Supply Chain Performance under Market Valuation: An Operational Approach to Restore Efficiency

Guoming Lai
McCombs School of Business, University of Texas at Austin, Austin TX 78731, guoming.lai@mccombs.utexas.edu

Wenqiang Xiao
Stern School of Business, New York University, New York 10012, wxiao@stern.nyu.edu

Jun Yang
School of Management, Huazhong University of Science and Technology, Wuhan, Hubei, China, jun.yang@mail.hust.edu.cn

Based on a supply chain framework, we study the stocking decision of a downstream buyer who receives private demand information and has the incentive to influence her capital market valuation. We first characterize a market equilibrium under a general, single buy-back contract. We show that the buyer’s stocking decision can be distorted in equilibrium. Such a downstream stocking distortion hurts the buyer firm’s own performance, but it might either benefit or hurt the supplier and the supply chain, depending on the contract terms. We further reveal scenarios where full supply chain efficiency cannot be reached under any single buy-back contract. Then, focusing on contract design, we characterize conditions under which a menu of buy-back contracts can prevent downstream stocking distortion and restore full efficiency in the supply chain. Our study demonstrates that in a supply chain context, a firm’s incentive to undertake real economic activities to influence capital market valuation can potentially be resolved through operational means.

Key words: Supply Chain, Newsvendor, Capital Market Valuation

1. Introduction

Firms, when making their decisions, might consider not only their long-term profitability but also their short-term valuation in the capital market. If the market is able to correctly assess a firm’s performance, the short-term focus on market value would be aligned with the long-term goals. Nevertheless, given their information advantage, firms might undertake real economic activities to purposely influence their market valuation. Prior research on project investments by Bebchuk and Stole (1993) demonstrates that inefficient overinvestment in projects can arise in a market equilibrium when information asymmetry is present. They show that with a short-term interest in market value, the firms with less productive investment opportunities have incentives to mimic the more efficient ones; then, the latter, in order to reveal their information, might choose to suboptimally overinvest. Such activities hurt firms’ true performance.

In this paper, we study a firm’s inventory stocking decision in the presence of a short-term interest in market value. Although the stocking decision does not directly reveal a firm’s value, it conveys information about its expectations regarding potential sales. Investors might thus react to those stocking decisions that can have significant effects on a firm’s performance.\(^1\) Given that firms might anticipate a market response, it is interesting to investigate whether a firm, when it has a short-term interest in market value, might purposely distort its stocking level. We study this problem based on the newsvendor and supply chain framework. In our model, a downstream buyer who receives private information of her potential demand (either optimistic or pessimistic) decides on the stocking level of a product supplied by an upstream supplier. The buyer cares not only about the firm’s long-term profitability but also about the firm’s market value in the short term.

We first study a scenario where the buyer orders according to a single contract offered by the supplier. The contract contains a wholesale price and may also include a per-unit buy-back price and a lump-sum transfer payment. A capital market that consists of homogenous, rational investors values the buyer firm. Although the market valuation will be accurate after the sales are observed, a discrepancy in the valuation can arise in the shorter term when the market observes neither the sales nor the demand signal but only the stocking level. The buyer might distort her stocking decision as a means to influence the market valuation. Similar to Bebchuk and Stole (1993), we characterize a separating market equilibrium in which overstocking can arise under certain conditions. We show that a stocking distortion, if it occurs, hurts the buyer firm’s true performance; it can also influence the supplier’s profit as well as the total supply chain surplus, which, for a given contract, may either increase or decrease in the magnitude of the buyer’s short-term interest in market value.

\(^1\) For instance, when it was revealed on May 26 that Gome, China’s second-biggest electronics retailer, signed a distribution contract with LG that targeted $1.4 billion (9.3 billion RMB) of sales of LG products through Gome’s retail stores in 2010 (a 90 percent increase from 2009), Gome’s stock price rose 13.7 percent in Hong Kong trading (the benchmark Hang Seng Index rose 1.1 percent), the most in 10 months. Ashley Cheung, an analyst at BOCI Research Ltd. in Hong Kong commented on the news, “With this contract, Gome is going to see a material positive impact on its revenue” (Longid 2010). In contrast, when it was revealed on December 18, 2009 that Zales, the second-largest U.S. jewelry retailer at that time, refused to accept tens of millions of dollars of inventory at the end of November 2009, its stock price plunged 12.7 percent. Both the Dow Jones and S&P 500 indices closed up that day. Cancellation of orders was generally permitted for Zales’ contracts with the suppliers; however, “the cancellation of orders at a busy time of year is an ominous sign for Zales’ sales prospects,” Milton Pedraza, Chief Executive of Luxury Institute said of the cancellation. “Anyone who thinks Christmas will be dramatically up is fooling themselves. It [cancellation] means they are in trouble, that they’re not expecting sales to be as good as expected” (Wahba 2009).
Furthermore, we find situations where the maximum supply chain surplus that could be achieved under the classical supply chain framework (i.e., in the absence of the buyer’s short-term interest in market value) becomes no longer achievable in our model.

In the second part, we seek ways to prevent such a system-wise inefficient stocking distortion. To do so, we focus on the design of trade contracts between the supplier and the buyer. We characterize conditions under which a menu of buy-back contracts can successfully prevent downstream stocking distortion in equilibrium. A typical menu would contain one contract that has a premium wholesale price but a generous buy-back term and another contract that has a discounted wholesale price but a stringent buy-back term. The buyer prefers the first contract if the true demand outlook is pessimistic and the second otherwise, even if she takes her market valuation into account. Providing alternative contracting choices serves as an operational means to overcome the buyer’s incentive to influence the market valuation. Our findings extend to the setting with a continuous demand signal. In such an environment, the menu of buy-back contracts can be alternatively implemented using a specific single contract that has a quantity discount scheme and a buy-back schedule.

Hence, our study reveals how a buyer with a short-term interest in market value may distort her operational decision, the stocking level, and what the consequent effects on the supply chain are. Although such a short-term interest in market value is common in reality, it is relatively new to the operations management literature. More importantly, we show that such a stocking distortion might be prevented by implementing an appropriately designed supply chain contract scheme. This finding indicates that a firm’s incentive to use real economic activities to influence the capital market valuation can potentially be resolved not just at the regulatory level but through operational means.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the model. Section 4 analyzes the game with a single buy-back contract, which reveals the potential distortion of the buyer’s stocking decision. Section 5 explores the design of a menu of buy-back contracts to restore supply chain efficiency. We analyze the extension with a continuous demand signal in section 6 and conclude in section 7.
2. Literature

Our work relates to the supply chain literature. Based on the newsvendor model, the supply chain literature has studied various supply contracts, including wholesale price contracts (Lariviere and Porteus 2001), buy-back contracts (Pasternack 1985, Emmons and Gilbert 1998), and revenue sharing contracts (Cachon and Lariviere 2005). Some research has also been conducted on contract design in settings with asymmetric demand information (Cachon and Lariviere 2001, Özer and Wei 2006, Taylor and Xiao 2009), asymmetric cost information (Ha 2001, Zhang 2010), asymmetric inventory information (Zhang et al. 2010), and strategic consumers (Su and Zhang 2008). However, the element that we explore, the interest in market value, and its interplay with asymmetric demand information have been little investigated. We enrich the above literature by revealing the possibility of stocking distortion in a setting like ours and providing a mitigating mechanism based on supply contract design.

The capital market interaction is not new in the earnings management and signaling literature. Stein (1989) reveals that a firm that cares about the market value might inflate the current earnings by pulling future cash flows forward. Based on a two-period inventory model, Lai et al. (2011) demonstrate that a firm after obtaining true sales may apply different magnitudes of channel stuffing (i.e., pushing excess inventory to the downstream channel) to inflate the first-period reported sales as well as the prospect of the second-period demand. Firms might also use investment to influence the capital market valuation. Bebchuk and Stole (1993) show that high productivity firms might overinvest in long-term projects to signal their productivity to the capital market. In certain environments, firms with different quality investment opportunities also might invest in the same amount, which results in a pooling outcome, as revealed in Kedia and Philippon (2009) and Gaur et al. (2011). This literature, however, has not investigated approaches that can be used to mitigate such inefficient distortions. Several accounting works do discuss that aspect, but they focus on accounting policies. For instance, Dye and Sridhar (2004) and Liang and Wen (2007) examine the magnitude of investment distortions under different accounting regimes and discuss the advantages
and disadvantages of different accounting policies. Our work extends to a supply chain setting and reveals the significant role that a supplier can play in restoring system efficiency.

Finally, our work relates to several empirical research in operations management. Chen et al. (2005) and Hendricks and Singhal (2009) reveal that excess inventory (e.g., inventory writedowns) often negatively affects stock returns. Some of our results are qualitatively aligned with their empirical observations. In our study, when the sales are realized, for the same initial stocking level, high leftovers suggest a low firm value. Lai (2006) discusses that, anticipating the market response, firms may have incentive to maintain less inventory to signal their competencies measured by fill rate to inventory ratios. Lai uses aggregated inventory data from financial reports across firms and industries. Our study shares a similar motivation but has a more specific focus. We study the stocking decision of an individual firm for a specific selling event where the stocking level implies the prospect of the demand and the investors need to value the firm before any sales are realized.

3. Model

**Problem Description.** We consider a buyer (she) who procures a product from a supplier (he) for a selling event. The selling price is fixed at $p$ per unit and the unit production cost for the supplier is $c$. The stocking decision and the production need to be carried out before the demand is realized. There is no replenishment opportunity afterwards; thus, any excess demand is lost. In the case of overage, the leftover inventory has zero value. This setting is common knowledge.

Before deciding at what level to stock, the buyer is able to observe a signal of the demand. Ex ante, the signal is uncertain, denoted by $i$, that is high ($H$) with probability $\lambda \in (0, 1)$ and low ($L$) with probability $1 - \lambda$. The demand conditional on $i$ is a nonnegative random variable $X_i$ with a strictly increasing distribution $F_i(\cdot)$ (density $f_i(\cdot)$) over $\mathbb{R}^+$. A high-value signal implies a stochastically larger demand, with $F_H(x) < F_L(x)$ for all $x > 0$. Let $\bar{F}_i(\cdot) \equiv 1 - F_i(\cdot)$. We assume that the prior distribution of the signal and the conditional distributions of the demand are all common knowledge, but only the buyer observes the realization of the signal, and she is not able to credibly communicate this information to external parties.
The trade between the buyer and the supplier is carried out through a general form of buy-back contract. We use \((w, b, t)\) to denote a single contract in which \(w\) is the per-unit wholesale price, \(b\) is the per-unit buy-back price, and \(t\) is the transfer payment. If \(b\) and \(t\) are zero, the contract reduces to a wholesale price contract. Given a single contract, the buyer procures \(q\) units from the supplier. The trade can also be carried out through a menu of two buy-back contracts, denoted as \(\{(w_H, b_H, t_H), (w_L, b_L, t_L)\}\), with the subscripts corresponding to the possible values of the signal. Given a menu of two contracts, the buyer chooses one contract \((w_i, b_i, t_i)\) and procures \(q\) units from the supplier. We impose no constraint on the choice of \(q\). That is, the buyer can select any \(q \in \mathbb{R}^+\) that maximizes her own payoff. The contract, once taken, is legally binding and is not renegotiable.

This information, including the implemented contract(s) and the stocking level, is accessible to the capital market (which we introduce below).

Deviating from the classical supply chain framework, we include a capital market that values the buyer. The capital market consists of homogenous, rational investors. Their valuation of the buyer firm is the expectation of the buyer’s ending-period profit conditioned on the information they can access. As the demand signal is private to the buyer, a discrepancy of the valuation may arise in the short term when the sales have not been realized. We use \(j\) to denote the market belief of the signal to formulate the short-term market value. The buyer cares not only about the true profit the firm will make but also about the market value in the short term. To model the buyer’s incentive scheme, we apply a simple objective function (which has been similarly applied in the literature; see, e.g., Stein 1989, Liang and Wen 2007): the buyer places a weight \(\beta \in (0, 1)\) on the short-term market value and a weight \(1 - \beta\) on the long-term true profit in her consideration. We consider no time discount. The buyer’s incentive scheme (captured by \(\beta\)) is common knowledge.

Such an objective function can be motivated, for instance, if the buyer firm’s executive managers receive market-based incentives (e.g., options) or if, as discussed in Stein (1989) and Liang and Wen (2007), the buyer bears some liquidation pressure and needs to sell a fraction of her shares to

\(^2\)To simplify the model, we limit our study to the setting where the buyer cares about her market value but the supplier does not. We discuss this assumption in more detail in section 7.
the capital market. Finally, we consider no accounting manipulation. That is, all the information accessible to the market is precise.

**Timeline.** Figure 1 details the timeline of the model. First, the supplier offers a single contract or a menu of contracts to the buyer. The buyer privately observes the demand signal and chooses a contract and the corresponding stocking level. The supplier produces and delivers the products to meet the buyer’s order, and the buyer pays the wholesale and transfer prices. Then, the capital market observes the buyer’s contract choice and stocking level and assesses the expected profit the buyer can make, which forms the short-term market value. The buyer realizes a short-term payoff equal to the market value multiplied by the weight $\beta$. After that, the demand is realized and then the leftover inventory is returned and the payments between the buyer and the supplier are made according to the chosen contract. Finally, the buyer firm’s true value is realized, which equals the true profit, and the buyer realizes another payoff equal to the profit multiplied by the weight $1 - \beta$.

**Information Structure.** Similar to the settings adopted in the signaling and the supply contract design literatures (see, e.g., Stein 1989, Bebchuk and Stole 1993, Cachon and Lariviere 2001), we assume a single source of information asymmetry in our model. Such a stylized modeling approach lends tractability to the analysis and also captures the major qualitative insights. In particular, the realization of the demand signal, which we assume is private to the buyer, represents information that is most difficult for external parties to obtain. Furthermore, compared to the selling price, the production cost, and the details of a contract (for which invoices and other legal proofs could exist), a signal of the potential demand is also difficult to communicate credibly (as cheap talks...
could arise). Finally, to simplify the model, we assume the same information setting for the supplier and the investors.$^3$

**Benchmark.** Notice that without the interaction with the capital market, the problem we have described follows the classical selling to newsvendor problem. An appropriately designed single contract would be sufficient to maximize the total supply chain surplus. That is, when $\frac{w-b}{p-b} = \xi$, the buyer would stock $q_{i}^o \equiv \hat{F}_{i}^{-1}(\frac{\xi}{p})$, which maximizes the total supply chain surplus for each signal $i \in \{H, L\}$—a classical result in the supply chain literature (see, e.g., Pasternack 1985). Nevertheless, when the buyer cares about her capital market valuation, her decision can deviate from these quantities. Hence, we use the classical selling to newsvendor problem as our benchmark and call $q_{i}^{*} \in \{H, L\}$ the first-best stocking level.

4. Analysis with A Single Contract

In this section, we analyze the model with a single contract offer $(w, b, t)$. We first derive the downstream market equilibrium and analyze the impact of the buyer’s short-term interest in market value on her payoff in subsection 4.1; then we analyze the supplier’s profitability and the supply chain efficiency in subsections 4.2 and 4.3.

4.1. Downstream Market Equilibrium

Given a contract offer $(w, b, t)$, for each signal $i \in \{H, L\}$, the expected profit of the buyer firm with a stocking level $q$ follows:

$$
\pi^{B}(q; i) = p\mathbb{E}[\min(q, X_{i})] + b\mathbb{E}[\max(q - X_{i}, 0)] - wq - t
$$

$$
= (p - b) \int_{0}^{q} \hat{F}_{i}(x)dx - (w - b) q - t.
$$

(1)

However, the buyer’s payoff depends partially on the firm’s real profit and partially on the firm’s short-term market value. In the following, we formulate the buyer firm’s market value.

$^3$It would be more realistic to assume that the supplier has more information about the buyer firm’s demand outlook than the investors. For instance, the supplier might receive a noisy signal about the buyer firm’s information and then updates his belief of the demand outlook to high (low) with some probability $\lambda'(1-\lambda')$. However, note that the separating equilibrium as well as the menu of contracts we characterize does not depend on the probability of the demand outlook. Such a change of the model would not qualitatively change the main findings.
Because the signal is private to the buyer, the market needs to hold a belief to infer the signal. We focus on pure-strategy separating equilibrium in the following analysis\(^4\) and formulate the market belief as:

\[
j(q) = \begin{cases} 
H & \text{if } q \in Q_H, \\
L & \text{otherwise},
\end{cases}
\]

where \(Q_H\) is a subset of \(\mathbb{R}^+\). That is, if the observed stocking level \(q \in Q_H\), then the market believes that the realization of the signal is high; otherwise, the market believes that the signal is low. Given this belief, the buyer firm’s market value follows:

\[
P(q) = (p - b) \int_0^q \bar{F}_{j(q)}(x)dx - (w - b)q - t.
\]

(2)

Hence, to maximize her own payoff, the buyer, for each signal \(i \in \{H, L\}\), solves:

\[
\max_{q \in \mathbb{R}^+} \beta P(q) + (1 - \beta) \pi_B(q; i),
\]

(3)

where the weight \(\beta\) represents the buyer’s interest in her market value. Before carrying out the equilibrium analysis, we first analyze the buyer’s objective function. We use the definition:

\[
\bar{F}_{ij}(x) \equiv \beta \bar{F}_j(x) + (1 - \beta) \bar{F}_i(x), \forall i, j \in \{H, L\},
\]

where the subscript \(i\) \((j)\) indicates the true \(\text{(market believed)}\) signal value; in addition:

\[
G_{ij}(q) \equiv (p - b) \int_0^q \bar{F}_{ij}(x)dx - (w - b)q - t, \forall i, j \in \{H, L\},
\]

which is the buyer’s expected payoff if, given \(q\), the market believes the value of the signal is \(j\) while the true signal value is \(i\).

**Lemma 1.** \(G_{ij}(q)\) is concave in \(q\) for any \(i, j \in \{H, L\}\).

Lemma 1 establishes the concavity result of the possible payoff functions of the buyer (see Figure 2 for an illustration). Therefore, a unique maximizer of the buyer’s problem exists in any scenario. Define \(q_{ij}^* \equiv \bar{F}_{ij}^{-1}\left(\frac{w - b}{p - b}\right)\), which maximizes \(G_{ij}(q)\). To simplify the notation, we reduce the subscript
Figure 2 Demonstration of the buyer’s possible payoff functions and the thresholds \( q \) and \( \overline{q} \). The parameters are: \( \beta = 0.4, p = 20, c = 5, w = 8, b = 4, t = 0, \lambda = 0.5 \), and the demand follows the gamma distribution with density \( f_i(x) = \frac{\Gamma(x_i)^{\kappa_i - 1}}{\kappa_i \theta_i^{x_i}} e^{-x_i/\theta_i} \) for \( i \in \{H, L\} \) with \( (\kappa_H, \theta_H) = (1.5, 5) \) and \( (\kappa_L, \theta_L) = (1.5) \) such that \( F_H(x) < F_L(x), \forall x > 0 \).

If the buyer’s optimal stocking decision and the market belief satisfy \( j(q(i)) = i \) so that \( P(q(i)) = \pi^B(q(i); i) \) for each signal \( i \in \{H, L\} \).

We derive Lemma 2 which will be useful for the equilibrium characterization.

**Lemma 2.** There exists a unique \( q > q_{LH}^* \) that satisfies \( G_{LH}(q) = G_L(q_{L}^*) \) and a unique \( \overline{q} > q_{HL}^* \) that satisfies \( G_{HL}(\overline{q}) = G_{HL}(q_{HL}^*) \); \( q < \overline{q} \).

We depict \( q \) and \( \overline{q} \) in Figure 2 that equate \( G_{LH}(q) \) to \( G_L(q_{L}^*) \) and \( G_{HL}(q) \) to \( G_{HL}(q_{HL}^*) \), respectively. In the following, we explain the implications of Lemma 2 by assuming some given market belief.

\[4\]We have described a typical signaling game which can have multiple pooling and separating equilibria. Stocking distortion occurs also in pooling equilibrium because the buyer stocks the same quantity for either demand outlook. We focus only on separating equilibrium in the paper because any pooling equilibrium in our model cannot survive the intuitive criterion refinement (Cho and Kreps 1987; see Appendix B). However, note that we have not considered any constraint on the stocking level. Specific constraints may exist in practice, for which pooling equilibrium might survive the intuitive criterion. Pooling equilibrium might also survive the intuitive criterion if there are more than two states for the demand signal.
(\mathcal{Q}_H) that is known to the buyer.

After stocking a quantity \( q \), the worst outcome for the buyer with a high signal is to be valued by the market as if she has a low signal, which leads to an expected payoff \( G_{HL}(q) \). Maximizing \( G_{HL}(q) \) would result in a payoff \( G_{HL}(q^*_{HL}) \) that the buyer observing a high signal can at least secure. With the understanding of this reservation payoff, \( \overline{q} \), as defined in Lemma 2, represents the largest quantity the buyer with a high signal is willing to stock, to achieve a correct market recognition. This result implies that, for the buyer’s strategy to be consistent with the given market belief, there must be some quantity in \( \mathcal{Q}_H \) that is no larger than \( \overline{q} \). By a similar reason, \( G_L(q^*_{L}) \) serves as the reservation payoff for the buyer when she has a low signal. \( \underline{q} \), as defined in Lemma 2, represents the largest quantity the buyer with a low signal is willing to stock, to be considered as having a high signal. Therefore, the strategy of the buyer with a low signal is consistent with the market belief if \( \mathcal{Q}_H \) contains quantities all above \( \underline{q} \).

The result that \( \underline{q} < \overline{q} \) guarantees the existence of a market belief under which the buyer’s strategy with either signal is consistent with the market belief, thereby resulting in a separating equilibrium. In fact, multiple equilibria exist in our model. We focus on the one presented in Proposition 1 that uniquely survives the intuitive criterion (Cho and Kreps 1987).

**Proposition 1.** Given any single contract offer \((w, b, t)\), a unique separating market equilibrium exists that survives the intuitive criterion in which the stocking level follows

\[
q(i) = \begin{cases} 
\hat{q} & \text{if } i = H, \\
\underline{q} & \text{if } i = L,
\end{cases}
\tag{4}
\]

with \( \hat{q} = \max\{\underline{q}, q^*_{HL}\} \) and the market belief can be specified as \(^5\)

\[
j(q) = \begin{cases} 
H & \text{if } q = \hat{q}, \\
L & \text{otherwise}.
\end{cases}
\tag{5}
\]

In this equilibrium, the buyer stocks \( \hat{q} \) with a high signal and \( q^*_{L} \) with a low signal, consistent with the market belief. Notice that if the signal is low, stocking distortion does not occur. In contrast,

\(^5\)The market belief on the off-equilibrium path could be specified in other ways as long as the stocking strategies on the off-equilibrium path are dominated by the equilibrium strategy for the buyer. For instance, instead of the singleton \( \mathcal{Q}_H = \{\hat{q}\} \), we can alternatively specify \( \mathcal{Q}_H = \{q \in \mathbb{R}^+ : q \geq \hat{q}\} \). The equilibrium will not change.
if the signal is high, overstocking occurs when \( q^*_H < q \). Proposition 2 establishes a further result of the buyer’s equilibrium stocking decision.

**Proposition 2.** Given any single contract offer \((w, b, t)\), a threshold \( \hat{\beta} = \frac{G_L(q^*_H) - G_L(q^*_L)}{\bar{c}_H(q^*_H) - \bar{c}_L(q^*_H)} \) exists such that \( q(H) = q^*_H \), which is fixed, when \( \beta \leq \hat{\beta} \), and \( q(H) = q \), which increases in \( \beta \), when \( \beta > \hat{\beta} \).

In the presence of a short-term interest in market value, the buyer observing a low demand signal may find it advantageous to mimic the order quantity associated with a high demand signal to gain from market valuation. It may thus become necessary for the buyer when truly observing a high demand signal to inflate her order to an extent (i.e., \( q \)) beyond which mimicking with a low signal would not be profitable, thereby credibly signaling her demand outlook. Such an overstocking will not arise only when \( \beta \) is small (\( \beta \leq \hat{\beta} \)). In such circumstances, the buyer with a low signal will never attempt to mimic, even without quantity inflation because the potential gain from market valuation does not outweigh the cost associated with profit loss from a suboptimal quantity. The left plot of Figure 3 illustrates the buyer’s equilibrium stocking levels. We see that when \( \beta \leq \hat{\beta} \), \( q(H) = q^*_H \), which is fixed, and when \( \beta > \hat{\beta} \), \( q(H) = q \), which is larger than \( q^*_H \) and increases in \( \beta \). (It is intuitive that the larger the \( \beta \) is, the more the buyer with a high signal would need to inflate her order to credibly signal her demand information.) Overstocking, if it occurs, apparently will hurt the buyer firm’s true performance, given that the quantity deviates from the truly optimal. This is concluded in Corollary 1 and depicted in the right plot of Figure 3.

**Corollary 1.** The buyer’s expected payoff follows \( \Pi^B = \lambda G_H(q^*_H) + (1 - \lambda)G_L(q^*_L) \), which is fixed, when \( \beta \leq \hat{\beta} \) and follows \( \Pi^B = \lambda G_H(q) + (1 - \lambda)G_L(q^*_L) \), which decreases in \( \beta \), when \( \beta > \hat{\beta} \).

Thus far, we have derived the results of the game between the buyer and the investors that value the buyer. In the following subsection, we analyze how this downstream market game influences the performance of the supplier and the supply chain.

### 4.2. Supplier Profit

We have discussed that stocking distortion could occur in equilibrium which hurts the buyer firm’s true performance. However, whether such a stocking distortion is detrimental or beneficial for the
The parameters are: $p = 20$, $c = 5$, $w = 8$, $b = 4$, $t = 0$, $\lambda = 0.5$, and the demand follows the gamma distribution with density $f_i(x) = \left(\frac{\kappa_i}{\theta_i}\right)^{\theta_i-1} \frac{x^{\theta_i-1}}{\Gamma(\theta_i)}$ for $i \in \{H, L\}$, where $(\kappa_H, \theta_H) = (1.5, 5)$ and $(\kappa_L, \theta_L) = (1, 5)$.

supplier is not straightforward. If the buyer overstocks, on the one hand, the supplier would seem to benefit because more revenues could be collected; on the other hand, when a buy-back term is provided, more returns could occur, which is costly for the supplier. In the following, we investigate this impact for a given contract offer $(w, b, t)$.

From the results of Propositions 1-2, the supplier’s expected profit follows:

$$
\Pi^S = \begin{cases} 
\lambda \left[ (w - c - b) q^*_H + b \int_0^{q^*_H} F_H(x)dx \right] + (1 - \lambda) \left[ (w - c - b) q^*_L + b \int_0^{q^*_L} F_L(x)dx \right] + t & \text{if } \beta \leq \hat{\beta}, \\
\lambda \left[ (w - c - b) q + b \int_0^{q} F_H(x)dx \right] + (1 - \lambda) \left[ (w - c - b) q^*_L + b \int_0^{q^*_L} F_L(x)dx \right] + t & \text{if } \beta > \hat{\beta}.
\end{cases}
$$

(6)

Based on (6), we derive the following proposition:

**Proposition 3.** When $\beta \leq \hat{\beta}$, the supplier’s expected profit $\Pi^S$ is independent of $\beta$; when $\beta > \hat{\beta}$, the following properties hold:

(i) If $b \geq \frac{w - c}{p - c} p$, then $\Pi^S$ strictly decreases in $\beta$;

(ii) If $b < \frac{w - c}{p - c} p$, then a threshold $\beta' \in [\hat{\beta}, 1]$ exists such that $\Pi^S$ increases in $\beta$ when $\hat{\beta} < \beta < \beta'$ and decreases in $\beta$ when $\beta > \beta'$.

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6In principle, if the supplier has the full bargaining power in the supply chain, he can optimize the contract offer anticipating how the downstream market equilibrium is formed. The buyer’s incentive to overstock may lead to a lower optimal buy-back price. We do not provide the detailed analysis here for two reasons: first, a supplier may not have full bargaining power, and thus the insights we reveal with a general contract would be useful; and second, such an optimization would become superfluous when, as we reveal in section 5, menus of buy-back contracts exist under which full supply chain efficiency can be restored and the supplier can be better off compared to any single contract.
In Proposition 3, when $\beta$ is less than $\hat{\beta}$, the supplier’s profit does not change in $\beta$ because stocking distortion does not occur in this region (see the straight lines in Figure 4).

In addition, stocking distortion, when it occurs ($\beta > \hat{\beta}$), could either hurt or benefit the supplier, depending on the contract terms. When $b \geq \frac{w-c}{p-c}$, which can be rewritten as $\frac{p-w}{p-b} \geq w-c$, stocking distortion always hurts the supplier (see the left plot in Figure 4). In particular, $\frac{p-w}{p-b}$, which equals $F_H(q_H)$, represents the probability that a unit product will be returned to the supplier when the buyer stocks $q_H$, and $\frac{p-w}{p-b}b$ captures the marginal refund cost. Hence, if $\frac{p-w}{p-b}b \geq w-c$, then the marginal refund cost ($F_H(q)b$) will always outweigh the marginal revenue ($w-c$) when the buyer stocks any $q$ beyond $q_H$ (given that $F_H(q)$ increases in $q$), which is costly for the supplier.

In contrast, if $b < \frac{w-c}{p-c}p$, some amount of overstocking might benefit the supplier in that it mitigates double marginalization. A threshold $\beta'$ can be determined such that the downstream overstocking will benefit (hurt) the supplier when $\beta < (>)\beta'$ (see the middle plot in Figure 4). Note that the value of $\beta'$ can possibly reach one if the double marginalization effect is strong (see the right plot in Figure 4). Therefore, for a given contract offer, the downstream stocking distortion is beneficial for the supplier if it mitigates double marginalization in scenarios where $\beta$ is intermediate, and it is detrimental if the downstream stocking level is distorted to a large extent when $\beta$ is large.
We further note that if $p$ becomes larger relative to $c$, then the term $\frac{w-c}{p-c}p$ becomes smaller and the region where the downstream stocking distortion is detrimental for the supplier becomes wider. In other words, for high-margin products (e.g., jewelry or other luxury products), the buyer’s short-term interest in market value is more likely to be detrimental for the supplier.

4.3. Supply Chain Efficiency

The previous two subsections have shown that, for a given single contract offer, the buyer’s short-term interest in market value hurts the firm’s performance, while it can be either detrimental or beneficial for the supplier. Because the market value always coincides with the buyer firm’s true expected profit in equilibrium, the total supply chain surplus is, in essence, the “pie” that the supplier and the buyer are sharing through the trade contract, even though the latter realizes a part of that value from the investors. Thus, examining the effect of the buyer’s short-term interest in market value on supply chain efficiency is useful.

Summing up the two parties’ expected profits leads to the total supply chain surplus as

$$
\Pi_{SC} = \begin{cases} 
\lambda \left[ p \int_0^{q_H} \bar{F}_H(x) \, dx - cq_H^* \right] + (1 - \lambda) \left[ p \int_0^{q_L} \bar{F}_L(x) \, dx - cq_L^* \right] & \text{if } \beta \leq \hat{\beta}, \\
\lambda \left[ p \int_0^{q_L} \bar{F}_H(x) \, dx - cq_L^* \right] + (1 - \lambda) \left[ p \int_0^{q_L} \bar{F}_L(x) \, dx - cq_L^* \right] & \text{if } \beta > \hat{\beta}.
\end{cases}
$$

The following proposition details the impact of the buyer’s short-term interest in market value on the performance of the supply chain.

**Proposition 4.** When $\beta \leq \hat{\beta}$, the expected supply chain surplus $\Pi_{SC}$ is independent of $\beta$; when $\beta > \hat{\beta}$, the following properties hold:

(i) If $b \geq \frac{w-c}{p-c}p$, then $\Pi_{SC}$ strictly decreases in $\beta$;

(ii) If $b < \frac{w-c}{p-c}p$, then a threshold $\beta'' \in [\hat{\beta}, 1]$ exists such that $\Pi_{SC}$ increases in $\beta$ when $\hat{\beta} < \beta < \beta''$ and decreases in $\beta$ when $\beta > \beta''$.

We see that the supply chain surplus is not affected when $\beta \leq \hat{\beta}$, and it may decrease or increase in $\beta$ when $\beta > \hat{\beta}$, depending on the contract terms. In particular, when $b \geq \frac{w-c}{p-c}p$, the downstream stocking distortion deteriorates the supply chain efficiency (see the left plot in Figure 5). To explain the intuition, we rewrite the condition $b \geq \frac{w-c}{p-c}p$ to $c \geq \frac{w-b}{p-b}b \left( = \bar{F}_H(q_H^*)p \right)$. Hence, stocking beyond
\[ \text{Expected Total Supply Chain Surplus} \ \beta > \beta' \]

\[ \text{Expected Total Supply Chain Surplus} \ \beta > \beta'' \]

Figure 5 Demonstration of the expected total supply chain surplus \( \Pi^{SC} \) as a function of \( \beta \). The common parameters are: \( p = 20 \), \( c = 5 \), \( t = 0 \), \( \lambda = 0.5 \), and the demand follows the gamma distribution with density 
\[ f_i(x) = \left( \frac{x^{i-1} e^{-x/\kappa_i}}{\kappa_i \Gamma(i)} \right) \text{ for } i \in \{H, L\}, \]

where \( (\kappa_H, \theta_H) = (1.5, 5) \) and \( (\kappa_L, \theta_L) = (1.5) \). In the left plot, \( w = 8 \) and \( b = 4 \); in the middle plot, \( w = 8 \) and \( b = 0 \); and in the right plot, \( w = 14 \) and \( b = 0 \).

\( q_{it}^* \) is costly because the marginal cost \( c \) exceeds the marginal revenue \( \bar{F}_H(q) p \). In contrast, when \( b < \frac{w-c}{p-c} p \), downstream overstocking could benefit the supply chain if \( \beta \) is less than a threshold \( \beta'' \) and it is detrimental otherwise (see the middle plot in Figure 5). \( \beta'' \) may reach one for some given contract if the double marginalization effect is strong (see the right plot in Figure 5).

We note that when \( b = \frac{w-c}{p-c} p \) (or identically, \( \frac{w-b}{p-b} = \frac{c}{p} \)), the supply chain would be coordinated in the benchmark. Therefore, the intuition we have just explained suggests that, for a given contract offer, if the buyer’s short-term interest in market value leads the stocking level closer to the level that would coordinate the supply chain in the benchmark, then such an interest improves the performance of the supply chain; otherwise, it hurts the supply chain.

More importantly, Proposition 4(i) shows that when \( b = \frac{w-c}{p-c} p \) (i.e., \( \frac{w-b}{p-b} = \frac{c}{p} \), the necessary condition for coordination), the supply chain surplus decreases in \( \beta \) when \( \beta > \hat{\beta} \); this result implies that supply chain coordination is not achieved. In fact, we can define 
\[ \hat{\beta}^0 = \frac{G_L(q^*_L) - G_L(q^*_H)}{G_H(q^*_H) - G_L(q^*_H)}, \]

which depends only on the system parameters, and obtain the following proposition:

**Proposition 5.** When \( \beta > \hat{\beta}^0 \), supply chain coordination cannot be achieved by any single contract offer in the form of \( (w, b, t) \).

As shown in Figure 6, \( \hat{\beta}^0 \) decreases as \( p \) (or equivalently, the margin \( p - c \)) increases. Hence, the larger the margin is, the more likely the supply chain efficiency will be affected. Although
similar operations distortions have been revealed in the literature in either separating or pooling equilibrium (see, e.g., Bebchuk and Stole 1993, Gaur et al. 2011), little discussion appears in the literature about how to restore system efficiency. Notice that in our model, improving supply chain efficiency is in both the buyer’s and the supplier’s interest because the buyer’s market value always coincides with her true profit in equilibrium. Maximizing the total supply chain surplus thus maximizes the “pie” that the two parties can divide through their trade. Naturally, one approach to improve system efficiency would be to reduce $\beta$, for instance, structuring executive compensation to include fewer market-based incentives; regulation policies might also help. In the following section, we investigate whether operational approaches exist that can improve system efficiency.

5. Design of Menus of Buy-back Contracts

In this section, we explore menus of buy-back contracts that can restore full supply chain efficiency. Notice that the contract design in our study differs from those in the traditional adverse selection context because our problem involves a third-party, the capital market; as a result, we need to establish a downstream market equilibrium. This equilibrium must be a separating equilibrium because full efficiency would not be achieved otherwise.

Let $(w_i, b_i, t_i)$ denote the menu of contracts corresponding to the signal $i \in \{H, L\}$. We use
\( \tau \in \{H, L\} \) to denote the buyer’s contract choice. The market infers the signal from the buyer’s decisions by a belief denoted by:

\[
j(\tau, q) = \begin{cases} H & \text{if } q \in Q^H_H, \\ L & \text{otherwise}, \end{cases}
\]

where \( Q^\tau_{\tau \in \{H, L\}} \) is the set of stocking levels corresponding to the contract \( \tau \), for which the market believes the signal to be high. Recall from section 3 that \( q^o_i = F_i^{-1}(\frac{i}{p}) \) is the first-best stocking level.

Under a menu of contracts, full efficiency can be achieved in the supply chain if and only if the first-best stocking level can be implemented and, at the same time, the market is able to correctly infer the signal from the buyer’s decisions. Formally, we define the following concept.

**Definition 2.** A market equilibrium with a menu of two buy-back contracts is system-wise efficient if the buyer’s decisions follow \((\tau, q)(i) = (i, q^o_i)\) and the market belief satisfies \( j((\tau, q)(i)) = i \) for each signal \( i \in \{H, L\} \).

Notice from Definition 2 that in a system-wise efficient market equilibrium, the set \( Q^H_H \) must contain \( q^o_H \) so that a high signal can be correctly inferred if the buyer with a high signal chooses the \( H \) contract and stocks \( q^o_H \); in contrast, \( Q^L_H \) must not contain \( q^o_L \) so that a low signal can be correctly inferred if the buyer with a low signal selects the \( L \) contract and stocks \( q^o_L \). Given the structure of the game, any design of the contracts needs to be associated with the characterization of a market belief (i.e., \( Q^\tau_{\tau \in \{H, L\}} \)). To directly solve this contract design problem could be challenging. However, the result of the following lemma will greatly reduce its complexity.

**Lemma 3.** Given any menu of two buy-back contracts, if a system-wise efficient market equilibrium is achieved with a market belief \( (Q^H_H, Q^L_L) \), then the equilibrium can also be achieved with the market belief \( (\{q^o_H\}, \emptyset) \).

Lemma 3 indicates that if a system-wise efficient market equilibrium of our problem exists, then it must be achievable under a restrictive market belief in which the set \( Q^H_H \) is a singleton that contains just \( q^o_H \) and \( Q^L_H \) is an empty set. Under this market belief, if the buyer chooses the \( H \) contract, then she must stock \( q^o_H \) to be recognized as having a high signal; if the buyer chooses the
L contract then she is automatically considered to have a low signal. This result is powerful but also very intuitive, as we explain in the following.

Suppose there is a general belief \((Q_H^H, Q_L^H)\) with which a system-wise efficient market equilibrium is achieved. Then, choosing the \(H(L)\) contract and stocking \(q_H^o (q_L^o)\) is the buyer’s best strategy given a high (low) signal.

Now, suppose we keep \(Q_L^H\) fixed but shrink the set \(Q_H^H\) to the singleton containing only \(q_H^o\). Such a modification obviously does not change the buyer’s payoff with a high signal from choosing the \(H\) contract and stocking \(q_H^o\); however, the modification will reduce the payoff for the buyer if she stocks other quantities because, for any deviation from \(q_H^o\), she would be considered to have a low signal. Thus, the buyer’s best strategy if she has a high signal remains the same. It is also clear that the buyer will not mimic if the signal is low because succeeding from mimicking becomes more difficult (the buyer now would have to stock \(q_H^o\) to succeed; before, she could select an ideal quantity from the original set \(Q_H^H\)).

Next, we replace \(Q_L^H\) by the empty set \((\emptyset)\). This modification only reduces the buyer’s payoff from choosing the \(L\) contract because she would be directly considered as having a low signal. Hence, the buyer with a high signal does not deviate from her original best strategy. The buyer also does not deviate if the signal is low given that she cannot benefit from any deviation.

Hence, replacing \((Q_H^H, Q_L^H)\) with \((\{q_H^o\}, \emptyset)\) changes neither the buyer’s contract choice nor the stocking level in the market equilibrium. The market belief with \((\{q_H^o\}, \emptyset)\) serves as the most conservative market belief in that the buyer is believed to have a high signal. This result implies that any system-wise efficient market equilibrium, if it exists, can always be achieved with the market belief \((\{q_H^o\}, \emptyset)\); thus, we can design the mechanism directly using this market belief, which greatly shrinks the searching space.

Notice that for the buyer to stock the first-best quantity for each signal \(i \in \{H, L\}\), the contract terms must satisfy:

\[
\frac{w_H - b_H}{p - b_H} = \frac{w_L - b_L}{p - b_L} = \frac{c}{p}
\]
With this condition, we can determine the wholesale price $w_i$ once the buy-back price $b_i$ is given, and vice versa. Further, let

$$g_{ij}(q) = \int_0^q \bar{F}_{ij}(x) dx - \frac{c}{p} q, \forall i, j \in \{H, L\},$$

where the subscript $ij$ is reduced to $i$ when $i = j$, and $q_{ij}^o \equiv \bar{F}_{ij}^{-1}\left(\frac{z}{p}\right)$ for $i \neq j$. Proposition 6 establishes conditions, under which a menu of buy-back contracts can restore full efficiency.

**Proposition 6.** With the market belief $((q_{H}^o), \emptyset)$, a system-wise efficient market equilibrium can be achieved if the menu of buy-back contracts $(w_i, b_i, t_i)$ satisfies:

$$w_i - b_i p - b_i = c_{pq} \text{ for each } i \in \{H, L\},$$

$$b_L \geq \frac{1}{K} b_H + p \frac{K-1}{K}, \text{ and } t_H - t_L \in [\Delta_t, \overline{\Delta_t}]$$

where

$$K = \max \left\{ \frac{g_{HL}(q_{HL}^o) - g_L(q_L^o)}{g_H(q_H^o) - g_{HL}(q_{HL}^o)}, \frac{g_{HL}(q_{HL}^o) - g_L(q_L^o)}{g_H(q_H^o) - g_{HL}(q_{HL}^o)} \right\};$$

$$\Delta_t = (p - b_H) \max\{g_{HL}(q_H^o), g_L(q_L^o)\} - (p - b_L) g_L(q_L^o);$$

$$\overline{\Delta_t} = (p - b_H) g_H(q_H^o) - (p - b_L) g_{HL}(q_{HL}^o).$$

In equilibrium, the buyer observing the high (low) signal shall prefer the $H$ ($L$) contract and stock the associated first-best quantity even if she takes the market value into account. A key condition for achieving this result is the buy-back prices chosen for the contracts. In general, the buyer favors a generous return term when the demand outlook is pessimistic. Thus, the buy-back price $(b_L)$ of the $L$ contract shall be attractive enough for the buyer with a low signal to choose this contract. Specifically, $b_L$ shall be no less than a particular threshold level $(\frac{1}{K} b_H + p \frac{K-1}{K})$ that is contingent on $b_H$. When this condition is satisfied, a pair of transfer payments can always be chosen (with their difference bounded by the two thresholds $\Delta_t$ and $\overline{\Delta_t}$ depending on the buy-back prices) that provides sufficient incentives for the buyer having each signal to take the truth-telling contract and stock the first-best quantity under the coordination condition.

The result of Proposition 6 indicates that in a supply chain context, the supplier might be able to “correct” the downstream stocking distortion by offering alternative contract choices. The buyer credibly reveals her information through her choice of contract, in contrast to the overstocking that would otherwise be necessary under a single contract offer.
A remaining issue to implement the mechanism described is to determine how the surplus can be divided between the parties in the supply chain. Notice that the mechanism could be difficult to implement if the resulting payoff is not satisfactory for one party (e.g., compared with the payoff she or he can obtain under an existing single contract offer). This issue is addressed in the following.

**Proposition 7.** With the market belief \( \{q_{oH}\}, \emptyset \), there exists a menu of buy-back contracts

\[
\begin{align*}
   &b_H = 0, \quad w_H = c \quad \text{and} \quad t_H = pg_H(q_{oH}) - \varepsilon [g_H(q_{oH}) - g_L(q_{oL})] - T \\
   &b_L = p - \varepsilon, \quad w_L = p - \varepsilon(1 - \frac{c}{p}) \quad \text{and} \quad t_L = \varepsilon g_L(q_{oL}) - T,
\end{align*}
\]

with any constant \( \varepsilon \leq \frac{p}{K} \) and \( T \), under which a system-wise efficient market equilibrium can be reached. The supplier’s profit goes to the total supply chain surplus as \( \varepsilon \) and \( T \) go to zero.

Proposition 7 provides a special menu of buy-back contracts that can achieve a system-wise efficient market equilibrium. In particular, under this menu of contracts, the supplier is able to obtain almost all of the supply chain surplus as \( \varepsilon \) and \( T \) go to zero. Because \( T \) is a constant appearing in both \( t_H \) and \( t_L \), any specific allocation of the supply chain surplus can always be achieved by adjusting \( T \). As a result, Proposition 7 demonstrates that both parties can be feasibly made better off compared to a single contract scenario (given that the total supply chain surplus is enlarged). That is, pareto improvement can be achieved. Note, however, that it is not always necessary to use the menu of contracts provided in Proposition 7, unless the supplier intends to capture full supply chain surplus. Alternative menus of contracts that can achieve a system-wise efficient market equilibrium and a specific allocation of the supply chain surplus might exist.

6. **Extension with A Continuous Signal**

The model analyzed in the previous sections has a two-state demand signal, either high or low. This section extends the model to a continuous signal setting. With a little abuse of notation, we assume that the signal \( i \), ex ante, is distributed on a continuous support \([i_L, i_H]\) by a nonnegative density function \( \varphi(i) \). The demand conditional on the signal is a nonnegative random variable \( X(i) \) that has a strictly increasing distribution function \( F(x, i) \) (density \( f(x, i) \)) over \( \mathbb{R}^+ \). We assume that a larger signal implies a stochastically (strictly) larger demand (i.e., \( \frac{\partial F(x, i)}{\partial i} < 0 \)).
6.1. Analysis with A Single Contract

Given a single contract offer \((w, b, t)\), the buyer, without a short-term interest in market value, would stock
\[
\bar{F}^{-1}\left(\frac{w-b}{p-b}, i\right),
\]
where \(\bar{F}^{-1}\) is the inverse of \(\bar{F}\) with respect to \(x\). When the contract satisfies
\[
\frac{w-b}{p-b} = \frac{c}{p},
\]
the buyer’s stocking level \(q^c(i) = \bar{F}^{-1}\left(\frac{c}{p}, i\right)\) would maximize the total supply chain surplus for any given signal (denoted as the first-best stocking level).

In the following, we analyze the downstream market game when the buyer has a short-term interest in market value. Similar to the previous sections, we focus on pure-strategy separating equilibrium. We again use \(j(q)\) to denote the investors’ belief that maps a stocking level \(q\) to a signal \(i \in [i_L, i_H]\). The formulations of the expected profit and market value of the buyer firm and the buyer’s objective function remain the same as in (1), (2), and (3), except that the complementary distribution functions \(\bar{F}_i(x)\) and \(\bar{F}_j(q)\) are replaced by \(\bar{F}(x, i)\) and \(\bar{F}(x, j(q))\) in the equations.

We again use \(q(i)\) to denote the buyer’s optimal stocking level and apply the equilibrium concept in Definition 1 with the extension to a continuous signal. That is, a separating market equilibrium is reached if the buyer’s optimal stocking decision is consistent with the market belief (i.e., \(j(q(i)) = i\) and \(P(q(i)) = \pi^B(q(i); i)\), for any signal \(i \in [i_L, i_H]\)).

**Proposition 8.** Given any single contract offer \((w, b, t)\), a unique fully separating equilibrium exists where the buyer’s stocking level \(q(i)\) follows an ordinary differential equation
\[
[(p-b)\bar{F}(q(i), i) - (w-b)]q'(i) + \beta(p-b)\int_0^{q(i)} \frac{\partial \bar{F}(x, i)}{\partial i} \, dx = 0 \tag{8}
\]
with the initial condition \(q(i_L) = \bar{F}^{-1}\left(\frac{w-b}{p-b}, i_L\right)\), and the investors’ belief \(j(q)\) is characterized by the inverse function \(q^{-1}(\cdot)\) for \(q(i_L) \leq q \leq q(i_H)\). The off-equilibrium belief can be specified as \(j(q) = i_L\) for \(q < q(i_L)\) and \(j(q) = i_H\) for \(q > q(i_H)\).

Proposition 8 characterizes a fully separating equilibrium. In particular, when \(q(i)\) guided by the ordinary differential equation is strictly increasing, the investors can perfectly infer the buyer’s signal from the stocking level. The market belief on the equilibrium path can be characterized by the inverse function of the buyer’s optimal stocking decision. On the off-equilibrium path, the market
would believe that the signal is the smallest (largest) if the observed stocking level is lower (greater) than the smallest (largest) stocking level that can appear in equilibrium. Such an off-equilibrium belief supports the equilibrium outcome. We further derive Corollary 2 from Proposition 8.

**Corollary 2.** When $\beta > 0$, $q(i) > F^{-1}\left(\frac{w-b}{p-b}, i\right)$ for any $i > i_L$, and $q(i)$ increases in $\beta$.

With a continuous signal, the buyer can pretend to have a higher (infinitesimally) signal by very slightly overstocking, at minimal cost to her operations. This incentive always exists, which results in overstocking in the equilibrium even for very small $\beta$s, and the distortion increases in $\beta$. It becomes apparent that only if $\beta$ goes to zero would the buyer’s stocking level coincide with the first-best level under the coordination condition $\frac{w-b}{p-b} = \frac{c}{p}$. That is, full supply chain efficiency is not achievable under a single contract offer for any $\beta > 0$. The managerial implications of the downstream stocking distortion to the buyer firm’s true performance, the supplier’s profitability, and the supply chain surplus remain largely similar to those of the two-state signal case. Thus, in the following subsection, we move directly to the analysis to resolve the stocking distortion using menus of buy-back contracts.

### 6.2. Design of Menus of Buy-back Contracts

A menu of buy-back contracts can be specified as a continuum of $(w(i), b(i), t(i))$ corresponding to each signal $i \in [i_L, i_H]$. Given a menu of contracts, the buyer, observing signal $i$, chooses one contract, denoted by $\tau$, and decides on the stocking level, $q$. The investors then infer the buyer’s signal from her decisions, with a belief denoted by a function $j(\tau, q)$.

We apply the equilibrium concept in Definition 2 with the extension to a continuous signal. Full supply chain efficiency can be ensured if the buyer always chooses the contract that matches her signal and stocks the first-best quantity and, at the same time, the investors can correctly infer the true signal. That is, given a menu of buy-back contracts $(w(i), b(i), t(i))$, a system-wise efficient market equilibrium is reached if the buyer’s decision follows $(\tau, q)(i) = (i, q^*(i))$ and the market belief satisfies $j((\tau, q)(i)) = i$ for any signal $i \in [i_L, i_H]$. Notice, however, that this equilibrium concept only specifies the market belief on the equilibrium path with respect to $q$. The market
belief is not specified for any input \((\tau, q)\) with \(q \neq q^o(\tau)\). In general, the off-equilibrium belief can be specified flexibly as long as it supports the equilibrium. However, such freedom can increase the complexity of the mechanism design problem. We thus impose the following specific off-equilibrium belief, which does not eliminate any equilibrium solution to the original problem.

**Lemma 4.** Given any menu of buy-back contracts \((w(i), b(i), t(i))\), if a system-wise efficient market equilibrium is achieved with a market belief \(j(\tau, q)\), then the equilibrium can also be achieved with the market belief:

\[
\tilde{j}(\tau, q) = \begin{cases} 
\tau, & \text{if } q = q^o(\tau) \\
\tilde{i}_L, & \text{o/w.}
\end{cases}
\]

Lemma 4 parallels Lemma 3 in section 5 for the two-state signal case. \(\tilde{j}(\tau, q)\) specifies the most conservative belief for any off-equilibrium action with respect to \(q\). That is, the investors believe that the true signal equals the buyer firm’s contract choice only if the stocking level matches the corresponding first-best level; otherwise, they believe that the true signal is the worst. As a result, given a menu of contracts, if the buyer is induced to choose the truth-telling contract and stock the first-best quantity under a specific equilibrium market belief \(j(\tau, q)\), then the buyer will also do so if we replace \(j(\tau, q)\) with \(\tilde{j}(\tau, q)\). With Lemma 4, we derive the conditions for a menu of contracts that induces a system-wise efficient market equilibrium.

**Proposition 9.** With the market belief \(\tilde{j}(\tau, q)\), a system-wise efficient market equilibrium can be achieved if the menu of buy-back contracts \((w(i), b(i), t(i))\) satisfies:

\[
\frac{w(i) - b(i)}{p - b(i)} = \frac{c}{p}, \quad b'(i) < 0, \quad t(i) = (p - b(i)) \int_0^{\mu(i)} F(x, i)dx - (w(i) - b(i))q^o(i) - (1 - \beta) \int_{\tilde{i}_L}^i \left[ (p - b(y)) \int_0^{q^o(y)} \frac{\partial}{\partial y} F(x, y)dx \right] dy - T
\]

for any constant \(T\). The supplier’s expected profit goes to the total supply chain surplus when the function \(b(\cdot)\) that specifies the return schedule converges to \(p\) and the constant \(T\) goes to zero.

Proposition 9 shows that menus of buy-back contracts exist that can restore full supply chain efficiency. In particular, the return schedule decreases in the value of the signal, which implies that the wholesale price also decreases in the value of the signal, by the condition \(\frac{w(i) - b(i)}{p - b(i)} = \frac{c}{p}\).

Such a result is intuitive because the buyer will favor a generous return term to a lesser degree but a low wholesale price to a greater degree when the demand outlook improves. This is aligned...
with the discussion for the two-state signal case in section 5. When \( b'(i) < 0 \), a proper transfer payment scheme \( t(i) \) can be designed that ensures the buyer always takes the truth-telling contract. Furthermore, notice that \( t(i) \) contains a constant \( T \) that provides the flexibility to divide the supply chain surplus among the two parties. Given menus of contracts exist by which the supplier can obtain almost all of the supply chain surplus, we can thus adjust \( T \) in those contracts to achieve any specific allocation of the surplus. Hence, pareto improvement is achievable, compared to the case using a single contract.

Thus far, we have extended our findings in the previous sections to the continuous signal case. In the following, we demonstrate that, with an additional condition, we can transform a menu of buy-back contracts to a specific single contract that consists of a quantity discount scheme and a return schedule. Notice that, given a menu of buy-back contracts \( (w(i), b(i), t(i)) \) that achieves full supply chain efficiency, the total initial payment from the buyer to the supplier is \( M(i) = t(i) + w(i)q^0(i) \). The following lemma shows the relationship between \( M(i) \) and \( q^0(i) \).

**Lemma 5.**
\[
\frac{dM(i)}{dq^0} > 0 \text{ and } \frac{d^2M(i)}{dq^0^2} < 0 \text{ if } b(i) \text{ satisfies the following condition:}
\]
\[
\beta (p - b(i)) \int_0^{q^0(i)} \frac{\partial}{\partial i} F(x, i)dx + b'(i) \int_0^{q^0(i)} F(x, i)dx = 0. \quad (9)
\]

Therefore, if (9) is satisfied, then the total initial payment \( M(i) \) increases in the stocking level \( q^0(i) \) at a decreasing rate, which satisfies the property of a quantity discount contract. (9) implies that \( b'(i) < 0 \), and thus it is aligned with the condition specified in Proposition 9. Note that such a return schedule \( b(i) \) can be expressed as a monotonically decreasing function of \( q^0(i) \). Hence, the following proposition holds.

**Proposition 10.** A system-wise efficient market equilibrium can be achieved by a single contract that consists of a proper pair of quantity discount scheme and return schedule \( (q, M(q), b(q)) \).

Prior research has explored different purposes for using a menu of contracts, such as, to share and improve demand forecasting (Cachon and Lariviere 2001, Özer and Wei 2006, Taylor and Xiao 2009), or to elicit cost and inventory information (Ha 2001, Zhang 2010, Zhang et al. 2010).
study reveals that offering a menu of contracts might also be helpful to mitigate stocking distortion when the downstream party has private demand information and, at the same time, cares about her market value. However, a menu of contracts is more complicated to implement than a single contract, especially when there are many underlying states. Proposition 10 indicates that it is possible to design a specific single contract to achieve full supply chain efficiency in our model. Although such a contract still contains substantial complexity, it avoids the difficulties of dealing with a menu of contracts. Finally, notice that to design the contracts in our context, knowing the parameter \( \beta \) (which captures the buyer’s short-term focus) is important for the supplier. The effectiveness of the mechanism could be affected when the information of \( \beta \) is inaccurate.

7. Conclusion

In this paper, we explore how a downstream buyer’s short-term interest in her market value might influence the performances of the parties in the supply chain. First, we show that under a single buy-back contract the buyer might purposely distort the stocking level in equilibrium. Such a stocking distortion hurts the buyer firm’s profitability, and it either benefits or hurts the supplier, depending on the contract terms. We reveal scenarios where full supply chain efficiency cannot be achieved by any single buy-back contract offer. These findings enrich the supply chain literature. An interest in the capital market valuation is not uncommon for firms in practice; however, it has been little explored in the supply chain literature. Second, aiming to prevent stocking distortion, we investigate providing a menu of buy-back contracts in such a context. We derive conditions under which a menu of buy-back contracts can restore full efficiency in the supply chain. This finding enriches the literature that explores real earnings management. We demonstrate that in a supply chain context, operational means can possibly be designed to resolve the distortions.

We conclude by discussing several assumptions in our study. First, we have assumed that the buyer cares about her market value but the supplier does not. In practice, both firms might be interested in their market value. Notice that if the information of the supplier’s operations is complete, our results continue to hold because the market can correctly assess the supplier’s
performance. However, if the supplier also possesses private information, he may have an incentive to induce the buyer to order more (and potentially report a false return allowance) to gain from market valuation. The mechanism proposed in our study is effective in resolving the distortion caused by a downstream buyer but may not be effective for distortions triggered by an upstream supplier. Exploring operational approaches to mitigate the distortions caused by upstream suppliers is an interesting direction for future research. Second, we have assumed that the information of the contracts and stocking level is accessible to the market. In other words, the investors are familiar with the firms’ operations and are able to gather specific supply chain information. Such an assumption is important for characterizing the equilibrium, in which the market is able to perfectly infer the signal. In scenarios where information is noisy, to hold some prior belief of the contracts and stocking level would be necessary, and the market would need to update its belief relying on the distribution of the noises. Our study is limited from this perspective because of the significant analytical complexity that could arise. Finally, our study focuses on some specific selling event which can have a significant impact on a firm’s performance, and we apply a one-period model. Extending our study to more general inventory decision problems with a longer time horizon is interesting for future research.

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Appendix A: Proofs

Proof of Lemma 1. \( G_L(q) \) and \( G_H(q) \) follow the classical newsvendor objective function and are thus concave. Notice that \( G_{LH}(q) \) and \( G_{HL}(q) \) are linear combinations of \( G_L(q) \) and \( G_H(q) \). Therefore, they are also concave. \( \blacksquare \)

Proof of Lemma 2. From the definitions of \( G_{LH}(q) \), \( q_{LH}^* \) and \( q_{HL}^* \), it is direct to see that \( G_{LH}(q_{LH}^*) > G_{LH}(q_{HL}^*) > G_L(q_{HL}^*) \). Given \( G_{LH}(q) \) is concave, there exists a unique \( \bar{q} > q_{LH}^* \) that satisfies \( G_{LH}(\bar{q}) = G_L(q_{HL}^*) \). By a similar argument, we can show that there exists a unique \( \bar{q} > q_{HL}^* \) that satisfies \( G_{HL}(\bar{q}) = G_{HL}(q_{HL}^*) \).

In the following, we show \( 0 < \bar{q} \) by contradiction. Suppose \( q > \bar{q} \). Then, we have

\[
G_H(q) - G_{LH}(\bar{q}) = \left( p - b \right) \int_0^{\bar{q}} F_H(x) dx - (w - b)\bar{q} - \left( p - b \right) \int_0^q F_{LH}(x) dx - (w - b)q
\]

\[
= \left( p - b \right) \int_0^{\bar{q}} (1 - \beta) [F_H(x) - \bar{F}_L(x)] dx + (w - b)(\bar{q} - q) - \left( p - b \right) \int_{\bar{q}}^q F_{LH}(x) dx
\]

\[
= \left( p - b \right) \int_0^q (1 - \beta) [F_H(x) - \bar{F}_L(x)] dx + (p - b) \int_{\bar{q}}^q [\bar{F}_{LH}(q_{LH}) - \bar{F}_{LH}(x)] dx
\]

\[
\geq \left( p - b \right) \int_0^q (1 - \beta) [F_H(x) - \bar{F}_L(x)] dx.
\]

The third equality holds because \( q_{LH}^* = \bar{F}_{LH}^{-1}(w - b) \) and thus \( \bar{F}_{LH}(q_{LH}^*) = \frac{w - b}{p - b} \). The last inequality holds because \( \bar{q} > q_{HL}^* > q_{LH}^* \) and thus \( \bar{F}_{LH}(q_{LH}^*) > \bar{F}_{LH}(x) \) for any \( x > \bar{q} \).

Further, we can obtain

\[
G_{HL}(q_{HL}^*) - G_L(q_{HL}^*) = \left( p - b \right) \int_0^{q_{HL}^*} \bar{F}_{HL}(x) dx - (w - b)q_{HL}^* - \left( p - b \right) \int_0^{q_{HL}^*} \bar{F}_L(x) dx - (w - b)q_{HL}^*
\]

\[
= \left( p - b \right) \int_0^{q_{HL}^*} (1 - \beta) [\bar{F}_H(x) - \bar{F}_L(x)] dx + (p - b) \int_{q_{HL}^*}^{q_{HL}^*} [\bar{F}_L(x) - \bar{F}_L(q_{HL}^*)] dx
\]

\[
\leq \left( p - b \right) \int_0^{q_{HL}^*} (1 - \beta) [\bar{F}_H(x) - \bar{F}_L(x)] dx.
\]

Notice that \( \bar{q} > q_{HL}^* > q_{HL}^* \). Consequently, the above two inequalities lead to \( G_H(\bar{q}) - G_{LH}(\bar{q}) > G_{HL}(q_{HL}^*) - G_L(q_{HL}^*) \) which contradicts with the definitions that \( G_H(\bar{q}) = G_{HL}(q_{HL}^*) \) and \( G_{LH}(q_{HL}^*) = G_L(q_{HL}^*) \). Hence we conclude \( q < \bar{q} \), which completes the proof. \( \blacksquare \)

Proof of Proposition 1. From Lemma 2, it is clear that if the market holds a belief as

\[
j(q) = \begin{cases} H & \text{if } q = \bar{q}, \\ L & \text{o/w}, \end{cases}
\]
where \( \hat{q} = \max\{q_L, q_H^*\} \), then the buyer’s strategy follows

\[
q(i) = \begin{cases} 
\hat{q} & \text{if } i = H, \\
q_L & \text{if } i = L.
\end{cases}
\]

That is, under this market belief, when observing a low signal, the buyer stocks \( q_L \) and has no incentive to stock \( \hat{q} \) to mimic the high signal strategy; the buyer also has no incentive to deviate from \( \hat{q} \) if she observes a high signal. The market belief is consistent with the buyer’s strategies. Thus, a separating equilibrium holds. One can easily apply the standard procedure in Appendix B to show that this equilibrium is the unique equilibrium that survives the intuitive criterion in our model when the demand signal has two states. The detailed proof is omitted. ■

**Proof of Proposition 2.** We first prove \( \underline{q} \) increases in \( \beta \). As \( \underline{q} \) is determined by \( G_{LH}(\underline{q}) = G_L(q_L^*) \), we define

\[
H(\underline{q}, \beta) \equiv G_{LH}(\underline{q}) - G_L(q_L^*)
\]

\[
= \left[ (p - b) \int_0^{\underline{q}} \bar{F}_{LH}(x) dx - (w - b)\underline{q} \right] - \left[ (p - b) \int_0^{q_L^*} \bar{F}_L(x) dx - (w - b)q_L^* \right] = 0.
\]

Recall \( \bar{F}_{LH}(q) = \beta \bar{F}_H(q) + (1 - \beta) \bar{F}_L(q) \). We can easily verify that \( \frac{\partial H(\underline{q}, \beta)}{\partial \beta} > 0 \) as \( G_{LH}(\underline{q}) \) increases in \( \beta \) and \( G_L(q_L^*) \) is independent of \( \beta \). On the other hand, given \( \underline{q} > q_{LH}^* \), by its definition, \( G_{LH}(\underline{q}) \) decreases in \( \underline{q} \); thus, \( \frac{\partial H(\underline{q}, \beta)}{\partial \underline{q}} < 0 \). Consequently, \( \frac{\partial \underline{q}}{\partial \beta} = -\frac{\partial H(\underline{q}, \beta)}{\partial \underline{q}} > 0 \).

In the following we show a unique threshold \( \beta = \hat{\beta} \) exists at which \( \underline{q} = q_H^* \). First, when \( \beta = 0 \), from the definition of \( \underline{q} \) (i.e., \( G_{LH}(\underline{q}) = G_L(q_L^*) \)), we learn \( \underline{q} = q_L^* < q_H^* \). Second, when \( \beta = 1 \), \( q_{LH}^* = q_H^* \). The definition of \( \underline{q} \) asserts \( \underline{q} > q_{LH}^* = q_H^* \). Therefore, a unique threshold \( \beta = \hat{\beta} \) exists at which \( \underline{q} = q_H^* \).

To characterize \( \hat{\beta} \), we know \( \underline{q} = q_H^* \) at \( \hat{\beta} \) and thus the condition \( G_{LH}(\underline{q}) = G_L(q_L^*) \) which determines \( \underline{q} \) is equivalent to \( G_{LH}(q_H^*) = G_L(q_L^*) \), or identically, \( G_{LH}(q_H^*) = \hat{\beta} G_H(q_H^*) + (1 - \hat{\beta}) G_L(q_H^*) = G_L(q_L^*) \). By rearranging the terms we obtain \( \hat{\beta} = \frac{G_L(q_L^*) - G_L(q_H^*)}{G_H(q_H^*) - G_L(q_H^*)} \), which completes the proof. ■

**Proof of Corollary 1.** When \( \beta \leq \hat{\beta} \), the buyer’s stocking decision is fixed and thus his expected payoff is also fixed; when \( \beta > \hat{\beta} \), \( q(H) = \underline{q} \) increases in \( \beta \), which implies that the buyer overstocks more units and thus his expected payoff decreases. ■
Proof of Proposition 3. The result for $\beta \leq \hat{\beta}$ is straightforward. To show the results of the two properties (i-ii) for $\beta > \hat{\beta}$, we take the derivative

$$\frac{d\Pi^S}{d\beta} = \lambda [w - c - bF_H(q)] \frac{dq}{d\beta}.$$  

Recall from Proposition 2 that $\frac{dq}{d\beta} > 0$. Therefore, to show (i-ii), we only need to determine the sign of $w - c - bF_H(q)$.

(i) Note that $F_H(q^*_H) = \frac{w - c}{p - b}$. When $\beta > \hat{\beta}$, $q > q^*_H$ and thus $F_H(q) > F_H(q^*_H) = \frac{w - c}{p - b}$. Therefore, if $b \geq \frac{p - w}{p - b}(w - c)$, then we have $w - c - bF_H(q) < w - c - \frac{p - w}{p - b}(w - c)\frac{w - c}{p - b} = 0$, which asserts $\Pi^S$ is strictly decreasing in $\beta$. Hence, (i) holds.

(ii) Given $q$ increases in $\beta$, $\bar{q}$ reaches the largest, denoted as, $\bar{q}^{\max}$, when $\beta = 1$. Note that when $\beta = 1$, $q_{LH} = q^*_H$. Therefore, by the definition of $q$, $\bar{q}^{\max}$ is the solution of $G_H(q) = G_L(q^*_L)$ that satisfies $\bar{q}^{\max} > q^*_H$.

Given that $F_H(q)$ reaches the largest at $q = \bar{q}^{\max}$, if $w - c - bF_H(\bar{q}^{\max}) \geq 0$, then $\Pi^S$ is increasing in $\beta$ when $\beta \in [\hat{\beta}, 1]$. That is, $\beta' = 1$.

Now, suppose $w - c - bF_H(\bar{q}^{\max}) < 0$. When $\beta > \hat{\beta}$, $F_H(q)$ reaches the least as $\beta \rightarrow \hat{\beta}$ (and $q \rightarrow q^*_H$). At $q = q^*_H$, we have $w - c - bF_H(q^*_H) = w - c - \frac{p - w}{p - b} > 0$. Therefore, there exists a $\beta' \in [\hat{\beta}, 1]$ and a corresponding $q \in [q^*_H, \bar{q}^{\max}]$ at which $w - c - bF_H(q) = 0$, and $\Pi^S$ is increasing in $\beta$ when $\hat{\beta} < \beta < \beta'$ and decreasing in $\beta$ when $\beta > \beta'$. This completes the proof. \[\square\]

Proof of Proposition 4. The result for $\beta \leq \hat{\beta}$ is straightforward. To show the results of the two properties (i-ii) for $\beta > \hat{\beta}$, we take the derivative

$$\frac{d\Pi^{SC}}{d\beta} = \lambda [p\bar{F}_H(q) - c] \frac{dq}{d\beta}.$$  

Recall from Proposition 2 that $\frac{dq}{d\beta} > 0$. Therefore, to show (i-ii), we only need to determine the sign of $p\bar{F}_H(q) - c$.

(i) Note that $\bar{F}_H(q^*_H) = \frac{w - b}{p - b}$. When $\beta > \hat{\beta}$, $q > q^*_H$ and thus $\bar{F}_H(q) < \bar{F}_H(q^*_H) = \frac{w - b}{p - b}$. Therefore, if $\frac{w - b}{p - b} \leq \frac{c}{p}$, then we have $p\bar{F}_H(q) - c < p\bar{F}_H(q^*_H) - c \leq 0$, which asserts $\Pi^{SC}$ is strictly decreasing in $\beta$. Hence, (i) holds.
(ii) When $\beta > \tilde{\beta}$, $F_H(q)$ reaches the largest as $\beta \to \tilde{\beta}$ (and thus $q \to q_H^*$. If $\frac{w - b}{p - b} > \frac{\tilde{\epsilon}}{p}$, then at $q = q_H^*$, we have $pF_H(q) - c = p\frac{w - b}{p - b} - c > 0$, which implies that $\Pi^{SC}$ is increasing at $\beta = \tilde{\beta}$.

Given $q$ increases in $\beta$, $q$ reaches the largest, denoted as, $q^{max}$, when $\beta = 1$, while $F_H(q)$ reaches the least at $q = q^{max}$. Therefore, if $pF_H(q^{max}) - c \geq 0$, then $\Pi^{SC}$ is increasing in $\beta$ when $\beta \in [\hat{\beta}, 1]$. That is, $\beta'' = 1$. In case $pF_H(q^{max}) - c < 0$, it is obvious that a $\beta'' \in [\tilde{\beta}, 1]$ exists at which the corresponding $q$ satisfies $pF_H(q) - c = 0$. Hence, there exists a $\beta'' \in [\tilde{\beta}, 1]$ such that $\Pi^{SC}$ is increasing in $\beta$ when $\hat{\beta} < \beta < \beta''$ and decreasing in $\beta$ when $\beta > \beta''$. This completes the proof.

Proof of Proposition 5. The proof is straightforward. Notice that in order to coordinate the supply chain, the contract must satisfy: $\frac{w - b}{p - b} = \frac{\epsilon}{p}$. From Proposition 4(i), we notice that if $\frac{w - b}{p - b} = \frac{\epsilon}{p}$, then $\Pi^{SC}$ is strictly decreasing in $\beta$ when $\beta > \hat{\beta}$. Hence, the supply chain will not be coordinated by any single contract offer $(w, b, t)$. The definition of $\hat{\beta}'$ has been explained in the text.

Proof of Lemma 3. If the system can achieve the first-best with a market belief $(Q_H^H, Q_L^H)$, then these two sets must satisfy: $q_H^o \in Q_H^H$ and $q_L^o \notin Q_L^H$ (otherwise the buyer would need to distort the stocking levels in order for the market to correctly infer the signal). To achieve the maximum efficiency, we need $\frac{w_H - b_H}{p - b_H} = \frac{w_L - b_L}{p - b_L} = \frac{\epsilon}{p}$. Under such a menu of buy-back contracts, when the signal is high, the buyer’s decision follows

$$
\max_{(\tau, q)} \begin{cases} 
\Pi^B(H, H; H) = \max_{q \in Q_H^H} (p - b_H) \left[ \int_0^q F_H(x) dx - \frac{\epsilon}{p} q \right] - t_H & \text{if } \tau = H, \\
\Pi^B(H, L; H) = \max_{q \notin Q_H^L} (p - b_H) \left[ \int_0^q F_H(x) dx - \frac{\epsilon}{p} q \right] - t_H & \text{if } \tau = H, \\
\Pi^B(L, H; H) = \max_{q \in Q_L^H} (p - b_L) \left[ \int_0^q F_H(x) dx - \frac{\epsilon}{p} q \right] - t_L & \text{if } \tau = L, \\
\Pi^B(L, L; H) = \max_{q \notin Q_L^H} (p - b_L) \left[ \int_0^q F_H(x) dx - \frac{\epsilon}{p} q \right] - t_L & \text{if } \tau = L;
\end{cases}
$$

when the signal is low, the buyer’s decision follows

$$
\max_{(\tau, q)} \begin{cases} 
\Pi^B(H, H; L) = \max_{q \in Q_H^H} (p - b_H) \left[ \int_0^q F_L(x) dx - \frac{\epsilon}{p} q \right] - t_H & \text{if } \tau = H, \\
\Pi^B(H, L; L) = \max_{q \notin Q_H^L} (p - b_H) \left[ \int_0^q F_L(x) dx - \frac{\epsilon}{p} q \right] - t_H & \text{if } \tau = H, \\
\Pi^B(L, H; L) = \max_{q \in Q_L^H} (p - b_L) \left[ \int_0^q F_L(x) dx - \frac{\epsilon}{p} q \right] - t_L & \text{if } \tau = L, \\
\Pi^B(L, L; L) = \max_{q \notin Q_L^H} (p - b_L) \left[ \int_0^q F_L(x) dx - \frac{\epsilon}{p} q \right] - t_L & \text{if } \tau = L.
\end{cases}
$$

If a system-wise efficient market equilibrium is reached with the market belief $(Q_H^H, Q_L^L)$, then the buyer takes the contract $(w_H, b_H, t_H)$ and stocks $q_H^o$ when the signal is high and she takes the
contract \((w_L, b_L, t_L)\) and stocks \(q^o_L\) when the signal is low, which implies the following (IC) and (IR) constraints:

\[
\begin{align*}
\Pi^B(L; L; L) & \geq \max\{\Pi^B(H; H; L), \Pi^B(H, L; L), \Pi^B(L; H; L)\} \quad (IC1), \\
\Pi^B(H; H; H) & \geq \max\{\Pi^B(H, L; H), \Pi^B(L, H; H), \Pi^B(L; H; H)\} \quad (IC2), \\
\Pi^B(L; L; L) & \geq 0 \quad (IR1), \\
\Pi^B(H; H; H) & \geq 0 \quad (IR2).
\end{align*}
\]

When the thresholds in the market belief \((Q^H_L, Q^L_H)\) are replaced by \((\{q^o_H\}, \emptyset)\), \(\Pi^B(H, H; H)\) and \(\Pi^B(L; L, L)\) will not change and it is not difficult to find out that the right hand sides of the IC constraints will only become smaller since the market belief becomes stricter in terms of recognizing a high signal. Hence, the same separating equilibrium will be achieved. ■

**Proof of Proposition 6.** We start the proof by analyzing the (IC) and (IR) constraints. With the market belief \((\{q^o_H\}, \emptyset)\), the (IC1) constraint in (10) can be captured by the following two inequalities:

\[
(p - b_H) \left[ \int_0^{q^o_L} F_L(x) dx - \frac{c}{p} q^o_L \right] - t_L \geq (p - b_H) \left[ \beta \int_0^{q^o_H} F_H(x) dx + (1 - \beta) \int_0^{q^o_L} F_L(x) dx - \frac{c}{p} q^o_H \right] - t_H, \tag{11}
\]

and

\[
(p - b_L) \left[ \int_0^{q^o_L} F_L(x) dx - \frac{c}{p} q^o_L \right] - t_L \geq (p - b_H) \left[ \int_0^{q^o_L} F_L(x) dx - \frac{c}{p} q^o_H \right] - t_H. \tag{12}
\]

The (IC2) constraint can be rewritten as

\[
(p - b_H) \left[ \int_0^{q^o_H} F_H(x) dx - \frac{c}{p} q^o_H \right] - t_H \geq (p - b_L) \left[ \beta \int_0^{q^o_H} F_L(x) dx + (1 - \beta) \int_0^{q^o_L} F_H(x) dx - \frac{c}{p} q^o_L \right] - t_L. \tag{13}
\]

The (IR) constraints are

\[
(p - b_H) \left[ \int_0^{q^o_H} F_H(x) dx - \frac{c}{p} q^o_H \right] - t_H \geq 0, \tag{14}
\]

\[
(p - b_L) \left[ \int_0^{q^o_L} F_L(x) dx - \frac{c}{p} q^o_L \right] - t_L \geq 0. \tag{15}
\]

From (11-13), we can obtain

\[
t_H - t_L \geq (p - b_H) g_{LH}(q^o_H) - (p - b_L) g_L(q^o_L); \tag{16}
\]
Therefore, the right hand side of (18) must be larger than or equal to the maximum of those of (16-17) and hence we obtain the condition \( \frac{p-b_H}{p-b_L} \geq K \) where

\[
K = \max \left\{ \frac{g_{HL}(q_{HL}) - g_L(q^*_L)}{g_H(q^*_H) - g_{HL}(q_{HL})}, \frac{g_{HL}(q_{HL}) - g_L(q^*_H)}{g_H(q^*_H) - g_{HL}(q_{HL})} \right\}.
\]

When \( \frac{p-b_H}{p-b_L} \geq K \), it is straightforward from the above reasoning that we can always find a pair of transfer payments, \( t_H \) and \( t_L \), which satisfy the (IC) and (IR) constraints. \( K \) is obviously positive. Furthermore, it is clear that \( \frac{g_{HL}(q_{HL}) - g_L(q^*_L)}{g_H(q^*_H) - g_{HL}(q_{HL})} < 1 \). For \( \frac{g_{HL}(q_{HL}) - g_L(q^*_H)}{g_H(q^*_H) - g_{HL}(q_{HL})} \), we can find that

\[
g_H(q^*_H) - g_{HL}(q_{HL}) = (1-\beta) g_H(q^*_H) - (1-\beta) g_L(q^*_H)
\]

\[
> (1-\beta) g_H(q^*_H) - (1-\beta) g_L(q^*_H)
\]

\[
> (1-\beta) g_H(q^*_H) + \beta g_L(q^*_H) - g_L(q^*_H)
\]

\[
= g_{HL}(q_{HL}) - g_L(q^*_H).
\]

Thus, \( 0 < K < 1 \). Rearranging the terms results in the condition \( b_L \geq \frac{b_H}{K} + p \frac{K-1}{K} \).

Now, we analyze the range of the supplier’s profit. First, it is obvious that there always exist such \( t_H \) and \( t_L \) under which the supplier obtains zero profit (one can always reduce \( t_H \) and \( t_L \) by the same amount while keeping their difference fixed until the supplier’s profit reaches zero). Therefore, the supplier’s profit has a lower bound that is zero. Second, the upper bound of the supplier’s profit is reached if the buyer’s profit is minimized. From (14),(15) and (16), we can find the largest \( t_H \) and \( t_L \) under which the equilibrium can held are

\[
t_L = (p-b_L) g_L(q^*_L),
\]

\[
t_H = t_L + (p-b_H) g_H(q^*_H) - (p-b_L) g_{HL}(q^*_HL).
\]

Hence, the largest profit that the supplier can obtain is \( \hat{\Pi}^s = p \{ \lambda g_H(q^*_H) + (1-\lambda) g_L(q^*_L) \} - \lambda (p-b_L) [g_{HL}(q^*_HL) - g_L(q^*_L)] \), which completes the proof. \( \blacksquare \).
Proof of Proposition 7. When \( b_H = 0 \) and \( b_L = p - \varepsilon, \frac{p-b_H}{p-b_L} = \frac{\varepsilon}{p} \to \infty \) as \( \varepsilon \to 0 \). Hence the condition for efficiency, \( \frac{p-b_H}{p-b_L} \geq K \), is satisfied. By the condition \( \frac{w_H}{p-b_H} = \frac{w_L}{p-b_L} = \frac{\varepsilon}{p} \), we can obtain \( w_H = c \) and \( w_L = p - \varepsilon (1 - \frac{\varepsilon}{p}) \). We can easily verify that with \( t_H = p g_H (q_H^c) - \varepsilon [g_H (q_H^c) - g_L (q_L^c)] - T \) and \( t_L = \varepsilon g_L (q_L^c) - T \), a system-wise efficient equilibrium can be reached and the supplier obtains almost all of the supply chain surplus as \( \varepsilon \) and \( T \) go to zero. 

Proof of Proposition 8. We denote the buyer’s objective function as

\[
G(q,i) = \beta \left[ (p-b) \int_0^q F(x,j(q))dx - (w-b)q - t \right] + (1-\beta) \left[ (p-b) \int_0^q F(x,i)dx - (w-b)q - t \right].
\]

The optimal stocking level \( q(i) \) shall satisfy \( \frac{\partial G(q,i)}{\partial q} = 0 \) and \( \frac{\partial^2 G(q,i)}{\partial q^2} \leq 0 \), where \( \frac{\partial G(q,i)}{\partial q} = 0 \) follows

\[
\beta \left[ (p-b) \left[ F(q,j(q)) + \int_0^q \frac{\partial F(x,j(q))}{\partial i} \frac{dj(q)}{dq} dx \right] - (w-b) \right] + (1-\beta) \left[ (p-b) \left[ F(q,i) - (w-b) \right] \right] = 0.
\]

To verify \( \frac{\partial^2 G(q,i)}{\partial q^2} \leq 0 \) when \( \frac{\partial G(q,i)}{\partial q} = 0 \) holds, we derive from the latter

\[
\frac{\partial^2 G(q,i)}{\partial q \partial i} + \frac{\partial^2 G(q,i)}{\partial q^2} \frac{dq}{di} = 0.
\]

Given \( \frac{\partial F(q,i)}{\partial i} < 0 \), it can be easily verified from (20) that \( \frac{\partial^2 G(q,i)}{\partial q \partial i} > 0 \). Therefore, \( \frac{\partial^2 G(q,i)}{\partial q^2} < 0 \) if and only if \( \frac{dq}{di} > 0 \). Notice that in order for a fully separating equilibrium to hold in this game, the equilibrium stocking level must be increasing in the signal value. Hence, \( \frac{\partial^2 G(q,i)}{\partial q^2} < 0 \) can be assured as long as a separating equilibrium exists, which we validate in the following.

In equilibrium, the market belief must be consistent with the true signal for any given \( q \) that the buyer firm optimally stocks. In other words, \( j(q) = i \) must hold in equilibrium. We thus impose this equilibrium condition over the buyer firm’s first-order condition

\[
\beta \left[ (p-b) \left[ F(q,i) + \int_0^q \frac{\partial F(x,i)}{\partial i} \frac{dq}{dq} dx \right] - (w-b) \right] + (1-\beta) \left[ (p-b) F(q,i) - (w-b) \right] = 0.
\]

Rearranging the terms, we obtain

\[
(p-b) F(q,i) - (w-b) \frac{dq}{di} + \beta (p-b) \int_0^q \frac{\partial F(x,i)}{\partial i} dx = 0,
\]

(21)
which serves as the equilibrium condition of the buyer’s stocking level. Finally, as the signal can be
perfectly inferred in a separating equilibrium, the best strategy for the buyer receiving a signal \(i_L\)
is to stock \(q(i_L) = \bar{F}^{-1} \left( \frac{w-b}{p-b}, i_L \right) \). Given that the buyer has no incentive to pretend to have a lower
signal, stocking \(q(i_L) = \bar{F}^{-1} \left( \frac{w-b}{p-b}, i_L \right) \) can credibly reveal her signal to the market. Consequently,
\(q(i_L) = \bar{F}^{-1} \left( \frac{w-b}{p-b}, i_L \right) \) serves as the initial condition to (21). Given this initial condition, we can
easily verify that \(\frac{dq}{di} > 0\) and thus the first-order approach we use to characterize the equilibrium
is validated. The same proof procedure is applied in Bebchuk and Stole (1993).

Proof of Corollary 2. Rearranging (21) we obtain \(\bar{F}(q, i) = \frac{w-b}{p-b} - \beta \int_0^q \frac{\partial F(x,i)}{\partial i} dx \frac{di}{\partial i} \). Given both
\(\frac{\partial F(x,i)}{\partial i} > 0\) and \(\frac{di}{\partial i} > 0\), the solution \(q(i)\) must be larger than \(\bar{F}^{-1} \left( \frac{w-b}{p-b}, i \right) \). Furthermore, when \(\beta\)
increases, \(\bar{F}(q, i) = \frac{w-b}{p-b} - \beta \int_0^q \frac{\partial F(x,i)}{\partial i} dx \frac{di}{\partial i} \) decreases and thus the solution \(q(i)\) will increase.

Proof of Lemma 4. The intuition of this lemma is discussed in the main body of the paper. If a
system-wise efficient market equilibrium holds with a given market belief \(j(\tau, q)\), according to the
definition, this market belief must take the chosen contract as the signal value. In equilibrium, the
buyer with signal \(i\) always chooses the truth-telling contract \(\tau = i\) and stocks the first-best quantity
\(q^o(i)\). Notice that the market belief \(\bar{j}(\tau, q)\) will also take the chosen contract as the signal value if
the stocking level matches the first-best quantity and assume the signal is the least otherwise. As a
result, \(q^o(i)\) will continue to be the buyer’s best strategy as the buyer’s payoff of stocking the other
quantities will only decrease if \(j(\tau, q)\) is replaced by \(\bar{j}(\tau, q)\). Hence, the equilibrium will continue
to hold.

Proof of Proposition 9. The buyer’s objective function, given a menu of contracts \((w(i), b(i), t(i))\)
and a true signal \(i\), follows:

\[
G(\tau, q; i) = (p - b(\tau)) \left[ \beta \int_0^q \bar{F}(x, j(\tau, q)) dx + (1 - \beta) \int_0^q \bar{F}(x, i) dx \right] - (w(\tau) - b(\tau))q - t(\tau).
\]

First, to ensure the first-best stocking level will be implemented, we must have the supply chain
coordination condition \(\frac{w(i) - b(i)}{p - b(i)} = \frac{c}{p}\) for any \(i \in [i_L, i_H]\).

Second, we need to ensure that the buyer always selects the truth-telling contract; i.e., \(\tau = i\).
Taking the derivative with respect to \( t \), the objective function holds. Substituting \( \tau \), which serves as a necessary condition for the buyer to select the truth-telling contract.

\[
\mathcal{G}(\tau, i) \equiv G(\tau, q^\prime(\tau); i) = (p - b(\tau)) \left[ \beta \int_0^{q^\prime(\tau)} \tilde{F}(x, \tau) dx + (1 - \beta) \int_0^{q^\prime(\tau)} \tilde{F}(x, i) dx \right] - (w(\tau) - b(\tau))q^\prime(\tau) - t(\tau).
\]

When \( \tau = i \), we have

\[
\mathcal{G}(i, i) = (p - b(i)) \int_0^{q^\prime(i)} \tilde{F}(x, i) dx - (w(i) - b(i))q^\prime(i) - t(i). \tag{22}
\]

In order for the buyer to select the truth-telling contract, the first-order condition of the buyer’s objective function, \( \frac{\partial \mathcal{G}(\tau, i)}{\partial \tau} \bigg|_{\tau = i} = 0 \), must hold, with which we have

\[
\frac{d\mathcal{G}(i, i)}{di} = \left[ \frac{\partial \mathcal{G}(\tau, i)}{\partial i} + \frac{\partial \mathcal{G}(\tau, i)}{\partial \tau} \frac{\partial \tau}{\partial i} \right] \bigg|_{\tau = i} = (1 - \beta) (p - b(i)) \int_0^{q^\prime(i)} \frac{\partial}{\partial y} \tilde{F}(x, i) dx.
\]

Integrating the above equation yields

\[
\mathcal{G}(i, i) = T + (1 - \beta) \int_{i_L}^i \left( p - b(y) \right) \int_0^{q^\prime(y)} \frac{\partial}{\partial y} \tilde{F}(x, y) dx \ dy, \tag{23}
\]

where \( T \) is a constant. (This procedure is classical for mechanism design problems.)

Therefore, if the first-order condition holds, then, comparing (22) with (23), we have

\[
t(i) = (p - b(i)) \int_0^{q^\prime(i)} \tilde{F}(x, i) dx - (w(i) - b(i))q^\prime(i) - (1 - \beta) \int_{i_L}^i \left( p - b(y) \right) \int_0^{q^\prime(y)} \frac{\partial}{\partial y} \tilde{F}(x, y) dx \ dy - T,
\]

which serves as a necessary condition for the buyer to select the truth-telling contract.

Now, we derive the sufficient condition under which the first-order condition of the buyer’s objective function holds. Substituting \( t(i) \) into the original \( \mathcal{G}(\tau, i) \) function, we obtain

\[
\mathcal{G}(\tau, i) = (1 - \beta) (p - b(\tau)) \left[ \int_0^{q^\prime(\tau)} \tilde{F}(x, i) dx - \int_0^{q^\prime(\tau)} \tilde{F}(x, \tau) dx \right] + (1 - \beta) \int_{i_L}^\tau \left[ (p - b(y)) \int_0^{q^\prime(y)} \frac{\partial}{\partial y} \tilde{F}(x, y) dx \ dy \right] dy + T.
\]

Taking the derivative with respect to \( \tau \) yields

\[
\frac{\partial \mathcal{G}(\tau, i)}{\partial \tau} = -(1 - \beta) b(\tau) \left[ \int_0^{q^\prime(\tau)} \tilde{F}(x, i) dx - \int_0^{q^\prime(\tau)} \tilde{F}(x, \tau) dx \right] \]
which completes the proof.

\[\frac{\partial q^o(\tau)}{\partial \tau} > 0; \text{ by assumption, } \frac{\partial \tilde{F}(x, i)}{\partial x} > 0. \text{ Therefore, we will have } \frac{\partial C(\tau, i)}{\partial \tau} |_{\tau = 0} = 0 \text{ and } \frac{\partial C(\tau, i)}{\partial \tau} |_{\tau < 0} > (\tau > 0) \text{ when } \tau < b'(\tau) < 0 (\text{it is implicitly assumed that } b(i) < p; \text{ we derive the return term and the fixed payment of the menu of contracts influence the supplier’s expected profit. It is obvious that if } b(i) \text{ converges to } p \text{ and } T \text{ goes to zero, then } \Pi^S \text{ goes to the maximum expected total surplus of the supply chain (i.e., } \int_{i^H}^H \varphi(i) \left[ p \int_0^{q^o(i)} \tilde{F}(x, i)dx - cq^o(i) \right] di), \text{ which completes the proof.} \]

**Proof of Lemma 5.** The total initial payment from the buyer to the supplier equals

\[M(i) = t(i) + w(i)q^o(i) = (p - b(i)) \int_0^{q^o(i)} \tilde{F}(x, i)dx + b(i)q^o(i) - (1 - \beta) \int_{i^H}^H \left[ (p - b(y)) \int_0^{q^o(y)} \frac{\partial \tilde{F}(x, y)dx}{\partial y} \right] dy - T. \]

We derive

\[\frac{dM}{di} = -b'(i) \int_0^{q^o(i)} \tilde{F}(x, i)dx + b'(i)q^o(i) - (1 - \beta) (p - b(i)) \int_0^{q^o(i)} \frac{\partial \tilde{F}(x, i)dx}{\partial i} + b(i) \frac{dq^o}{di} + (p - b(i)) \int_0^{q^o(i)} \frac{\partial \tilde{F}(x, i)dx}{\partial i} + b(i) \frac{dq^o}{di} = \beta (p - b(i)) \int_0^{q^o(i)} \frac{\partial \tilde{F}(x, i)dx}{\partial i} + b'(i) \int_0^{q^o(i)} \tilde{F}(x, i)dx + (p - b(i)) \tilde{F}(q^o(i), i) \frac{dq^o}{di} + b(i) \frac{dq^o}{di} = \beta (p - b(i)) \int_0^{q^o(i)} \frac{\partial \tilde{F}(x, i)dx}{\partial i} + b'(i) \int_0^{q^o(i)} \tilde{F}(x, i)dx + (p - b(i)) \frac{p - c}{p} \frac{dq^o}{di} + b(i) \frac{dq^o}{di}. \]
Therefore, if the sum of the first two terms equals zero (i.e., \( \beta(p-b(i))\int_0^{q^p(i)} \frac{\partial}{\partial i} F(x,i)dx + b'(i)\int_0^{q^p(i)} F(x,i)dx = 0 \)), we will have
\[
\frac{dM}{dq} = \frac{\partial M}{\partial i} \frac{dq}{di} = \frac{\partial M}{\partial q} \frac{dq}{dq} = \left( \frac{b(i)}{p} \right) > 0 \quad \text{and} \quad \frac{d^2M}{dq^2} = b'(i) \frac{dq}{dq} < 0
\]
given \( b'(i) < 0 \) and \( \frac{dq}{dq} > 0 \), which completes the proof.\[\blacksquare\]

**Proof of Proposition 10.** This proposition follows from Lemma 5. The seller can specify the contract with \( q \) from \( q^o(i_L) \) to \( q^o(i_H) \) and \( M(q) \) and \( b(q) \) accordingly. \[\blacksquare\]

**Appendix B: The Intuitive Criterion and Pooling Equilibria**

**B1. The Intuitive Criterion**

In the following, we describe the intuitive criterion developed by Cho and Kreps (1987) and its application to our model when the demand signal has two states (note that the intuitive criterion may not be effective when there are more than two states). The intuitive criterion uses two steps to examine an equilibrium of a signaling game between a signal sender and a signal receiver.

(i) **The first step** of the intuitive criterion derives a set \( \Theta \) of the types of the sender, with which the highest utility that the sender can obtain by taking a specific off-equilibrium strategy is lower than that by keeping the equilibrium strategy. That is, under those types, the off-equilibrium strategy is dominated by the equilibrium strategy for the buyer.

Specifically, in our model, suppose we have an equilibrium in which the buyer stocks \( q^e_H \) when observing a high signal and stocks \( q^e_L \) when observing a low signal. If \( q^e_H = q^e_L \), then the equilibrium is pooling; otherwise, the equilibrium is separating. In the first step of the intuitive criterion refinement, for any off-equilibrium stocking level \( q \), we derive a set of the signals:

\[
\Theta(q) = \{i \in \{H, L\} : G(q^e; i) > \hat{G}(q; i)\}
\]

where \( G(q^e; i) \) denotes the buyer's equilibrium payoff while \( \hat{G}(q; i) \) denotes the highest payoff that the buyer can obtain by stocking the off-equilibrium \( q \). Note that the highest payoff for a given stocking level \( q \) is achieved if the market believes the buyer observes a high signal; that is,

\[
\hat{G}(q; i) = (p-b) \int_0^{q} \bar{F}_{iH}(q)dx - (w-b)q - t.
\]

Therefore, \( \Theta(q) \) contains those signals under which the off-equilibrium strategy \( q \) is dominated by the equilibrium strategy \( q^e \) for the buyer.
If the set $\Theta^C$, the complement of $\Theta$, is an empty set, the second step becomes unnecessary since for all types the off-equilibrium strategy is always dominated by the equilibrium strategy and the sender will not deviate at all. In this case, the intuitive criterion imposes no constraint on the solution space. If $\Theta^C$ is nonempty, then we need to carry out the second step.

(ii) The second step of the intuitive criterion checks if there exists a specific type in $\Theta^C$ such that the equilibrium utility of the sender with this type is lower than the lowest utility that she can obtain by taking a specific off-equilibrium strategy given that the receiver restricts his belief to $\Theta^C$ after observing such a deviation. If there does exist such a type, the equilibrium fails the intuitive criterion; otherwise, the intuitive criterion imposes no constraint on the solution space.

Specifically, in our model, the second step of the intuitive criterion checks, for any stocking level $q$, if there exists a signal $i \in \Theta^C(q)$, the complement of $\Theta(q)$, such that with this signal the buyer’s equilibrium payoff $G(q^e_i; i)$ is lower than the lowest payoff that the buyer can obtain by deviating to the stocking level $q$ when the market belief is restricted to $\Theta^C(q)$ for such a deviation. Let $\hat{G}(q; i)$ denote this lowest payoff, and it follows

$$\hat{G}(q; i) = \begin{cases} 
(p - b) \int_0^q F_{iL}(q) dx - (w - b) q - t & \text{if } L \in \Theta^C(q), \\
(p - b) \int_0^q F_{iH}(q) dx - (w - b) q - t & \text{o/w}.
\end{cases}$$

That is, if the low signal $L$ is contained in $\Theta^C(q)$ (i.e., the strategy to deviate to $q$ is not dominated by the equilibrium strategy for the buyer with a low signal), then the lowest payoff the buyer would obtain to deviate to $q$ is achieved under the market belief that the buyer observed a low signal for such a deviation. If the low signal $L$ is not contained $\Theta^C(q)$, then $\Theta^C(q)$ contains only the high signal and thus the lowest payoff the buyer would obtain to deviate to $q$ is achieved under the market belief that the buyer observed a high signal. If there exists such a signal $i \in \Theta^C(q)$ that $G(q^e_i; i) < \hat{G}(q; i)$, then the equilibrium fails the intuitive criterion in our model; otherwise, the intuitive criterion imposes no constraint on the solution space.

We use the above procedure of the intuitive criterion to refine the equilibria in our model.
B2. Refinement over Pooling Equilibria

Suppose there is a pooling equilibrium, in which the buyer stocks $q_{PL}$ for each signal $i \in \{H, L\}$ and the market values the buyer firm at

$$P(q_{PL}) = (p - b) \int_{0}^{q_{PL}} (\lambda F_H(x) + (1 - \lambda) F_L(x)) \, dx - (w - b) q_{PL} - t$$

which equals the expected true profit of the firm. Given this market value, the buyer’s payoff in this pooling equilibrium follows, for each signal $i \in \{H, L\}$,

$$G_i(q_{PL}) \equiv \beta P(q_{PL}) + (1 - \beta) \pi^B(q_{PL}; i)$$

$$= (p - b) \int_{0}^{q_{PL}} (\beta (\lambda F_H(x) + (1 - \lambda) F_L(x)) + (1 - \beta) F_i(x)) \, dx - (w - b) q_{PL} - t.$$ 

Now, we apply the intuitive criterion. In particular, we search for a stocking level $q'$ different from $q_{PL}$ such that: the buyer does not have any incentive to deviate from $q_{PL}$ to $q'$ when observing a low signal even if such a deviation would lead the market to believe that the signal is high; whereas, she does have incentive to do that with a high signal if this deviation would lead the market to believe that the signal is high. If such a $q'$ exists, then in the first step of the intuitive criterion (see Appendix B1), we will have a set $\Theta(q') = \{L\}$; that is, for the low signal, the off-equilibrium strategy $q'$ is dominated by the equilibrium strategy $q_{PL}$ for the buyer, while it is not for the high signal. The complement of $\Theta(q')$ is: $\Theta^C(q') = \{H\}$. Then, in the second step, given $\Theta^C(q') = \{H\}$, the lowest payoff that the buyer can obtain by deviating to the off-equilibrium stocking level $q'$ is derived under the market belief that the buyer observed a high signal. Hence, if the buyer does observe a high signal, she would deviate to $q'$ since in the first step we have already verified that this off-equilibrium strategy is not dominated by the equilibrium strategy for the buyer with a high signal. Consequently, the equilibrium fails the intuitive criterion. If such a $q'$ does not exist, the equilibrium survives.

In the following, we search for such a $q'$. Assume that the market believes the signal is high when observing a stocking level $q$ different from $q_{PL}$. Then, the market value of the buyer firm follows

$$P(q) = (p - b) \int_{0}^{q} F_H(x) \, dx - (w - b) q - t.$$
To stock $q$, the buyer’s payoff function follows

$$G_H(q) = \beta P(q) + (1 - \beta)\pi^B(q; H)$$

$$= (p - b) \int_0^q \bar{F}_H(x) dx - (w - b) q - t$$

with a high signal and

$$G_{LH}(q) = \beta P(q) + (1 - \beta)\pi^B(q; L)$$

$$= (p - b) \int_0^q (\beta \bar{F}_H(x) + (1 - \beta)\bar{F}_L(x)) dx - (w - b) q - t$$

with a low signal.

Given $F_H(x) \leq F_L(x)$ for any $x \geq 0$ and $F_H(x) < F_L(x)$ for some $x > 0$, $\bar{F}_H(x) \geq \bar{F}_L(x)$ for any $x \geq 0$ and $\bar{F}_H(x) > \bar{F}_L(x)$ for some $x > 0$. Thus $G_H(q) > G_{H\bullet}(q_{PL})$ at $q = q_{PL}$. Since $G_H(q)$ is concave, there exists a unique $\hat{q} > q_{PL}$ such that $G_H(\hat{q}) = G_{H\bullet}(q_{PL})$; furthermore, $G_H(q)$ is decreasing at $\hat{q}$.

Now, we verify that $G_{LH}(\hat{q}) < G_{L\bullet}(q_{PL})$. In particular, we have

$$G_{LH}(\hat{q}) = (p - b) \int_0^{\hat{q}} (\beta \bar{F}_H(x) + (1 - \beta)\bar{F}_L(x)) dx - (w - b) \hat{q} - t$$

$$= G_H(\hat{q}) - (p - b) \int_0^{\hat{q}} ((1 - \beta) \bar{F}_H(x) - (1 - \beta)\bar{F}_L(x)) dx$$

and

$$G_{L\bullet}(q_{PL}) = (p - b) \int_0^{q_{PL}} (\beta (\lambda \bar{F}_H(x) + (1 - \lambda)\bar{F}_L(x)) + (1 - \beta) \bar{F}_L(x)) dx - (w - b) q_{PL} - t$$

$$= G_{H\bullet}(q_{PL}) - (p - b) \int_0^{q_{PL}} ((1 - \beta) \bar{F}_H(x) - (1 - \beta)\bar{F}_L(x)) dx.$$

Given $G_H(\hat{q}) = G_{H\bullet}(q_{PL})$, we have

$$G_{L\bullet}(q_{PL}) = G_{LH}(\hat{q}) + (1 - \beta) (p - b) \int_0^{\hat{q}} (\bar{F}_H(x) - \bar{F}_L(x)) dx$$

$$- (1 - \beta) (p - b) \int_0^{q_{PL}} (\bar{F}_H(x) - \bar{F}_L(x)) dx.$$

Since $\hat{q} > q_{PL}$ and $\bar{F}_H(x) \geq \bar{F}_L(x)$ for any $x \geq 0$ and $\bar{F}_H(x) > \bar{F}_L(x)$ for some $x > 0$, we have $G_{L\bullet}(q_{PL}) > G_{LH}(\hat{q})$. 

\[ \text{Supply Chain Performance Under Market Valuation} \]

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Figure 7 Demonstration of the intuitive criterion refinement over the pooling equilibria. The parameters are: \( \beta = 0.4, p = 20, c = 5, w = 8, b = 4, t = 0, \lambda = 0.5, \) and the demand follows the gamma distribution with density 
\[
f_i(x) = \left(\frac{x_i}{\nu_i}\right)^{\kappa_i - 1} e^{-\frac{x_i}{\nu_i}} \text{ for } i \in \{H, L\}, \text{ where } (\kappa_H, \theta_H) = (1.5, 5) \text{ and } (\kappa_L, \theta_L) = (1, 5).
\]

Given both \( G_H(q) \) and \( G_{LH}(q) \) are continuous and \( \frac{dG_H(q)}{dq} > \frac{dG_{LH}(q)}{dq} \) at any \( q \), there must exist an \( \varepsilon \) such that at \( q' = \bar{q} - \varepsilon, G_H(q') > G_{H}\bullet(q_{PL}) \) and \( G_{LH}(q') < G_{L}\bullet(q_{PL}) \). We illustrate this result in Figure 7. Hence, for the low signal, the off-equilibrium strategy \( q' \) is dominated by the equilibrium strategy \( q_{PL} \) for the buyer, while it is not for the high signal. The existence of such a \( q' \) asserts that any pooling equilibrium fails the intuitive criterion.

The above result continues to hold if a menu of two contracts is offered since the above result can be proved corresponding to the contract where the pooling equilibrium arises.

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