beta(\cdot)\}$. Recall from §2 that after accepting the menu $w(\cdot, \cdot)$, the seller first decides the signal precision σ, then observes the signal s and thus μ_{ps}, then chooses a contract (by reporting $\widehat{\mu}_{ps}$), and finally makes the sales effort decision. Next we derive the seller's optimal decisions using backward induction.

We begin with the seller's sales effort decision. The seller who observed μ_{ps} is called the type-μ_{ps} seller. Recall that $x|\mu_{ps} \sim N(\mu_{ps}+a, \sigma_\theta^2\sigma^2+\sigma_\varepsilon^2)$. Thus, the type-$\mu_{ps}$ seller's expected compensation is equal to $\alpha(\widehat{\mu}_{ps})(\mu_{ps}+a)+\beta(\widehat{\mu}_{ps})$, if he reports $\widehat{\mu}_{ps}$ and exerts sales effort a. Subtracting the cost of sales effort $V(a) = a^2/2$ from the expected compensation, we have the type-μ_{ps} seller's expected profit (excluding the sunk information-acquisition cost). Maximizing this profit over a, we obtain

$$\pi(\mu_{ps}, \widehat{\mu}_{ps}) = \max_{a\geq 0}[\alpha(\widehat{\mu}_{ps})(\mu_{ps} + a) + \beta(\widehat{\mu}_{ps}) - a^2/2]$$

which is the type-μ_{ps} seller's maximum expected profit if he reports $\widehat{\mu}_{ps}$. From the revelation principle, we can without loss of generality restrict to the menus that induce the seller to truthfully report his type, i.e., $\pi(\mu_{ps}, \mu_{ps}) \geq \pi(\mu_{ps}, \widehat{\mu}_{ps})$ for any μ_{ps} and $\widehat{\mu}_{ps}$. Let $\pi(\mu_{ps}) \equiv \pi(\mu_{ps}, \mu_{ps})$. Clearly, with truth telling, the type-μ_{ps} seller's optimal sales effort, denoted by $a(\mu_{ps})$, is equal to the commission rate of the chosen contract, i.e., $a(\mu_{ps}) = \alpha(\mu_{ps})$.

Now consider the seller's decision on σ, the signal precision. Let $\widehat{\pi}(\sigma)$ be the seller's expected profit as a function of σ. Thus

$$\widehat{\pi}(\sigma) = E_{\mu_{ps}}[\pi(\mu_{ps})] - \Gamma(\sigma),$$

where $\mu_{ps} \sim N(\mu_\theta, \sigma_\theta^2(1-\sigma^2))$ (see (2)). Let $\sigma = \arg\max_{\sigma'\in(0,1]} \widehat{\pi}(\sigma')$, the optimal signal precision.

We are now ready to consider the producer's optimization problem. Let I represent the producer's information related to the market demand at the time of the production decision. Because of truth telling, $I = \{\mu_{ps}, \sigma\}$, where σ is inferred from the seller's optimization problem. We can then derive the minimum expected demand and supply mismatch cost $\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}$ (similar to the first-best solution). Consequently, the producer's expected profit under the menu $\{\alpha(\cdot),\beta(\cdot)\}$ can be written as

$$\Pi(\alpha(\cdot),\beta(\cdot)) = E_{\mu_{ps}}[\mu_{ps}+\alpha(\mu_{ps})] - E_{\mu_{ps}}[\alpha^2(\mu_{ps})/2] - \Gamma(\sigma) - \rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2} - \widehat{\pi}(\sigma),$$

which can be interpreted as the supply chain's expected profit (the first four terms: gross profits, cost of sales effort, cost of information-acquisition, and cost of demand and supply mismatch) minus the seller's expected profit (the last term). The optimal menu of linear contracts is the solution to:

$$\text{(P2).}\quad \max_{\alpha(\cdot)\geq 0,\beta(\cdot)} \; \Pi(\alpha(\cdot),\beta(\cdot))$$

$$\text{s.t. } \pi(\mu_{ps}) \geq \pi(\mu_{ps},\widehat{\mu}_{ps}),\ \forall \mu_{ps},\widehat{\mu}_{ps} \tag{IC1}$$

$$\sigma = \arg\max_{\sigma'\in(0,1]} \widehat{\pi}(\sigma') \tag{IC2}$$

$$\widehat{\pi}(\sigma) \geq 0. \tag{IR}$$

Note that (IC1) and (IC2) are incentive-compatibility constraints, whereas (IR) is the participation constraint. Denote the optimal menu by $\{\alpha^*(\cdot),\beta^*(\cdot)\}$.

Before we present the optimal menu, define

$$\phi(z,\sigma) \equiv \frac{1}{\sigma_\theta\sqrt{1-\sigma^2}}\phi(\frac{z-\mu_\theta}{\sigma_\theta\sqrt{1-\sigma^2}}) \text{ and } \overline{\Phi}(z,\sigma) \equiv \overline{\Phi}(\frac{z-\mu_\theta}{\sigma_\theta\sqrt{1-\sigma^2}}),$$

which are the density and the residual distribution of the normal random variable μ_{ps} with mean μ_θ and standard deviation $\sigma_\theta\sqrt{1-\sigma^2}$. Define $\lambda(\sigma) \equiv \{\lambda \mid \lambda\overline{\Phi}(-1/\lambda) = -\Gamma'(\sigma)\sqrt{1-\sigma^2}/(\sigma_\theta\sigma)\}$ for $\sigma\in(0,1]$. Note that $\lambda\overline{\Phi}(-1/\lambda)$ strictly increases from 0 to infinity as λ increases from 0 to infinity. Because $\Gamma'(\sigma)<0$ and $\Gamma''(\sigma)>0$, $-\Gamma'(\sigma)\sqrt{1-\sigma^2}/(\sigma_\theta\sigma)$ is positive and strictly decreases in σ for $\sigma\in(0,1]$. Therefore, $\lambda(\sigma)$ is well defined and strictly decreasing in σ. Let $x^+ \equiv \max\{x,0\}$.

Proposition 2. *The optimal menu of linear contracts is*

$$\alpha^*(\mu_{ps}) = \left[1+\lambda(\sigma^*)\frac{\mu_{ps}-\mu_\theta}{\sigma_\theta\sqrt{1-\sigma^{*2}}}\right]^+$$

and $\beta^*(\mu_{ps}) = -\frac{1}{2}[\alpha^*(\mu_{ps})]^2 - \mu_{ps}\alpha^*(\mu_{ps}) + \Gamma(\sigma^*) - \int_{-\infty}^{+\infty}\overline{\Phi}(z,\sigma^*)\alpha^*(z)dz + \int_{-\infty}^{\mu_{ps}}\alpha^*(z)dz$, *where*

$$\sigma^* = \arg\max_{\sigma\in(0,1]}\left\{\int_{-\infty}^{+\infty}\left[\alpha(z,\sigma)-\frac{\alpha^2(z,\sigma)}{2}\right]\phi(z,\sigma)dz - \Gamma(\sigma) - \rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\right\}, \tag{6}$$

where $\alpha(\mu_{ps},\sigma) \equiv [1+\lambda(\sigma)\frac{\mu_{ps}-\mu_\theta}{\sigma_\theta\sqrt{1-\sigma^2}}]^+$. *The seller will always accept this contract offer, choose signal precision σ^*, collect a market signal (and thus observe μ_{ps}), reveal μ_{ps} to the producer (and thus* *choose the linear contract with commission rate* $\alpha^*(\mu_{ps})$ *and fixed transfer* $\beta^*(\mu_{ps})$*), and finally, exert sales effort* $a^*(\mu_{ps}) = \alpha^*(\mu_{ps})$*. The producer's maximum expected profit is*

$$\Pi^*_{MLC} = \mu_\theta + \int_{-\infty}^{+\infty} \left[\alpha^*(z) - \frac{[\alpha^*(z)]^2}{2}\right] \phi(z, \sigma^*)dz - \Gamma(\sigma^*) - \rho\sqrt{\sigma_\theta^2 \sigma^{*2} + \sigma_\varepsilon^2}. \tag{7}$$

The following corollaries point out the key differences between the optimal MLC and the first-best solution. Recall that $a_o(= 1)$ and σ_o are the first-best sales effort and information-acquisition effort, respectively.

Corollary 1. *If* $\rho > 0$*, the optimal MLC leads to underinvestment in information acquisition, i.e.,* $\sigma^* > \sigma_o$*. Moreover,* $\lim_{\rho \to 0} \sigma^* = \lim_{\rho \to 0} \sigma_o = 1$*.*

Corollary 2. *If* $\rho > 0$*, the optimal MLC leads to overinvestment in sales effort when the market signal is favorable, and underinvestment in sales effort otherwise. That is,* $a^*(\mu_{ps}) > a_o$ *for* $\mu_{ps} > \mu_\theta$ *and* $a^*(\mu_{ps}) < a_o$ *for* $\mu_{ps} < \mu_\theta$*. Moreover,* $\lim_{\rho \to 0} a^*(\mu_{ps}) = \lim_{\rho \to 0} a_o = 1$*.*

Corollary 3. Let $\Gamma_k(\sigma) \equiv k\Gamma(\sigma)$ *for* $k > 0$ *and* $\sigma \in (0, 1]$*. Let* $\Pi_o(k)$ *and* $\Pi^*_{MLC}(k)$ *be the producer's expected profit under the first-best solution and the optimal MLC, respectively, assuming that the seller's cost of information acquisition is* $\Gamma_k(\cdot)$*. Then* $\lim_{k \to 0} \Pi^*_{MLC}(k) = \lim_{k \to 0} \Pi_o(k)$*. In other words, as the cost of information acquisition decreases, the producer's optimal expected profit under the menu of linear contracts approaches to her first-best profit.*

Unlike the findings in prior literature, the above corollaries clearly show that the optimal menu of linear contracts does not achieve the first best in our context. To understand the intuition, it is helpful to remind that the producer needs to motivate the seller to exert two hidden efforts: gathering information and generating sales. Fundamentally, the MLC loses efficiency because these two goals are in conflict with each other. To motivate the seller to exert the first-best sales effort, all the linear contracts should have the same slope because that gives the marginal return for the seller's sales effort. On the other hand, to motivate the seller to gather information, the producer however must offer a *broader* menu, i.e., the linear contracts in the menu should offer a wide range of slopes so that the optimal contract choice for the seller varies greatly depending on the market signal. In other words, the contract the seller picks after receiving good news from the market should be very different from the contract chosen when the news is bad. It is thus beneficial to gather precise market information to make more accurate contract choice. As a result, under the form of linear contracts, to motivate optimal sales effort calls for a *narrower* menu, whereas to motivate optimal information-acquisition effort requires a *broader* menu. It is this conflict that

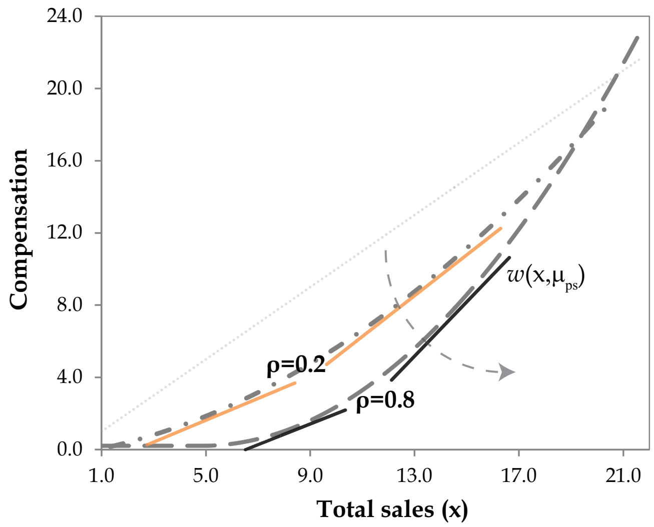

Figure 2: Illustration of the characteristics of the optimal menu of linear contracts. The dash-dotted and the dashed curves characterize the menus of linear contracts, where any tangent line is a linear contract in the menu. The parameters are: $\mu_\theta = 10$, $\sigma_\theta = 3$, $\sigma_\varepsilon = 0$, and $\Gamma(\sigma) = 0.2(\frac{1}{\sigma} - 1)$.

leads to the efficiency loss of the menu of linear contracts, which we call the *conflicted moral hazard effect*. This conflicted moral hazard effect is more significant if the demand and supply mismatch cost is bigger, when the producer wants to motivate a larger information-acquisition effort. To have a vivid understanding, we illustrate the characteristics of the optimal menu of linear contracts in Figure 2. We plot the curves characterized by the incentive compatible condition that defines the menu of linear contracts (any tangent line of the curve is a linear contract in the menu). The dotted diagonal line represents the commission rate that would induce the first-best sales effort. We can see that due to the conflicted moral hazard effect, the slopes of the linear contracts in the optimal menu deviate from that of the dotted diagonal line, and both downward and upward deviations can arise. Moreover, we can observe that the curvature of the curve becomes larger as ρ increases (which drives the demand and supply mismatch cost).

The above discussions readily suggest that if motivating information acquisition becomes either less useful or easier, then the producer's profit will approach the first best. The intuition is clear because both of these scenarios make it unnecessary to have a broad menu. The theoretical arguments are provided in Corollaries 1 and 2 when the market information becomes less useful as the demand and supply mismatch cost decreases, and in Corollary 3 when the cost of information acquisition decreases.

6 The Forecast-based Contract

In this section, we consider the forecast-based contract that is obtained by adding a penalty term to a linear contract. Specifically, the seller is required to submit a sales forecast and his compensation is reduced if the actual sales turn out to be different from the forecast. Mathematically, the seller's compensation is determined by the following formula:

$$w(x, F) = \alpha x + \beta - \gamma h(x - F) \tag{8}$$

where x and F are the actual sales and the submitted forecast respectively, $\{\alpha, \beta, \gamma\}$ are contract parameters chosen by the producer, and $h(\cdot)$ is a penalty function with $h(0) = 0$, $h'(z) > 0$ for $z > 0$, and $h'(z) < 0$ for $z < 0$. Examples of the penalty function include: $h(z) = |z|$, $h(z) = z^2$, etc. While it appears as a single contract, this forecast-based contract in fact is also a menu contract because the seller essentially chooses one contract from the menu by specifying his forecast F. Notice that the Gonik (1978) scheme is a special case of the above contract form by specifying $h(z) = -uz$ for $z < 0$ and $h(z) = vz$ for $z > 0$ for some positive constants u and v. We seek to characterize the optimal contract parameters $\{\alpha^*, \beta^*, \gamma^*\}$ that maximize the producer's expected profit for any general form of $h(\cdot)$.[3]

Suppose the producer offers the contract (8) to the seller. Assume that the seller accepts the contract. He then faces a two-stage decision problem. The first stage is to decide the signal precision σ or equivalently the effort for improving the accuracy of the signal. The second stage is after collecting the market signal. Here the seller decides a sales forecast F and determines the sales effort a. Obviously, the forecast and the sales effort can both be functions of the market signal. As before, we substitute μ_{ps} for the market signal. At the second stage, given σ and μ_{ps}, the seller reports the forecast F and exerts sales effort a to maximize his expected profit (excluding the sunk information-acquisition cost). That is,

$$\pi(\mu_{ps}) = \max_{F, a \geq 0} [\alpha(\mu_{ps} + a) + \beta - \gamma E_x[h(x - F)|\mu_{ps}] - a^2/2] \tag{9}$$

where $\pi(\mu_{ps})$ is thus the seller's maximum expected profit going forward after observing μ_{ps}. Recall that $x|\mu_{ps} \sim N(\mu_{ps} + a, \sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2)$. Thus we can rewrite $x = \mu_{ps} + a + \xi\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}$, where $\xi \sim N(0, 1)$. Using this expression in (9) and defining $\Delta = F - \mu_{ps} - a$, we have

$$\pi(\mu_{ps}) = \max_{a \geq 0} [\alpha(\mu_{ps} + a) + \beta - a^2/2] - \min_{\Delta} \gamma E_\xi[h(\xi\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2} - \Delta)]. \tag{10}$$

[3]It will been shown in Proposition 3 that the specific form of the penalty function does not affect the performance of the optimized forecast-based contract.

Notice that the optimization is greatly simplified because the objective function is separable in a and Δ. The optimal solution is $\tilde{a} = \alpha$ and $\tilde{\Delta} = \arg\min_{\Delta} E_{\xi}[h(\xi\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2} - \Delta]$. It is important to note that $\tilde{a}$ is independent of the first-stage decision (σ) and the market signal, and that $\tilde{\Delta}$ is independent of the market signal (but may depend on σ). Consequently, the seller's optimal forecast is $\tilde{F} = \mu_{ps} + \alpha + \tilde{\Delta}$. Having characterized the seller's decisions at stage 2, we step back to stage 1, where the decision is σ. Denote by $\widehat{\pi}(\sigma)$ the seller's expected profit as a function of σ. Clearly,

$$\widehat{\pi}(\sigma) = E_{\mu_{ps}}[\pi(\mu_{ps})] - \Gamma(\sigma), \tag{11}$$

where $\mu_{ps} \sim N(\mu_\theta, \sigma_\theta^2(1 - \sigma^2))$. Maximizing the above expression over σ leads to the optimal precision level.

We proceed to consider the producer's optimization problem. Let I be the information the producer has that is related to the demand during the sales season at the time of the production decision. Because $\tilde{F} = \mu_{ps} + \alpha + \tilde{\Delta}$, the producer can first solve the seller's first- and second-stage problems to determine the value of $\tilde{\Delta}$, from which she infers the value of μ_{ps} from the seller's submitted forecast $\tilde{F}$. Therefore, $I = \{\mu_{ps}, \sigma\}$. Given this information, the minimum expected demand and supply mismatch cost is $\rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}$ (see §3). This, together with the fact that the seller exerts sales effort $\tilde{a} = \alpha$, implies that the producer's expected profit, denoted by $\Pi(\alpha, \beta, \gamma)$, can be written as:

$$\Pi(\alpha, \beta, \gamma) = \mu_\theta + \alpha - \frac{\alpha^2}{2} - \Gamma(\sigma) - \rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2} - \widehat{\pi}(\sigma).$$

The producer's optimization problem can thus be formulated as:

$$\begin{aligned} \text{(P3).} \quad & \max_{\alpha \geq 0, \beta, \gamma \geq 0} \ \Pi(\alpha, \beta, \gamma) \\ & \text{s.t. } \sigma = \arg\max_{\sigma' \in (0,1]} \widehat{\pi}(\sigma') \\ & \widehat{\pi}(\sigma) \geq 0. \end{aligned}$$

Let $\alpha^* = 1$, γ^* be the solution to

$$\arg\min_{\sigma \in (0,1]} \left\{ \gamma^* \min_{\Delta} E_{\xi}[h(\xi\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2} - \Delta)] + \Gamma(\sigma) \right\} = \sigma_o$$

where $\xi \sim N(0, 1)$ and σ_o is the first-best signal precision, and

$$\beta^* = -\mu_\theta - 1/2 + \gamma^* \min_{\Delta} E_{\xi}[h(\xi\sqrt{\sigma_\theta^2\sigma_o^2 + \sigma_\varepsilon^2} - \Delta)] + \Gamma(\sigma_o).$$

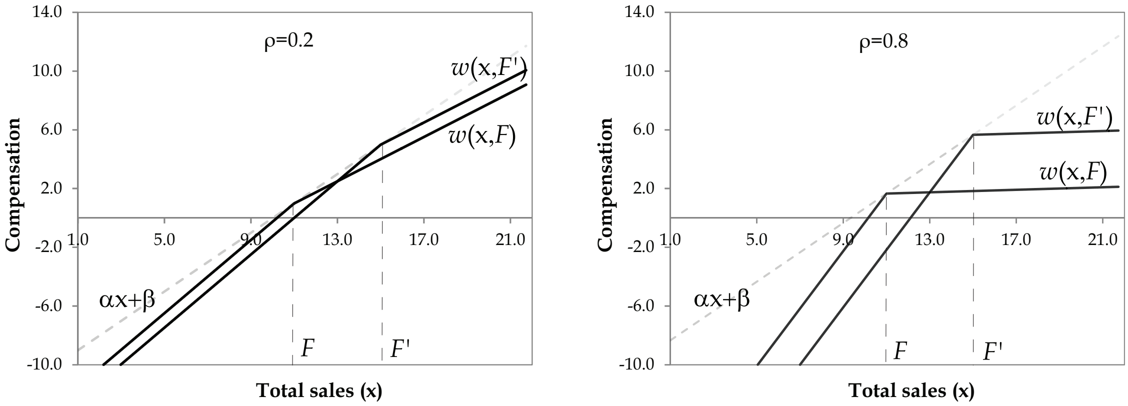

Figure 3: Illustration of the characteristics of the optimal forecast-based contract. The parameters are: $\mu_\theta = 10$, $\sigma_\theta = 3$, $\sigma_\varepsilon = 0$, $\Gamma(\sigma) = 0.2(\frac{1}{\sigma} - 1)$, $h(x - F) = |x - F|$, $\alpha = 1, \gamma = 0.246$ and $\beta = -10.01$ (left plot) and $\gamma = 0.957$ and $\beta = -9.34$ (right plot).

Proposition 3. *The forecast-based contract with parameters* $\{\alpha^*, \beta^*, \gamma^*\}$ *characterized in the above is acceptable to the seller. Under this contract, the seller's optimal decisions on the signal precision and the sales effort are:* $\sigma^* = \sigma_o$ *and* $a^* = 1$*. Moreover, for any given* μ_{ps}*, the seller's optimal forecast is* $F^* = \mu_{ps} + a^* + \Delta^*$*, where* $\Delta^* = \arg\min_\Delta E_\xi[h(\xi\sqrt{\sigma_\theta^2\sigma_o^2 + \sigma_\varepsilon^2} - \Delta)]$ *with* $\xi \sim N(0,1)$*. Therefore, the producer can fully infer the value of* μ_{ps} *from the seller's forecast. Finally, under this contract, the seller's expected profit is equal to his reservation profit and the producer's expected profit, denoted by* Π^*_{FC}*, is equal to the first-best profit, i.e.,* $\Pi^*_{FC} = \Pi_o$*.*

We saw in the previous section that the menu of linear contracts suffers from the conflicted moral hazard effect and loses efficiency at optimum compared to the first-best solution. Therefore, it is all the more interesting to see that the forecast-based contract can achieve the first best and thus outperforms the menu of linear contracts. To understand the intuition, it is important to notice that the forecast-based contract consists of two parts: a linear commission plan based on the realized sales and a penalty term based on the difference between the forecast and the realized sales. These two parts can successfully *decouple* the producer's two objectives in our context. In particular, the first part can act to motivate the seller to exert sales effort, while the second part can motivate the seller to improve the quality of information and convey it to the producer. When the forecast provided by the seller includes the sales effort that he plans to exert, the potential deviation of the realized sales from the forecast will not be affected by the sales effort under the additive demand structure, and thus, the penalty term in the contract will have no influence on the seller's decision of his sales effort. This makes it possible for the single linear commission plan to motivate the seller to exert the first-best effort irrespective of the signal he receives. On the other hand, it is clearly in the seller's interest to always include his sales effort in his demand forecast because that will minimize the potential deviation of the realized sales from the forecast and hence minimize the penalty he will incur. It is also in his interest to include the posterior mean of the market condition (μ_{ps}) in the forecast to minimize the potential penalty, which conveys the signal he receives to the producer. Furthermore, through the penalty term, the producer can control the seller's information-acquisition decision (σ) by adjusting the coefficient γ, to achieve an ideal balance between the demand and supply mismatch cost and the information-acquisition cost. Such a decoupling of two objectives overcomes the conflicted moral hazard effect suffered by the menu of linear contracts and makes it possible to achieve the first best for the forecast-based contract. We illustrate the characteristics of the optimal forecast-based contract in Figure 3. We can observe that a change of the seller's response (i.e., his forecast) only shifts the position of the compensation plan but does not alter its shape. Furthermore, a change of the demand and supply mismatch cost changes the penalty coefficient term (γ) and the fixed payment (β) but does not affect the slope of the linear commission term (α).

To compare the performances of the two contract forms, we conduct a numerical study. We consider 675 instances of the following parameters: $\mu_\theta = 10$, $\sigma_\theta = \{1, 2, 3\}$, $\sigma_\varepsilon = 0$, $\rho = \{0.1, 0.3, 0.5, ..., 2.9\}$, $\Gamma(\sigma) = k(\frac{1}{\sigma} - 1)$ with $k = \{0.1, 0.3, 0.5, ..., 2.9\}$, and the penalty function in the forecast-based contract takes the form of $h(x - F) = |x - F|$. For each instance, we derive the optimal menu of linear contracts and then compute the producer's profits under this menu of linear contracts and the optimal forecast-based contract (the latter is essentially the first-best profit). The numerical study reveals several useful insights. First, we observe that the forecast-based contract can substantially outperform the menu of linear contracts, and the improvement enlarges as ρ (or equivalently, the demand and supply mismatch cost) increases. Figure 4 shows that the average relative improvement across the instances can reach 22.7 percent when $\rho = 2.9$. Second, we observe that the forecast-based contract outperforms the menu of linear contracts the most when the information-acquisition cost is intermediate (see the second panel in Figure 4). This is because: If the information-acquisition cost is very small, it will be convenient for the producer to induce the seller to exert the information-acquisition effort, and if the information-acquisition cost is very large, the producer will not have strong incentive to induce the information-acquisition effort. Hence, the conflicted moral hazard effect in these scenarios will not be as significant as when

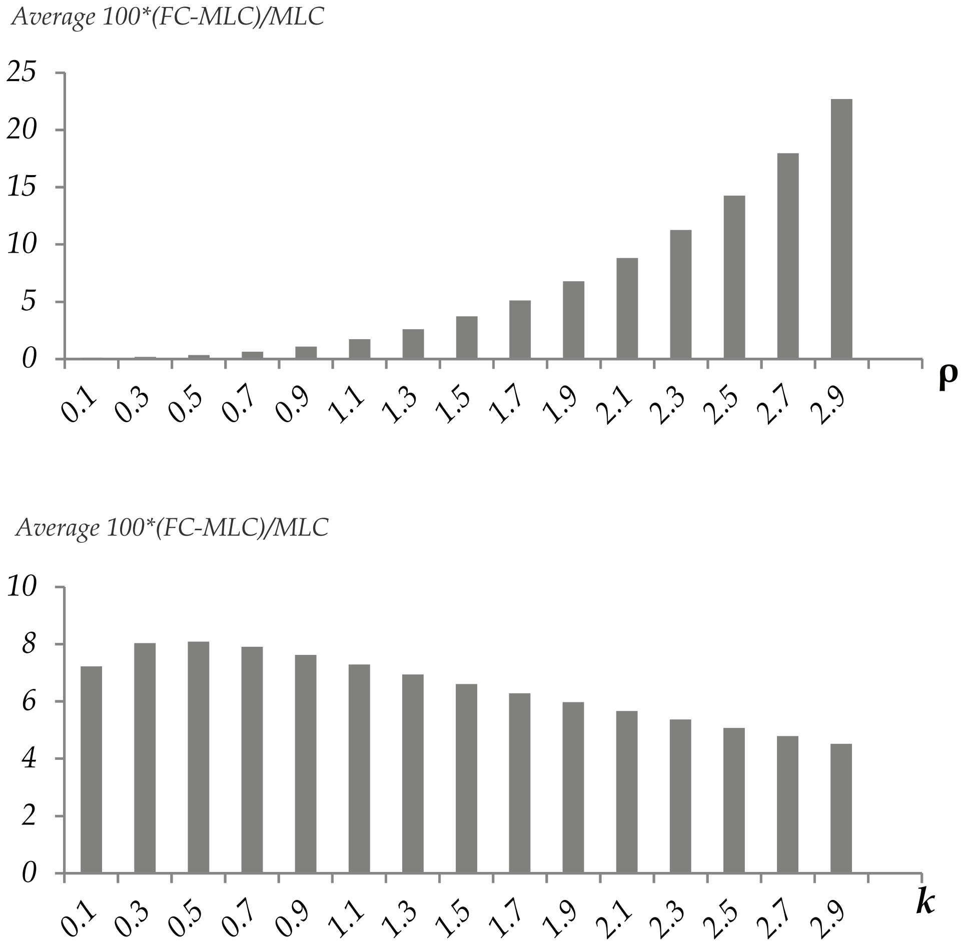

Figure 4: The comparison between the performances of the optimal menu of linear contracts and the forecast-based contract.

the information-acquisition is intermediate.

7 Discussion and Conclusion

7.1 Interim Participation Constraint

It is intriguing to observe that the forecast-based contract can achieve the optimality and outperform the menu of linear contracts in our context, while it is generally shown in prior literature that the menu of linear contracts is either the optimal contract form or simply better than the forecast-based contract (see, e.g., Laffont and Tirole 1986, Rao 1990, Chen 2005). To understand the contrast, it is helpful to note that in our main model the seller needs to decide whether or not to accept the producer's contract offer before he collects any information of the market condition, and once he agrees, he cannot renege regardless of the realized market signal he receives. These assumptions are reasonable in the supply chain context that our study focuses on. First, to conduct demand forecast may require significant investment in market study and might also be time consuming, which makes it necessary for the two parties to first agree on the contract terms before the seller takes any action. Second, the penalties for breaching the contract in such business relationships can be sufficiently severe so that the seller will always honor the contract. The penalties may take various forms such as a damage to the seller's reputation, a court-enforced monetary payment, the termination of a long-term relationship etc. In such circumstances, the producer only needs to guarantee the seller to receive an expected profit no less than his reservation profit in order for him to accept the contract. This is often referred to as the "ex ante participation constraint" in the principal-agent literature. A consequence is that the producer's problem is in essence immune to the *adverse selection effect*. As a result, once a contract resolves the conflicted moral hazard effect, it becomes feasible to implement the first-best solution as does the forecast-based contract.

Differently, the existing studies assume that the agent's private information is exogenously endowed and, on the other hand, his participation needs to be ensured for any received information (which is often referred to as the "interim participation constraint"). This would correspond to a setting in our context if the seller does not have the capability to improve the quality of the information he obtains but he can renege after observing a bad signal. Clearly, in such a situation, the conflicted moral hazard effect is absent, while the adverse selection effect will play a role since the seller can have a misreporting incentive and his interim participation condition needs to be satisfied. The menu of linear contracts is effective to deal with the adverse selection effect, which can contain a complete set of commission and fixed payment pairs, corresponding to the set of signals the seller may receive. One can adjust the commission rates and the fixed payments (a high commission rate is paired with a low fixed payment, and vice versa) to hinder the seller's misreporting incentive (see the tangent lines in Figure 2 which correspond to the linear contracts). The information rent paid to the seller is the least under the menu of linear contracts. The forecast-based contract, however, will suffer from the adverse selection effect. Given it contains only one single commission and fixed payment pair, the seller will obtain a higher expected profit when he receives a good signal than a bad one. With the interim participation constraint, the producer needs to ensure the participation of the seller with the most negative signal, which implies that the seller will receive information rent whenever he observes a better signal. This adverse selection effect deteriorates the performance of the forecast-based contract.

Hence, it becomes clear that in the contexts of prior literature, without the conflicted moral hazard effect, the adverse selection effect hurts the performance of the forecast-based contract, which leads to the dominance of the menu of linear contracts. In contrast, in our main model, the adverse selection effect is absent and the menu of linear contracts suffers from the conflicted moral hazard effect, which results in the dominance of the forecast-based contract.

While the setting of our main model is mostly reasonable in a supply chain context, it is still useful to shed light on understanding the performances of these two contracts when both effects are present. For this purpose, we relax our original setting by allowing the seller to renege after receiving the signal, while keeping the assumption that it is always profitable for the producer to engage the seller regardless of the value of the signal. As a result, in addition to the ex ante participation constraint, the producer also needs to guarantee that the expected profit the seller will obtain under any signal he receives is no less than his reservation profit. In such a scenario, the producer needs to trade off the costs associated with both moral hazard and adverse selection. We shall point out that this is a significantly challenging problem as it nests two moral hazard subproblems with adverse selection. Deriving the optimal contract is difficult even numerically. However, we are able to obtain some structural property which provides useful insights.

Proposition 4. *With the addition of the interim participation constraint, there exist two thresholds $\underline{\rho} < \overline{\rho}$ such that the forecast-based contract dominates the menu of linear contracts when $\rho > \overline{\rho}$ and it is the reverse when $\rho < \underline{\rho}$.*

Specifically, we find that when the demand and supply mismatch cost is substantial (i.e., ρ is large), the producer wants to induce a large information-acquisition effort, which leads to the dominance of the conflicted moral hazard effect over the adverse selection effect. Hence, the forecast-based contract is superior to the menu of linear contracts. In contrast, if the demand and supply mismatch cost is small (i.e., ρ is small), the accuracy of the signal the seller receives becomes less important. In such a situation, the producer will be content with a small information-acquisition effort but keen on the reduction of the information rents. As a result, the menu of linear contracts is preferred to the forecasted-based contract with the dominance of the adverse selection effect.

Besides the demand and supply mismatch cost, the insight we discussed in the previous section with respect to the information-acquisition cost will be preserved. The forecast-based contract will have the best chance of outperforming the menu of linear contracts, when the information-acquisition cost is intermediate and the conflicted moral hazard effect is the most significant. In case the information-acquisition effort is either costless or extremely costly, the conflicted moral hazard effect will varnish and thus the menu of linear contracts will dominate the forecast-based contract in the presence of the interim participation constraint. We summarize the comparison between these two contract forms in the following table.

Interim participation?	Intermediate cost of information acquisition	Costless or extreme cost of information acquisition
No	FC $\geq$ MLC	FC $=$ MLC
Yes	FC $\leq$ MLC if ρ is small FC $\geq$ MLC if ρ is large	FC $\leq$ MLC

Table 1. MLC vs. FC

7.2 Concluding Remarks

Most studies of decentralized supply chains assume that the actors are simply endowed with information (some may be private) about the operating environment. But reality is often that valuable information needs to be gathered and doing so is costly. This paper fills this important gap by considering a producer-seller relationship where the seller, besides his selling role, is uniquely positioned to gather market information, which can be used to improve production planning by the producer. We study two popular contract forms for the incentive alignment problem in such a context. One is the forecast-based contract, under which the seller is required to report a demand forecast, he gets commissions from the sales but also needs to pay a penalty if the sales differ from the forecast. The other is the menu of linear contracts, each of which specifies a unique commission rate and a fixed payment. Our analysis reveals an important difference between these two contract forms in their abilities to manage the tension between motivating for information acquisition and motivating for sales effort. Specifically, the menu of linear contracts suffers from the conflicted moral hazard effect that creates friction between these two tasks: To motivate information acquisition calls for a broader menu of contracts, but for motivating the sales effort, it is better to have a narrower menu. In contrast, we find that the forecast-based contract can effectively decouple these two motivation tasks. As a result, when it is not critical to ensure the interim participation condition for the seller (i.e., once the seller accepts the contract offer, he has to participate regardless of any information he receives), the forecast-based contract is superior, which can substantially outperform the menu of linear contracts in certain circumstances. However, when the producer has to ensure the seller's interim participation condition (e.g., renegotiation is difficult to prevent), the forecast-based contract might suffer from the adverse selection effect as it is unable to effectively separate different types, in which the menu of linear contracts excels. In such an environment, if the demand and supply match cost is substantial and the information-acquisition cost is neither costless nor extremely costly, the conflicted moral hazard effect dominates the adverse selection effect and hence the forecast-based contract is more efficient to align the seller's incentives than the menu of linear contracts, and it is the converse otherwise. These observations significantly enrich the conventional understanding of these two contract forms and can provide useful insights for sales and operations planning in practice.

References

[1] Chen, F. (2005). Salesforce incentives, market information, and production/inventory planning. *Management Science* 51(1), 60-75.

[2] Chu, L. Y., N. Shamir and H. Shin (2013). Strategic communication for capacity alignment with pricing in a supply chain. Working Paper, University of California at San Diego.

[3] Fu, Q. and K. Zhu (2010). Endogenous information acquisition in supply chain management. *European Journal of Operational Research* 201(2), 454-462.

[4] Gonik, J. (1978). Tie salesmen's bonuses to their forecasts. *Harvard Business Review*, May-June 1978.

[5] Guo, L. (2009). The benefits of downstream information acquisition. *Marketing Science* 28(3), 457-471.

[6] Ha, A.Y. and S. Tong. (2008). Contracting and information sharing under supply chain competition. *Management science* 54(4), 701-715.

[7] Ha, A.Y., S. Tong and H. Zhang. (2011). Sharing demand information in competing supply chains with production diseconomies. *Management Science* 57(3), 566-581.

[8] Khanjari, N., S. Iravani and H. Shin (2014). The impact of the manufacturer-hired sales-agent on a supply chain with information asymmetry. *Manufacturing & Service Operations Management*, Forthcoming.

[9] Laffont, J.J. and J. Tirole (1986). Using cost observation to regulate firms. *Journal of Political Economy* 94(3), 614-641.

[10] Lal, R. and V. Srinivasan (1993). Compensation plans for single- and multi-product salesforces: an application of the Holmstrom-Milgrom model. *Management Science* 39(7), 777-793.

[11] Lewis, T.R. and D.E.M. Sappington (1997). Information management in incentive problems. *Journal of Political Economy* 105(4), 796-821.

[12] Li, L. (2002). Information sharing in a supply chain with horizontal competition. *Management Science* 48(9), 1196–1212.

[13] Li, T., S. Tong and H. Zhang (2014). Transparency of information acquisition in a supply chain. *Manufacturing & Service Operations Management*, Forthcoming.

[14] Mantrala, M.K. and K. Raman. (1990). Analysis of a sales force incentive plan for accurate sales forecasting and performance. *International Journal of Research in Marketing* 7(2), 189-202.

[15] Mishra, B.K. and A. Prasad (2004). Centralized pricing versus delegating pricing to salesforce under information asymmetry. *Marketing Science* 23(1), 21-28.

[16] Picard, P. (1987). On the design of incentive scheme under moral hazard and adverse selection. *Journal of Public Economics* 33, 305-331.

[17] Rao, R. (1990). Compensating heterogeneous salesforces: some explicit solutions. *Marketing Science* 9(4), 319-341.

[18] Shamir, N. and H. Shin (2013). Public forecast information sharing in a market with competing supply chains. Working Paper, University of California at San Diego.

[19] Shin H. and T. Tunca (2010). The Effect of competition on demand forecast investments and supply chain coordination. *Operations Research* 58(6), 1592-1610.

[20] Turner, R., C. Lasserre and P. Beauchet (2007). Innovation in field force bonuses: Enhancing motivation through a structured process-based approach. *Journal of Medical Marketing* 7(2), 126-135.

[21] Winkler, R.L. (1981). Combining probability distributions from dependent information sources. *Management Science* 27(4), 479-488.

[22] Zhang, H. (2002). Vertical information exchange in a supply chain with duopoly retailers. *Production and Operations Management* 11(4), 531-546.

Appendix

Proof of Proposition 1. Given the problem is concave, the optimal solutions of the effort levels are unique and can be derived: $a_o = \arg\max_{a\geq 0}\{a - V(a)\} = 1$ and $\sigma_o = \arg\min_{\sigma\in(0,1]} \left\{\rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2} + \Gamma(\sigma)\right\}$. The producer can also specify a compensation plan contingent on the efforts to motivate the seller. ■

Proof of Proposition 2. The proof is carried out in two main steps. First, we derive a constraint that must be satisfied if (IC1) and (IC2) are both satisfied. The producer's objective function, together with this new constraint, forms a relaxed optimization problem, whose optimal objective is clearly an upper bound of that of (P2). Second, we solve the relaxed problem and prove that its optimal solution satisfies all the constraints of (P2) and thus is also an optimal solution to (P2).

It follows from the Envelope Theorem and (IC1) that

$$\begin{aligned}\pi'(\mu_{ps}) &= \frac{\partial\pi(\mu_{ps},\widehat{\mu}_{ps})}{\partial\mu_{ps}}|_{\widehat{\mu}_{ps}=\mu_{ps}} \\ &= \alpha(\mu_{ps}),\end{aligned}$$

which by integration, leads to

$$\pi(\mu_{ps}) = \int_{-\infty}^{\mu_{ps}} \alpha(z)dz + \pi(-\infty). \tag{12}$$

Substituting the above expression of $\pi(\mu_{ps})$ into (IC2), we can rewrite (IC2) as follows:

$$\begin{aligned}\sigma &= \arg\max_{\sigma'\in(0,1]}\{E_{\mu_{ps}}\int_{-\infty}^{\mu_{ps}} \alpha(z)dz - \Gamma(\sigma')\} \\ &= \arg\max_{\sigma'\in(0,1]}\{\int_{-\infty}^{+\infty}\int_{-\infty}^{y} \alpha(z)\phi(x,\sigma')dzdy - \Gamma(\sigma')\} \\ &= \arg\max_{\sigma'\in(0,1]}\{\int_{-\infty}^{+\infty} \alpha(z)\overline{\Phi}(z,\sigma')dz - \Gamma(\sigma')\},\end{aligned}$$

where the second equality is due to $\mu_{ps} \sim N(\mu_\theta, \sigma_\theta^2(1-\sigma'^2))$ and the last equality is by integration by part. Therefore, the first-order necessary condition of (IC2) is

$$\int_{-\infty}^{+\infty} \alpha(z)\frac{(z-\mu_\theta)\sigma}{(1-\sigma^2)}\phi(z,\sigma)dz + \Gamma'(\sigma) = 0,$$

or equivalently,

$$\int_{-\infty}^{+\infty} \alpha(z)\frac{(z-\mu_\theta)}{\sigma_\theta\sqrt{1-\sigma^2}}\phi(z,\sigma)dz + \Gamma'(\sigma)\sqrt{1-\sigma^2}/(\sigma_\theta\sigma) = 0. \tag{13}$$

Note that at the optimal solution, $\widehat{\pi}(\sigma)=0$, because otherwise the solution can be improved by decreasing $\beta(\cdot)$ by a small value. The new derived constraint (13), together with the objective function of (P2), forms the following relaxed problem, denoted by (P2'):

$$\begin{aligned}&\text{(P2')}. \quad \max_{\alpha(\cdot)\geq 0,\sigma\in(0,1]} E_{\mu_{ps}}[\alpha(\mu_{ps})-\alpha^2(\mu_{ps})/2]-\Gamma(\sigma)-\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\\ &\text{s.t. (13),}\end{aligned}$$

where we remove from the objective function the constant terms that are independent of the decision variables.

Because the objective function of (P2') is quadratic and the constraint is linear in $\alpha(\cdot)$, we can solve (P2') by using the KKT condition. Let $\lambda(\sigma)$ be the Lagrange multiplier associated with the constraint. It follows from the KKT condition (taking derivative of the Lagrange function with respect to $\alpha(z)$) that the optimal value of $\alpha(z)$ for any given σ, denoted by $\alpha(z,\sigma)$, is

$$\alpha(z,\sigma)=[1+\lambda(\sigma)\frac{z-\mu_\theta}{\sigma_\theta\sqrt{1-\sigma^2}}]^+.$$

Substituting the above expression for $\alpha(z)$ in the constraint and after some algebra, we have $\lambda(\sigma)=\{\lambda|\lambda\overline{\Phi}(-1/\lambda)=-\Gamma'(\sigma)\sqrt{1-\sigma^2}/(\sigma_\theta\sigma)\}$, which is the same as the notation $\lambda(\sigma)$ introduced before Proposition 2. The results then follow because we can substitute the above expression for $\alpha(z)$ in the objective function of (P2') so as to reduce (P2') to the following unconstrained optimization problem:

$$\max_{\sigma\in(0,1]}\left\{\int_{-\infty}^{+\infty}\left[\alpha(z,\sigma)-\frac{\alpha^2(z,\sigma)}{2}\right]\phi(z,\sigma)dz-\Gamma(\sigma)-\rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\right\}.$$

Consequently, the optimal solution to (P2'), denoted by $\{\sigma^*,\alpha^*(\cdot)\}$, takes the form as given in Proposition 2.

Next we construct $\beta^*(\cdot)$ based on (12) and the binding (IR) constraint. It follows from the definition of $\pi(\mu_{ps})$ that

$$\pi(\mu_{ps})=\alpha(\mu_{ps})(\mu_{ps}+\frac{\alpha(\mu_{ps})}{2})+\beta(\mu_{ps}).$$

This, together with (12), leads to

$$\beta(\mu_{ps})=-\alpha(\mu_{ps})(\mu_{ps}+\frac{\alpha(\mu_{ps})}{2})+\int_{-\infty}^{\mu_{ps}}\alpha(z)dz+\pi(-\infty). \tag{14}$$

It follows from the binding (IR) constraint that

$$\begin{aligned}0 &= \widehat{\pi}(\sigma)\\ &= E_{\mu_{ps}}\int_{-\infty}^{\mu_{ps}}\alpha(z)dz+\pi(-\infty)-\Gamma(\sigma)\\ &= \int_{-\infty}^{+\infty}\alpha(z)\overline{\Phi}(z,\sigma)dz+\pi(-\infty)-\Gamma(\sigma),\end{aligned}$$

which implies that

$$\pi(-\infty) = -\int_{-\infty}^{+\infty} \alpha(z)\overline{\Phi}(z,\sigma)dz + \Gamma(\sigma). \tag{15}$$

By (14) and (15), we have

$$\beta(\mu_{ps}) = -\alpha(\mu_{ps})(\mu_{ps} + \frac{\alpha(\mu_{ps})}{2}) + \Gamma(\sigma) - \int_{-\infty}^{+\infty} \alpha(z)\overline{\Phi}(z,\sigma)dz + \int_{-\infty}^{\mu_{ps}} \alpha(z)dz.$$

Therefore, we can construct $\beta^*(\cdot)$ from the above equation by replacing $\{\sigma, \alpha(\cdot)\}$ with $\{\sigma^*, \alpha^*(\cdot)\}$.

Now it remains to verify that the constructed solution $\{\alpha^*(\cdot), \beta^*(\cdot)\}$ satisfies (IC1) and (IC2). First, it follows from the standard result in adverse selection that the sufficiency of (IC1) is ensured if $\alpha^*(\cdot)$ is monotone. The monotonicity property is easily verifiable from the definition of $\alpha^*(\cdot)$. Thus, (IC1) is satisfied. Second, to verify (IC2), we need show that

$$\sigma^* \in \arg\max_{\sigma\in(0,1]} \left\{ \int_{-\infty}^{+\infty} \alpha^*(z)\overline{\Phi}(z,\sigma)dz - \Gamma(\sigma)\right\}.$$

Because σ^* satisfies the first-order necessary condition of the above maximization problem, it suffices to show that the above objective function is concave in σ, which follows because its first part is concave (by Lemma 1) and its second part $\Gamma(\sigma)$ is convex.■

Lemma 1. Let $H(\sigma) \equiv \int_{-\infty}^{+\infty}(1+kz)^+\overline{\Phi}(\frac{z}{\sqrt{1-\sigma^2}})dz$ with $k \geq 0$. Then $H(\sigma)$ is nonincreasing and concave, i.e., $H'(\sigma) \leq 0$ and $H''(\sigma) \leq 0$ for every $\sigma \in (0,1]$.

Proof of Lemma 1. Note that

$$\begin{aligned} H'(\sigma) &= -\int_{-\infty}^{+\infty}(1+kz)^+\phi(\frac{z}{\sqrt{1-\sigma^2}})\frac{z\sigma}{(1-\sigma^2)\sqrt{1-\sigma^2}}dz \\ &= -\int_{-\infty}^{+\infty}(1+k\sqrt{1-\sigma^2}y)^+\phi(y)\frac{y\sigma}{\sqrt{1-\sigma^2}}dy \\ &= -\int_{-\frac{1}{k\sqrt{1-\sigma^2}}}^{+\infty}(1+k\sqrt{1-\sigma^2}y)\phi(y)\frac{y\sigma}{\sqrt{1-\sigma^2}}dy \\ &= -k\sigma\overline{\Phi}(-\frac{1}{k\sqrt{1-\sigma^2}}). \end{aligned}$$

where the last equality follows from the fact that $\int_{\Delta}^{+\infty}\phi(y)ydy = \phi(\Delta)$ and $\int_{\Delta}^{+\infty}\phi(y)y^2dy = \Delta\phi(\Delta) + \overline{\Phi}(\Delta)$ for any Δ. Clearly, $H'(\sigma) \leq 0$ and $H''(\sigma) \leq 0$.■

Proof of Corollary 1. By definition of σ^* in Proposition 2,

$$\begin{aligned} \sigma^* &= \arg\max_{\sigma\in(0,1]}\left\{\frac{1}{2}\int_{-1/\lambda(\sigma)}^{+\infty}(1-\lambda^2(\sigma)y^2)\phi(y)dy - \Gamma(\sigma) - \rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\right\} \\ &= \arg\min_{\sigma\in(0,1]}\left\{-\frac{1}{2}\int_{-1/\lambda(\sigma)}^{+\infty}(1-\lambda^2(\sigma)y^2)\phi(y)dy + \Gamma(\sigma) + \rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\right\} \\ &\geq \arg\min_{\sigma\in(0,1]}\left\{\Gamma(\sigma) + \rho\sqrt{\sigma_\theta^2\sigma^2+\sigma_\varepsilon^2}\right\} = \sigma_o, \end{aligned}$$

where the inequality follows from the fact that $-\frac{1}{2}\int_{-1/\lambda(\sigma)}^{+\infty}(1-\lambda^2(\sigma)y^2)\phi(y)dy$ strictly decreases in σ (because $\lambda(\sigma)$ strictly decreases in σ). The latter part of Corollary 1 follows directly from the definition of σ^* and σ_o.■

Proof of Corollary 2. The result follows directly from the fact that $a^*(\mu_{ps}) = \alpha^*(\mu_{ps})$ and the definition of $\alpha^*(\mu_{ps})$ in Proposition 2.■

Proof of Corollary 3. As $k \to 0$, $\lambda(\sigma)$ goes to 0 by its definition and thus σ^* goes to 0 by (6). It then follows from (5) and (7) that $\lim_{k\to 0}\Pi^*_{MLC}(k) = \lim_{k\to 0}\Pi_o(k) = \mu_\theta + 1/2 - \rho\sigma_\varepsilon$.■

Proof of Proposition 3. Under the forecast-based contract $\{\alpha^*, \beta^*, \gamma^*\}$, it follows from the analysis prior to Proposition 3 that the type-μ_{ps} seller's optimal sales effort and forecast are

$$a(\mu_{ps}) = \alpha^* = 1$$

and

$$F(\mu_{ps}) = \mu_{ps} + a(\mu_{ps}) + \Delta(\sigma) = \mu_{ps} + 1 + \Delta(\sigma),$$

where $\Delta(\sigma) = \arg\min_\Delta E_\xi[h(\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\xi - \Delta)]$.

Now consider the seller's decision on the signal precision. Given the seller's optimal decisions on sales effort and forecast, we can simplify (11) as follows

$$\begin{aligned} \sigma &= \arg\min_{\sigma\in(0,1]}\left\{\gamma^* E_\xi[h(\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\xi - \Delta(\sigma))] + \Gamma(\sigma)\right\} \\ &= \arg\min_{\sigma\in(0,1]}\left\{\gamma^* \min_\Delta E_\xi[h(\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\xi - \Delta)] + \Gamma(\sigma)\right\} \\ &= \sigma_o \end{aligned}$$

where the second equality is by definition of $\Delta(\sigma)$, and the last equality follows from the definition of γ^*.

Finally, it remains to show the existence of γ^*. Let $H(\gamma,\sigma) \equiv \gamma\min_\Delta E_\xi[h(\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\xi - \Delta)] + \Gamma(\sigma)$. The existence of γ^* is ensured by the following two observations. First, as γ goes to 0, $\arg\min_{\sigma\in(0,1]} H(\gamma,\sigma)$ goes to 1. This observation follows from the fact that $\Gamma(\sigma)$ is decreasing in σ. Second, as γ goes to infinity, $\arg\min_{\sigma\in(0,1]} H(\gamma,\sigma)$ goes to 0. To prove the second observation, it suffices to show that $K(\sigma) \equiv \min_\Delta E_\xi[h(\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\xi - \Delta)]$ strictly increases in σ. By the Envelope Theorem, we have that

$$\begin{aligned} K'(\sigma) &= E_\xi[\frac{\sigma_\theta^2\sigma}{\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}}\xi h'(\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\xi - \Delta(\sigma))] \\ &= E_{\widetilde{\xi}}[\frac{\sigma_\theta^2\sigma}{\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}}\widetilde{\xi}h'(\widetilde{\xi})] \end{aligned}$$

where $\widetilde{\xi} = \sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\xi - \Delta(\sigma)$ and the last equality follows from the first-order necessary condition that $E_\xi[h'(\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\xi - \Delta(\sigma))] = 0$. Because $h'(z) < 0$ for $z < 0$ and $h'(z) > 0$ for $z > 0$, the above equation implies that $K'(\sigma) > 0$ for $\sigma \in (0, 1]$.■

Proof of Proposition 4. When $\rho = 0$, the optimal FC takes the form of a single linear contract, which is weakly dominated by the menu of linear contracts. This suggests that there exists a threshold $\underline{\rho}$ such that the menu of linear contracts is better than the forecast-based contract when $\rho \leq \underline{\rho}$.

Consider the other extreme case where ρ is sufficiently large. Because the inclusion of the interim participation constraint reduces the producer's expected profit, Π^*_{MLC} (defined in (7)) is an upper bound on the producer's expected profit under the optimal MLC. On the other hand, under the forecast-based contract $\{0, \beta^* + 1/2, \gamma^*\}$ where β^* and γ^* are defined prior to Proposition 3, the seller chooses the first-best signal precision and exerts zero sales effort regardless of the signal. Thus the seller's interim participation constraint is ensured by his ex ante participation constraint. The producer's expected profit under this contract, denoted by $\underline{\Pi}_{FC}$, is $\underline{\Pi}_{FC} = \mu_\theta - \min_\sigma\{\Gamma(\sigma) + \rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\}$. Clearly, $\underline{\Pi}_{FC}$ is a lower bound on the producer's expected profit under the optimal FC. It remains to show that $\underline{\Pi}_{FC} > \Pi^*_{MLC}$ when ρ is sufficiently large.

It follows from the definition of $\alpha(z, \sigma)$ (in Proposition 2) that there exists $\overline{\sigma}$ such that

$$\int_{-\infty}^{+\infty} \left[\alpha(z,\sigma) - \frac{[\alpha(z,\sigma)]^2}{2}\right] \phi(z,\sigma)dz < 0, \tag{16}$$

for any $\sigma \in (0, \overline{\sigma})$. Let ρ be sufficiently large so that

$$\Gamma(\sigma) + \rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2} - \min_\sigma\{\Gamma(\sigma) + \rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\} > 1/2 \tag{17}$$

for any $\sigma \in [\overline{\sigma}, 1]$. If $\sigma^* \in (0, \overline{\sigma})$, then

$$\begin{aligned}
\Pi^*_{MLC} &= \mu_\theta + \int_{-\infty}^{+\infty} \left[\alpha^*(z) - \frac{[\alpha^*(z)]^2}{2}\right] \phi(z,\sigma^*)dz - \Gamma(\sigma^*) - \rho\sqrt{\sigma_\theta^2\sigma^{*2} + \sigma_\varepsilon^2} \\
&< \mu_\theta - \Gamma(\sigma^*) - \rho\sqrt{\sigma_\theta^2\sigma^{*2} + \sigma_\varepsilon^2} \text{ (by (16))} \\
&\leq \mu_\theta - \min_\sigma\{\Gamma(\sigma) + \rho\sqrt{\sigma_\theta^2\sigma^2 + \sigma_\varepsilon^2}\} \\
&= \underline{\Pi}_{FC}.
\end{aligned}$$

If $\sigma^* \in [\overline{\sigma}, 1]$, then

$$\begin{aligned}
\Pi^*_{MLC} &= \mu_\theta + \int_{-\infty}^{+\infty} \left[\alpha^*(z) - \frac{[\alpha^*(z)]^2}{2} \right] \phi(z, \sigma^*) dz - \Gamma(\sigma^*) - \rho \sqrt{\sigma_\theta^2 \sigma^{*2} + \sigma_\varepsilon^2} \\
&\leq \mu_\theta + \frac{1}{2} - \Gamma(\sigma^*) - \rho \sqrt{\sigma_\theta^2 \sigma^{*2} + \sigma_\varepsilon^2} \\
&\leq \mu_\theta - \min_\sigma \{ \Gamma(\sigma) + \rho \sqrt{\sigma_\theta^2 \sigma^2 + \sigma_\varepsilon^2} \} \text{ (by (17))} \\
&= \underline{\Pi}_{FC}.
\end{aligned}$$

■