“Fulfilled by Amazon”: A Strategic Perspective of Competition at the E-commerce Platform

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Abstract

Problem definition: This paper examines the economic effects of the fulfillment services offered by Amazon (FBA) to the third-party sellers on its retail platform. Academic/Practical relevance: Logistics is critical for e-commerce. It is intriguing that the third-party sellers that use FBA may directly compete with Amazon (Amazon [2018]). By FBA, their service levels are substantially improved, and so is their competitiveness. Such a phenomenon has been little investigated in prior literature. Methodology: We develop a strategic competition model where Amazon and a representative third-party seller engage in both price and service competition. Results: Interestingly, we find that although FBA may intensify the service competition between Amazon and the third-party sellers, it can mitigate their price competition besides the direct revenue Amazon collects from the services. The latter effects dominate the former when the cross-service sensitivity of the consumers is either sufficiently low or sufficiently high. Therefore, not only the third-party sellers but also Amazon may benefit from FBA. Moreover, since the sales of Amazon’s products may increase due to FBA, its OEM supplier may gain from FBA too. These findings extend to the settings where the third-party seller is Amazon’s OEM supplier, cross-price and cross-service sensitivities are correlated, or the parties set their retail prices sequentially. Managerial implications: Our analysis provides insights about when FBA is more likely to be beneficial and when it can be harmful. They are useful for understanding the impacts of FBA on both Amazon and the third-party sellers.

1 Electronic copy available at: https://ssrn.com/abstract=3939085
1 Introduction

As one of the largest retail platforms, hundreds of millions of merchandises are listed on Amazon. Among them, about half are sold by the third-party sellers. For online retailing, logistics is the backbone of the business. An efficient logistics system not only saves the costs of processing, storage and transportation, but also adds value to the transaction and increases the consumers’ willingness to purchase online. Studies show that about 93% of the shoppers consider shipping options to be an important factor in their overall shopping experience (PitneyBowes 2015). To provide fast and reliable delivery services, Amazon has invested tremendous resources in recent years in its logistics system, from air cargo hubs to last mile delivery fleets. As such, a growing number of locations in the U.S. are now eligible for same-day, one-hour and Sunday delivery services. Besides delivery, the reverse logistics is also critical for online retailing. For the merchandises purchased through Amazon, consumers can use its free return shipping labels with fast processing or return the merchandises to Amazon’s local partner stores with instantaneous refunds. Amazon’s efficient and hassle free return services dramatically ease consumers’ concern of erroneous purchases and boost their confidence in online shopping.

While the advanced logistics system benefits Amazon’s own sales, it is also made available to the third-party sellers by the so-called “Fulfillment by Amazon” (FBA) program, despite the fact that they might compete directly against Amazon. Through FBA, these third-party sellers can store their merchandises at Amazon’s fulfillment centers; when an order is placed, similar as its own merchandises, Amazon ships the items, provides customer services and processes possible returns on behalf of these sellers (Amazon 2018). Although the FBA program contributes to Amazon’s revenue as these third-party sellers pay storage and fulfillment fees, it can substantially improve their competitiveness and increase their sales by allowing them to access Amazon’s advanced logistics system and extensive distribution network. It has been reported that almost half of Amazon’s customer base would refuse to buy from sellers who do not use FBA (Mathes 2017). In essence, the subscription of the FBA program can elevate the merchandises “sold by third-party sellers” to a similar competitive level as those “sold by Amazon itself” from the delivery and reverse logistics perspective.

The competition between Amazon and the third-party sellers has been widely documented in
the literature (Jiang et al. 2011; Shannon 2014; Rankin 2015). Most of the prior discussions focus on the competitive advantages of Amazon against the third-party sellers, such as, Amazon’s data intelligence, distribution network and company image. The third-party sellers were reported to have love-hate feelings about Amazon since their market shares can be easily eroded if Amazon launches similar products. While certain gaps between Amazon and the third-party sellers have been enlarged in recent years as Amazon continues its growth, the FBA program on the contrary might reduce the competitive advantage of Amazon given that the products sold by the third-party sellers with FBA will become more attractive. Furthermore, enabling the third-party sellers may also cause reactions from Amazon’s upstream OEM manufacturers who often want to restrain the competition from the third-party sellers as they may sell unauthorized or imitative competing products (Mantin et al. 2014). As to why Amazon continually promotes the FBA program, practitioners often point to the fees that Amazon collects from the FBA subscribers and the fact that FBA may allow small third-party sellers to overcome the barrier of selling on Amazon (Kanter 2017), but neglect the effects that might be caused by the changes of the competition. While it seems to be a conventional understanding that enabling a competitor would make the competition become more intense, it is however unclear whether it is truly so in this scenario since improving the third-party sellers’ delivery services might boost the consumers’ willingness-to-pay. Furthermore, the change of the downstream competition might also induce Amazon’s OEM suppliers to strategically alter their wholesale pricing strategies. Both forces contribute to the complexity of the problem, and the equilibrium outcome cannot be readily observed.

To fill this gap, we develop a strategic competition model among Amazon, Amazon’s OEM supplier, and a representative third-party seller in this paper. Amazon’s product is supplied by its OEM supplier based on a wholesale price only contract, while the third-party seller sells a (partially or perfectly) substitutable product. The two firms compete on price as well as fulfillment service. That is, both the price and the fulfillment service have a competing effect in the competition. Without FBA, the third-party seller’s fulfillment service is inferior to Amazon’s; with FBA, by paying a unit fee, the third-party seller achieves the same fulfillment service quality as Amazon’s. To examine the impact of the FBA program, we first solve two subgames with and without FBA. In each case, Amazon’s OEM supplier first sets the wholesale price, followed by the simultaneous price competition between Amazon and the third-party seller. Based on the outcomes of these two
subgames, Amazon endogenizes the FBA fee in anticipation of the third-party seller’s participation decision.

From our analysis, we find that as long as the FBA fee does not exceed a threshold, the FBA program always benefits the third-party seller, and the benefit is higher when the cross-service sensitivity is lower. More interestingly, we find that instead of intensifying competition, the FBA program may actually soften it in certain scenarios. The third-party seller’s retail price may increase with FBA, which can lower the price pressure on Amazon. As a result, we find that when the consumers’ cross-service sensitivity (with respect to the opponent’s fulfillment service) is less than a threshold, the FBA program benefits Amazon too, which leads to a “win-win” outcome for the two parties. We call this effect the price competition mitigation effect of FBA. However, when the cross-service sensitivity exceeds the threshold to fall into a medium range, the improvement of the third-party seller’s fulfillment service by FBA will significantly undermine Amazon’s fulfillment advantage, forcing Amazon to reduce its selling price while possibly losing its market share. In this scenario, Amazon is hurt by offering FBA to the third-party seller, and the profit reduction may increase as the cross-service sensitivity gets stronger. We call this effect the service competition intensification effect. Nevertheless, when the cross-service sensitivity is sufficiently large, the FBA fee as well as the improvement of the sales commission that Amazon collects from the third-party seller may become dominant, as the above two effects are weakened. As a result, the FBA program may benefit Amazon again. We also find that the third-party seller’s market size and the cross-price sensitivity may play different roles in those regions. In particular, when the cross-service sensitivity is low, the FBA program is more beneficial for Amazon when the third-party seller has a larger market size or when the cross-price sensitivity is stronger. However, when the cross-service sensitivity is high, the effects reverse. The FBA program becomes more beneficial for Amazon when the third-party seller has a smaller market size or when the cross-price sensitivity is weaker. Lastly, we find that the FBA program may allow Amazon’s OEM supplier to charge a higher wholesale price while boosting Amazon’s sales. Therefore, the FBA program may benefit Amazon’s OEM supplier too and may hence lead to a win-win-win outcome for all parties. These findings are robust in various extensions.

Our study can offer useful insights for the practice of FBA. The cross-service sensitivity, which plays an important role in our model, reflects not only the substitutability of the products but
also the product characteristics (e.g., expensiveness, fragility, perishability, fashionability). Our results suggest that FBA offers a win-win resolution for Amazon and the third-party sellers when the cross-service sensitivity is low, which can arise in practice, for instance, when the parties do not sell the same product (as also suggested by an empirical study, Zhu and Liu 2018) or when the products are not very expensive, fragile, perishable or fashionable so that the consumers do not excessively compare the fulfillment services across the parties. Our results also suggest that when the cross-service sensitivity is very high, for instance, when the parties are direct competitors and their products fall into the expensive, fragile, perishable or fashionable categories, FBA can be mutually beneficial too. While guaranteeing the benefit for the third-party sellers, Amazon is able to obtain extra revenues from the service fee and the improved sales commission by FBA. Our results further imply that in an environment with low cross-service sensitivity, it is more beneficial for Amazon to offer the FBA program to a third-party seller that has a larger market size or whose product has a stronger cross-price sensitivity related to Amazon’s. On the contrary, in an environment with high cross-service sensitivity, it is more beneficial for Amazon to offer the FBA program to a third-party seller that has a smaller market size or whose product has a weaker cross-price sensitivity related to Amazon’s. Regarding the selling price, our results suggest that, accompanied with the offering of FBA, Amazon shall increase (decrease) the price when the cross-service sensitivity is low (high), and the subsequent change of the demand also implies the need of inventory adjustment.

The remainder of the paper is organized as follows. We review the related literature in section 2 and describe the model in section 3. Section 4 analyzes the main model. We discuss the extensions in section 5 and conclude in section 6.

2 Related Literature

First, our study is related to the literature that investigates the optimal selling formats for online platforms. Abhishek et al. (2015) study the selling decisions of online retail platforms that can either sell the products on behalf of manufacturers (i.e., agency selling) or resell the products. The authors find that when the platform channel has a negative effect on the traditional channel, the platforms prefer agency selling; otherwise, reselling is more beneficial. Kwark et al. (2017) study a similar problem but they focus on the role of third-party information (such as product reviews) that
consumers can access from the platforms. They show that platforms prefer reselling if the third-party information is on the quality dimension that can intensify upstream competition, whereas they prefer agency selling if the third-party information is on the fit dimension that can mitigate upstream competition. Tian et al. (2018) focus on upstream competition and order fulfillment costs. They show that when both of the order fulfillment costs and the upstream competition intensity are high, reselling is more beneficial than agency selling, and it is the reverse when both of the order fulfillment costs and the upstream competition intensity are low. They also show that a hybrid mode can be preferred when the market scenarios are in between. Hagiu and Wright (2015) investigate the roles of non-contractible marketing activities and variable costs. They show that reselling is more beneficial when the marketing activities have spillover effects across the products and when the network effects may negatively affect supplier participation. They also find that when reselling has a variable cost advantage, reselling is preferred for short-tail products while agency selling is preferred for long-tail products. Different from these studies, in our paper, the platform has both of the reselling function and the agency selling function which compete for a common consumer pool. We focus on the effects of the fulfillment service offered by the platform on the parties’ selling decisions and their profits.

Second, like us, there are several studies that investigate the strategic interactions between online platforms and third-party sellers. Jiang et al. (2011) study a scenario where a platform like Amazon may encroach into a third-party seller’s market segment if the demand information it learns from the third-party seller’s sales is favorable. The authors show that this may cause the reseller to exert less sales effort. In line with the above study, Zhu and Liu (2018) empirically investigate the factors that relate to Amazon’s encroachment decision. They find positive (negative) correlation between the success (the market growth effort) of the third-party sellers’ products and Amazon’s entry decision. They also show that it is less often that Amazon sells the same products when the third-party sellers use the FBA service than when they do not. Ryan et al. (2012) study a retailer’s decision of whether to sell through a platform in addition to its own channel amid participation fee or revenue sharing requirement. They analyze the optimal decisions for both parties and characterize the system equilibrium. Mantin et al. (2014) study a related problem but focus on the effect of selling through a platform on the retailer’s bargaining position with its manufacturer. They find that the manufacturer will be hurt by such an outside option of the retailer. While we focus on a
slightly different problem where the platform resells the manufacturer’s product as well as hosts a competing product sold by a third-party seller, we find that the manufacturer might be better off if its downstream platform can sell an additional fulfillment service to the third-party seller. [Xiao and Xu (2018) study the contract decisions between an online retailer and a third-party seller who has superior demand information. They develop a lost-sale penalty contract to incentivize the third-party seller to install optimal capacity and to extract full surplus. In parallel with us, [Qin et al. (2020) also study the phenomenon that an online platform may share its logistics service system with a seller. They focus on the impacts of the total market potential and the third-party logistics service provider’s service level. They find that platform sharing the logistics service leads to a lose-win (win-lose) outcome for the platform and the seller if the market potential and the third-party logistics service provider’s service level are both low (if either of them is very high), and a win-win outcome is possible if the market potential and the third-party logistics service provider’s service level are both intermediate. Differently, we study Amazon’s supply chain system that includes a strategic OEM supplier where Amazon endogenizes its FBA policy taking into account the reactions of both the third-party seller and the OEM supplier. We investigate the effects of the cross-service sensitivity which is related to product features, and thus our results can be used to discuss the implications of FBA for different product categories. This is useful as we reveal that the benefit of FBA is not the same for different product features while anecdotal evidence shows that the FBA adoption ratio in practice varies across product categories. Moreover, we consider asymmetric market sizes, simultaneous and sequential price competition, correlated cross-price and cross-service sensitivities, and the case where the OEM supplier and the third-party seller belong to the same firm. Therefore, our study reveals more (general) insights. For instance, we find that FBA can lead to a win-win-win outcome for the supply chain system not only when the third-party seller’s own service level is intermediate but also when its service level is low, and the former can arise if the cross-service sensitivity is high, while the latter can arise if the cross-service sensitivity is low. We also show that a larger market size does not necessarily make FBA more beneficial for Amazon. In fact, it is the reverse if the third-party seller’s market size is larger, when the cross-service sensitivity is high and Amazon’s market size is fixed. These results offer different insights than those in [Qin et al. (2020). Sun et al. (2000, 2021) empirically study a cross-border E-retailer’s logistics service decision. [Sun et al. (2000) develop machine learning tools to guide the choice of whether to use
FBA, while Sun et al. (2021) reveal that FBA can result in more sales but also more returns for the cross-border E-retailer.

Third, our paper is related to the studies on channel competition. Tsay and Agrawal (2000) study a distribution system where a manufacturer sells through two retailers. Similar as in our study, the demands of the retailers depend on both of their prices and service levels. They find that at times the retailers may benefit from a higher competitive intensity, and they investigate wholesale pricing mechanisms to coordinate the system. Bernstein and Federgruen (2004) develop inventory models in which the demands depend on the competing retailers’ prices and service levels. They investigate the equilibrium behaviors of simultaneous games that involve the retailers’ service, pricing and inventory decisions. Several other studies (Tsay and Agrawal 2004; Cattani et al. 2006; Arya et al. 2007; Li et al. 2014, 2015; Ha et al. 2016) focus on the competition between a manufacturer’s direct channel and its reseller. They show that the launch of such a direct channel can sometimes lead to a win-win outcome for the two parties by mitigating double marginalization but sometimes it may also result in a lose-lose outcome in the presence of information asymmetry or quality differentiation. Differently, our study considers the price and service competition between a manufacturer’s platform resale channel and an independent third-party seller. We investigate the effects of the strategy in practice that the platform firm sells its superior fulfillment service to the third-party seller, thereby upgrading the opponent’s service level. We reveal that such practice can sometimes mitigate price competition and improve the profits of all the parties.

Lastly, our paper is related to prior studies on production outsourcing and coopetition. Among them, Wang et al. (2013) study a supply chain where an OEM manufacturer outsources the production to a contract manufacturer that sells a competing product. The authors explore the two firms’ preferences over the contracting procedure as well as the impact of their bargaining power on the contract. In a similar supply chain setting, Niu et al. (2019) investigate the two firms’ technology specification and production timing strategies. Chen et al. (2019) study a setting where two competing manufacturers may collaborate through component production outsourcing with wholesaling or licensing. The authors show that the optimal collaboration strategy is affected by the two firms’ competition intensity, production efficiency, as well as negotiation power.
3 Model

Consider Amazon and a representative third-party seller that sell two substitutable products on Amazon at prices $p_A$ and $p_S$, respectively. The third-party seller pays a commission $r p_S$ to Amazon and earns $(1 - r) p_S$ for each unit sold. We assume that Amazon’s product is procured from its OEM supplier that incurs a production cost $c_0$ per unit. The supplier sets the wholesale price $w$ per unit in anticipation of Amazon’s sales. Different from Amazon, the origin of the third-party sellers’ products can be more complex. Some of their products might be salvage or discontinued items, some might be obtained from small overseas manufacturers, while some might be through the gray market. Hence, we do not explicitly consider the third-party seller’s supply chain in the main model, but assume that its product is obtained at an exogenous cost $g$ per unit. We consider an extension in Section 5.1 where Amazon’s OEM supplier and the third-party seller are the same company.

Besides the prices of the two products, the services provided by Amazon and the third-party seller can also influence the consumers’ purchasing decisions. Based upon the general observations from practice, we assume that the quality of Amazon’s fulfillment service, $s_A$, is superior to the quality, $s_S$, of the service that the third-party seller can provide itself; i.e., $s_A > s_S$. In our model, we assume for conciseness that although the service qualities differ, the fulfillment cost is $c$ per unit regardless of whether it is fulfilled by Amazon or the third-party seller (our structural results hold with asymmetric fulfillment costs). The third-party seller can also subscribe to Amazon’s FBA program, paying Amazon a fee $T(>c)$ for every unit of product fulfilled, which will make the quality of the fulfillment service for the third-party seller’s products identical to Amazon’s. The FBA fee $T$ is endogenized by Amazon in anticipation of the supplier’s wholesale price decision and the competition with the third-party seller. Although theoretically Amazon can also endogenize the FBA service quality, to differentiate it from the service quality for its own products, individually altering the logistics and warehousing services for different third-party sellers may be practically cumbersome. As such, we do not investigate endogenous FBA service quality in this study.

Following prior literature (Banker et al. 1998; Tsay and Agrawal 2000; Bernstein and Federgruen 2004), we adopt a reduced-form price and service competition model. In particular, without FBA,
the sales quantities of Amazon \((q_A)\) and the third-party seller \((q_S)\) follow:

\[
q_A = Q_A - p_A + \beta p_S + \alpha s_A - \eta s_S, \\
q_S = Q_S - p_S + \beta p_A + \alpha s_S - \eta s_A,
\]

and with FBA, the sales quantities become:

\[
q_A = Q_A - p_A + \beta p_S + \alpha s_A - \eta s_A, \\
q_S = Q_S - p_S + \beta p_A + \alpha s_A - \eta s_A.
\]

In the above model, \(Q_A\) and \(Q_S\) represent the market bases of the two products, \(\alpha \in (0, 1)\) reflects the unit service value with respect to quality and differentiates the effect of the service value from that of the price, while \(\beta \in (0, 1)\) and \(\eta \in (0, \alpha)\) are the cross-price and cross-service sensitivities. Here, \(\beta\) reflects the level of substitutability between the two products; whereas, the cross-service sensitivity \(\eta\) may reflect not only the product substitutability but also the general product characteristics. On the one hand, the comparison of the fulfillment services may play a more critical role in consumers’ decision making when the products they face are more similar, which is aligned with the rationale behind \(\beta\). On the other hand, even for the same level of \(\beta\), the cross-service sensitivity \(\eta\) can still vary for different product categories. For instance, consumers may pay more attention to the fulfillment services and compare them more excessively, when the products are expensive or fragile such as jewelries, watches and electronics than when they are not. The fulfillment services are also more important when the products are fashionable or perishable so that the consumers want to receive them quicker. Taking return into consideration, consumers may also pay more attention to the fulfillment services when the products are difficult to fully assess remotely such as apparel. In contrast, for some products such as office supplies, garden and tools, the fulfillment services may not be as critical as for the aforementioned ones. Therefore, the cross-service sensitivity \(\eta\) can be the largest when the two products are identical and fall into the categories of expensive, fragile, fashionable, perishable, or virtually-difficult-to-assess products. The cross-service sensitivity \(\eta\) decreases as the substitutability of the two products decreases or the product characteristics drift away from the above categories. The cross-service sensitivity \(\eta\) may be the least when the two
products are very different and the fulfillment service is not critical at all.

Similar to most of prior research on price and service competitions, we assume that the parameters in our model are complete information for all the involved parties. In what follows, we will analyze the two cases without FBA (the benchmark case) and with FBA separately. The sequence of events without FBA is as follows. The OEM supplier of Amazon first sets the wholesale price, followed by Amazon and the third-party seller setting their retail prices simultaneously. Whereas, for the case with FBA, Amazon first sets the FBA fee $T$, assuring that the third-party seller will subscribe to the FBA service against the benchmark case. Then, the OEM supplier of Amazon sets the wholesale price accordingly. Lastly, Amazon and the third-party seller decide their retail prices simultaneously. The mathematical notation of the model is summarized in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$\Pi_A$</td>
<td>Amazon’s profit</td>
</tr>
<tr>
<td>$\Pi_S$</td>
<td>Third-party seller’s profit</td>
</tr>
<tr>
<td>$\Pi_M$</td>
<td>OEM supplier’s profit</td>
</tr>
<tr>
<td>$p_A$</td>
<td>Amazon’s selling price</td>
</tr>
<tr>
<td>$p_S$</td>
<td>Third-party seller’s selling price</td>
</tr>
<tr>
<td>$c_0$</td>
<td>OEM’s production cost</td>
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<tr>
<td>$w$</td>
<td>OEM wholesale price (Amazon’s procurement cost)</td>
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<tr>
<td>$g$</td>
<td>Third-party seller’s product cost</td>
</tr>
<tr>
<td>$q_A$</td>
<td>Amazon’s sales quantity</td>
</tr>
<tr>
<td>$q_S$</td>
<td>Third-party seller’s sales quantity</td>
</tr>
<tr>
<td>$c$</td>
<td>Fulfillment service cost</td>
</tr>
<tr>
<td>$s_A$</td>
<td>Fulfillment service level of Amazon</td>
</tr>
<tr>
<td>$s_S$</td>
<td>Fulfillment service level of the third-party seller</td>
</tr>
<tr>
<td>$r$</td>
<td>Sales commission rate</td>
</tr>
<tr>
<td>$T$</td>
<td>FBA fee</td>
</tr>
<tr>
<td>$Q_A$</td>
<td>Market base of Amazon’s product</td>
</tr>
<tr>
<td>$Q_S$</td>
<td>Market base of third-party seller’s product</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Unit service value</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Cross-price sensitivity</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Cross-service sensitivity</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Correlation factor between cross-price and -service sensitivities</td>
</tr>
</tbody>
</table>
4 Analysis

In this section, we analyze the main model. We first describe the equilibrium characterization of the two cases in subsection 4.1, followed by the comparisons of the equilibrium selling prices, sales quantities and wholesale prices in subsection 4.2 and of the equilibrium profits in subsection 4.3.

4.1 Equilibrium Characterization

We start with the benchmark case without FBA. For any given selling prices \( p_A \) and \( p_S \), the sales quantities of Amazon and the third-party seller, i.e., \( q_A(p_A, p_S) \) and \( q_S(p_A, p_S) \), follow (1) and (2). Consequently, the third-party seller earns a profit \([ (1 - r)p_S - g - c]q_S(p_A, p_S)\) from its product, whereas Amazon earns not only a profit \((p_A - w - c)q_A(p_A, p_S)\) from its own product but also a commission \(r p_S q_S(p_A, p_S)\) from the third-party seller’s product. Since we assume that Amazon and the third-party seller set their selling prices simultaneously, their decisions can be characterized in the Nash equilibrium of a simultaneous game with the following objectives (the subscript “\( N \)” denotes the association with the No-FBA case):

\[
\Pi_{A,N} = (p_A - w - c)q_A(p_A, p_S) + r p_S q_S(p_A, p_S),
\]

\[
\Pi_{S,N} = [(1 - r)p_S - g - c]q_S(p_A, p_S).
\]

Anticipating the subgame price competition, the OEM supplier of Amazon makes the wholesale price decision to maximize its profit:

\[
\Pi_{M,N} = (w - c_0)q_A(p_A, p_S).
\]

As we show in the appendix, this game has a unique equilibrium. Given that the solutions are mainly algebraic, we relegate them to the appendix. We let \( w_N^* \) denote the equilibrium wholesale price charged by the OEM supplier, \((p_A^*, p_S^*)\) and \((q_A^*, q_S^*)\) denote the equilibrium selling prices and sales quantities of the two products, and \((\Pi_{A,N}^*, \Pi_{S,N}^*, \Pi_{M,N}^*)\) be the three parties’ corresponding profits.

Next, for the case with FBA, the third-party seller gains the same service as that of Amazon so that their sales quantities, \( q_A(p_A, p_S) \) and \( q_S(p_A, p_S) \), follow (3) and (4). Given that the third-party
seller’s product is delivered by Amazon under the FBA program, its profit changes to 
\[(1 - r)p_S - g - T]q_S(p_A, p_S)\) with the delivery fee replaced by the FBA fee. For Amazon, while the profit from its own product follows the same form \((p_A - w - c)q_A(p_A, p_S)\) as that without FBA, the profit from the third-party seller’s product now includes both sales commission and the FBA fee less the service cost, that is, \((rp_S + T - c)q_S(p_A, p_S)\). Hence, the two parties’ optimal pricing decisions can be obtained by solving the Nash equilibrium of a simultaneous game with the following objectives (the subscript “F” denotes the association with the FBA case):

\[
\Pi_{A,F} = (p_A - w - c)q_A(p_A, p_S) + (rp_S + T - c)q_S(p_A, p_S),
\]

\[
\Pi_{S,F} = [(1 - r)p_S - g - T]q_S(p_A, p_S).
\]

Anticipating the subgame price competition, the OEM supplier of Amazon makes the wholesale price decision to maximize its profit:

\[
\Pi_{M,F} = (w - c_0)q_A(p_A, p_S).
\]

As we show in the appendix, this subgame has a unique solution. Let \(\Pi_{A,F}(T)\) and \(\Pi_{S,F}(T)\) denote Amazon’s and the third-party seller’s corresponding profits derived in the subgame as functions of the FBA fee \(T\). Then, anticipating the subgame outcome, Amazon optimizes the FBA fee by solving the following problem:

\[
\max_T \Pi_{A,F}(T)
\]

\[
s.t. \Pi_{S,F}(T) \geq \Pi^*_{S,N}.
\]

As shown in the appendix, the above problem has a unique solution. Let \(T^*\) denote the equilibrium FBA fee, \(w^*_F\) the equilibrium wholesale price charged by the OEM supplier, \((p^*_{A,F}, p^*_{S,F})\) and \((q^*_{A,F}, q^*_{S,F})\) the equilibrium selling prices and sales quantities of the two products, and \((\Pi^*_{A,F}, \Pi^*_{S,F}, \Pi^*_{M,F})\) be the three parties’ corresponding profits. We shall note that to solve the above problem, we assume that Amazon is committed to offering FBA even if it might be worse off compared to the case without FBA. The solution, so derived, allows us to reveal insights regarding how the FBA program may influence the prices and the sales quantities. We will compare the profits of Amazon
and the OEM supplier of these two cases to derive the conditions of when FBA can benefit Amazon and the OEM supplier in the end.

### 4.2 Comparisons of Selling/Wholesale Prices and Sales Quantities

In this subsection, we use the equilibrium results derived in the above to address the questions of how each party should adjust its selling price and what to expect for the sales quantities of the two products after the use of the FBA program.

**Lemma 1.** The third-party seller’s equilibrium selling price is always higher with FBA than without, i.e., \( p^*_S,F > p^*_S,N \).

From the third-party seller’s perspective, FBA elevates its inferior fulfillment service level on par with the superior service level possessed by Amazon, which not only makes its product more appealing to consumers but also eliminates its disadvantage in the service comparison with Amazon. Hence, FBA provides extra room for the third-party seller to raise the selling price. Furthermore, to the extent that Amazon can charge an FBA fee greater than the third-party seller’s own fulfillment service cost, i.e., \( T \geq c \) (a result that will be confirmed later), FBA raises the third-party seller’s total cost of selling its product, thereby providing incentives for the third-party seller to raise its selling price. The combination of these forces derived from FBA, by enhancing product value, mitigating competitive disadvantage in service and raising the total selling cost, result in a higher selling price of the third-party seller’s product. This explains Lemma 1.

How does FBA impact the price of Amazon’s product? We find that, in contrast to the above result that FBA always raises the third-party seller’s selling price, the impacts of FBA on Amazon’s selling price are mixed. On the one hand, to offer FBA, Amazon loses its competitive advantage in fulfillment by sharing the superior service with the third-party seller, which relinquishes its leverage to charge a premium for its own product. We call this “the service competition intensification effect.” Such an effect results in downward pressure on Amazon’s selling price. On the other hand, as aforementioned, the FBA program can boost the third-party seller’s selling price, thereby mitigating the price competition faced by Amazon. We call this “the price competition mitigation effect.” Unlike the former effect, this effect provides room for Amazon to raise its selling price. Whether Amazon should adjust its selling price higher or lower after offering FBA clearly depends
on which effect is stronger. Naturally, the former effect derived from service competition becomes stronger as the cross-service sensitivity $\eta$ increases, whereas the latter effect on price competition is independent of $\eta$. This implies that after offering FBA, Amazon should adjust the selling price of its own product higher for small values of $\eta$, where the service competition intensification effect is insignificant and thus dominated by the price competition mitigation effect. In contrast, when $\eta$ is large, Amazon should reverse the course to lower the price of its own product after offering FBA. The above intuitive arguments are formally shown by the following lemma.

**Lemma 2.** There exists a threshold $\eta_1$ such that Amazon’s equilibrium selling price is higher with FBA than without, i.e., $p_{A,F}^* \geq p_{A,N}^*$, when $\eta \leq \eta_1$, and lower otherwise.

Next, we investigate how the FBA program may influence the sales quantities of the two competing products. We first focus on the third-party seller.

**Lemma 3.** The equilibrium sales quantity of the third-party seller’s product is higher with FBA than without, i.e., $q_{S,F}^* \geq q_{S,N}^*$.

The above lemma shows that, despite the increase of the third-party seller’s selling price, its sales quantity in equilibrium is always higher with FBA than without, even when its competitor, Amazon, lowers the selling price under FBA. In other words, the improvement of the third-party seller’s fulfillment service by the FBA program allows the third-party seller not only to increase its price but also to sell more of its product. It is worth noting that, although the result that FBA increases the third-party seller’s selling price does not depend on the condition that the FBA fee Amazon charges guarantees the third-party seller’s participation, the result that FBA increases the third-party seller’s sales quantity does depend on this condition. That is, as long as the FBA program is offered, it will increase the third-party seller’s selling price; but, it is the boundary condition over the FBA fee that allows the third-party seller to have sufficient competitiveness to always sell more of its product under FBA.

Different from the above result, the impact of the FBA program on Amazon’s own sales quantity is more sophisticated. We obtain the following result.

**Lemma 4.** There exist a threshold $\hat{s}_A$ and two thresholds $\eta_2$ and $\eta_3$ such that when $s_A \in [s_S, \hat{s}_A]$, the equilibrium sales quantity of Amazon’s product is lower with FBA than without, i.e., $q_{A,F}^* \leq q_{A,N}^*$,
if $\eta \leq \eta_2$ and higher otherwise; and when $s_A \in (\hat{s}_A, +\infty)$, the equilibrium sales quantity of Amazon’s product is higher with FBA than without, i.e., $q^*_A,F > q^*_A,N$, if $\eta < \eta_3$ and lower otherwise.

The above lemma shows that the impact of FBA on Amazon’s sales quantity depends on both the FBA service level and the cross-service sensitivity. In particular, when the FBA service level is not significantly higher than the third-party seller’s own service level, the third-party seller’s adoption of the FBA program does not change its pricing substantially. As such, the sales quantity of Amazon is affected more by its own price than by the price of the third-party seller. Recall from Lemma 2 that if the third-party seller subscribes to the FBA program, Amazon’s selling price increases when the the cross-service sensitivity is low and decreases when the cross-service sensitivity is high. This explains the result in the above lemma that Amazon’s sales quantity decreases for small $\eta$ and increases for large $\eta$ with FBA if $s_A$ is below the threshold $\hat{s}_A$. In contrast, if the FBA service level exceeds this threshold, the third-party seller’s adoption of the FBA program can significantly change the competition between the two parties. Now, the change of the third-party seller’s price by FBA plays a more critical role in determining Amazon’s sales quantity. Recall that the FBA program induces the third-party seller to charge a higher price for its product, which mitigates the price competition faced by Amazon. As such, for small values of $\eta$, where the price competition mitigation effect plays the dominant role, FBA can boost the sales quantity of Amazon’s product. However, for large values of $\eta$, the service competition intensification effect becomes dominant, which may result in a decline of Amazon’s sales. This explains the result in the second part of Lemma 4.

As we discussed early, the cross-service sensitivity $\eta$ in our model reflects not only the product substitutability but also the product characteristics. The above lemmas show that in equilibrium, the third-party seller’s subscription of the FBA service boosts the third-party seller’s selling price as well as its sales quantity irrespective of the substitutability between the products or the categories that the products fall in. That is, the third-party seller always hikes price while obtaining more sales, after subscribing to the FBA service. Whereas, for the selling price and the sales quantity of Amazon’s product, the effects depend on the substitutability and characteristics of the products and may also depend on the specific level of the FBA service. In particular, for small $\eta$, which may arise when the two products differ substantially and the products are not expensive, fragile, fashionable,
perishable, or difficult to assess virtually, Amazon’s selling price will also increase as the third-party seller subscribes to the FBA service. For these types of products, the subscription of the FBA service mitigates the price competition more than intensifying the service competition between the third-party seller and Amazon, which boosts Amazon’s selling price. In contrast, for the products that are highly substitutable or the products fall into the aforementioned categories, the third-party seller’s subscription of the FBA service will intensify the service competition more than the mitigation of the price competition, triggering Amazon to lower its selling price. The effect on Amazon’s sales is more sophisticated. Specifically, if the FBA service level is not significantly higher than that of the third-party seller’s own service, the conventional pattern occurs that as the Amazon’s price increases, its sales quantity largely decreases. The pattern is, however, fundamentally reversed if the FBA service level is significantly higher than that of the third-party seller’s own service.

Now, after the third-party seller subscribes to the FBA service, for different product characteristics and substitutability, Amazon’s sales may change in the same direction as its price, breaking the conventional principle. That is, as Amazon’s price increases/decreases, so may its sales.

Lastly, anticipating the above effects of the FBA program, Amazon’s OEM supplier will adjust its wholesale price to adapt to the change of the downstream competition. In particular, the anticipated increase of Amazon’s sales with FBA will give the OEM supplier leverage to increase its wholesale price, while the anticipated decrease of Amazon’s sales will prompt the OEM supplier to cut the wholesale price to boost the sales. We establish this result in the following lemma.

**Lemma 5.** When \( s_A \in [s_S, \hat{s}_A] \), the equilibrium wholesale price set by Amazon’s OEM supplier is lower with FBA than without, i.e., \( w^*_F \leq w^*_N \), if \( \eta \leq \eta_2 \) and higher otherwise. When \( s_A \in (\hat{s}_A, +\infty) \), the equilibrium wholesale price is higher with FBA than without, i.e., \( w^*_F > w^*_N \), if \( \eta < \eta_3 \) and lower otherwise.

Therefore, in addition to the service competition intensification effect and the price competition mitigation effect as identified in the above, the FBA program will introduce another effect on the downstream competition, as Amazon’s OEM supplier strategically adjusts its wholesale price. We call this effect “the wholesale price effect” which counters the above two effects.
4.3 Comparisons of Parties’ Profits

In this subsection, we examine how the FBA program impacts the equilibrium profits of the third-party seller, Amazon, as well as Amazon’s OEM supplier. The results established in the previous lemmas are useful for building the intuitions for the analytical results on profit comparisons. We first focus on the third-party seller.

**Proposition 1.** (i) The third-party seller benefits from FBA if and only if $T < T$, where $T \equiv c + \frac{(1-r)(s_A-s_S)(8a-(3+r)a\beta^2-6\beta\eta+(2+r)\beta^3\eta)}{(1-\beta^2)(8-(3+r)\beta^2)}$.

(ii) The third-party seller’s maximum willingness-to-pay for every unit fulfilled by Amazon, i.e., $\bar{T}$, increases in $s_A - s_S$, decreases in $r$ and $\eta$, and first decreases and then increases in $\beta$.

Part (i) of Proposition 1 formally establishes the threshold ($\bar{T}$) of the FBA service fee that acts as the boundary condition for Amazon’s problem in (5). Note from the above that $\bar{T} > c$. Therefore, part (i) asserts that the third-party seller will always be better off by FBA if Amazon sets the service fee at its cost, i.e., $T = c$, despite Amazon’s possible strategic reaction to reduce its selling price. Alternatively, if Amazon desires, it is able to obtain a margin as high as $\bar{T} - c$ from the FBA service.

Part (ii) of Proposition 1 characterizes how the third-party seller’s maximum willingness-to-pay ($\bar{T}$) is influenced by the model parameters. Such sensitivity results are useful for understanding the drivers of the benefits of FBA for the third-party seller, serving as guidelines for promoting the use of FBA and properly setting FBA fees to reflect its benefits. Specifically, the third-party seller is willing to pay more for FBA, if it faces a larger gap between its inferior service and Amazon’s superior service, i.e., a larger value of $s_A - s_S$, or if it obtains a larger proportion of the revenue from its sales, i.e., a lower commission rate $r$. The result that $\bar{T}$ decreases in $\eta$ is not obvious. At the first glance, consumers who are more fulfillment-service-sensitive make FBA more valuable for the third-party seller, given that it can elevate its inferior service on par with Amazon’s superior service, and hence one may expect that the third-party seller is willing to pay more as $\eta$ increases. However, Proposition 1(ii) shows that the opposite is true. Given the loss of its service advantage, Amazon may react to reduce its selling price more aggressively as $\eta$ increases, which undermines the benefit of FBA for the third-party seller. Finally, the result that $\bar{T}$ first decreases and then increases in $\beta$ implies that the FBA program is more likely to benefit the third-party seller when the
substitutability between its product and Amazon’s product is either low or high than intermediate. Intuitively, when the product substitutability is low, the price competition is weak and thus the third-party seller can retain most of the gain from the improvement of its service quality. While this benefit fades when the product substitutability becomes high, another benefit of FBA arises from the mitigation of the price competition as it becomes more and more intense. As a result, the combined benefits of the FBA program are more pronounced for the third-party seller when the product substitutability is either very low or very high.

For Amazon, although Proposition 1 indicates that Amazon can charge a premium for its FBA service, it remains unclear whether or not FBA can improve Amazon’s overall profitability, given that it influences the price competition. In the following, we address this question.

**Proposition 2.** Amazon benefits from FBA if the cross-service sensitivity η is either sufficiently small or large, and the opposite is true when η is moderate. That is, there exist two thresholds η and η̄ such that Π∗_{A,F} ≥ Π∗_{A,N} if η ≤ η or η ≥ η̄, and Π∗_{A,F} < Π∗_{A,N} otherwise.

When the cross-service sensitivity η is sufficiently small, Lemma 2 shows that the FBA program allows Amazon to sell its product at a higher price. In this scenario, although, according to Lemma 4, Amazon’s sales quantity may decrease if the FBA service level is not significantly higher than the level of the third-party’s own fulfillment service, its overall effect is dominated by the increase of the price; if the FBA service level is significantly higher than the level of the third-party’s own fulfillment service, FBA can also increase Amazon’s sales. As such, when the cross-service sensitivity is small, the price competition mitigation effect dominates the service competition intensification effect. This, together with the fact that Amazon makes a direct profit from selling the FBA service, implies that Amazon is better off under FBA for small values of η. However, as η grows, the service competition intensification effect is strengthened. In particular, for intermediate values of η, this negative effect of FBA outweighs the price competition mitigation effect, to result in a net loss for Amazon. Interestingly, when η becomes sufficiently large, the comparison among these effects can be reversed so that the FBA program benefits Amazon again. This is because, as η continues increasing, the profit margin of Amazon’s sales may decrease, which makes the loss caused by the service competition intensification effect decline relative to the direct profit Amazon obtains from selling the FBA service. It is worth noting that the wholesale price effect identified in Lemma 5

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generally works in the opposite direction of the net effect of price competition mitigation and service competition intensification. Therefore, either the benefit or the harm caused by FBA is mitigated to certain extent by the wholesale price effect. A numerical illustration of Proposition 2 is provided in Figure 1.

Given the implications of $\eta$, the result of Proposition 2 indicates that if the third-party seller subscribes to the FBA service, it may be more beneficial for Amazon when the products the two parties sell are either substantially different or highly substitutable so that $\eta$ falls into the small or large region. Similarly, the benefit may also be higher for Amazon for the products that have extreme characteristics, i.e., either very expensive, fragile, fashionable, perishable, virtually-difficult-to-assess, or not at all. Together with the result of Proposition 1, we see that the FBA program can lead to win-win outcomes for the third-party seller and Amazon when $\eta$ is either small or large. The presence of mutual benefits implies that the FBA program is more likely to be used in such regions. This implication is in line with some empirical and anecdotal observations. For instance, the empirical finding in Zhu and Liu (2018) suggests that there exists a positive correlation between the subscription of the FBA program and the difference among the products sold by third-party sellers and Amazon. We also find from Amazon’s website that for products such as cell phones that are relatively expensive and fragile, the majority of the third-party sellers use Amazon’s FBA service (e.g., about 83% of cell phone sellers use FBA based on our collected data).

Figure 1: Illustration of Amazon’s profit changes by FBA. The parameters are: $c_0 = 12$, $g = 18$, $c = 12.5$, $s_A = 22$, $s_S = 5$, $r = 0.06$, $Q_A = 14.8$, $Q_S = 7$, $\beta = 0.45$, $\alpha = 0.9$. 

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Proposition 3. There exists a threshold $\hat{\eta}$ such that $\frac{\partial (\Pi_{A,F}^* - \Pi_{A,N}^*)}{\partial Q_S} > 0$ if $\eta < \hat{\eta}$, and $\frac{\partial (\Pi_{A,F}^* - \Pi_{A,N}^*)}{\partial Q_S} \leq 0$ otherwise. When $\eta$ is small, Amazon benefits more from the FBA program if the third-party seller is stronger with a larger market size. Whereas, when $\eta$ is large, Amazon benefits more from the FBA program if the third-party seller has a smaller market size.

The above proposition shows how the market size of the third-party seller influences the benefit of the FBA program for Amazon. Interestingly, the proposition reveals a non-monotone pattern. When the cross-service sensitivity is relatively small, the FBA program is more beneficial for Amazon if a stronger competitor subscribes to the FBA program; in contrast, when $\eta$ is large, the FBA program becomes more beneficial if a weaker competitor does so. The intuition is as follows. As we have discussed above, for small values of $\eta$, the price competition mitigation effect dominates the service competition intensification effect. We find that this dominance is more significant if the competitor is stronger, implying that the benefit of the FBA program is more substantial. In contrast, for large values of $\eta$, the comparison between these two effects reverses, and the sales and service commissions that Amazon obtains from the third-party seller start to play an important role. The service competition intensification effect is weaker if a weaker competitor subscribes to the FBA program, while the relative gain of the commissions that Amazon obtains with the FBA program, compared to without, are more significant if the third-party seller has a smaller market size. This finding is illustrated in the left subplot of Figure 1 with three different levels of $Q_S$. Although it is theoretically challenging to establish, we numerically find that an analogous pattern may arise with respect to the cross-price sensitivity $\beta$. That is, when the cross-service sensitivity is relatively small, the FBA program might be more beneficial for Amazon if $\beta$ is larger, i.e., the intensity of the price competition between the two parties is stronger; whereas, when $\eta$ is large, the FBA program may become more beneficial for Amazon if the price competition between the two parties is weaker. This is shown numerically in the right subplot of Figure 1 with three different levels of $\beta$. These two findings offer meaningful implications about what types of third-party sellers/competing products can be more appealing to attract for Amazon’s FBA program according to the extent of service and price competitions between them.

Moreover, the benefit of the FBA program is also affected by the sales commission rate ($r$) since it influences the sales incomes of the two parties and thus their competition. In our model, we assume...
that the sales commission rate is exogenous. Although in theory the platform could customize the commission rate according to the platform-seller competition and the use of FBA, in practice, sales commission rate decisions are often long-term strategies of platforms, set at the product category level, and are determined by many factors such as platform-to-platform competition and platform-seller relationships. Therefore, we evaluate the effect of the sales commission rate \( r \) as an exogenous variable, while the subsequent competition and the endogenous FBA fee are solved accordingly as in the above. Figure 2 presents a numerical example. It shows that as \( r \) increases, the benefit of FBA for Amazon first decreases and then increases. In particular, when \( r \) is intermediate, the FBA program can hurt Amazon. The intuition is as follows. When \( r \) is small, the third-party seller has a high sales margin while Amazon obtains little from the third-party seller’s sales. As such, both of them have a strong incentive to compete selling their own products. The use of the FBA program allows Amazon to gain from the third-party seller’s sales and thus can alleviate the price competition. This effect is particularly strong when \( r \) is small. When \( r \) increases, the third-party seller becomes weaker in the competition; on the other hand, as Amazon obtains more commissions from the third-party seller’s sales, its price competition incentive against the third-party seller decreases. Therefore, the price competition mitigation benefit of the FBA program weakens and can be dominated by the detriment of the program to boost the third-party seller’s service capability, when \( r \) is intermediate. When \( r \) is large, the third-party seller is in a substantially disadvantageous position in the competition. Both the price competition mitigation effect and the service competition intensification effect of the FBA program become weak, which makes the FBA service fee that the program allows Amazon to charge become a dominant factor. As such, the benefit of the FBA program increases again in \( r \), as the two competition effects become weaker. Since the sales commission rate is typically set at the product category level and is influenced by external environment, this finding shows that the FBA program is particularly beneficial for Amazon in the scenarios where it has to charge a low sales commission rate or where it is able to charge a high sales commission rate. The FBA program is not as beneficial for product categories or in a specific business environment where Amazon charges intermediate sales commission rates.

In the above, we discussed the effects of the FBA program from the perspectives of the third-party seller and Amazon. For the OEM supplier of Amazon, its profit is also affected by the FBA program given that Amazon’s sales quantity is changed by the program. From Lemmas 4 and 5, we
can readily observe that the FBA program increases Amazon’s sales, while Amazon’s OEM supplier can charge a higher wholesale price, if the FBA service level is not significantly higher than that of the third-party seller \((s_A \in [s_S, \hat{s}_A])\) and the cross-service sensitivity \(\eta\) is greater than the threshold \(\eta_2\). In such a scenario, apparently, Amazon’s OEM supplier will benefit from the FBA program. This can also arise when the FBA service level is higher than the threshold \(\hat{s}_A\) and the cross-service sensitivity \(\eta\) is less than the threshold \(\eta_3\). In the other scenarios, the OEM supplier is worse off by Amazon’s FBA program. Interestingly, we find that as long as the OEM supplier is better off by the FBA program, Amazon is better off too, although the converse may not be true. Since the FBA program as constructed in (5) guarantees the third-party seller’s participation, the above discussion reveals the conditions of a Pareto-improvement by FBA for all of the three parties. This is stated in the following proposition and demonstrated in Figure 3.

**Proposition 4.** The FBA program can achieve a win-win-win outcome for the third-party seller, Amazon and its OEM supplier when \(\eta > \eta_2\) and \(s_A \in [s_S, \hat{s}_A]\), or when \(\eta < \eta_3\) and \(s_A \in (\hat{s}_A, +\infty)\).

In the above, when we analyzed the case with FBA, we assumed that Amazon is committed to offering the FBA program. If there is a super game in which Amazon is allowed to choose whether to offer FBA, it is clear from Proposition 2 that Amazon will do so if \(\eta \leq \eta_2\) or \(\eta \geq \eta_1\) in the context of our model. Note that the comparative results we obtained above regarding Amazon’s selling price and sales quantity, the OEM supplier’s wholesale price, and the sensitivities of Amazon’s profit function all have a single threshold with respect to \(\eta\). Therefore, those properties (characterized in
Figure 3: Illustration of the profit changes of the three parties by FBA. The parameters are: $c_0 = 30$, $g = 20$, $c = 12.5$, $s_A = 20$, $s_S = 5$, $r = 0.08$, $Q_A = 30$, $Q_S = 5$, $\beta = 0.5$, $\alpha = 0.9$.

the regions either below or above the corresponding thresholds) will continue to arise in the super game as we vary $\eta$ although the exact regions now need to take into account the thresholds $\eta$ and $\bar{\eta}$ where Amazon switches its choice on FBA.

5 Extensions

In this section, we discuss several alterations of our main model. We first consider the case where the third-party seller and Amazon’s OEM supplier belong to the same company in subsection 5.1. Next, we consider the case where the cross-price and cross-service sensitivities are related in subsection 5.2 and lastly, we consider the case where the price competition between Amazon the third-party seller arises sequentially in subsection 5.3.

5.1 When the Third-party Seller is Amazon’s OEM Supplier

In our main model, we assume that the third-party seller and Amazon’s OEM supplier are different entities. In practice, OEM manufacturers, such as Apple and Samsung, may open their own stores on Amazon, while supplying products to Amazon. That is, the third-party seller can be Amazon’s OEM supplier at the same time. To analyze such a case, we slightly modify the sequences of events of the two models with and without FBA. Now, after the OEM supplier makes the wholesale price decision, the OEM supplier and Amazon engage in a simultaneous price competition. If the FBA
program is not offered, the profit functions of Amazon and the OEM supplier follow:

\[
\Pi_{A,N} = (p_A - w - c)q_A(p_A, p_S) + rpsq_S(p_A, p_S),
\]

\[
\Pi_{S,N} = [(1 - r)p_S - c]q_S(p_A, p_S) + wq_A(p_A, p_S) - c_0(q_A(p_A, p_S) + q_S(p_A, p_S)).
\]

Differently, if the FBA program is offered and the OEM supplier subscribes to the program, then the profit functions of Amazon and the OEM supplier are:

\[
\Pi_{A,F} = (p_A - w - c)q_A(p_A, p_S) + rpsq_S(p_A, p_S) + (T - c)q_S(p_A, p_S),
\]

\[
\Pi_{S,N} = [(1 - r)p_S - T]q_S(p_A, p_S) + wq_A(p_A, p_S) - c_0(q_A(p_A, p_S) + q_S(p_A, p_S)).
\]

The price competition between the two parties and the OEM supplier’s wholesale price decision of the above two games can be solved by backward induction. The solutions are provided in the appendix. For the case with FBA, for simplicity, we assume that the FBA fee is exogenous, and we can find the OEM supplier’s maximum willingness-to-pay for the FBA service, denoted by \(\tilde{T}\). To compare the two cases with and without FBA, we assume that the FBA fee \(T \leq \tilde{T}\), which implies \(\Pi^*_{S,F} \geq \Pi^*_{S,N}\). We obtain the following result.

**Lemma 6.** There exist thresholds \(\eta_4, \eta_5, \eta_6\) and \(\eta_7\) such that:

(i) The equilibrium wholesale price set by the OEM supplier is higher with FBA than without, i.e., \(w^*_F \geq w^*_N\), if \(\eta \leq \eta_4\) and lower otherwise;

(ii) The OEM supplier’s equilibrium selling price is higher with FBA than without, i.e., \(p^*_{S,F} \geq p^*_{S,N}\), if \(\eta \leq \eta_5\), and lower otherwise, while its corresponding sales quantity is higher with FBA than without, i.e., \(q^*_{S,F} \geq q^*_{S,N}\), if \(\eta \leq \eta_6\), and lower otherwise;

(iii) Amazon’s equilibrium selling price is higher with FBA than without, i.e., \(p^*_{A,F} \geq p^*_{A,N}\), if \(\eta \leq \eta_7\), and lower otherwise, while its corresponding sales quantity is always lower with FBA than without, i.e., \(q^*_{A,F} < q^*_{A,N}\).

The above lemma shows that when the third-party seller is also Amazon’s OEM supplier, the equilibrium outcome may differ from that under our main model. FBA can either boost the third-party seller’s selling price and sales quantity or reduce them. The former occurs when the cross-service sensitivity is low, while the latter occurs when the cross-service sensitivity is high. This is
attributed to the change of the wholesale price given that the OEM supplier receives revenues not only from the wholesale but also from the retail sale. We can notice from the above lemma that although FBA now always reduces Amazon’s sales, Amazon’s OEM supplier (or the third-party seller) may still charge a higher wholesale price with FBA than without when the cross-service sensitivity is low. However, the effects of FBA that we discussed under our main model continue to exist. FBA boosts the third-party seller’s service level, which can mitigate the price competition between the two parties when the cross-service sensitivity is low and intensify the service competition when the cross-service sensitivity is high. Therefore, we find that the comparison of Amazon’s profits with and without FBA remains largely similar to that under our main model, as shown in the following proposition.

**Proposition 5.** There exist two thresholds $\eta^o$ and $\bar{\eta}^o$ such that $\Pi^*_{A,F} \geq \Pi^*_{A,N}$ if $\eta \leq \eta^o$ or $\eta \geq \bar{\eta}^o$, and $\Pi^*_{A,F} < \Pi^*_{A,N}$ otherwise.

Recall that due to analytical complexity, we set the FBA fee exogenous in this extension. From our numerical analysis, we find that with the same FBA fee, Amazon can benefit either more or less from the FBA program when the third-party seller and Amazon’s OEM supplier are the same firm than when they are different entities. But, the difference is not significant. Therefore, the main insights obtained above are robust in this respect.

### 5.2 Correlation between Cross-price and Cross-service Sensitivities

Although the cross-price and cross-service sensitivities are typically assumed to be independent in prior research, certain relationships between them may exist in practice. For instance, some consumers may compare the prices and the fulfillment services in an equal manner. To capture such dependencies, we conduct an analysis with the following revised demand models: without FBA,

$$q_A = Q_A - p_A + \beta \theta p_S + \alpha s_A - \eta \theta s_S,$$

$$q_S = Q_S - p_S + \beta \theta p_A + \alpha s_S - \eta \theta s_A,$$
and with FBA,

\[ q_A = Q_A - p_A + \beta \theta p_S + \alpha s_A - \eta \theta s_A, \]
\[ q_S = Q_S - p_S + \beta \theta p_A + \alpha s_A - \eta \theta s_A, \]

where \( \theta > 0 \) reflects a certain positive relationship between the cross-price and cross-service sensitivities.

The competition games can be solved similarly as for the main model. Our analysis shows that the structural properties revealed in the previous section continue to hold. Specifically, given any \( \theta \), Amazon benefits from FBA if \( \eta \) is either small or large, and the opposite is true when \( \eta \) is moderate. We also conduct a numerical analysis for the effect \( \theta \). Figure \[ \text{4} \] shows that as \( \theta \) increases, the benefit of the FBA program for Amazon may first increase and then decrease. That is, as both the cross-price and the cross-service sensitivities increase, the FBA program may first become more beneficial for Amazon and then the benefit may decrease. Recall that in our main model where the two cross sensitivities are unrelated, the benefit of the FBA program for Amazon first decreases and then increases in \( \eta \). However, the analysis of this extension shows that if the cross-price sensitivity increases alongside the cross-service sensitivity, the effect of the change of the cross-service sensitivity \((\eta \theta)\) may be dominated by the effect of the change of the cross-price sensitivity \((\beta \theta)\). As a consequence, the benefit of the FBA program for Amazon would be the most substantial when the intensities of the two types of competition are jointly intermediate.

5.3 Sequential Price Competition

In our main model, we assume that Amazon and the third-party seller set their prices simultaneously. In practice, the technology of electronic platforms enables the sellers to adjust their prices quickly, which makes simultaneous price competition more practical. Nevertheless, scenarios may exist where either Amazon or the third-party seller leads the price competition, followed by the opponent. While such a sequential price competition can be solved by converting the simultaneous procedure to the sequential procedure, the solution becomes complex without explicit expressions. Numerically, we find that compared to the simultaneous price competition, the first mover in the

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sequential price competition always gains an advantage, either with or without FBA. The second mover in the sequential price competition does not necessarily gain less than in the simultaneous price competition, since the sequential price competition may be less intensive than the simultaneous price competition. Regarding the benefits of FBA, we find that the structural patterns we identified for the simultaneous price competition as illustrated in Figure 1 remain the same for the sequential price competition, although quantitatively the benefits can become either larger or smaller. That is, the price setting sequence between the parties does not alter the managerial insights with respect to the effects of FBA.

6 Conclusion

This paper studies the phenomenon observed in practice that Amazon, the leading online retail platform, offers its superior fulfillment services (FBA) to the third-party sellers that may sell competing products on its retail platform. When the third-party sellers subscribe to the FBA program, they will achieve the same service level as Amazon’s, and it is explicitly noted on the web page of their products that the fulfillment including return is processed by Amazon. Apparently, the FBA program will make the third-party sellers more appealing to the consumers and improve their competitiveness potentially against Amazon.
To better understand the effects of the FBA program, we develop a strategic competition model between Amazon, Amazon’s OEM supplier and a representative third-party seller. The demands of Amazon and the third-party seller depend on both of their selling prices and their fulfillment services. Interestingly, the fact that the FBA program will boost the third-party seller’s service level not just makes its product more appealing, but allows the seller to increase its selling price, which can mitigate the price competition between the two parties. As a result, although the FBA program intensifies the service competition and reduces Amazon’s relative competitiveness, this price competition mitigation effect together with the service revenue can benefit Amazon. Specifically, Amazon is better off by FBA when the cross-service sensitivity is either sufficiently low or sufficiently high. The former is mainly because of the dominant effect of price competition mitigation, while the latter is due to weakened service competition intensification. Moreover, Amazon’s OEM supplier can gain from the FBA program too. There are scenarios where Amazon’s sales increase although the OEM supplier charges a higher wholesale price, and thus a Pareto-improvement for all the parties is possible. We also examine the effects of the third-party seller’s market size and the cross-price sensitivity. We find that when the cross-service sensitivity is low, the FBA program is more beneficial for Amazon when the third-party seller has a larger market size or when the cross-price sensitivity is stronger. However, when the cross-service sensitivity is high, the effects reverse. The FBA program becomes more beneficial for Amazon when the third-party seller has a smaller market size or when the cross-price sensitivity is weaker. These findings extend to the settings where the third-party seller and Amazon’s OEM supplier are the same firm, the cross-service and cross-price sensitivities are correlated, and the parties set their prices in different sequences.

We conclude by discussing several limitations of our study and the directions for future research. First, as we focus on the strategic interactions between the parties with respect to their price competition, our model assumes that the information regarding the parameters is complete. While the product prices, the fulfillment service qualities and costs can often be observed or estimated, the information of some other factors such as the market sizes and the procurement costs may sometimes be uncertain or asymmetric in practice. It will be interesting to investigate how information asymmetry and uncertainty for such factors may influence the effects of FBA. Second, we assume that the FBA service quality and cost are perfectly aligned with those for Amazon’s own products and are not influenced by the third-party seller’s sales volume. It may be useful to investigate what
service quality Amazon shall offer to the third-party sellers and how the dependence of the service quality and cost on a third-party seller’s sales volume may influence the effects of FBA. Third, in this study, we focus on the competition between Amazon and an existing third-party seller that have both already launched their products. It will also be interesting to examine how the FBA program may influence the parties’ market entry decisions.

References


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Online Appendix: Proofs

We first present Lemmas A.1-A.3 that solve the equilibria of the two cases without and with FBA.

**LEMMA A.1:** For the case without FBA, there exists a unique equilibrium, in which the three parties’ decisions follow:

\[ w^*_N = (g \beta + Q_A(2 - r \beta^2) + Q_S(\beta - r \beta - c(2 - \beta - \beta^2) + s_A(2 \alpha - \beta \eta - r \alpha \beta^2 + r \beta \eta) + s_S(\alpha \beta - r \alpha \beta - 2 \eta + r \beta^2 \eta)) / (2(2 - \beta^2)) + c_0/2, \]

\[ p^*_{A,N} = (g \beta (3 + r - \beta^2 - r \beta^2) + c((1 - r)(2 - \beta)(1 + \beta) + (1 + r)\beta(2 - \beta^2)) + c_0(1 - r)(2 - \beta^2) + Q_A(1 - r) \]

\[ (6 - (2 + r) \beta^2) + Q_S(3 - 2r - \beta^2 - r^2 + r^2 \beta^2) + s_A(1 - r)(\alpha (6 - (2 + r) \beta^2 - \beta(3 + r - (1 + r) \beta^2) \eta) + s_S(1 - r)((\alpha \beta(3 + r - (1 + r) \beta^2) - 6 \eta + (2 + r) \beta^2 \eta)) / ((1 - r)(4 - (1 + r) \beta^2)(2 - \beta^2)), \]

\[ p^*_{S,N} = (g(8 - (3 + r) \beta^2) + c(8 + 2(1 - r) \beta - (3 + r) \beta^2 - (1 - r) \beta^3) + c_0 \beta(1 - r)(2 - \beta^2) + Q_A(1 - r) \beta(6 - (2 + r) \beta^2) + Q_S(1 - r)(8 - (3 + r) \beta^2) + s_A(1 - r)(6 \alpha \beta - (2 + r) \alpha \beta^3 - 8 \eta + (3 + r) \beta^2 \eta) \]

\[ + s_S(8 \alpha - (3 + r) \alpha \beta^2 - 6 \beta \eta + (2 + r) \beta^3 \eta)) / (2(1 - r)(4 - (1 + r) \beta^2)(2 - \beta^2)). \]

**PROOF.** Given the OEM supplier’s wholesale price, Amazon and the third-party seller’s best responses in sale prices can be obtained as \( p^*_{A,N}(w) \) and \( p^*_{S,N}(w) \). Substituting them into \( \Pi_{M,N} = (w - c_0) q^*_{A,N}(w) \), we have the profit function of OEM supplier with respect to \( w \). Since \( \frac{\partial^2 \Pi_{M,N}}{\partial w^2} = -\left(\frac{2 - \beta^2}{(1 + \beta \eta)(2 - \beta^2)}\right)^2 < 0 \), \( \Pi_{M,N} \) is a concave quadratic function of \( w \). By solving the first-order condition \( \frac{\partial \Pi_{M,N}}{\partial w} = 0 \), we obtain the unique equilibrium wholesale price. Substituting it into \( p^*_{A,N}(w) \) and \( p^*_{S,N}(w) \), we can obtain the third-party seller’s and Amazon’s equilibrium selling prices. \( \square \)

**LEMMA A.2:** For the case with FBA, there exist unique best responses of the three parties in the subgame that follow:

\[ w^*_F = (g \beta + Q_A(2 - r \beta^2) + Q_S(1 - r) \beta - c(2 - 2 \beta - \beta^2 + \beta^3) - T(\beta - \beta^3) \]

\[ + s_A\left(2 + (1 - r) \beta - r \beta^2\right)(\alpha - \eta) / (2(2 - \beta^2)) + c_0 / 2, \]

\[ p^*_{A,F} = (g \beta (3 + r - \beta^2 - r \beta^2) + c_0(1 - r) \beta - c(1 - r)(1 - \beta)(2 - \beta^2) + s_A(\alpha - \eta)(1 - r)(6 + \beta(1 - \beta)(3 + \beta + r \beta(1 - \beta - \beta^2)) + Q_A(1 - r)(6 - (2 + r) \beta^2) \]

\[ + Q_S(1 - r)(3 + r - \beta^2 - r \beta^2 - T \beta(5 - 2 \beta^2)) / ((1 - r)(2 - \beta^2)(4 - (1 + r) \beta^2)), \]

\[ p^*_{S,F} = (g(8 - (3 + r) \beta^2) + s_A(\alpha - \eta)(1 - r)(8 + (6 \beta - \beta^2)(3 + r + (2 + r) \beta)) \]

\[ + c(1 - r)(2 - 2 \beta - \beta^2 + \beta^3) + Q_A(1 - r)(6 - (2 + r) \beta^2) \beta + Q_S(1 - r)(8 - (3 + r) \beta^2 \]

\[ + c_0 \beta(1 - r)(2 - \beta^2) + T(8 - (1 + 3r) \beta^2 - (1 - r) \beta^4)) / ((2(1 - r)(2 - \beta^2)(4 - (1 + r) \beta^2)). \]

**PROOF.** Given the OEM supplier’s wholesale price, Amazon and the third-party seller’s best responses in sale prices can be obtained as \( p^*_{A,F}(w) \) and \( p^*_{S,F}(w) \). Substituting them into \( \Pi_{M,F} = (w - c_0) q^*_{A,F}(w) \), we
have the profit function of OEM supplier with respect to $w$. Since $\frac{\partial^2 \Pi_\text{M,F}}{\partial w^2} = -\frac{(2-\beta^2)}{4(1+r)^2} < 0$, $\Pi_\text{M,F}$ is a concave quadratic function of $w$. By solving the first-order condition $\frac{\partial \Pi_\text{M,F}}{\partial w} = 0$, we obtain the unique best responses of the three parties in the subgame. □

LEMMA A.3: There exists a unique FBA fee $T^*$ that maximizes Amazon’s profit subject to the third-party seller’s participation constraint.

PROOF. Substituting the best responses in LEMMA A.2 into $\Pi_{A,F} = (p^*_A,F(T)-w^*_F-c)q^*_A,F(T)+(rp^*_S,F(T)+T-c)q^*_S,F(T)$, we have the profit function of Amazon with respect to $T$. Since $\frac{\partial^2 \Pi_{A,F}}{\partial T^2} < 0$, $\Pi_{A,F}$ is a concave quadratic function of $T$. By solving the first-order condition $\frac{\partial \Pi_{A,F}}{\partial T} = 0$, we obtain the unique solution $T = \tilde{T}$, where $\tilde{T}$ is a decreasing linear function of $\eta$. The maximum willingness-to-pay of the third-party seller is defined as $\bar{T}$ (see the proof of Proposition 1). Therefore, Amazon will set the FBA fee as $T^* = \min\{\tilde{T}, \bar{T}\}$. The equilibrium can be obtained by substituting this optimal FBA fee into the subgame best response functions characterized in the above lemma. □

PROOF OF LEMMA 1\[1\] By comparing the third-party seller’s equilibrium selling prices with and without FBA, we have

$$p^*_{S,F} - p^*_{S,N} = \frac{(T^*-c)(8-(1+3r)\beta^2-(1-r)\beta^4)+(1-r)(s_A-s_S)(8-(3+r)\beta^2)\alpha-(1-r)(s_A-s_S)\beta(6-(2+r)\beta^2)\eta}{2(1-r)(2-\beta^2)(4-(1+r)\beta^2)}$$

which is always positive with $\alpha > \eta$ and $T^* > c$. Therefore, $p^*_{S,F} - p^*_{S,N} > 0$ holds for any $\eta \in (0, \alpha)$. □

PROOF OF LEMMA 2\[2\] By comparing Amazon’s equilibrium selling prices with and without FBA, we have

$$p^*_{A,F} - p^*_{A,N} = \frac{(T^*-c)(5\beta-r\beta-2\beta^3)+(1-r)(s_A-s_S)\alpha(3+r)\beta-(1+r)\beta^2)-(s_A-s_S)(1-r)(6-(2+r)\beta^2)\eta}{(1-r)(2-\beta^2)(4-(1+r)\beta^2)}$$

The fact that both $\tilde{T}$ and $\bar{T}$ linearly decrease in $\eta$ implies that $T^*$ also decreases in $\eta$. It’s easy to check that $p^*_{A,F} - p^*_{A,N}$ decreases in $\eta$ and is positive when $\eta = 0$. There exists a $\eta_1$, such that $p^*_{A,F} \geq p^*_{A,N}$ with $\eta \leq \eta_1$ and $p^*_{A,F} < p^*_{A,N}$ with $\eta > \eta_1$, where $\eta_1 > 0$. □

PROOF OF LEMMA 3\[3\] By comparing the third-party seller’s equilibrium sales quantities with and without FBA, we have

$$q^*_{S,F} - q^*_{S,N} = \frac{-((T^*-c)(1-\beta^2)(8-(3+r)\beta^2)+(1-r)(s_A-s_S)\alpha(8-(3+r)\beta^2)-(s_A-s_S)(1-r)\beta(6-(2+r)\beta^2)\eta}{2(1-r)(2-\beta^2)(4-(1+r)\beta^2)}$$

$q^*_{S,F} \geq q^*_{S,N}$ iff $T^* \leq c + \frac{(1-r)(s_A-s_S)(8\alpha-(3+r)\beta^2-6\beta\eta+(2+r)\beta^2)\eta}{(1-r)(2-\beta^2)(8-(3+r)\beta^2)}$, where the threshold is equal to $\bar{T}$. By definition of $T^*$, $T^* \leq \bar{T}$. Thus, we have $q^*_{S,F} \geq q^*_{S,N}$ holds for any $\eta \in (0, \alpha)$. □
PROOF OF LEMMA [3] By comparing Amazon’s equilibrium sales quantities with and without FBA, we have \( q_{A,F}^* - q_{A,N}^* = \frac{(1-r)(s_A-s_S)\alpha - (T^*-c)\beta(1-\beta^2)-(s_A-s_S)(2-r\beta^2)}{2(4-(1+r)\beta^2)} \).

First, we check the monotonicity of \( q_{A,F}^* - q_{A,N}^* \) with respect to \( \eta \).

(i) If \( T^* = T \), we can show that 
\[
\frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} = -\frac{(2-\beta^2)(s_A-s_S)}{s-\beta^2(r+3)} < 0.
\]

(ii) If \( T^* = \tilde{T} \), we can show that 
\[
\frac{\partial^2(q_{A,F}^*-q_{A,N}^*)}{\partial \eta \partial s_A} < 0.
\]
We also have 
\[
\frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} > 0 \text{ with } s_A = s_S.
\]
Define \( \tilde{s}_A \) as the solution of 
\[
\frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} = 0.
\]
Thus, we obtain 
\[
\frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} \geq 0 \text{ if } s_A \in [s_S, \tilde{s}_A] \text{ and } \frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} < 0 \text{ otherwise.}
\]

We then compare \( T \) and \( \tilde{T} \) to check the value of \( T^* \) at the point of \( \eta = 0 \). It can be shown that \( \frac{\partial(T-T^*)}{\partial s_A} < 0 \) at the point of \( \eta = 0 \). Define \( \tilde{s}_A \) as the solution of \( T - \tilde{T} = 0 \) at the point of \( \eta = 0 \). Thus, \( T \leq \tilde{T} \) which implies \( T^* = T \), for \( s_A \in [s_S, \tilde{s}_A] \), and \( T > \tilde{T} \) which shows that \( T^* = \tilde{T} \), for \( s_A \in (\tilde{s}_A, +\infty) \) at the point of \( \eta = 0 \). Furthermore, it’s easy to check that \( q_{A,F}^* - q_{A,N}^* = 0 \) with \( T^* = T \), and \( q_{A,F}^* - q_{A,N}^* > 0 \) with \( T^* = \tilde{T} \) at the point of \( \eta = 0 \).

Next we examine the following three cases, \( s_A \leq \min\{\tilde{s}_A, \tilde{s}_A\} \), \( s_A \in (\min\{\tilde{s}_A, \tilde{s}_A\}, \max\{\tilde{s}_A, \tilde{s}_A\}) \), and \( s_A \geq \max\{\tilde{s}_A, \tilde{s}_A\} \), respectively.

Case 1. \( s_A \leq \min\{\tilde{s}_A, \tilde{s}_A\} \)

\( T^* = T \) at the point of \( \eta = 0 \) and may switch to \( \tilde{T} \) as \( \eta \) increases. \( \frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} < 0 \) if \( T^* = T \), and \( \frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} \geq 0 \) if \( T^* = \tilde{T} \). The above imply that there exists one threshold \( \eta_2 \) such that \( q_{A,F}^* \leq q_{A,N}^* \) for \( \eta \leq \eta_2 \) and \( q_{A,F}^* > q_{A,N}^* \) otherwise, where \( \eta_2 > 0 \).

Case 2. \( s_A \in (\min\{\tilde{s}_A, \tilde{s}_A\}, \max\{\tilde{s}_A, \tilde{s}_A\}) \)

We discuss two subcases with \( \tilde{s}_A \geq 0 \) or \( \tilde{s}_A < 0 \)

(i) Subcase 2.1. \( \tilde{s}_A \geq 0 \) and \( s_A \in (\tilde{s}_A, \tilde{s}_A) \)

\( T^* = T \) at the point of \( \eta = 0 \) and may switch to \( \tilde{T} \) as \( \eta \) increases. \( \frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} < 0 \) if \( T^* = T \), and \( \frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} \geq 0 \) if \( T^* = \tilde{T} \). \( \frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} < 0 \) if \( T^* = \tilde{T} \).

(ii) Subcase 2.2. \( \tilde{s}_A < 0 \) and \( s_A \in (\tilde{s}_A, \tilde{s}_A) \)

\( T^* = \tilde{T} \) at the point of \( \eta = 0 \) and may switch to \( \tilde{T} \) as \( \eta \) increases. \( \frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} \geq 0 \) if \( T^* = \tilde{T} \), and \( \frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} \geq 0 \) if \( T^* = T \).

Case 3. \( s_A \geq \max\{\tilde{s}_A, \tilde{s}_A\} \)

\( T^* = \tilde{T} \) at the point of \( \eta = 0 \) and may switch to \( \tilde{T} \) as \( \eta \) increases. \( \frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} \geq 0 \) if \( T^* = \tilde{T} \), and \( \frac{\partial(q_{A,F}^*-q_{A,N}^*)}{\partial \eta} \geq 0 \) if \( T^* = T \).

Combining Case 2 and Case 3, there exists one threshold \( \eta_3 \) such that \( q_{A,F}^* > q_{A,N}^* \) for \( \eta < \eta_3 \) and \( q_{A,F}^* \leq q_{A,N}^* \) otherwise, where \( \eta_3 \geq 0 \).

To conclude, we have the following result. Define that \( \hat{s}_A = \min\{\tilde{s}_A, \tilde{s}_A\} \). When \( s_A \leq \hat{s}_A \), there exist one threshold \( \eta_2 \) such that \( q_{A,F}^* \leq q_{A,N}^* \) if \( \eta \leq \eta_2 \) and \( q_{A,F}^* > q_{A,N}^* \) otherwise, where \( \eta_2 > 0 \). When \( s_A > \hat{s}_A \),
there exist one threshold η3 such that \( q^*_A,F > q^*_A,N \) if \( η < η_3 \) and \( q^*_A,F ≤ q^*_A,N \) otherwise, where \( η_3 ≥ 0 \). □

PROOF OF LEMMA 5 By comparing the equilibrium wholesale price with and without FBA, we have
\[
\begin{align*}
w^*_F - w^*_N &= \frac{(1-r)(s_A-s_S)αβ-(T^*-c)β(1-β^2)-(s_A-s_S)(2-βr)η}{2(2-βr)^2}.
\end{align*}
\]

Note that \( w^*_F - w^*_N = \left(q^*_A,F - q^*_A,N\right) \frac{4-(1+r)β^2}{2-βr} \). Thus, \( w^*_F - w^*_N \) has the same sign as \( q^*_A,F - q^*_A,N \). □

PROOF OF PROPOSITION 1 By taking the first derivative of \( T = [(1-r)p_S - g - c]q_S(p_A,p_S) \) with respect to \( p_S \), the third-party seller’s best response function in \( p_S \) should satisfy \([(1-r)p_S^N - g - c] = (1-r)q^*_S,N \), which implies that in equilibrium \( Π^*_S,N = [(1-r)p^*_S,N - g - c]q^*_S,N = (1-r)^2 \left(q^*_S,N\right)^2 \). Similar to the above non-FBA scenario, we can obtain that after using FBA, \( Π^*_S,F = (1-r)^2 \left(q^*_S,F\right)^2 \) in equilibrium. Consequently, \( Π^*_S,F - Π^*_S,N = (1-r)^2 \left((q^*_S,F)^2 - (q^*_S,N)^2\right) \). Hence, \( Π^*_S,F ≥ Π^*_S,N \) iff \( q^*_S,F ≥ q^*_S,N \).

\[
\begin{align*}
q^*_S,F - q^*_S,N &= -\frac{(T^*-c)(1-β^2)(3+3r)(3+3r)β^2(1-β)}{2(1-r)(2-βr)^2(4-βr)^2},
\end{align*}
\]

It’s easy to check that \( q^*_S,F ≥ q^*_S,N \) iff \( T ≤ c + \frac{(1-r)(s_A-s_S)(6α+3αβ+6β^2+6β^3η)(8+2r)β^3η}{(1-β^2)(8-3r)β^2} \). Define \( T \) as the maximum willingness-to-pay of the third-party seller, where \( T = c + \frac{(1-r)(s_A-s_S)(6α+3αβ+6β^2+6β^3η)(8+2r)β^3η}{(1-β^2)(8-3r)β^2} \).

Thus, \( Π^*_S,F ≥ Π^*_S,N \) iff \( T ≤ T^* \).

Next, we examine the monotonicity of \( T \) with respect to \( s_A-s_S, η, r, \) and \( β \). Taking the first derivative of \( T \) with respect to \( s_A-s_S \), we have \( \frac{dT}{ds_A-s_S} = \frac{(1-r)(8α+3αβ+6β^2+6β^3η)(8+2r)β^3η}{(1-β^2)(8-3r)β^2} \), which is positive for \( r ∈ (0,1) \) and \( β ∈ (0,1) \) and \( η ∈ (0,α) \). Therefore, \( T \) increases in \( s_A-s_S \). Because \( \frac{dT}{dη} = -\frac{(1-r)(s_A-s_S)(6+5r+r^2)β^3η}{(1-β^2)(8-3r)β^2} < 0 \), \( T \) decreases in \( η \).

Further, \( \frac{dT}{dr} = \frac{(s_A-s_S)(6+5r+r^2)β^3η}{(1-β^2)(8-3r)β^2} \), which is negative for \( r ∈ (0,1) \) and \( β ∈ (0,1) \) and \( η ∈ (0,α) \). Thus, \( T \) decreases in \( r \).

Taking the first derivative of \( T \) with respect to \( β \), we have
\[
\begin{align*}
\frac{dT}{dβ} &= \frac{(1-r)(s_A-s_S)\left(128αβ-32(3+r)αβ^3+2(3+r)^2αβ^5-48η\right)}{(1-β^2)(8-3r)β^2}.
\end{align*}
\]

Define \( Γ = 128αβ-32(3+r)αβ^3+2(3+r)^2αβ^5-48η \). \( Γ \) has the same sign as \( Γ \). Because \( \frac{Γ}{dβ} < 0 \), \( Γ \) is a concave function of \( β \). At the point of \( β=0 \), \( Γ = -48η ≤ 0 \). At the point of \( β=1 \), \( Γ = 2(5-r)(5α-r(α-η)-4η) > 0 \). Thus, there exists a \( β ∈ [0,1] \), \( Γ < 0 \) when \( β ∈ [0,β] \), and \( Γ ≥ 0 \) when \( β ∈ [β,1] \). Equivalently, \( \frac{dT}{dβ} < 0 \) when \( β < β \), and \( \frac{dT}{dβ} > 0 \) when \( β > β \). Therefore, \( T \) first decreases and then increases in \( β \). □

PROOF OF PROPOSITION 2 First, we prove that with either \( T^* = T \) or \( T^* = T^* \), \( Π^*_A,F - Π^*_A,N \) is always a convex function with respect to \( η \).
(1) In the case that $T^* = T$, taking the second derivative of $\Pi^*_{A,F} - \Pi^*_{A,N}$ with respect to $\eta$, we have

\[
\frac{\partial^2 (\Pi^*_{A,F} - \Pi^*_{A,N})}{\partial \eta^2} = \left\{ \begin{array}{ll}
s_A \left( 4\beta^2 + \beta^8(-r+1) - \beta^7(r+3) + \beta^6(4r+7) + \beta^5(r(3r+20) + 9) \right) \\
-s_A(-2\beta^4(2r+7) - \beta^3(r(r+34) + 45) + 8\beta(r+7) + 8) \\
-s_S\left( \beta^6(r(r+2) - 1) + \beta^0(1-3r(r+5)) \right) \\
-s_S(\beta^4(r(r+27) + 14) - 2\beta^2(3r+11) - 8)
\end{array} \right.
\]

which is positive with $s_A > 0$, $s_S > 0$, $r \in (0, 1)$ and $\beta \in (0, 1)$. Thus, we have $\frac{\partial^2 (\Pi^*_{A,F} - \Pi^*_{A,N})}{\partial \eta^2} > 0$.

(2) In the case that $T^* = \tilde{T}$, taking the second derivative of $\Pi^*_{A,F} - \Pi^*_{A,N}$ with respect to $\eta$, we have

\[
\frac{\partial^2 (\Pi^*_{A,F} - \Pi^*_{A,N})}{\partial \eta^2} = \frac{\phi\psi^\ast}\left( \frac{\phi\psi^\ast - \phi\psi^\ast - \psi\psi^\ast}{s_A - s_S} \right)
\]

where $\phi, \varphi$ and $\psi$ are functions of $r$ and $\beta$, and the denominator is positive with $r \in (0, 1)$ and $\beta \in (0, 1)$. The facts that $\phi > 0$, $\varphi > 0$, $\psi > 0$, and $\phi - \varphi - \psi > 0$ hold with $r \in [0, 1]$ and $\beta \in (0, 1]$, imply that $\phi S^2_A - \varphi S_A s_S - \psi S^2_S > \phi S^2_A - \varphi S_A S_A - \psi S^2_S > 0$ with $s_A > s_S$. Therefore, $\frac{\partial^2 (\Pi^*_{A,F} - \Pi^*_{A,N})}{\partial \eta^2} > 0$.

Based on the above, we obtain that $\Pi^*_{A,F} - \Pi^*_{A,N}$ is a convex function with respect to $\eta$.

Next we prove that when $\eta = 0$, $\Pi^*_{A,F} - \Pi^*_{A,N} > 0$. In equilibrium, Amazon’s profits without and with FBA are as follows: $\Pi_{A,N} = (p^*_{A,N} - w^*_N - c) q^*_A,N + r p^*_{S,N} q^*_S,N$ and $\Pi_{A,F} = (p^*_{A,F} - w^*_F - c) q^*_A,F + r p^*_{S,F} q^*_S,F + (T^* - c) q^*_S,F$.

Therefore, $\Pi^*_{A,F} - \Pi^*_{A,N} = ((p^*_{A,F} - w^*_F - c)q^*_A,F - (p^*_{A,N} - w^*_N - c)q^*_A,N) + r(p^*_{S,F}q^*_S,F - p^*_{S,N}q^*_S,N) + (T^* - c) q^*_S,F$.

Based on Lemmas 1 and 2 when $\eta = 0$, we have $p^*_{S,F} > p^*_{S,N}$ and $q^*_S,F \geq q^*_S,N$. This leads to that $p^*_{S,F}q^*_S,F - p^*_{S,N}q^*_S,N > 0$. From the first-order conditions $\frac{\partial \Pi^*_{A,F}}{\partial p_{A,F}} = 0$ and $\frac{\partial \Pi^*_{A,N}}{\partial p_{A,F}} = 0$, we have $p^*_{A,F} - w^*_F - c = q^*_A,F + \beta r p^*_{S,F}$ and $p^*_{A,N} - w^*_N - c = q^*_A,N + \beta r p^*_{S,N}$, implying that $p^*_{A,F} - w^*_F - c$ and $p^*_{A,N} - w^*_N - c$ are both positive in equilibrium. It’s easy to check that $q^*_A,F - q^*_A,N = 0$ with $T^* = T$, and $q^*_A,F - q^*_A,N > 0$ with $T^* = \tilde{T}$ at the point of $\eta = 0$. Thus, we have $q^*_A,F \geq q^*_A,N$ holds at the point of $\eta = 0$. This, combining the facts that $p^*_{S,F} > p^*_{S,N}$ and $T^* > c$, leads to that $(p^*_{A,F} - w - c)q^*_A,F - (p^*_{A,N} - w - c)q^*_A,N > 0$. Clearly, $(T^* - c) q^*_S,F > 0$. To conclude, $\Pi^*_{A,F} - \Pi^*_{A,N} > 0$ holds for $\eta = 0$.

We have proved that $\Pi^*_{A,F} - \Pi^*_{A,N}$ is a convex quadratic function with respect to $\eta$ and that $\Pi^*_{A,F} - \Pi^*_{A,N} > 0$ with $\eta = 0$. Consequently, there exist two thresholds $\bar{\eta}$ and $\bar{\eta}$ such that $\Pi^*_{A,F} \geq \Pi^*_{A,N}$ if $\eta \leq \bar{\eta}$ or $\eta \geq \bar{\eta}$ and $\Pi^*_{A,F} < \Pi^*_{A,N}$ otherwise. □

**PROOF OF PROPOSITION 3** We examine the two cases $T^* = T$ and $T^* = \tilde{T}$, respectively.

(1) If $T^* = T$, 

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By Lemmas 1 and 3, we have
\begin{align*}
\frac{\partial (\Pi^*_{A,F}-\Pi^*_{A,N})}{\partial q_S} &= (s_A - s_S)(\alpha (\beta^6 - (r^2 + 2r - 3)) - \beta^4 (-3r^2 - 18r + 5) - 32\beta^2 (r + 1) + 64) + \\
&= 2\beta\eta (-3\beta^3 + 19\beta^2 + (\beta^6 - 3\beta^4 + \beta^2) r^2 + (\beta^6 - 10\beta^4 + 20\beta^2 - 4) r - 28) \] 
\end{align*}
which is positive, due to that \((\beta^6 - (r^2 + 2r - 3)) - \beta^4 (-3r^2 - 18r + 5) - 32\beta^2 (r + 1) + 64 > 0, \alpha > \eta,\) and
\[(s_A - s_S)(\eta (\beta^6 - (r^2 + 2r - 3) - \beta^4 (-3r^2 - 18r + 5) - 32\beta^2 (r + 1) + 64) + \\
\frac{\partial (\Pi^*_{A,F}-\Pi^*_{A,N})}{\partial q_S} > 2\beta\eta (-3\beta^3 + 19\beta^2 + (\beta^6 - 3\beta^4 + \beta^2) r^2 + (\beta^6 - 10\beta^4 + 20\beta^2 - 4) r - 28) \] 
> 0.

This implies that \(\Pi^*_{A,F} - \Pi^*_{A,N}\) increases as \(Q_S\) grows.

(2) If \(T^* = \hat{T},\)
\[
\frac{\partial^2 (\Pi^*_{A,F}-\Pi^*_{A,N})}{\partial Q_S \partial \eta} = \frac{\chi s_A - \Phi s_S }{2(\beta - 1)(\beta^2 - 2)(\beta^2 + 1)(\beta^2 + 3)(\beta^4 + \beta^2 + 3)} - \frac{\chi s_A - \Phi s_S }{2(\beta^2 (r + 1) - 4)^2 (\beta^8 - 5\beta^6 - 20\beta^4 + 116\beta^2 + (\beta^4 - 4\beta^2 + 3) \beta^4 r^3 - (\beta^6 + 5\beta^4 - 30\beta^2 + 28) \beta^2 r^2 - (\beta^8 - 14\beta^6 + 29\beta^4 + 24\beta^2 - 64) r - 128) - (\beta^6 - 3\beta^4 + \beta^2) r^2 + (\beta^6 - 10\beta^4 + 20\beta^2 - 4) r - 28)}
\]
where \(\chi\) and \(\Phi\) are functions of \(r\) and \(\beta\), and the denominator is positive with \(r \in (0, 1)\) and \(\beta \in (0, 1)\). The fact that \(\chi > \Phi > 0\) holds with \(r \in [0, 1]\) and \(\beta \in [0, 1]\), implies that \(\chi s_A - \Phi s_S > \chi s_A - \Phi s_A > 0\) with \(s_A > s_S\). Therefore, \(\frac{\partial^2 (\Pi^*_{A,F}-\Pi^*_{A,N})}{\partial Q_S \partial \eta} < 0\).

To sum up, when \(T^* = \hat{T}, \frac{\partial (\Pi^*_{A,F}-\Pi^*_{A,N})}{\partial Q_S} > 0;\) and when \(T^* = \hat{T}, \frac{\partial (\Pi^*_{A,F}-\Pi^*_{A,N})}{\partial Q_S}\) decreases with \(\eta\). Thus, there exists a threshold \(\hat{\eta}\) such that \(\frac{\partial (\Pi^*_{A,F}-\Pi^*_{A,N})}{\partial Q_S} > 0\) for \(\eta < \hat{\eta}\) and \(\frac{\partial (\Pi^*_{A,F}-\Pi^*_{A,N})}{\partial Q_S} \leq 0\) otherwise. \(\square\)

**PROOF OF PROPOSITION 4** First, we examine the change of OEM supplier’s profit by FBA. \(\Pi^*_{M,F} - \Pi^*_{M,N} = (w_F - c_0)\) \(q^*_A,F - (w_N - c_0)\) \(q^*_A,N\). Since \(w_F - w_N = (q^*_A,F - q^*_A,N) \frac{4 - (1 + r)\beta^2}{2 + \beta^2}\), \(w_F - w_N\) has the same sign as \(q^*_A,F - q^*_A,N\). Consequently, \(\Pi^*_{M,F} \geq \Pi^*_{M,N}\) iff \(q^*_A,F \geq q^*_A,N\). This, combining Lemma 4, shows the following result. When \(s_A \leq \hat{s}_A\), we have \(\Pi^*_{M,F} \leq \Pi^*_{M,N}\) if \(\eta \leq \eta_2\) and \(\Pi^*_{M,F} > \Pi^*_{M,N}\) otherwise. When \(s_A > \hat{s}_A\), we have \(\Pi^*_{M,F} > \Pi^*_{M,N}\) if \(\eta < \eta_3\) and \(\Pi^*_{M,F} \leq \Pi^*_{M,N}\) otherwise.

Next, we examine whether Amazon also benefits from FBA with \(\eta > \eta_2\) if \(s_A \leq \hat{s}_A\), and \(\eta < \eta_3\) if \(s_A > \hat{s}_A\). As shown in Proof of Proposition 2
\[
\Pi^*_{A,F} = \Pi^*_{A,N} = (p^*_{A,F} - w^*_F - c)q^*_A,F - (p^*_{A,N} - w^*_N - c)q^*_A,N + r(p^*_{S,F}q^*_S,F - p^*_{S,N}q^*_S,N) + (T^* - c) q^*_S,F.
\]
And in equilibrium we have \(p^*_{A,F} - w^*_F - c = q^*_A,N + \beta rp^*_{S,N}\) and \(p^*_{A,F} - w^*_F - c = q^*_A,F + \beta(rp^*_{S,F} + T^* - c).\)

By Lemmas 1 and 3 we have \(p^*_{S,F} > p^*_{S,N}\) and \(q^*_S,F > q^*_S,N\). By Lemma 4 \(q^*_A,F > q^*_A,N\) holds with \(\eta > \eta_2\) if \(s_A \leq \hat{s}_A\) and with \(\eta < \eta_3\) if \(s_A > \hat{s}_A\). Thus, we have \(r(p^*_{S,F}q^*_S,F - p^*_{S,N}q^*_S,N) > 0\) and
\[
(p^*_{A,F} - w^*_F - c)q^*_A,F = (q^*_A,F + \beta(rp^*_{S,F} + T^* - c))q^*_A,F > (q^*_A,N + \beta rp^*_{S,N})q^*_A,N = (p^*_{A,F} - w^*_N - c)q^*_A,N.
\]
The above, combining the fact that \((T^* - c) q^*_S,F > 0,\) implies that \(\Pi^*_{A,F} - \Pi^*_{A,N} > 0\) holds with \(\eta > \eta_2\) if \(s_A \leq \hat{s}_A\), and \(\eta < \eta_3\) if \(s_A > \hat{s}_A\).

It follows from Proposition 4 that \(\Pi^*_{S,F} \geq \Pi^*_{S,N}\) with \(T = T^*\). Consequently, a win-win-win outcome is achieved for all the three parties if \(\eta > \eta_2\) when \(s_A \leq \hat{s}_A\), and \(\eta < \eta_3\) when \(s_A > \hat{s}_A\). \(\square\)
LEMMA A.4: When the third-party seller is Amazon’s OEM supplier, for the case without FBA, there exists a unique equilibrium in which the decisions of the two parties follow:

\[
\begin{align*}
\alpha^* &= \frac{w_N}{Q_A(1-r) + \beta^2(1 - r)} = \frac{(\alpha^* + \beta^2)(r^2 - 1) + 2\beta^2r^2 + 2(\beta^2 + 4)r + 8}{Q_A(1-r)}
\end{align*}
\]

For the case with FBA, there exists a unique equilibrium in which the decisions of the two parties follow:

\[
\begin{align*}
\alpha^* &= \frac{w_F}{Q_A(1-r) + \beta^2(1 - r)} = \frac{4(\beta^2 + 2\beta^2r^2 + 2(\beta^2 + 4)r + 8)}{Q_A(1-r)}
\end{align*}
\]

PROOF. The proof for this lemma is largely similar to those for Lemmas A.1-A.2. Given the third-party
seller’s wholesale price charged to Amazon, we solve the best response sales prices of Amazon and the third-party seller, from the simultaneous price competition, and the corresponding sales quantities, all as functions of the wholesale price $w$. Substituting them into the third-party seller’s profit function, we find that the third-party seller’s profit follows a concave quadratic function of $w$. Therefore, a unique optimal wholesale price exists, which is used to obtain the equilibrium selling prices shown in the above. □

PROOF OF LEMMA 6. (i) By comparing the equilibrium wholesale price with and without FBA, we have

$$w^*_F - w^*_N = \frac{1}{2} \left( \frac{\alpha \beta (1-r)(s_A-s_S)}{1-\beta^2} - \beta (T-c) - \eta \frac{1-\beta^4(r+1)^2-8\beta^2 r+8)(s_A-s_S)}{1-\beta^2}(1-\beta^2)(r+1)^2-8\beta r+8) \right)$$

which decreases in $\eta$. Denote $\eta_4$ as the solution to $w^*_F - w^*_N = 0$, i.e., $\eta_4 = \frac{1-\beta^4(r+1)^2-8\beta^2 r+8)(s_A-s_S)}{1-\beta^2}(1-\beta^2)(r+1)^2-8\beta r+8)$. We next examine $w^*_F - w^*_N$ at the point of $\eta = 0$. Note that for $\Pi_{S,F}^* \geq \Pi_{S,N}^*$ to hold, we have $T \leq T^*$. When $\eta = 0$, $T \leq T^*$ is equivalent to that $(1-\beta^2)(c+2c_0+T)-2(1-r)(\beta Q_A+Q_S)-\alpha(2\beta+1)(1-r)s_1-\alpha(1-r)(s_A(2\beta+1)+s_S) < 0$ and $(1-\beta^2)(T-c) - \alpha(1-r)(s_A-s_S) < 0$. This implies that $T^* \leq c + \frac{(1-r)(s_A-s_S)\alpha}{1-\beta^2}$ at the point of $\eta = 0$. The leads to that $w^*_F \geq w^*_N$ at the point of $\eta = 0$.

Consequently, $w^*_F \geq w^*_N$ for $\eta \leq \eta_4$ and $w^*_F < w^*_N$ for $\eta > \eta_4$, where $\eta_4 \geq 0$.

(ii) By comparing the OEM supplier’s equilibrium selling prices with and without FBA, we have

$$p^*_S,F - p^*_S,N = \frac{1}{2} \left( T-c \frac{(s_A-s_S)\alpha}{1-\beta^2} + \frac{(s_A-s_S)\beta(\beta^2+\beta^2(-r^2)+6r-10)}{(1-\beta^2)(1-\beta^2)(r+1)^2-8\beta r+8)} \right)$$

decreases in $\eta$. Denote $\eta_5$ as the solution to $p^*_S,F - p^*_S,N = 0$, i.e., $\eta_5 = \frac{(s_A-s_S)\beta(\beta^2+\beta^2(-r^2)+6r-10)}{(1-\beta^2)(1-\beta^2)(r+1)^2-8\beta r+8)}$, which is positive. Thus, $p^*_S,F \geq p^*_S,N$ with $\eta \leq \eta_5$ and $p^*_S,F < p^*_S,N$ with $\eta > \eta_5$.

By comparing the OEM supplier’s equilibrium sales quantities with and without FBA, we have $q^*_S,F - q^*_S,N = \frac{1}{2} \left( -\frac{(T-c)(1-\beta^2)}{1-r} + \frac{(s_A-s_S)\alpha}{1-\beta^2} \right)$. It’s easy to check that $q^*_S,F \geq q^*_S,N$ at the point of $\eta = 0$. Denote $\eta_{temp}$ as the solution to $q^*_S,F - q^*_S,N = 0$, i.e., $\eta_{temp} = \frac{(1-r)(s_A-s_S)\alpha}{(s_A-s_S)(1-\beta^2)(1-\beta^2)(r+1)^2-8\beta r+8)}$.

If $r \leq \frac{3-\beta^2-\sqrt{9-8\beta^2}}{\beta^2}$, we have $\beta^2 + \beta^2r^2 + 2(\beta^2 - 3)r + 2 \geq 0$ and $q^*_S,F - q^*_S,N$ decreases in $\eta$. This shows that $q^*_S,F \geq q^*_S,N$ with $\eta \leq \eta_{temp}$, and $q^*_S,F < q^*_S,N$ otherwise.

If $r > \frac{3-\beta^2-\sqrt{9-8\beta^2}}{\beta^2}$, we have $\beta^2 + \beta^2r^2 + 2(\beta^2 - 3)r + 2 < 0$ and $q^*_S,F - q^*_S,N$ increases in $\eta$. This implies that $q^*_S,F \geq q^*_S,N$ with any $\eta \in [0,\alpha]$.

To conclude, there exists a threshold $\eta_6$ such that $q^*_S,F \geq q^*_S,N$ with $\eta \leq \eta_6$ and $q^*_S,F > q^*_S,N$ otherwise, where $\eta_6 \geq 0$.

(iii) By comparing Amazon’s equilibrium selling prices with and without FBA, we have

$$p^*_A,F - p^*_A,N = \frac{1}{2} \left( \frac{\beta(T-c)}{1-r} + \frac{(s_A-s_S)\alpha}{(1-\beta^2)} - \frac{\eta(-\beta^4-2\beta^2+2(\beta^2-4\beta^2+6)r+12)(s_A-s_S)}{(1-\beta^2)(1-\beta^2)(r+1)^2-8\beta r+8)} \right)$$

which decreases in $\eta$.

Denote $\eta_7$ as the solution to $p^*_A,F - p^*_A,N = 0$, i.e., $\eta_7 = \frac{\beta(\beta^2+\beta^2r^2+2(\beta^2-4)r+8)((1-\beta)^2(T-c)+\alpha(1-r)(s_A-s_S))}{(1-\beta^2)(1-\beta^2)(r+1)^2-8\beta r+8)}$.  

Electronic copy available at: https://ssrn.com/abstract=3939085
Thus, $p_{A,F}^* \geq p_{A,N}^*$ with $\eta \leq \eta_7$ and $p_{A,F}^* < p_{A,N}^*$ with $\eta > \eta_7$. It’s easy to check that $\eta_5 > \eta_7 > 0$.

By comparing Amazon’s equilibrium sales quantities with and without FBA, we have $q_{A,F}^* - q_{A,N}^* = -\frac{q(2\delta^2(\beta^2 - 2) + 2(\beta^2 - 4)\eta + 8)}{2\beta + 2\beta^2 + 2(\beta^2 - 4)\eta + 8} < 0$. Therefore, $q_{A,F}^* < q_{A,N}^*$. □

**PROOF OF PROPOSITION 5.**

By Lemma 6, when $q_s > q_p$, we have

$$\frac{\partial^2 (\Pi_{A,F} - \Pi_{A,N})}{\partial q_p^2} = \frac{s_2^2}{4(1 - \beta^2)(\beta^2 + 2\beta^2 + 2(\beta^2 - 4)\eta + 8)} \left(\phi' \left(\frac{s_2}{s_\beta} + \psi'\right) - \phi' \frac{s_2}{s_\beta} - \psi\right),$$

where $\phi'$, $\phi'$ and $\psi'$ are functions of $r$ and $\beta$, with $\varphi' > 0$ and $\phi' - \varphi' + \psi = 0$. Define $s_A = s_A'$ as the other solution of $\phi' \left(\frac{s_2}{s_\beta} + \psi'\right) - \phi' \frac{s_2}{s_\beta} - \psi = 0$, besides $s_A = s_s$. It can be shown that there exists two thresholds $r_1$ and $r_2$ such that $\phi' \geq 0$ if $r \leq r_1$ and $\phi' < 0$ otherwise, and $\psi' \leq 0$ if $r \leq r_2$ and $\psi' > 0$ otherwise, where $r_2 < r_1$.

Based on the above, there are four cases in total.

Case 1. If $r \leq r_2$, we have $\phi' > 0$ and $\psi' \leq 0$, which implies that

$$\frac{\partial^2 (\Pi_{A,F} - \Pi_{A,N})}{\partial q_p^2} > 0.$$

Case 2. If $r_2 < r \leq r_1$ and $s_A \leq s_A'$, we have $\phi' \geq 0$, $\psi' > 0$ and

$$\frac{\partial^2 (\Pi_{A,F} - \Pi_{A,N})}{\partial q_p^2} \leq 0.$$

Case 3. If $r_2 < r \leq r_1$ and $s_A > s_A'$, we have $\phi' \geq 0$, $\psi' > 0$ and

$$\frac{\partial^2 (\Pi_{A,F} - \Pi_{A,N})}{\partial q_p^2} > 0.$$

Case 4. If $r > r_1$, we have $\phi' < 0$ and $\psi' > 0$, which implies that

$$\frac{\partial^2 (\Pi_{A,F} - \Pi_{A,N})}{\partial q_p^2} < 0.$$

Next we prove that when $\eta = 0$, $\Pi_{A,F}^* - \Pi_{A,N}^* > 0$. In equilibrium, Amazon’s profits without and with FBA are as follows:

$$\Pi_{A,N} = (p_{A,N}^* - w_{N}^* - c) q_{A,N}^* + rp_{S,N}^* q_{S,N}^* \quad \text{and} \quad \Pi_{A,F} = (p_{A,F}^* - w_{F}^* - c) q_{A,F}^* + rp_{S,F}^* q_{S,F}^* + (T - c) q_{S,F}^*.$$  

Therefore,

$$\Pi_{A,F} - \Pi_{A,N} = ((p_{A,F}^* - w_{F}^* - c) q_{A,F}^* - (p_{A,N}^* - w_{N}^* - c) q_{A,N}^*) + r(p_{S,F}^* q_{S,F}^* - p_{S,N}^* q_{S,N}^*) + (T - c) q_{S,F}^*.$$  

We first examine the term $(p_{A,F}^* - w_{F}^* - c) q_{A,F}^* - (p_{A,N}^* - w_{N}^* - c) q_{A,N}^*$. The point of $\eta = 0$.

$$(p_{A,F}^* - w_{F}^* - c) q_{A,F}^* - (p_{A,N}^* - w_{N}^* - c) q_{A,N}^* = (Q_A + \alpha s_A - (1 - \beta)(c + c_0)) \frac{\beta(\beta^2 + 2(\beta^2 - 4)(\beta^2 - 4)\eta + 8)}{2^{(2r + 1)}(r + 1) + 2\beta(\beta^2 + 2(\beta^2 - 4)\eta + 8)}.$$  

For the sales quantity of Amazon to be positive, $w < Q_A - c$ must hold. Since $w \geq c_0$, we have $Q_A > c + c_0$.

This implies that $(p_{A,F}^* - w_{F}^* - c) q_{A,F}^* - (p_{A,N}^* - w_{N}^* - c) q_{A,N}^* > 0$ at the point of $\eta = 0$.

By Lemma 6, when $\eta = 0$, we have $p_{S,F}^* > p_{S,N}^*$ and $q_{S,F}^* \geq q_{S,N}^*$. This leads to $p_{S,F}^* q_{S,F}^* - p_{S,N}^* q_{S,N}^* > 0$.

Clearly, $(T - c) q_{S,F}^* > 0$. Consequently, $\Pi_{A,F}^* - \Pi_{A,N}^* > 0$ holds for $\eta = 0$.

To conclude, we have the following results.

In Case 1 and 3, $\frac{\partial^2 (\Pi_{A,F} - \Pi_{A,N})}{\partial q_p^2} > 0$. There exist two thresholds $\eta'$ and $\eta''$ such that $\Pi_{A,F}^* \geq \Pi_{A,N}^*$ if $\eta \leq \eta'$ or $\eta \geq \eta''$, and $\Pi_{A,F}^* < \Pi_{A,N}^*$ otherwise.

In Case 2 and 4, $\frac{\partial^2 (\Pi_{A,F} - \Pi_{A,N})}{\partial q_p^2} \leq 0$. There exist one thresholds $\eta^*$ such that $\Pi_{A,F}^* \geq \Pi_{A,N}^*$ if $\eta \leq \eta^*$, and $\Pi_{A,F}^* < \Pi_{A,N}^*$ otherwise.

Combining the above, there exist two thresholds $\eta^o$ and $\eta^o$ such that $\Pi_{A,F}^* \geq \Pi_{A,N}^*$ if $\eta \leq \eta^o$ or $\eta \geq \eta^o$, and $\Pi_{A,F}^* < \Pi_{A,N}^*$ otherwise. □