Prior literature has shown that managerial short-termism causes only over-investment at the more efficient type firms for the signaling purpose. Two fundamental assumptions are typically made: the manager’s short-term incentive is exogenous and the setting of information asymmetry guarantees the manager’s possible payoff functions satisfy the single-crossing condition. In an operations context, this study investigates how a manager’s short-term interest in the firm’s market value influences his inventory decision when the aforementioned assumptions do not hold. We consider the case where the firm’s demand uncertainty can be either high or low, ex ante. The true uncertainty is known to the manager while the investors only know the prior distribution. In contrast to prior literature, we show that both over- and understocking can arise at either type of firms. We reveal the critical roles of the magnitude of the manager’s short-term incentive and the firm’s newsvendor critical ratio in determining the impacts of the manager’s short-termism on his inventory decision. Furthermore, we find that when the manager’s short-term incentive is endogenous, it is possible that the inefficient inventory distortions can be resolved for both types of firms. Specifically, in our setting, if the manager has the flexibility to decide the timing and the amount of his stocks to sell, then scenarios exist under the characterized condition, where his private demand uncertainty information can be signaled to the investors through his choice of stock selling instead of inventory distortions. These findings can provide useful insights for understanding managers’ short-termist behaviors.

**Key words**: Short-termism, Market Valuation, Newsvendor, Signaling

1. **Introduction**

There has been continued interest in research about the effects of managers’ short-term objectives on their decision making with partially informed investors. Early studies typically focus on the strategic investment decisions represented by abstracted settings (see, e.g., Stein 1988, 1989, Bebchuk and Stole 1993, Liang and Wen 2007). As predicted by the theories, distortions can arise due to the lack of information for the investors to form precise valuation. It can be either for those firms that are more efficient whose managers overinvest to signal their characteristics to the investors, or a rather passive one in a signal jamming situation where managers simply follow the investors’ expectations to make their investments. While these findings are insightful, they are often bounded by their abstracted capital investment contexts. On the other hand, recent empirical studies as well as anecdotal evidence suggests that the operational decisions are just equally important. For instance, it is shown that inventory information is useful to predict firms’ future performance (Gaur et al. 2005, Kesavan et al. 2010, Kesavan and Mani 2013, Alan et al. 2014),
and investors are paying close attention to firms’ inventory metrics (Raman et al. 2005, Monga 2012). Firms can be penalized for having excess inventory (Chen et al. 2005, Hendricks and Singhal 2009), as well as for not carrying enough (Fisher and Raman 2010, The Associated Press 2006, Trefis Team 2013). Such outcomes can cause investors to doubt their operating environments as well as management capabilities. As a result, managers with an interest in their firms’ stock performance may proactively manage operations to provide investors “promising prospects”, sometimes sacrificing their firms’ long-term profits (Graham et al. 2005, Roychowdhury 2006, Lai 2006, 2008, Monga 2012). It is thus important to understand the managers’ incentives and their consequences.

Operational decisions are often associated with more detailed factors, which naturally triggers the question whether the insights revealed from the early research remain intact. Two recent studies investigate inventory decisions under managerial short-termism (Lai et al. 2012, Schmidt et al. 2014). While they follow the classical assumption for the setting of information asymmetry (to assure the single-crossing regularity condition), they include new factors that are specific for operations. Schmidt et al. (2014) show that some physical operational constraints can lead to a non-standard pooling equilibrium that yields Pareto improvement compared to the classical signaling equilibrium with over-investment. Differently, Lai et al. (2012) reveal the existence of endogenized supply contracts that can resolve over-investment in the signaling equilibrium. However, these findings rely on their specific information setting which may not always apply to general operations environments. Moreover, prior literature typically assumes the managers’ short-term incentives are exogenous. While this assumption is supported by a variety of reasons (e.g., reputation and career concern, threat of takeovers; see, Narayanan 1985, Stein 1988, Holmstrom 1999), there are also various scenarios where the short-term incentives are endogenous. For instance, managers may receive stock-based payments with certain flexibility to decide the timing and the amount of their stocks to sell. Despite the prevalence, there is little investigation of how such endogenized short-term incentives may influence the equilibrium outcomes.

This paper seeks to understand the effects of managers’ short-termism in broader environments where the two aforementioned fundamental assumptions in the literature might be violated. We develop a two-period model where the first period represents a short-term horizon while the second period represents a longer horizon. The first-period inventory decision made by the manager is our focus. The demand is random and the manager needs to decide the regular stocking level. Unmet demand needs to be fulfilled by more expensive emergent supply. The manager is interested in the firm’s short-term profit as well as its long-term value. In the first part of our analysis, we keep the exogenous setting of the manager’s short-term incentive while studying private information about demand uncertainty. Specifically, we assume there can be two types of demand uncertainties, either high or low, which is privately known to the manager. This demand uncertainty represents the
manager’s management capability and the firm’s operating environment. As shown later in detail, such a simple while relatively general setting makes the manager’s possible payoff functions violate the single-crossing condition. In the second part, we provide the manager some flexibility to decide the proportion of his shares to sell, which hence endogenizes his short-term interest in the firm’s stock price. In both parts, the inventory decision once made is observable to the investors and thus a typical signaling game arises.

Our analysis yields interesting results. First, it has been a classical finding in prior literature that due to short-termism, the more efficient type firm will overinvest for the signaling purpose, while the less efficient type firm will invest optimally. Hence, a separating equilibrium is obtained and short-termism would only concern the more efficient type firm. Whereas, our analysis shows that in environments where the single-crossing condition does not hold, not only at the more efficient type firm, inventory distortion can also arise at the less efficient type firm. Hence, short-termism can impact the latter too, which offers a useful implication for managerial compensation design. Moreover, we find that besides the classical separating equilibrium with over-investment, there can exist other separating as well as pooling equilibria with under-investment in inventory. It has been observed in practice that public firms may purposely understock inventory (Lai 2006, Schmidt et al. 2014). Our finding can complement prior literature that aims to explain this phenomenon. We show that in addition to capacity constraints and specific equilibrium refinement that have been considered, inventory understocking can arise for other reasons. For instance, it can be in a scenario where the manager installs less safety stock to signal that his firm has a smaller demand uncertainty, or because it is too costly to signal his information and thus the manager always stocks at some low (pooling) inventory level irrespective of the firm’s type. These insights can shed light on understanding the managers’ incentives and the inventory decisions of the public firms.

Second, we find that in environments where the managers’ short-term incentives are endogenized, the operational distortions which would arise in the conventional settings can be potentially resolved. Specifically, we identify a threshold in our model with respect to the manager’s pressure to realize his stock-based payment early. When the actual pressure is below this threshold, an efficient equilibrium is present, in which the manager of either type firm will always stock the first-best inventory level. Instead of relying on operational distortions, the private information about the demand uncertainty is signaled to the investors through the manager’s endogenized choice of stock selling. The manager of the less efficient type firm realizes more stock-based payment in the short term, while the manager of the more efficient type firm will keep more stock-based payment to the long term. If the actual pressure of early stock payment realization exceeds this threshold, then inventory distortion will occur again. Hence, our study not only reveals an alternative signal
which can resolve inefficient operational distortions in certain scenarios, but also indicates that it can be beneficial to alleviate managers’ pressure on early stock payment realization.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature, and section 3 describe the model. We analyze the case with exogenous short-term objective in section 4 and investigate the endogenous case in section 5. We conclude in section 6.

2. Literature

The impacts of managers’ short-term objectives were first studied in the economics and finance literature. With partially informed investors, Ross (1977) and Miller and Rock (1985) show how short-term oriented managers of the more efficient type firms can use issuing more debts and dividends to signal their types and obtain consistent market valuation. Similarly, Stein (1988) shows that the manager of a more efficient firm can sell a part of the long-term asset to separate from a less efficient firm. In contrast, Stein (1989) portrays a signaling jamming scenario where only the current-period earnings are reported to the investors while the manager’s action is unobservable. He reveals that the manager’s short-term objective can drive him to borrow some long-term profits to inflate the current earnings (such as, liquidate unmature long-term investments) even though the investors correctly anticipate it. As further elaborated by Bebchuk and Stole (1993), the type of information asymmetry plays an important role. They demonstrate with a unified framework that: when the manager’s action is observable while the firm’s long-term productivity is private information, a typical signaling game arises in which the manager who cares about the firm’s short-term market value may overinvest in the long-term project to signal the firm’s type; conversely, the manager will simply follow the investors’ expectation about his action to make under-investment in the long-term project. Several accounting studies also investigate managerial short-termism but from the perspective of exploring more effective accounting policies (see, e.g., Dye and Sridhar 2004, Liang and Wen 2007).

In all these studies, the specification of the managers’ short-term objective is exogenous. Differently, Dybvig and Zender (1991) study a setting and show that the firm owner can design an efficient compensation contract to align the manager’s short-term objective with the first-best solution for the firm. However, it requires that the owner has the knowledge of all potential realizations, when designing the compensation contract. As argued by Bebchuk and Stole (1993), it is often infeasible in practice for the owner to have all of the information or redesign the contract in every period. Garvey et al. (1999) also show, based on a moral hazard setting, that when the manager has the flexibility to decide the amount of his shares to sell, the firm’s shareholders may choose an incentive contract which induces short-term bias.
The above studies mainly focus on firms’ strategic decisions by abstracting away detailed operational factors. Several recent research works investigate the impacts of managers’ short-term objectives on firms’ operations management. Lai et al. (2011) study the commonly observed channel-stuffing phenomenon. They show how short-term oriented managers may inflate their sales depending on their demand realization and examine the role that the firm’s inventory level plays in the managers’ channel-stuffing decision. Lai et al. (2012) and Schmidt et al. (2014) focus on managers’ ordering decisions when they care about their firms’ short-term market value. The classical signaling theory suggests that managers may overorder to signal their demand potential to the investors. However, Lai et al. (2012) show the existence of endogenized supply contracts that can resolve this distortion. The manager can signal the firm’s characteristics to the investors through the choice of contract. Differently, Schmidt et al. (2014) reveal that the physical constraints upon the ordering decisions can lead to a Pareto-improving pooling equilibrium. These studies also adopt the classical assumptions in this stream of literature: the manager’s possible payoff functions satisfy the single-crossing condition and his short-term objective is exogenous. Whereas, violations of these assumptions are not rare in practice. Hence, our work can complement the findings of prior literature with interesting new insights.

3. Model

To align our study with prior literature, we follow the classical framework to model managerial short-termism (see, e.g., Ross 1977, Miller and Rock 1985, Stein 1989, Bebchuk and Stole 1993, Liang and Wen 2007, Lai et al. 2011, 2012, Schmidt et al. 2014). We consider a firm that is operated by a representative risk-neutral and self-interested manager in two periods. The firm is publicly valued by the capital market that is perfectly competitive and consists of identical, risk-neutral investors. The time discount factor for the investors is normalized to one, and the firm does not provide any dividend within the time horizon. As such, the firm’s market value is its conditional expected value. The investors’ information will be updated twice in our model: At the end of the first period, the firm will report the operations outcomes of the first period, and at the end of the second period, the firm will be dissolved with its true value being revealed and distributed to the shareholders, as a typical simplification in the literature. The manager is interested in both the firm’s market value at the end of the first period and the firm’s true value at the end of the second period. Specifically, the manager has a payoff function which follows a linear combination of these two values with certain weights. In section 4, we consider the case same as in prior literature where these weights are exogenously specified, while, in section 5, they are endogenous. To provide a unified framework, we scale the weights to be \( \beta \in (0, 1) \) on the market value and \((1 - \beta)\delta\) on the true value. As such, in section 5, we can interpret \( \beta (1 - \beta)\) as the proportion of shares the manager...
plans to sell at the end of the first period (keep to the end of the second period), while $\delta(< 1)$ can represent the manager’s relative short-term capital pressure. As in prior literature, these weights (once determined) are known to the investors (an extension with $\beta$ being the manager’s private information is provided in Appendix B).

Now, we describe the firm’s operations. Our focus is on the first period which represents a short-term horizon. Similar to Lai et al. (2012) and Schmidt et al. (2014), we assume that the firm sells one representative product with uncertain demand. For analytical tractability, the single-crossing condition has been typically assumed in the literature. In the contexts of Lai et al. (2012) and Schmidt et al. (2014), they assume that the possible demand distributions satisfy the first-order stochastic dominance property which guarantees that the manager’s payoff functions satisfy the single-crossing condition. While this regularity condition facilitates analysis, it does not hold in various scenarios such as the following setting we consider. We assume the demand for the product follows: $D = \mu + \varepsilon/\tau$, where $\mu$ is the mean, $\varepsilon$ is a standard normal random variable with density $\phi(\cdot)$ and distribution function $\Phi(\cdot)$, and $1/\tau$ is the standard deviation. This is one of the classical demand settings for the newsvendor problem used in the operations literature (Porteus 2002, Cachon and Terwiesch 2012). Further, we assume that $\tau$ can be either $\tau_h$ with probability $\rho$ or $\tau(< \tau_h)$ with probability $1 - \rho$, ex ante. While this distribution is common knowledge, only the manager learns the true realization of $\tau$. Here, the demand uncertainty as controlled by $\tau$ can reflect the firm’s operating environment and management capability during this focal time window, about which the internal manager can often have better knowledge than the external investors. Clearly, a larger $\tau$ implies less uncertain demand and thus a better prospect. To isolate the effect from the mean and also facilitate analysis and exposition, we assume $\mu$ is public information (the qualitative insights we reveal carry over to broader settings with asymmetric information of both the mean and the standard deviation as long as the single-crossing condition does not hold). Given this demand setting, the manager decides the regular stocking level $q$ in the first period which can be procured at $c$ per unit. The selling price of the products is $p$ per unit. If the realized demand is greater than the inventory, the unmet portion is backordered and satisfied by more expensive emergent supply at $c_e$ per unit, where $c < c_e < p$. If the demand is less than the inventory, the leftover inventory is salvaged at $s(< c)$ per unit. All the inventory and sales information is reported to the investors at the end of the first period.

The second period represents a long-term horizon. In practice, short-termist behaviors arise often because managers wish to show a better prospect of their firm’s long-term performance. To embed this element in the model, it is sufficient to have a positive association between the firm’s long-term cash flow and the firm’s intrinsic characteristic in the short term. Hence, to facilitate exposition, we use a function $v(\tau)$ to represent the expected profit the firm will make in the second period and
assume \( v(\tau_h) > v(\tau_l) \). Naturally, a currently more efficient firm is likely to also perform better in the future. Let \( K \equiv v(\tau_h) - v(\tau_l) \) which will be the driver of short-termism in our model. This setting is common knowledge to the investors. Note that this future profit function can be characterized by operations similar as in the first period. More sophisticatedly, one might also incorporate the carryover effect of the leftover inventory from the first period. However, the analysis and exposition will become significantly more challenging, while qualitatively short-termism will be triggered as long as the aforementioned positive association exists.

The sequence of events are as follows: First, the manager decides the regular inventory level in the first period. Then, the demand and the sales are realized and reported. The investors value the firm, and the manager obtains his first-period payoff. After that, the firm’s second-period profit is obtained, the firm is dissolved with its true value revealed, and the manager obtains his second-period payoff. In these events, the investors obtain all the information about the firm except for \( \tau \).

While the model we have described shares the same stylized spirit as those in prior literature, it also incorporates distinct elements to reveal interesting new insights about the effects of managers’ short-termist behaviors in practice.

4. Operations Signal

This section analyzes the case where the weights \((\beta \text{ and } (1 - \beta)\delta)\) in the manager’s payoff function are exogenous. We first formulate the investors’ and the manager’s problems.

4.1. Problem Formulation

We focus on pure-strategy equilibria in this paper. At the end of the first period, the investors learn the realization of the firm’s profit, \( \pi(q, D) \equiv pD - cq - c_e(D - q)^+ + s(q - D)^+ \), and infer \( \tau \) from the reported inventory and sales information. Let \( \eta(q, D) \) denote the investors’ posterior belief of \( \tau \). Their inference can be either imperfect or perfect. In the former case, \( \eta(q, D) \) is a random variable, while, in the latter case, it degenerates to a number. The investors can obtain the firm’s expected value as:

\[
P(q, D) = \pi(q, D) + \mathbb{E}_\eta[v(\eta(q, D))].
\]

To make the inventory decision, the manager considers the firm’s market value at the end of the first period as well as the true value at the end of the second period. Suppose the manager knows how the investors value the firm. Then, the manager’s decision can be formulated as:

\[
\max_{q \in [0, \infty)} \beta \mathbb{E}_\varepsilon[P(q, \mu + \varepsilon/\tau_i)] + (1 - \beta) \delta \mathbb{E}_\varepsilon[\pi(q, \mu + \varepsilon/\tau_i) + v(\tau_i)], \; \forall i \in \{h, l\}.
\]

Let \( q^*(\tau_i) \) denote the manager’s optimal inventory decision. In equilibrium, the investors’ valuation for the firm needs to be consistent with the optimal decision the manager makes. We thus define the following equilibrium concept.
DEFINITION 1. A market equilibrium is reached if the following two conditions hold:

(i) The manager’s inventory decision, \( q^*(\tau) \), is the maximizer of (2).

(ii) The investors’ inference function \( \eta(q, D) \) that determines the market value \( P(q, D) \) in (1) satisfies: \( \Pr(\eta(q^*(\tau), D) = \tau_l) = 1 \) when \( q^*(\tau_h) \neq q^*(\tau_l) \); \( \Pr(\eta(q^*(\tau), D) = \tau_h) = 1 - \Pr(\eta(q^*(\tau), D) = \tau_l) \) when \( q^*(\tau_h) = q^*(\tau_l) \); and for any \( q \neq q^*(\tau_h) \) or \( q^*(\tau_l) \), \( \Pr(\eta(q, D) = \tau_h) = 1 - \Pr(\eta(q, D) = \tau_l) = 0 \).

Definition 1 follows the perfect Bayesian equilibrium concept. Notice that if the manager’s inventory decision is different for different \( \tau \), then a separating equilibrium arises, in which the investors infer \( \tau \) perfectly; otherwise, a pooling equilibrium occurs, in which the investors use Bayes rule to update their belief of \( \tau \) based on the demand realization. Bayes’ rule, however, does not apply to any off-equilibrium action. Definition 1 specifies that for any \( q \) that deviates from the equilibrium levels, the investors believe that the firm they face has large uncertainty. This is intuitive because, in our model, it is the manager who, observing large uncertainty, has an incentive to pretend to have small uncertainty through operations distortion.

4.2. The First-Best Benchmark

If the manager is not interested in the firm’s market value or \( \tau \) is publicly known, then the manager’s problem in the first period becomes a classical newsvendor problem, which we use as our first-best benchmark. The objective is to maximize the firm’s expected profit, \( \Pi_i(q) \equiv \mathbb{E}_\epsilon [\pi(q, \mu + \epsilon/\tau_i)] \). Let \( c_u = c_e - c \) and \( c_o = c - s \) denote the underage and overage costs, and \( CR = \frac{c_o}{c_u + c_o} \) be the newsvendor critical ratio. It is not difficult to derive the optimal inventory decision:

\[
q^c(\tau_i) = \mu + \Phi^{-1}(CR)/\tau_i, \forall i \in \{h, l\},
\]

and the corresponding expected profit:

\[
\Pi_i(q^c(\tau_i)) = (p - c)\mu - (c_u + c_o)\phi(\Phi^{-1}(CR))/\tau_i, \forall i \in \{h, l\}.
\]

PROPOSITION 1. (i) \( \Pi^*_h(q) > \langle \langle \Pi^*_l(q) \) when \( q < \langle \rangle \mu \);

(ii) When \( CR > 0.5 \), \( q^c(\tau_i) > q^c(\tau_h) > \mu \); when \( CR < 0.5 \), \( q^c(\tau_i) < q^c(\tau_h) < \mu \); and when \( CR = 0.5 \), \( q^c(\tau_h) = q^c(\tau_l) = \mu \).

We depict the results of Proposition 1 in Figure 1. Proposition 1 provides two important implications. First, it shows that the profit functions of the two firm types do not satisfy the first-order dominance property. Their first derivatives change the order at the mean of the distributions. As a result, the single-crossing condition is violated because it requires, for any change of the decision variable, there is a monotone ordering of the profit changes among different types (Sobel 2009). Signaling games typically have multiple equilibria. The intuitive criterion (Cho and Kreps 1987)
Figure 1  The expected short-term profit functions of the efficient and inefficient firm types. The parameters are: $p = 1$, $c = 0.6$ (left plot) and $0.4$ (right plot), $c_e = 0.8$, $s = 0.2$, $\mu = 10$, $\tau_h = 1$, and $\tau_l = 1/3$.

has been widely used in prior literature as a refinement to obtain a unique separating equilibrium under the single-crossing condition. Proposition 1(i) implies that the conventional outcome may not apply to our setting.

Second, the ordering of the first-best inventory levels switches when the critical ratio crosses 0.5. When $CR > 0.5$, a larger uncertainty calls for a higher inventory level, whereas it is the converse when $CR < 0.5$. This contrasts to the settings in prior literature where the first-best decisions always have the same ordering. A direct implication is that if one type of firm wants to mimic the other, it may cause either upward or downward distortion in the inventory decision. These intuitions are useful for understanding the insights we will reveal.

4.3. Market Equilibrium

In this subsection, we analyze our model with $\beta > 0$ and $\tau$ being private information of the manager. Notice that the two cases under $CR > 0.5$ and $CR < 0.5$ are symmetric in our model. Therefore, we only show the analysis for the large critical ratio case.

Given $v(\tau_h) > v(\tau_l)$, the manager will always like the investors to believe that the firm’s demand uncertainty is low (i.e., $\tau = \tau_h$). As a result, imitation or signaling may arise. To analyze the manager’s strategy, it is convenient to define:

$$V_{ij}(q) \equiv \beta \left( E_{\varepsilon} [\pi(q, \mu + \varepsilon/\tau_i)] + v(\tau_j) \right) + (1 - \beta) \delta \left( E_{\varepsilon} [\pi(q, \mu + \varepsilon/\tau_l)] + v(\tau_i) \right), \forall i, j \in \{h, l\}. \quad (5)$$

$V_{ij}(q)$ is the expected payoff of the manager when the firm’s true type is $i$ while the investors believe that the firm’s type is $j$. Hence, $V_{ih}(q) - V_{il}(q)$ represents the misvaluation gain (loss) of the manager if the less (more) efficient type firm is considered as the more (less) efficient type firm.
Lemma 1. (i) \( V_{ij}(q) \) is concave and maximized at \( q^o(\tau_i) \) for \( i, j \in \{h, l\} \); (ii) \( V_{il}(q) - V_{li}(q) = \beta K \) for \( i \in \{h, l\} \); (iii) \( V'_{ij}(q) > (\leq) V'_{ij}(q) \) when \( q < (\geq) \mu \) for \( j \in \{h, l\} \).

First, for Lemma 1(i), \( V_{ij}(q) \) is essentially a newsvendor objective function. Thus, it must be concave and has a unique maximizer. Given the composition of \( V_{ij}(q) \), its maximizer in fact coincides with the first-best solution \( q^o(\tau_i) \). We illustrate the curves of \( V_{ij}(q) \) in Figure 2. Second, Lemma 1(ii) shows that under the same inventory level, the gain for the manager if the firm is miscued when it is the less efficient type equals the loss if the firm is the more efficient type but miscued. Furthermore, these gain and loss linearly increase in \( \beta \) (recall \( K = v(\tau_h) - v(\tau_l) \) which is a constant). Finally, since \( q^o(\tau_i) > q^o(\tau_h) > \mu \) when \( CR > 0.5 \), the result of Lemma 1(iii) indicates that \( V_{hl}(q) \) will increase faster than \( V_{lh}(q) \) in \( q \) when \( q \) is below the mean; however, when \( q \) exceeds the mean, \( V_{hl}(q) \) will increase slower and then decrease faster than \( V_{lh}(q) \) in \( q \). Therefore, together with the result of Lemma 1(i), we can see that for any amount of inventory distortion upward from their corresponding optimums, the payoff of the manager with \( \tau_h \) will always decrease more than that with \( \tau_l \). In contrast, for a small amount of inventory distortion downward from their corresponding optimums, the payoff of the manager with \( \tau_h \) will decrease less than that with \( \tau_l \); but, this comparison can again be reversed when the downward distortion is significantly large. In other words, overstocking or substantial understocking will be more costly for the manager of the more efficient type firm than of the less efficient type, while moderate understocking can be less costly for the manager of the more efficient type firm. These findings imply that if it is needed to distort the inventory level to credibly signal his information, the manager must understock; however, there might also be scenarios where credibly signaling is not achievable since it can be too costly for the manager, which then gives rise to pooling outcomes (Debo et al. 2013 find similar scenarios in their consumer purchase context based on the queueing framework when the single-crossing condition does not hold).

With the above intuition, we characterize the equilibria of our model in the following. We first focus on the separating equilibrium.

Lemma 2. There exists a unique \( q_h(\beta) < q^o(\tau_h) \) such that \( V_{hh}(q_h(\beta)) = V_{hl}(q^o(\tau_h)) \) and a unique \( q_l(\beta) < q^o(\tau_l) \) such that \( V_{lh}(q_l(\beta)) = V_{ll}(q^o(\tau_l)) \). Both \( q_h(\beta) \) and \( q_l(\beta) \) decrease in \( \beta \); moreover, there exists \( \beta_H^{se} \) such that \( q_h(\beta) \leq q_l(\beta) \) when \( \beta \leq \beta_H^{se} \) and \( q_h(\beta) > q_l(\beta) \) when \( \beta > \beta_H^{se} \).

In the above lemma, \( q_l(\beta) \) represents the smallest amount of inventory that the manager with \( \tau_l \) is willing to stock if, by doing so, he can successfully mislead the market to believe that the firm is of the more efficient type. Similarly, \( q_h(\beta) \) represents the smallest amount of inventory that the manager with \( \tau_h \) is willing to stock to avoid the firm being miscued. These two threshold
inventory levels depend on $\beta$, and they both decrease as $\beta$ increases. This is intuitive because when the interest in the market value increases, the manager learning $\tau_i$ will have more incentive to imitate; likewise, he will also have more incentive to signal with $\tau_h$. Naturally, if $q_h(\beta) \leq q_l(\beta)$, then inventory levels between $q_h(\beta)$ and $q_l(\beta)$ exist such that the manager with $\tau_h$ is willing to stock to credibly reveal his information, while he has no incentive to imitate when $\tau = \tau_l$. Therefore, when this condition holds, a separating equilibrium can arise, and we can also show that it survives the intuitive criterion. (Note that since $q_h(\beta)$ decreases in $\beta$, inventory distortion might be needed to deter imitation, when $\beta$ becomes large.) However, this condition is not always warranted in our model. In fact, we can find a threshold $\beta_{se}^{H}$ such that $q_h(\beta) > q_l(\beta)$ when $\beta < \beta_{se}^{H}$. We demonstrate these two threshold inventory levels in Figure 2. Armed with these intuitions, we obtain the following result.

**Proposition 2.** When $\beta \leq \beta_{se}^{H}$, there exists a unique separating equilibrium that survives the intuitive criterion, in which the manager’s inventory decision follows:

$$q^*(\tau_i) = \begin{cases} q^o(\tau_i) & \text{if } i = l, \\ \min\{q^o(\tau_h), q_l(\beta)\} & \text{if } i = h, \end{cases} \quad (6)$$

and $q^*(\tau_h) = q^o(\tau_h)$ iff $\beta$ is no greater than a threshold $\beta_{se}^{H} \in (0, \beta_{se}^{H}]$. When $\beta > \beta_{se}^{H}$, there does not exist any separating equilibrium.

Proposition 2 provides an interesting contrast to the classical finding in the literature where the more efficient type firm always overinvests compared to the first best. The intuition of the difference
boils down to the source of asymmetric information. If it is about the expected demand (e.g., one firm has a larger expectation while the other has a smaller one), then to stock more would be less costly for the more optimistic firm since its demand has a larger mean. Therefore, overstocking can be a credible means for the manager to signal his information. In contrast, when asymmetric information arises about the demand uncertainty (e.g., one has a smaller variance while the other has a greater one), the scenario is more sophisticated. The more efficient type firm’s characteristic is that its potential demand is more closely distributed around the mean. Hence, when the underage cost outweighs the overage cost (i.e., \( CR > 0.5 \)), the manager of the more efficient type firm can no longer credibly signal his information by overstocking because this would be more appealing for the less efficient type firm whose demand has a large variance. Rather, reducing the safety stock to some extent will be more costly for the less efficient type to imitate, which hence can become a credible signal.

However, Proposition 2 also shows that there does not exist any separating equilibrium in our model when \( \beta \) is large. The intuition has been revealed in Lemma 2 as it can be too costly for the manager of the more efficient type firm to signal his information. In such a scenario, only pooling equilibria can arise. The following proposition shows that such pooling equilibria can survive the intuitive criterion once \( \beta \) exceeds a threshold.

**Proposition 3.** There exist two thresholds \( \beta_{pe}^H < \hat{\beta}_{pe}^H \). When \( \beta \geq \beta_{pe}^H \), there are multiple pooling equilibria that survive the intuitive criterion. Among these equilibria, there is one unique pooling equilibrium that Pareto dominates the others when \( \beta_{pe}^H \leq \beta < \hat{\beta}_{pe}^H \) and the equilibrium inventory level \( q^*(\tau_h) = q^*(\tau_l) < q^*(\tau_h) \); when \( \beta \geq \hat{\beta}_{pe}^H \), there is a continuum of Pareto-dominant pooling equilibria with \( q^*(\tau_h) \leq q^*(\tau_l) = q^*(\tau_l) < q^*(\tau_l) \). Furthermore, if \( \beta_{pe}^H < \beta_{se}^H \), then the separating equilibrium in (6) is Pareto dominated by at least one pooling equilibrium when \( \beta_{pe}^H \leq \beta < \beta_{se}^H \).

Note that pooling equilibria always exist in our model. However, they do not survive the intuitive criterion unless \( \beta \) is sufficiently large so that the manager’s mimicking incentive when \( \tau = \tau_l \) is very strong. In such a scenario, there will be pooling equilibria, for which one cannot find an alternative inventory level that the manager observing either \( \tau_h \) or \( \tau_l \) will unilaterally deviate to, even if such a deviation would make the investors believe \( \tau = \tau_h \), and thus they survive the intuitive criterion refinement. Proposition 3 further reveals that among these equilibria, some will dominate the others from the manager’s perspective under both \( \tau \) values. In these Pareto-dominant equilibria, the manager always understocks when \( \tau = \tau_l \) and can either overstock or understock when \( \tau = \tau_h \). The pooling equilibria and the separating equilibrium can sometimes coexist, however, the latter will always be Pareto dominated by at least one of the pooling equilibria. Finally, it is possible that \( \beta_{se}^H < \beta_{pe}^H \). In such a scenario, there is no equilibrium that survives the intuitive
4.4. Numerical Analysis

We conduct a numerical study to assess the effects of the system parameters on the possible equilibrium realizations. We consider 5,000 instances of the following parameters: $p = 1$, $c = 0.4$, $s = 0.2$, $c_e \in \{0.7, 0.75, 0.8, 0.85, 0.9\}$, $\mu = 10$, $\tau_l = 1/3$, $\tau_h \in \{2/5, 1/2, 2/3, 1\}$, $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, $\delta = 1$, and $v(\tau_h) - v(\tau_l) \in \{0.02, 0.04, ..., 1\}$. Note that the newsvendor critical ratios are controlled to be greater than 0.5 in all these instances, and the firm’s expected first-best profit in the first period ranges from 5.29 to 5.81. The difference of the two second-period profits ($v(\tau_h) - v(\tau_l)$) ranges from 0.34 to 18.9 percent of the firm’s expected first-best profit in the first period. Thus, the force that drives short-termism is relatively moderate in our experiments. Based on such a setting, we obtain the $\beta$ thresholds that divide the equilibrium regions.

Several observations are made from the numerical study. First, both the separating and pooling equilibrium regions are significant. Figure 3 shows three representative instances. In particular, the 25th to 90th percentiles of $\hat{\beta}_{se}$ (which divides the separating and pooling equilibrium regions) range from 0.055 to 0.641 in these instances, with an average at 0.23. In other words, both separating and pooling equilibria can arise quite frequently under a reasonable setting. Second, it is useful to notice that the 25th to 90th percentiles of $\hat{\beta}^H$ (below which no inventory distortion occurs) range from 0.007 to 0.124, with an average at 0.058. That is, inventory distortion can occur even under a relatively small $\beta$. Third, we find that as $\rho$ becomes smaller, the separating equilibrium region

---

### Table 1  Summary of the Equilibrium Structure

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Stable Dominant Equilibrium</th>
<th>Uniqueness</th>
<th>More Efficient type</th>
<th>Distortion</th>
<th>Less Inefficient type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[\beta^H_{se}, \beta^H_{pe}]$</td>
<td>Separating</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>$[\beta^C_{se}, \beta^C_{pe}]$</td>
<td>Separating</td>
<td>Yes</td>
<td>overstock</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>$[\beta^C_{pe}\beta^L_{pe}]$ (Unstable Pooling)</td>
<td>(Yes)</td>
<td>(overstock)</td>
<td>(overstock)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[\beta^C_{se}, \beta^C_{pe}]$</td>
<td>Pooling</td>
<td>Yes</td>
<td>overstock</td>
<td>overstock</td>
<td></td>
</tr>
<tr>
<td>$[\beta^L_{pe}, 1]$</td>
<td>Pooling</td>
<td>No</td>
<td>understock</td>
<td>overstock</td>
<td></td>
</tr>
</tbody>
</table>

(A) Large Critical Ratio ($CR > 0.5$)

(B) Small Critical Ratio ($CR < 0.5$)
expands. This result is reasonable because when the prior probability that the firm is efficient decreases, the gain for the manager of an inefficient type firm from the pooling equilibrium becomes smaller, which makes it easier to separate. We also observe in our experiments that a larger $\tau_h$ or $c_u$ widens the efficient separating equilibrium region. This finding mainly results from the fact that the two first-best inventory positions will become more distinct and thus it needs a stronger short-term incentive for distortion to occur. However, we notice that an increase of $\tau_h$ does not necessarily increase the entire separating equilibrium region because it changes the curvature of the manager’s payoff function so that pooling equilibria may arise earlier. The separating equilibrium region shrinks when the difference between the two long-term profits ($v(\tau_h) - v(\tau_l)$) increases, since it will enhance the manager’s mimicking incentive. Fourth, we also investigate the firm’s first-period profit loss with $\beta \in \{0.1, 0.2, ..., 0.9\}$. We observe in our experiments that compared to the first best, the firm suffers an average 1 percent profit loss in the first period due to the manager’s short-term incentive, with the 75th to 90th percentiles ranging from 1.3 to 2.8 percent. The loss increases in $\beta$ and the difference $v(\tau_h) - v(\tau_l)$ since the manager’s mimicking incentive will become stronger. Hence, the cost of managerial short-termism can be relatively significant for the firm. Note that
since the investors correctly value the firm in equilibrium, the manager incurs the same ratios of losses for the part linked to the firm’s first-period profit. Finally, we find that the region where no stable equilibrium exists (i.e., $(\beta_{se}^H, \beta_{pe}^H)$) does not occur in our entire experiments. In fact, for such a region to appear, the prior probability ($\rho$) that the firm is the more efficient type needs to be very small (e.g., 0.01) in our experiments. Hence, the main analytical results hold for quite common settings.

4.5. Managerial Implications

The above findings can offer useful insights. First, it has been typically shown in prior literature that only the more efficient type firm may distort its decisions, while the less inefficient type firm will follow the first-best solution. However, we find that when the single-crossing condition is not satisfied (which can arise in fairly general scenarios), both types of firms may distort their operational decisions. This finding complements prior literature. It indicates that managers’ short-termist behaviors are concerning not just when the firm is more efficient, they can also cause losses when the firm is less efficient. Hence, firms need to pay attention to managerial short-termism in both scenarios, when they structure the compensation packages with stock-based payments or design employee stock retention policies, which can influence managers’ incentive structure. This insight is also useful for the investigation of security and accounting policies or the design of operations mechanisms (e.g., supply chain contracts) to mitigate managerial short-termism (calling for attention to both types of firms).

Second, our results can shed light on understanding public firms’ inventory decisions. As discussed in Lai (2006) and Schmidt et al. (2014), it has been observed that firms purposely reduce their inventory levels in practice (in contrast to the prediction of the existing theories). Besides the explanations Schmidt et al. (2014) provide related to physical capacity constraints and specific equilibrium refinement, we show that the type of information asymmetry is also critical for managers’ inventory decisions. When asymmetric information arises about the firm’s demand uncertainty, understocking can occur in both separating and pooling equilibria. In particular, when the manager’s short-term incentive is intermediate and the newsvendor critical ratio is large, the manager may purposely reduce the firm’s safety stock to signal that his firm has a small demand uncertainty; or, when the manager’s short-term incentive is strong and it is too costly to reveal his information, the manager may stock at a low pooling inventory level irrespective of the observed demand uncertainty.

Our results also indicate that the newsvendor critical ratios (or service levels) that are often related to firms’ profit margins and cost structures are very important to understand the impacts of managerial short-termism on their inventory data. Managers’ short-termist behaviors can influence
their inventory decisions in different ways under different critical ratios. Specifically, in contexts similar to ours, when managers’ short-term incentives are intermediate, the inventory levels of the firms with small demand uncertainties might be deflated if their critical ratios are large and inflated if their critical ratios are small. In contrast, when their short-term incentives are strong, their inventory levels might be inflated with large critical ratios and deflated with small critical ratios. For the firms with large demand uncertainties, if there are distortions, then their inventory levels are always deflated if their critical ratios are large and inflated if their critical ratios are small. This suggests that to properly understand the public firms’ inventory decisions, we need to carefully control the short-term incentive factor together with the other important operations characteristics such as the gross margin and demand variance.

5. Stock-based Payment Realization

Thus far, we have considered the case where the weights \( \beta \) and \((1 - \beta)\delta \) in the manager’s payoff function are exogenous, same as in prior literature. In practice, however, beyond the exogenous factors, managers may also receive stock-based payments and they may have certain flexibility to decide both the proportion of their stocks to sell and the timing, after the granted stock-based payments are vested. That is, in some scenarios, the weight that managers place on their firm’s short-term market value in their decision making can be endogenous under certain constraint. On the other hand, investors may also pay close attention to the managers’ stock selling decisions, which is then reflected into their firms’ stock prices (Krantz 2013, Cahill 2013). For instance, the stock price of Facebook jumped up 4.8 percent on September 5th, 2012, when Mark Zuckerberg pledged not to sell his stocks at least for a year (Risberg 2012). Hence, it is also interesting to analyze the case with endogenous short-termism in our model. In this regard, we interpret \( \beta \in [0, \bar{\beta}] \) as the proportion of stocks that the manager decides to sell at the end of the first period and \((1 - \beta)\) as the proportion that the manager will keep to the second period. The upper limit \( \bar{\beta} \) reflects the constraint that might be in place, and \( \delta (<1) \) represents the manager’s short-term capital pressure.

When \( \beta \) becomes a decision variable, it may also provide information about \( \tau \). Thus, we use \( \eta(\beta, q, D) \) to denote the investors’ inference and rewrite the market value as:

\[
P(\beta, q, D) = \pi(q, D) + E[\eta(\beta, q, D)].
\]  

The manager’s decision can be reformulated as:

\[
\max_{\beta \in [0, \bar{\beta}], q \in (0, \infty)} \beta E[\xi[P(\beta, q, \mu + \varepsilon/\tau_i)]] + (1 - \beta) \delta(E[\xi[\pi(q, \mu + \varepsilon/\tau_i)] + v(\tau_i)], \forall i \in \{h, l\}.
\]  

Let \((\beta^*(\tau_i), q^*(\tau_i))\) denote the manager’s optimal decision. In equilibrium, it shall be consistent with the investors’ valuation for the firm. We redefine the equilibrium concept as follows.
The manager’s decision, \((\beta^*(\tau_i), q^*(\tau_i))\), is the maximizer of (8).

(ii) The investors’ inference function \(\eta(\beta, q, D)\) that determines the market value \(P(\beta, q, D)\) in (7) satisfies: \(\Pr(\eta(\beta^*(\tau_i), q^*(\tau_i), D) = \tau_i) = 1\) when \((\beta^*(\tau_h), q^*(\tau_h)) \neq (\beta^*(\tau_i), q^*(\tau_i))\) for one or both terms; \(\Pr(\eta(\beta^*(\tau_i), q^*(\tau_i), D) = \tau_h) = 1 - \Pr(\eta(\beta^*(\tau_i), q^*(\tau_i), D) = \tau_i) = \frac{\rho_{\tau_h} \phi((D - \mu)\tau_h)}{\rho_{\tau_i} \phi((D - \mu)\tau_i) + (1 - \rho_{\tau_i}) \phi((D - \mu)\tau_i)}\) when \((\beta^*(\tau_h), q^*(\tau_h)) = (\beta^*(\tau_i), q^*(\tau_i))\); and for any \((\beta, q) \neq (\beta^*(\tau_i), q^*(\tau_i))\), \(\Pr(\eta(\beta, q, D) = \tau_h) = 1 - \Pr(\eta(\beta, q, D) = \tau_i) = 0\).

Clearly, this reformulated game becomes a multi-dimensional signaling game. The manager can use both \(\beta\) and \(q\) to signal his information to the investors. With the flexibility on \(\beta\), it is natural to expect that the pressure on operational distortion will be alleviated. Technically, however, the equilibrium structure becomes significantly more complex as the action space expands. Definition 2 indicates that there can be different types of separating equilibria, either with same \(\beta\) but different \(q\), or with both different \(\beta\) and \(q\), and the pooling equilibria are also two dimensional which can be at possibly infinite \((\beta, q)\) pairs. Hence, instead of characterizing the complete equilibrium spectrum, we are specifically interested in those equilibria, in which operational efficiency can be achieved (i.e., the manager always stocks the inventory at the first-best levels), and the conditions for them to arise. Obviously, such equilibria must be separating equilibrium since the manager installs distinct inventory levels under different \(\tau\) values. Furthermore, in such equilibria, given that the information of \(\tau\) is perfectly revealed, the manager must sell the proportion of his stocks to the upper limit \(\bar{\beta}\) when \(\tau = \tau_i\), because of his short-term capital pressure reflected by the discount factor \(\delta\) imposed on the second-period payoff. Whereas, when \(\tau = \tau_h\), the manager may sell a smaller proportion to signal his information.

**Proposition 4.** There is a threshold \(\bar{\beta}\) such that if and only if \(\delta \geq \bar{\delta}\), an efficient separating equilibrium exists, in which \(\beta^*(\tau_i) = \bar{\beta}, \beta^*(\tau_h) \leq \bar{\beta}\), and \(q^*(\tau_i) = q^*(\tau_h)\) for \(i \in \{h, l\}\).

Proposition 4 shows that an efficient separating equilibrium exists as long as the manager’s short-term capital pressure is less than a threshold so that the cost of deferring the realization of some stock-based payment is not large. Specifically, if the manager learns a pessimistic prospect of the firm, he realizes all of his permitted stock-based payment early, and if he observes a promising prospect, he defers some payment to the future. The inventory decisions always follow the first-best solutions. Hence, Proposition 4 confirms that it is possible for the manager to use the proportion of early stock selling as a signal of his information rather than distorting the firm’s operations. In contrast, we have shown in section 4 that when \(\beta\) is exogenous, there does not exist any efficient separating equilibrium after \(\beta\) exceeds a threshold \(\beta^H\), irrespective of \(\delta\).
Proposition 4 provides several interesting implications. First, in practice, many corporate executives in the United States follow Rule 10b5-1 under the Securities Exchange Act of 1934 to pre-establish their stock-selling plans (which can be months ahead of the actual trading dates). After the plans are established, however, they can be terminated anytime before execution with no liability for insider trading even if the executives are aware of material nonpublic information (http://www.sec.gov/interps/telephone/phonesupplement4.htm). While our study is not investigating this security policy, the result of Proposition 4 implies that to offer managers some flexibility upon the realization of their stock-based payment (such as, to later cancel the selling plans) may help signal their newly obtained information and alleviate the pressure on operational distortions amid their short-term incentives.

Second, the short-term capital pressure represented by $\delta$ is an important factor in our model. Proposition 4 reveals that to achieve operational efficiency, the discount factor $\delta$ must not be smaller than a threshold. This result implies that to alleviate the managers’ short-term capital pressure is especially helpful when their desire of early stock-based payment realization is high. In practice, to alleviate the short-term capital pressure, firms can alternatively offer more cash payments or provide incentive plans to award employee stock retention.

6. Conclusion

This paper studies the impacts of managers’ short-term incentives on firms’ inventory decisions. First, we find that the type of information asymmetry is critical to predict managers’ short-termist behaviors. In a setting where the manager privately learns the firm’s demand uncertainty while the investors only know the prior distribution, we show that both separating and pooling equilibria can arise, in which the manager of either type of firms may over- and understock inventory compared to the first-best solutions. This substantially enriches the findings of prior literature that typically associates the private information of the manager with a regularity condition and identifies only separating equilibrium with possible overinvestment at the more efficient type firms. Second, we find that a firm’s operational characteristics such as the overage and underage costs are very important to understand the impacts of the manager’s short-termist behavior. The possible distortion of the inventory level under a large newsvendor critical ratio (or service level) can be completely opposite to that under a small newsvendor critical ratio. Finally, we study a case where the manager’s short-term objective is endogenized. Interestingly, we find that to provide the manager some flexibility of deciding how many stocks to sell early may introduce an alternative signal for his information and alleviate the pressure upon operational distortion. It is possible to reinstall the first-best inventory levels for all types of firms, even if the manager has short-term incentives and private information. These insights can be potentially useful not only for properly structuring compensation.
packages, but also for the investigation of security and accounting policies, the design of operations mechanisms, as well as for the understanding of public firms’ inventory data.

To conduct our analysis, we have assumed that the magnitude of the manager’s short-term incentive (i.e., $\beta$) is publicly known. However, the major insights can extend to the case where the manager possesses private information regarding his interest in the firm’s market value. In particular, we analyze a setting in Appendix B, in which $\beta$ can be either large or small, the exact level is known to the manager, and the investors only know a prior distribution. We can again find the existence of threshold $\beta$ levels, below and above which stable separating and pooling equilibria arise, respectively. We have also assumed in our model that the mean of the demand is common knowledge. The qualitative insights we have revealed can extend to settings where the mean is also private information of the manager, as long as the manager’s payoff functions do not satisfy the single-crossing condition. Finally, we have assumed in our model that both the manager and the investors are risk neutral. Some empirical research has investigated risk premiums in firm valuation and their effect on operations (e.g., Chen et al. 2005). Incorporating the risk factors in the model may yield more insights.

References


Appendix A: Proofs

Proof of Proposition 1. Part (i) and (ii) follow directly from the equations in (3) and (4). ■

Proof of Lemma 1. (i) By (5), we can rewrite $V_{ij}(q) = E_{c} [\pi(q, \mu + \varepsilon/\tau_j)] + \beta v(\tau_j) + (1 - \beta) \delta v(\tau_j)$. Note that only the first term on the right hand side depends on $q$. Therefore, $V_{ij}(q)$ is concave and maximized at $q = q^o(\tau_i)$ for $i, j \in \{h, l\}$. (ii) is straightforward. (iii) It is verifiable from (5) that $V_{ij}'(q) = (c_e - s) \overline{\phi}(\tau_i(q - \mu)) - c + s$. Thus, $V_{ih}'(q) - V_{lj}'(q) = (c_e - s)[\overline{\phi}(\tau_h(q - \mu)) - \overline{\phi}(\tau_l(q - \mu))], which is strictly positive (negative) when $q < (>) \mu$.

Proof of Lemma 2. By (5), $V_{lh}(q) = V_{ll}(q) + \beta K$, where $K = v(\tau_h) - v(\tau_l)$. As $q$ increases over $q \leq q^o(\tau_l)$, $V_{lh}(q)$ strictly increases from $-\infty$ to $V_{lh}(q^o(\tau_l)) + \beta K$. Hence, there exists a unique value $q_l(\beta) < q^o(\tau_l)$ such that $V_{lh}(q_l(\beta)) = V_{lh}(q^o(\tau_l))$, or equivalently $V_{lh}(q_l(\beta)) = V_{lh}(q^o(\tau_l)) - \beta K$. Clearly, as $\beta$ increases, $V_{lh}(q_l(\beta))$ decreases and hence $q_l(\beta)$ decreases (because $V_{lh}(q)$ is an increasing function over $q < q^o(\tau_l)$). Similarly, there exists a unique value $q_h(\beta) < q^o(\tau_h)$ such that $V_{hh}(q_h(\beta)) = V_{hh}(q^o(\tau_h)) - \beta K$, and $q_h(\beta)$ decreases in $\beta$.

The inverse function of $q_i(\beta)$, denoted by $\beta_i(q)$, for $i = l, h$, is

$$\beta_i(q) = \frac{V_{ii}(q^o(\tau_i)) - V_{ii}(q)}{K},$$

defined over $q \leq q^o(\tau_i)$. Because $q^o(\tau_i) < q^o(\tau_l)$, both $q_h(q)$ and $q_l(q)$ are well defined over $q \leq q^o(\tau_h)$. Consequently, for $q \leq q^o(\tau_h)$,

$$\beta_h(q) - \beta_l(q) = \frac{[-V_{hh}(q) + V_{ll}(q)]}{K} = (c_e - s)[\overline{\phi}(\tau_l(q - \mu)) - \overline{\phi}(\tau_h(q - \mu))]/K,$$

implying that $\beta_h(q) - \beta_l(q) > 0$ for $q \in (\mu, q^o(\tau_h))$ and $\beta_h(q) - \beta_l(q) < 0$ for $q < \mu$. This, together with the fact that $\beta_h(q^o(\tau_h)) = 0$ and $\beta_l(q^o(\tau_h)) > 0$, implies that there exists a threshold $\widehat{q} < \mu$ such that $\beta_h(q) < \beta_l(q)$ for $q \in (\widehat{q}, q^o(\tau_h))$ and $\beta_h(q) > \beta_l(q)$ for $q < \widehat{q}$. Let $\beta_{sl}^o = \beta_h(\widehat{q})$ (which is also equal to $\beta_l(\widehat{q})$). Note that $\beta_{sl}^o \in (0, 1]$. Because both $\beta(\cdot)$ and its inverse function $q_i(\cdot)$ are monotone, $q_h(\beta) < q_l(\beta)$ for $\beta \in [0, \beta_{sl}^o)$ and $q_h(\beta) > q_l(\beta)$ for $\beta > \beta_{sl}^o$. ■

Proof of Proposition 2. We first prove by contradiction that when $\beta > \beta_{se}^o$, there does not exist any separating equilibrium. Suppose $\{q_l, q_h\}$ is a separating equilibrium. Then $q_l = q^o(\tau_l)$ because otherwise the manager of the inefficient firm is better off by deviating to $q^o(\tau_l)$. Further, by definition of $q_h(\beta)$, $q_h \geq q_h(\beta)$ because otherwise the manager of the efficient firm is better off under $q^o(\tau_h)$. It follows from Lemma 2 that when $\beta > \beta_{se}^o$, $q_h(\beta) > q_l(\beta)$. Hence, $q_h > q_l(\beta)$, which, by definition of $q_l(\beta)$ and the fact that $q_l = q^o(\tau_l)$, suggests that the manager of the inefficient firm is better off by deviating to $q_h$. This is in contradiction with the assumption that $\{q_l, q_h\}$ is a separating equilibrium.
Next we prove that when $\beta \leq \beta^N_{se}$, $\{q^*(\tau_i), q^*(\tau_h)\}$ is the unique stable separating equilibrium. The proof is carried out in two steps. First, we show that $\{q^*(\tau_i), q^*(\tau_h)\}$ is a stable separating equilibrium. Second, we show that there does not exist any other stable separating equilibrium.

First, consider the following market belief: $\Pr(\eta=q,D) = \tau_h = 1$ if $q = q^*(\tau_h)$, and 0 otherwise, where $q^*(\tau_h) = \min\{q^*(\tau_h), q_l(\beta)\}$. It follows from Lemma 2 that the inefficient manager stocks $q^*(\tau_i)$ and has no incentive to stock $q^*(\tau_h)$ to mimic the efficient type; the efficient type also has no incentive to deviate from $q^*(\tau_h)$. The market belief is consistent with the manager’s strategies. Thus, we have proved that $\{q^*(\tau_i), q^*(\tau_h)\}$ is a separating equilibrium. This equilibrium survives the intuitive criterion because we make the following claim: There does not exist any quantity $q$ such that the market believes the manager ordering $q$ is of the efficient type, and that the efficient manager is strictly better off and the inefficient manager is worse off by choosing $q$ relative to their performance under the equilibrium $\{q^*(\tau_i), q^*(\tau_h)\}$. We prove this claim by contradiction. Suppose such a quantity $q$ exists. Recall that $q^*(\tau_h) = \min\{q^*(\tau_h), q_l(\beta)\}$. In order for the efficient manager to be strictly better off under $q$ relative to $q^*(\tau_h)$, $q^*(\tau_h)$ can not be equal to $q^*(\tau_h)$, and further $q > q^*(\tau_h) = q_l(\beta)$ because of the concavity property of the manager’s objective as a function of the order quantity. Thus, we have that $q > q_l(\beta)$, which, together with the definition of $q_l(\beta)$, suggests that the inefficient manager is strictly better off under $q$ relative to $q^*(\tau_i)$. This is in contradiction with the earlier statement that the inefficient manager is worse off by choosing $q$ relative to her performance under the equilibrium. Thus, we have proved the claim, and therefore the equilibrium $\{q^*(\tau_i), q^*(\tau_h)\}$ is stable.

Second, we show that there does not exist any other stable separating equilibrium. We prove by contradiction. Suppose there exists another stable separating equilibrium $\{q_l, q_h\}$, which is not the same as $\{q^*(\tau_i), q^*(\tau_h)\}$. Then $q_l = q^*(\tau_i)$ because otherwise the inefficient manager is better off by deviating to $q^*(\tau_i)$. Because the inefficient manager has no incentive to deviate to $q_h$ at which the market’s belief is equal to the efficient type, $q_h \leq q_l(\beta)$ (see the definition of $q_l(\beta)$). This suggests that the efficient (inefficient) manager has (no) incentive to deviate to $q^*(\tau_h) = \min\{q^*(\tau_h), q_l(\beta)\}$. This separating equilibrium survives the intuitive criterion if and only if $q_h = q^*(\tau_h)$. This is in contradiction with the assumption that $\{q_l, q_h\}$ is not the same as $\{q^*(\tau_i), q^*(\tau_h)\}$.

Finally, as $\beta$ increases from 0 to $\beta^N_{se}$, $q_l(\beta)$ decreases from $q^*(\tau_i)$ to some value that is lower than $q^*(\tau_h)$. Hence, there exists a threshold $\beta^N \in [0, \beta^N]$ such that $q_l(\beta^N) = q^*(\tau_h)$. This, together with the definition $q^*(\tau_h) = \min\{q^*(\tau_h), q_l(\beta)\}$, implies that the manager who faces a small demand uncertainty stocks at its first-best inventory level, i.e., $q^*(\tau_h) = q^*(\tau_h)$ if $0 < \beta \leq \beta^N$, and it understocks, i.e., $q^*(\tau_h) = q_l(\beta) < q^*(\tau_h)$, if $\beta^N < \beta \leq \beta^N_{se}$. ■

Next we present and prove four lemmas that will be useful for the proof of Proposition 3. Let $V_{im}(q)$ be the type $i$ manager’s expected profit given that the two types order the same quantity
q in the first period. Observing the order quantity q, the market’s belief remains the same as its prior belief. However, the market will update its belief after observing the realized demand D in the first period by using Bayes’ rule. That is, \( \Pr(q,D) = \tau_h = 1 - \Pr(q,D) = \tau_i = \frac{\rho \sigma \delta ((D - \mu) \tau_i / (1 - \rho \tau_i (D - \mu / \tau_i)))}{\rho \sigma \delta ((D - \mu) \tau_i / (1 - \rho \tau_i (D - \mu / \tau_i)))}. \) Let \( \rho_i = E_D \Pr(q,D) = \tau_h), where \( D = \mu + \varepsilon / \tau_i. \) Note that \( \rho_h > \rho_i \) and \( V_{im}(q) - V_{il}(q) = \beta K_i \) where \( K_i = \rho_i(v(\tau_h) - v(\tau_i)). \) Intuitively, \( \beta K_i \) is the type \( i \) manager’s profit gain from the market valuation due to the change of the market belief from the inefficient type to the mixed type.

Define \( q_h(\beta) = \{q < q^*(\tau_i)|V_{il}(q) = V_{il}(q^*(\tau_i)) - \beta K_i\} \) and \( \bar{q}_h(\beta) = \{q > q^*(\tau_i)|V_{il}(q) = V_{il}(q^*(\tau_i)) - \beta K_i\}. \) Intuitively, \( q_h(\beta) \) represents the smallest (largest) amount of inventory that the manager with \( \tau_i \) is willing to stock, if, by doing that, he can successfully lead the market to believe that the manager is of a mixed type. Define \( q(\beta) = \max\{q_h(\beta), \bar{q}_h(\beta)\} \) and \( \bar{q}(\beta) = \min\{q_h(\beta), \bar{q}_h(\beta)\}. \)

**Lemma A1.** The complete set of pooling equilibria is given by \([q(\beta), \bar{q}(\beta)]\).

**Proof of Lemma A1.** For any \( q \in [q(\beta), \bar{q}(\beta)], \) we can construct the following market belief: if the manager orders \( q, \) then the market’s belief remains the same as its prior; otherwise, the market believes the manager is of the inefficient type. Under such a belief, the type \( i \)’s best possible deviation is to choose \( q^*(\tau_i) \) and its expected profit is \( V_{il}(q^*(\tau_i)) \), which is less than \( V_{il}(q) + \beta K_i \) because \( q \in [q(h), \bar{q}(\beta)]. \) This, together with the fact that \( V_{im}(q) = V_{il}(q) + \beta K_i \), implies the type \( i \) manager is worse off by deviating from \( q. \) Hence, any \( q \in [q(\beta), \bar{q}(\beta)] \) is a pooling equilibrium.

Next we prove by contradiction that if \( q \notin [q(\beta), \bar{q}(\beta)], \) then \( q \) can not be a pooling equilibrium. If \( q < q_h(\beta), \) then either \( q < q_h(\beta) \) or \( q < q_h(\beta). \) In the former case, \( V_{im}(q) = V_{il}(q) + \beta K_i < V_{il}(q^*(\tau_i)), \) implying that the inefficient manager is better off by deviating to \( q^*(\tau_i), \) while in the latter case, \( V_{hm}(q) = V_{il}(q) + \beta K_h < V_{hl}(q^*(\tau_h)), \) implying that the efficient manager is better off by deviating to \( q^*(\tau_h). \) Thus, in either case, we find contradiction with the definition of pooling equilibrium. Using the similar arguments, we can show that if \( q > \bar{q}(\beta), q \) can not be the pooling equilibrium quantity.

**Lemma A2.** The pooling equilibria exist if and only if \( \beta \geq \beta_1 \) where \( \beta_1 = \{\beta|V_h(\beta) = q_h(\beta)\}. \)

**Proof of Lemma A2.** Note that the interval \([q(\beta), \bar{q}(\beta)]\) is nonempty if and only if \( \bar{q}_h(\beta) > q_h(\beta). \) Because \( q_h(\beta) \) increases and \( q_h(\beta) \) decreases in \( \beta, \) there exists a threshold \( \beta_1 = \{\beta|V_h(\beta) = q_h(\beta)\}\) such that the interval \([q(\beta), \bar{q}(\beta)]\) is nonempty if and only if \( \beta \geq \beta_1. \)

**Lemma A3.** For any pooling equilibrium \( q, \) there exists a unique \( \hat{\beta}(q) \) such that \( q \) is a stable pooling equilibrium if \( \beta \geq \hat{\beta}(q). \)

**Proof of Lemma A3.** Recall that a pooling equilibrium \( q \) does not survive the Intuitive Criterion if and only if there exists \( q' \) such that the efficient (inefficient) type is better (worse) off by deviating from \( q \) to \( q' \) assuming that the market belief under \( q' \) is of the efficient type. Therefore, to show
that \( q \) is stable, we need to show that such \( q' \) does not exist. We first show that such \( q' \) does not exist in the range \( q' > q \). We then show that such \( q' \) does not exist in the range \( q' < q \) if \( \beta \geq \hat{\beta}(q) \).

Define \( q^a_i(\beta) = \{ \hat{q} > q | V_{ai}(\hat{q}) = V_{ai}(q) - \beta \hat{K}_i \} \) where \( \hat{K}_i = K - K_i \). Intuitively, \( \beta \hat{K}_i \) is the type \( i \)'s manager’s profit gain from the market valuation due to the change of the market belief from the mixed type to the efficient type. Thus \( q^a_i(\beta) \) is the threshold quantity above which the type \( i \) manager can not make a profitable deviation from \( q \) even if such a deviation will alter the market belief from being the mixed type to being the efficient type. Next we show that \( q^a_i(\beta) > q^a_h(\beta) \) for any \( \beta \), which implies that there does not exist any \( q' > q \) such that the efficient (inefficient) type is better (worse) off by deviating from \( q \) to \( q' \) assuming that the market belief under \( q' \) is of the efficient type. We consider two cases. Case 1. \( q \geq \mu \). It follows from Lemma 1(iii) that \( V'_{ii}(q) > V'_{hh}(q) \) for \( q > \mu \). This, together with the fact that \( \hat{K}_h < \hat{K}_i \), implies that \( q^a_i(\beta) > q^a_h(\beta) \) for any \( \beta \). Case 2. \( q < \mu \). Let \( q' = \{ q > \mu | V_{ai}(\hat{q}) = V_{ai}(q) \} \). This is well defined because \( V_{ai}(\cdot) \) is concave and its maximizer is larger than \( \mu \). If \( q' > q^h \), then by following the proof in Case 1, we have \( q^a_i(\beta) > q^a_h(\beta) \) for any \( \beta \). Thus, it remains to prove \( q' > q^h \). It is verifiable that \( V'_{hh}(\mu + \Delta) < V'_{hh}(\mu - \Delta) \) for any \( \Delta > 0 \). This, together with the definition of \( q^h \), implies that \( q^h - \mu > q - \mu \), i.e., \( q^h \) is further away from \( \mu \) relative to \( q \). Recall that \( V'_{hh}(q) - V'_{ii}(q) = (c_e - s) | \Phi(\tau_h(q - \mu)) - \Phi(\tau_i(q - \mu)) | \), implying that \( V_{hh}(\mu + \Delta) - V_{ih}(\mu - \Delta) = V_{hh}(\mu) - V_{ih}(\mu) \) for any \( \Delta > 0 \) and \( V_{hh}(\mu + \Delta) - V_{ih}(\mu - \Delta) \) decreases as \( \Delta \) increases. This, together with the fact that \( q^h - \mu > q - \mu \), implies that \( V_{hh}(q^h) - V_{ih}(q^h) < V_{hh}(q) - V_{ih}(q) \) and \( V_{hh}(q^h) = V_{hh}(q) \), we have \( V_{ii}(q^h) > V_{ii}(q) = V_{ii}(q') \), implying that \( q' > q^h \) by the concavity property of \( V_{ii}(\cdot) \).

Next we show that there exists a threshold \( \hat{\beta}(q) \) so that such \( q' \) does not exist in the range \( q' < q \) if \( \beta \geq \hat{\beta}(q) \). Define \( q^\ast_i(\beta) = \{ \hat{q} < q | V_{ai}(\hat{q}) = V_{ai}(q) - \beta \hat{K}_i \} \). It suffices to show that \( q^\ast_i(\beta) < q^\ast_h(\beta) \) for \( \beta \geq \hat{\beta}(q) \). The inverse function of \( q^\ast_i(\beta) \) is \( \beta^\ast_i(q^\ast) = [V_{ii}(q) - V_{ii}(q^\ast)]/\hat{K}_i \).

We first show that \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) \) changes sign only once from negative to positive. Note that \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) = [(c_e - s) \Phi(\tau_h(q - \mu)) - \Phi(\tau_i(q - \mu)) - (c_e - s)/\hat{K}_i = (c_e - s) \Phi(\tau_h(q - \mu)) - \Phi(\tau_i(q - \mu)) - (c_e - s)/\hat{K}_i \).

Note that \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) = (c_e - s) \tau_i \Phi(\tau_h(q - \mu)) \hat{K}_h - (c_e - s) \tau_i \Phi(\tau_i(q - \mu)) \hat{K}_i \). It follows from the property of the standard normal density function \( \phi(\cdot) \) that the equation \( \beta^\ast_h(q^a) - \beta^\ast_i(q^a) = 0 \) has only one solution on each side of \( \mu \); say \( q_L \) and \( q_H \). Hence, when \( q^a < q_L \), \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) < 0 \), when \( q_L < q^a < q_H \), \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) > 0 \), and when \( q^a > q_H \), \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) < 0 \). This implies that \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) \) decreases when \( q^a < q_L \), increases in between \( q_L \) and \( q_H \), and then decreases when \( q^a > q_H \). Note that \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) < 0 \) when \( q^a \to -\infty \) and \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) > 0 \) when \( q^a \to +\infty \). Therefore, we have shown that \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) \) changes sign only once, and \( \beta^\ast_i(q^a) < \beta^\ast_i(q^\ast) \).

The result that \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) \) changes sign only once from negative to positive implies that there exists a threshold \( \hat{q} \leq q \) such that \( \beta^\ast_i(q) - \beta^\ast_i(q) = 0 \). Further when \( q^a < \hat{q} \), \( \beta^\ast_i(q^a) - \beta^\ast_i(q^\ast) > 0 \).
and when \( q > q^* > \hat{q}, \beta_h(q^*) - \beta_i(q^*) < 0 \). This, together with the fact that \( \beta_i(\cdot) \) is inverse function of \( q^*(\cdot) \) and the fact that \( \beta_i(\cdot) \) is a decreasing function for \( i = h, l \), implies that \( q_h^*(\beta) > q_l^*(\beta) \) for \( \beta > \beta_h(\hat{q}) \). Define \( \hat{\beta}(q) = \beta_h(\hat{q}) \). Therefore, we conclude that if \( \beta > \hat{\beta}(q) \), then \( q \) is stable.

**Lemma A4.** For any \( q^H > q^L, \hat{\beta}(q^H) > \hat{\beta}(q^L) \).

**Proof of Lemma A4.** Let

\[
\beta_i^H(q) = \frac{[V_i(q^H) - V_i(q)]}{K_i}
\]

for \( q \leq q^H \), and let

\[
\beta_i^L(q) = \frac{[V_i(q^L) - V_i(q)]}{K_i}
\]

for \( q \leq q^L \).

It follows from the arguments in the proof Lemma A3 that for \( i = H, L \), there exists a threshold \( \hat{q}(q^i) \leq q^i \) such that \( \beta_h^i(q) = \beta_i(q) = 0 \) when \( q = \hat{q}(q^i) \). Note that the two functions \( \beta_h^L(q) - \beta_i^H(q) \) and \( \beta_h^L(q) - \beta_i^L(q) \) differ only by a fixed constant for any \( q \), i.e., \( \beta_h^L(q) - \beta_i^H(q) - [\beta_h^L(q) - \beta_l^L(q)] \) is a negative constant value for any \( q \). Hence, the fact that \( q^H > q^L \) implies that \( \hat{q}(q^H) < \hat{q}(q^L) \). This, together with the definition of \( \hat{\beta}(\cdot) \) in the proof of Lemma A3, implies that \( \hat{\beta}(q^H) > \hat{\beta}(q^L) \).

**Proof of Proposition 3.** Because \( q(\beta) \) decreases in \( \beta \) and \( \hat{\beta}(q) \) increases in \( q, \hat{\beta}(q(\beta)) \) decreases in \( \beta \). Thus, we can define \( \beta_2 = \min\{\beta \in (0, 1]|\hat{\beta}(q(\beta)) \leq \beta\} \). Further, there exists a threshold \( \hat{q}(\beta) \in [q(\beta), q(\beta)] \) such that for all \( q \in [q(\beta), \hat{q}(\beta)] \), we have \( \hat{\beta}(q) \leq \beta \). By Lemmas A1 and A3, we have that the complete set of stable pooling equilibria is the interval \( [q(\beta), \hat{q}(\beta)] \). Such an interval exists if and only if \( \beta \geq \beta^H_{pe} \), where \( \beta^H_{pe} = \max\{\beta_1, \beta_2\} \).

If \( 0 < \beta \leq \min\{\beta^H_{sc}, \beta^H_{pe}\} \), then there does not exist any stable pooling equilibrium. Thus, the stable separating equilibrium characterized in Proposition 2 is the only stable equilibrium of this game.

If \( \min\{\beta^H_{sc}, \beta^H_{pe}\} < \beta \leq \beta^H_{pe} \), then there does not exist any stable equilibrium. Recall that \( \beta^H_{pe} = \max\{\beta_1, \beta_2\} \). Note that \( \beta^H_{sc} \geq \beta_2 \), which follows because when \( \beta = \beta^H_{pe} \), \( q(\beta) \) is stable. Further, \( \beta^H_{sc} \) is independent of \( \rho \) whereas \( \beta_1 \) decreases in \( \rho \), implying that there exists a threshold \( \rho^H \) such that \( \beta^H_{sc} < \beta_1 \) if and only if \( 0 < \rho < \rho^H \). Therefore, \( \beta^H_{sc} < \beta^H_{pe} \) and thus this case exists if and only if \( 0 < \rho < \rho^H \).

Let \( \beta_3 \) be the threshold such that \( \hat{q}(\beta_3) = q^*(\tau_h) \). Let \( \beta^H_{pe} = \max\{\beta_1, \beta_3\} \). Because \( \hat{q}(\cdot) \) is an increasing function, \( \hat{q}(\beta) < q^*(\tau_h) \) for \( \beta^H_{pe} < \beta < \beta^H_{pe} \). Hence, \( \hat{q}(\beta) \) Pareto dominates any other pooling equilibrium \( q \in [q(\beta), \hat{q}(\beta)] \). Further, \( \hat{q}(\beta) \) also Pareto dominates the unique separating equilibrium.
(if it exists) for the following reasons: the inefficient type is better off by choosing the pooling equilibrium \( \tilde{q}(\beta) \) than by choosing the first-best quantity \( q^o(\tau_l) \), which is in turn better off than mimicking the efficient type by choosing the efficient type’s quantity at the unique separating equilibrium; the efficient type is also better off at the pooling equilibrium \( \tilde{q}(\beta) \) than choosing the efficient type’s quantity at the unique separating equilibrium because otherwise the pooling equilibrium \( \tilde{q}(\beta) \) cannot be stable. Therefore, there exists a unique Pareto dominant and stable pooling equilibrium, at which \( q^*(\tau_h) = q^*(\tau_l) < q^o(\tau_h) \).

When \( \beta^*_P \leq \beta \), then \( \tilde{q}(\beta) \geq q^o(\tau_h) \) and thus the set of Pareto dominant and stable pooling equilibria is given by \( \{ q^o(\tau_l), \min(\tilde{q}(\beta), q^o(\tau_l)) \} \), in which \( q^o(\tau_h) \leq q^*(\tau_h) = q^*(\tau_l) \leq q^o(\tau_l) \). □

**Proof of Proposition 4.** We start by deriving the necessary condition for the existence of an efficient separating equilibrium, and then prove that the necessary condition is also sufficient.

Suppose \( \{ (\beta^*(\tau_l), q^*(\tau_l)), (\beta^*(\tau_h), q^*(\tau_h)) \} \) is an efficient separating equilibrium. By its definition, \( q^*(\tau_i) = q^o(\tau_i) \) for \( i = l, h \). Further, \( \beta^*(\tau_l) = \beta \) because otherwise the manager with \( \tau = \tau_l \) is better off by deviating to \( \beta \).

Let \( \pi_{ij} = E_x [\pi(q^o(\tau_j), \mu + \varepsilon/\tau_j)] \) which is the firm’s first-period profit if the manager with \( \tau = \tau_i \) chooses the stocking quantity \( q^o(\tau_j) \), for \( i, j \in \{ l, h \} \). To ensure that the manager with \( \tau = \tau_l \) has no incentive to deviate to \( (\beta^*(\tau_h), q^o(\tau_h)) \), the following condition must be satisfied:

\[
\overline{\beta}(\pi_{lh} + v(\tau_l)) + (1 - \overline{\beta})\delta(\pi_{lh} + v(\tau_l)) \geq \beta^*(\tau_l)(\pi_{lh} + v(\tau_l)) + (1 - \beta^*(\tau_l))\delta(\pi_{lh} + v(\tau_l)). \tag{ICl}
\]

To ensure that the manager with \( \tau = \tau_h \) has no incentive to deviate to \( (\beta, q^o(\tau_h)) \) even if doing so will lead the market to believe \( \tau = \tau_l \), the following condition must be satisfied:

\[
\beta^*(\tau_h)(\pi_{hh} + v(\tau_h)) + (1 - \beta^*(\tau_h))\delta(\pi_{hh} + v(\tau_h)) \geq \overline{\beta}(\pi_{hh} + v(\tau_l)) + (1 - \overline{\beta})\delta(\pi_{hh} + v(\tau_l)). \tag{ICh}
\]

Note that the condition (ICl) imposes an upper bound on \( \beta^*(\tau_l) \), which is \( \frac{\overline{\beta}(1 - \delta)(\pi_{lh} + v(\tau_l)) - \delta(\pi_{lh} + v(\tau_l))}{(1 - \delta)(\pi_{lh} + v(\tau_l)) - \delta(\pi_{lh} + v(\tau_l))} \), and the condition (ICh) imposes a lower bound on \( \beta^*(\tau_h) \), which is \( \frac{\overline{\beta}(1 - \delta)(\pi_{hh} + v(\tau_l)) - \delta(\pi_{hh} + v(\tau_l))}{(1 - \delta)(\pi_{hh} + v(\tau_l)) - \delta(\pi_{hh} + v(\tau_l))} \). The existence of \( \beta^*(\tau_h) \) implies that the upper bound must be no lower than the lower bound, i.e.,

\[
\frac{\overline{\beta}(1 - \delta)(\pi_{lh} + v(\tau_l)) - \delta(\pi_{lh} + v(\tau_l))}{(1 - \delta)(\pi_{lh} + v(\tau_l))} \leq \frac{\overline{\beta}(1 - \delta)(\pi_{lh} + v(\tau_l)) - \delta(\pi_{lh} + v(\tau_l))}{(1 - \delta)(\pi_{lh} + v(\tau_l)) - \delta(\pi_{lh} + v(\tau_l))},
\]

which after some algebra manipulations, is equivalent to

\[
(1 - \delta) \left[ \delta(1 - \overline{\beta})A + \frac{\delta}{1 - \delta} \overline{\beta}(v(\tau_h) - v(\tau_l))^2 - \overline{\beta}B \right] \geq 0,
\]

where \( A = (\pi_{lh} - \pi_{lh})(\pi_{hh} + v(\tau_l)) \) and \( B = \pi_{hh}(\pi_{lh} + v(\tau_l) + \pi_{lh}v(\tau_h) - \pi_{hh}(\pi_{lh} + \pi_{hh}v(\tau_l) - \pi_{lh}v(\tau_h)). \) Because the terms inside the square brackets increase in \( \delta \), there exists a threshold \( \overline{\delta} \) such that the
above necessary condition is equivalent to $\delta \geq \bar{\delta}$. The necessary condition $\delta \geq \bar{\delta}$ implies that if $\delta < \bar{\delta}$ then there does not exist any efficient separating equilibrium.

Next we prove that the necessary condition $\delta \geq \bar{\delta}$ is also sufficient for the existence of an efficient separating equilibrium. Suppose $\delta \geq \bar{\delta}$. It follows from the prior analysis that $\delta \geq \bar{\delta}$ implies that there exists a continuous range of $\beta^*(\tau_h)$ that satisfies both (ICl) and (ICh). In particular, we set $\beta^*(\tau_h)$ to be the upper bound, i.e., $\beta^*(\tau_h) = \frac{(\bar{\beta} + (1 - \bar{\beta})\delta)(\pi_{hh} + v(\tau_i)) - \delta (\pi_{hh} + v(\tau_i))}{(1 - \delta)^2 \pi_{lh} + v(\tau_i) - \delta v(\tau_i)}$. Consider the following market belief: $\Pr(\eta(\beta, q, D) = \tau_h) = 1$ if $\beta = \beta^*(\tau_h)$ and $q = q^*(\tau_h)$, and $0$ otherwise. Note that the manager with $\tau = \tau_i$ is always better off under $(\bar{\beta}, q^*(\tau_i))$ than under any other solution that leads the market to believe $\tau = \tau_i$. Under the constructed market belief, the only solution under which the market belief is not $\tau_i$ is $(\beta^*(\tau_h), q^*(\tau_h))$. By (ICl), the manager with $\tau = \tau_i$ prefers $(\bar{\beta}, q^*(\tau_i))$ to $(\beta^*(\tau_h), q^*(\tau_h))$. Hence, $(\bar{\beta}, q^*(\tau_i))$ is the best response of the manager with $\tau = \tau_i$.

It remains to prove that the best response of the manager with $\tau = \tau_h$ is $(\beta^*(\tau_h), q^*(\tau_h))$. If he deviates from $(\beta^*(\tau_h), q^*(\tau_h))$, then under the constructed market belief, the market will always believe $\tau = \tau_i$, implying that the manager’s best deviation is to set the stocking quantity to be $q^*(\tau_h)$ and to set $\beta$ to be $\arg \max_{\beta \in [0, \bar{\beta}]} \beta (\pi_{hh} + v(\tau_i)) + (1 - \beta)\delta (\pi_{hh} + v(\tau_i))$. Because the maximander is linear in $\beta$, the maximizer must be either 0 or $\bar{\beta}$. If the maximizer is $\bar{\beta}$, then (ICh) implies that the manager with $\tau = \tau_h$ is worse off by deviating from $(\beta^*(\tau_h), q^*(\tau_h))$ to $(\bar{\beta}, q^*(\tau_h))$. If the maximizer is 0, then the expected payoff of the manager with $\tau = \tau_h$ under $(\beta^*(\tau_h), q^*(\tau_h))$ is $\beta^*(\tau_h)(\pi_{hh} + v(\tau_i)) + (1 - \beta^*(\tau_h))\delta (\pi_{hh} + v(\tau_i))$, which is higher than his expected payoff $\delta (\pi_{hh} + v(\tau_i))$ under $(0, q^*(\tau_h))$. Hence, $(\beta^*(\tau_h), q^*(\tau_h))$ is the best response of the manager with $\tau = \tau_h$. ■

Appendix B: Asymmetric Information about the Manager’s Short-term Interest in Market Value

Here, we relax the assumption that the manager’s short-term interest $\beta$ is common knowledge to the investors. Specifically, consider the signaling game where $\beta$ may take one of the two values $\beta_h$ and $\beta_l$ ($\beta_h \geq \beta_l$), and the investors do not know the exact value of the manager’s short-term interest $\beta$. Depending on the values of the short-term interest $\beta$ and the forecasting accuracy $\tau$, the manager’s type can have the following four possibilities: $(\beta_h, \tau_h)$, $(\beta_h, \tau_l)$, $(\beta_l, \tau_h)$, and $(\beta_l, \tau_l)$, which we call type $hh$, $hl$, $lh$, $ll$ respectively.

We focus on the scenario where the critical fractile is higher than 0.5. Lemmas 1 and 2 continue to hold under this setting. In particular, there exists $\beta^*_s \in (0, 1]$ such that $q_h(\beta) \leq q_l(\beta)$ when $\beta \leq \beta^*_s$, and $q_h(\beta) > q_l(\beta)$ when $\beta > \beta^*_s$. This implies that if $\beta_i \leq \beta^*_s$, then the type $ih$ can successfully separate himself from the type $il$ by choosing the quantity $\min\{q^*(\tau_h), q_l(\beta_i)\}$, for $i = h, l$. However, the condition $\beta_i \leq \beta^*_s$ alone can not guarantee that the manager can successfully signal his forecast capability to the investors because this condition doe not ensure that the type $hl$ has no incentive to mimic the type $lh$. 
To this end, we need to impose a new condition: $\beta_h \leq \Gamma(\beta_l)$ where $\Gamma(\beta_l) = \{\beta | q_l(\beta) = q_h(\beta_l)\}$, which is well defined over $\beta_l \leq \beta_{se}^H$. The condition $\beta_h \leq \Gamma(\beta_l)$ ensures that the type $hl$ is worse off by choosing the quantity $\min\{q^o(\tau_h), q_h(\beta_l)\}$ even if doing so allows him to be perceived as the efficient type than choosing his first-best quantity $q^o(\tau_l)$. Note that $\Gamma(\beta_l) \leq \beta_{se}^H$. Thus, if (i) $\beta_l \leq \beta_{se}^H$ and (ii) $\beta_h \leq \Gamma(\beta_l)$, then the two efficient types $hh$ and $lh$ can successfully signal their more accurate forecast capability by choosing the quantity $\min\{q^o(\tau_h), q_l(\beta_h)\}$, and the other two inefficient types $hl$ and $ll$ are better off by staying at their first-best quantity $q^o(\tau_l)$. Specifically, condition (i) and (ii) ensure that the type $ll$ and the type $hl$ have no incentive to deviate from $q^o(\tau_l)$, respectively. Following the arguments in the proof of Proposition 3, we can show that under condition (i) and (ii), the game has a unique stable separating equilibrium at which both the two efficient types stock quantity $\min\{q^o(\tau_h), q_l(\beta_h)\}$ and the other two inefficient types stock their first-best quantity $q^o(\tau_l)$. Further, if any one of the two conditions is violated, then the game does not have any stable separating equilibrium. In such a case, stable partial separating or pooling equilibria may arise.

To see why stable pooling equilibria may arise, we consider the scenario where $\beta_l \geq \beta_{pe}^H$. Proposition 4 implies that there exists at least one stable pooling equilibrium, say $q_{pe}$, for the type $lh$ and the type $ll$, i.e., none of them has incentive to deviate to his first-best quantity at which the market belief is of the inefficient type. The fact that $\beta_h \geq \beta_l$ implies that the type $hh$ and $hl$ benefit more than the type $lh$ and $ll$ by altering the market belief of his type from the inefficient type to the mixed type. This further implies that both the type $hh$ and the type $hl$ have no incentive to deviate from the stable pooling equilibrium $q_{pe}$ to the first-best quantities. Further, it follows from Lemma A3 that if $q_{pe}$ is stable for the type $lh$ and the type $ll$ with short-term interest $\beta_l$, then $q_{pe}$ is also stable for the type $hh$ and the type $hl$ with a higher short-term interest $\beta_h$. Therefore, if $\beta_l \geq \beta_{pe}^H$, the stable pooling equilibria described in Proposition 4 and associated with $\beta_l$ is also the stable pooling equilibrium in this extended model.

To conclude, we have established the same result as in the base model that the stable equilibrium of this setting with asymmetric information about $\beta$ is separating equilibrium if both $\beta_h$ and $\beta_l$ are small (i.e., $\beta_l \leq \beta_{se}^H$ and $\beta_h \leq \Gamma(\beta_l)$). Otherwise, there is no stable separating equilibrium, and multiple stable pooling (or partial separating) equilibria may arise. In particular, we have shown that if $\beta_l \geq \beta_{pe}^H$, then the stable pooling equilibria characterized in Proposition 4 remain to be the stable pooling equilibria in this extension. Therefore, most of the qualitative insights (e.g., the equilibrium can be either separating or pooling, stocking quantity can be distorted upward or downward for both the efficient and inefficient type, insights on the value of input and output information) obtained under the base model continue to hold when the assumption of $\beta$ being common information is relaxed.