Selling to Heterogeneous Strategic Customers with Uncertain Valuations under Returns Policies

We consider a firm selling a fixed amount of inventory to customers who possess uncertain valuations on a product prior to purchase and realize their complete valuations only after purchase. The firm determines the returns policies over two periods; each of which targets for a group of brand loyal customers or regular shoppers. The customers are strategic, taking into account both the product availability risk and the product misfit risk when they decide when to purchase. We identify two effects of returns including the positive effect of delay mitigation and the negative effect of surplus reduction, resulting in no returns to brand loyal customers in the first period and a positive refund to regular shoppers in the second period. The result complements Su (2009)’s finding - returns itself is not beneficial for the firm when faced with homogeneous customers with uncertain valuations. However, when customers have distinct and uncertain valuations, returns offered in a later period mitigates the incentive of loyal customers to delay their purchases. Hence, returns provides the firm with an additional instrument to mitigate the negative consequences of strategic customer behavior. We find that, with returns, the markup can emerge as the optimal pricing policy when the firm holds a high inventory. We also investigate how the benefit of returns over no returns is affected by the firm’s initial inventory level and customer valuation uncertainty.

1 Introduction

Dynamic pricing has become a common tool in the retailing industry to price discriminate and extract higher revenues from customers with distinct product valuations. However, due
to the rapid growth of information technology, customers have become increasingly sophisticated in their purchase decisions. For example, customers may strategize over the timing of their purchases and are referred to as strategic customers. Obviously, their strategic purchase behavior jeopardizes a firm’s profit. To counteract the adverse impact of strategic purchase behavior, several useful approaches have been proposed and studied in the existing literature. For example, a firm can ration inventory to create shortages at a low price to induce early purchases from high-valuation customers at a high price (Liu and van Ryzin 2008). When a firm has a quick response capability, it can eliminate customers’ waiting incentive by reducing the chance of discounting the remaining inventory (Cachon and Swinney 2009). A seller may use an appropriate inventory display format, e.g., displaying one unit of product, to create an increased sense of availability risk, and hence induce purchases at a full price. A posterior price matching policy can discourage strategic delay behavior because customers who have made early purchase are compensated for price difference if the price is marked down later on (Lai, Debo, and Sycara 2010).

One assumption that is commonly made in this stream of literature is that customers precisely know their own valuations on the product. However, in many real-world examples, customers often do not know their exact valuations on the product when they make purchase decisions. Such valuation uncertainty may arise in a number of ways. For example, when customers purchase an innovative or highly fashionable product, they are not sure about their exact values because they have not experienced such a product before. When customers buy products (such as clothing, shoes) via an online channel, the shipped products may not fit for sizes, styles, or their expectations.

Faced with a group of customers who privately possess some initial yet incomplete valuations about the product before purchase and realize the complete valuations only after purchase, a firm can offer returns which allows the customers to return products and get refunds. Hence, a returns policy essentially provides customers with an insurance that alleviates their concerns about the ex ante uncertainty of product value. As a result, returns encourages more purchases from customers whose valuations are less certain. On the other hand, returns is costly for the firm since it has to pay a refund for the returned product. Can
a firm then benefit from an appropriately designed returns policy? Su (2009) answers this question by revealing that the cost of refund outweighs the benefit of demand enhancement and hence returns itself is not a useful tool to generate more profits from homogeneous customers who share the same mean value on the product. Does the result hold when customers have distinct and uncertain valuations? This is the main question to be address in the paper. The focus of our work is to investigate the role of returns in coping with strategic purchase behavior when customers possess heterogeneous and uncertain valuations.

We consider a stylized model in which a firm sells a fixed amount of inventory to a heterogeneous market with two segments of customers: the brand loyal customers and the regular shoppers. All customers are ex ante uncertain about the product value, and brand loyal customers value the product more than regular shoppers in the sense of first-order dominance. To price discriminate the two customer segments, the firm offers two distinct returns policies in sequel, each of which targets a specific customer segment. Each selling policy consists of a selling price and a returns refund. All the customers are strategic and present at the beginning of the selling horizon. They decide to purchase either now or later. Due to limited supply, customers who delay purchases may not obtain the product. Hence, taking shortage risk into account, strategic customers compare expected surpluses of purchasing the product under different returns policies offered in different periods, and choose the one that maximizes their expected surplus. We characterize the firm’s optimal returns policies, and obtain the following main findings.

First, with regard to the question of whether or not offering returns is useful for the firm, we find that the firm should not offer returns to brand loyal customers in the first period, and that the firm should offer positive returns refund to regular shoppers in the second period. The former result of zero refund to the loyal customers is consistent with Su (2009)’s finding, following the insight that returns is ineffective in profit extraction. The latter result that a positive refund is offered to the regular shoppers reveals the strategic role of returns in weakening customers’ strategic waiting behavior. Specifically, a loyal customer values a refund less than a regular shopper because she is less likely to return the product. Consequently, relative to zero refund, a positive refund allows the firm to charge a higher
selling price in the second period so that it is still appealing to the regular shoppers but less so for the loyal customers, thereby reducing the loyal customers’ incentive to wait. In determining the optimal refund level in the second period, the firm needs to balance the tradeoff between the positive effect of weakening the loyal customers’ strategic waiting and the negative effect of extracting less profits from the regular shoppers.

Second, we find that the returns might reverse the order of the two prices offered in two periods. Without returns, it is well known that the firm’s optimal pricing policy takes the form of markdown. In contrast, with returns, we find that the markup pricing is optimal when returns offered to regular shoppers in the second period is sufficiently generous, a scenario that occurs when there is a stronger need to diminish the incentive of strategic delay behavior of brand loyal customers. For example, for a sufficiently large amount of inventory, a threat of “sold out” is minimal and hence loyal customers have a strong incentive to delay purchase. Therefore, a generous refund is required to thwart the purchase delay of loyal customers so that the selling price in the second period may exceed the first-period selling price.

Third, we identify and explore two drivers that influence the benefits from using returns relative to no returns for the firm. The first driver is the inventory level. We show that the firm gains more profits under returns compared with no returns as the inventory level increases. The intuition is that a higher inventory level reduces the shortage risk and thus intensifies the loyal customers’ delay purchase incentive, and hence there is a stronger need for the firm to offer distinct returns refunds to counteract such waiting behavior. The second driver is the customers’ ex ante uncertainty. Contrary to the conventional wisdom that the firm benefits from a reduction in customer valuation uncertainty, we show that the firm is worse off when the customer valuation uncertainty is smaller.

2 Literature Review

This paper is closely related to the stream of papers on strategic customer behavior in the operations management literature, especially those focusing on strategic waiting behavior and its impact on a firm’s pricing and inventory decisions. Strategic waiting behavior weak-
ens a firm’s ability to use intertemporal price discrimination to extract more surpluses from customers. This result has been revealed by Aviv and Pazgal (2008), Zhang and Cooper (2008), and Cachon and Swinney (2009), to name a few. To counteract such an adverse consequence of strategic waiting behavior, several mechanisms and approaches are investigated under a variety of selling strategies. For example, Aviv and Pazgal (2008) examine the effectiveness of two classes of pricing strategies - inventory-contingent markdown and pre-announced fixed discount - in dealing with strategic waiting behavior. Liu and van Ryzin (2008) show that capacity rationing can be effectively used to mitigate the negative effect of strategic customer behavior because a shortage risk created by rationing induces early purchases at a higher price. Yin et al. (2009) find that a firm’s in-store display format can influence customers’ perception of availability risk and thus their decision on buying now or later. They find that an appropriately designed display format such as display one unit is generally useful to discourage strategic customers from waiting for price discount. Lai, Debo and Sycara (2010) establish that a posterior price matching policy can eliminate the waiting incentive of strategic customers and it is therefore especially effective for a market with a large number of strategic customers whose valuations decline moderately over time. Cachon and Swinney (2009) reveal that a quick response empowering the firm an ability to better match supply and demand leads to a low level of clearance sales, and hence reduces the delay incentive of strategic customers. Li and Zhang (2013) investigate the pre-order, a strategy used to obtain advance demand information so that improve product availability in the regular selling season. Because an increased product availability enhances the delay incentive of strategic customers, the value of the pre-order strategy is reduced in the presence of strategic customers. For a comprehensive review of the operations literature on strategic customer behavior, please refer to Shen and Su (2007) and Netessine and Tang (2009).

A common assumption in this stream of literature is that customers know their exact valuations before purchase. We relax this assumption by allowing customers to have uncertain valuations when they make purchase decisions. We find that a returns policy is useful in mitigating the customers’ strategic waiting incentive, which has not been investigated in the existing literature. It has been well established that the firm’s optimal pricing policy takes
the form of markdown in the presence of strategic customers. In contrast, we show that, under returns, the optimal pricing policy can be markup. The result of the markup policy being optimal is also found in Su (2007) but with a distinct reason. The firm considered in Su (2007) should increase the price over time because the high-valuation customers are more patient than low-valuation customers.

The work by Su (2009) is closely related to ours. He shows that the cost of refund outweighs the benefit of the increased customers’ willingness to pay and returns is thus not beneficial for the firm when faced with homogeneous strategic customers. Our work differs from his mainly in heterogeneity of customers. Su studies a homogeneous market in which all customers have the same mean value on the product, although their realized valuations can be different. In contrast, we consider a market consisting of heterogeneous customers who have distinct expected valuations. We show that the firm can benefit from returns that targets a group of selected customers. Particularly, no returns is offered to loyal customers who have higher valuations on average; the result is consistent with Su’s finding. However, returns should be offered to regular shoppers who have lower mean valuations. The purpose of offering returns to regular shoppers is to thwart inefficient waiting of loyal customers. Hence, our paper complements Su’s in that we reveal the role of returns in mitigating the adverse effects of strategic waiting behavior when customers possess heterogeneous and uncertain valuations.

When customers have uncertain valuations, Swinney (2011) investigates the impact of strategic purchase behavior on the value of quick response, and shows that quick response strengthens customers’ incentive to wait since they can learn more information about the product value with an increased product availability (compared with no adoption of quick response). In his paper, the valuation uncertainty is resolved over time and thus customers become well informed about their valuations in later periods before purchase. However, customers in our work can obtain the exact valuations only after purchase, the case when uncertainty comes from some hidden product attributes, and the product value is realized only after experiencing it.

As customers have distinct valuations which are not known by the firm, the firm faces
a screening problem of tailoring different selling policies to different groups of customers. In this sense, our work is also related to the paper by Courty and Li (2000), who show that the optimal selling strategy is to simultaneously offer a menu of returns contracts. The major difference between their work and ours is that they assume unlimited inventory while we consider limited inventory. With limited inventory and uncertain demand, product availability is not guaranteed for each customer. Hence, when to offer different returns policies that target different groups of customers becomes an important decision problem for the firm.

3 Model

Consider a firm that sells to a market consisting of customers with heterogeneous and uncertain valuations on the product. Specifically, there are two types of customers: brand loyal customers and regular shoppers. The type of customers is indexed by \( \theta, \theta \in \{L, S\} \); and the type \( L \) refers to brand loyal customers while the type \( S \) refers to regular shoppers. The customer of type \( \theta \) has an uncertain valuation on the product, denoted by \( v_\theta \), following a distribution function \( F_\theta \) over \([v, \bar{v}]\). We assume that the valuation of the brand loyal customer, \( v_L \), and the valuation of the regular shopper, \( v_S \), are independent. Furthermore, \( v_L \) first-order stochastically dominates \( v_S \); that is, \( F_L(v) < F_S(v) \) for all \( v \in [v, \bar{v}] \). This implies that the brand loyal customers, on average, value more than the regular shoppers; that is, \( E[v_L] > E[v_S] \). Before purchase, customers do not know their exact valuations on the product, which are realized only after purchase. Each type of customers has a random population size \( N_\theta \), and \( N_L \) and \( N_S \) are assumed to be independent, following distributions \( G_L \) and \( G_S \) respectively. Hence, in our model, there is uncertainty in both demand size and customer valuations.

A firm has a fixed amount of inventory, \( Q \) units of products, to sell in a finite time horizon. Products cannot be replenished during the selling horizon. If customer demand exceeds the available supply, demand is lost and goodwill cost is incurred. Otherwise, the leftover inventory is salvaged at the end of the selling horizon. Without loss of generality,
both goodwill cost and salvage value are normalized to zero. A returns policy is denoted by \((p, b)\), whereby the customer can purchase the product by paying price \(p\) with the option of returning the product back to the firm and receiving refund \(b\). For a customer with uncertain valuation, the refund is valuable because it essentially provides a lower bound on the value that the customer places on the product. To see this, in case if the customer’s realized valuation is less than \(b\), she still gets \(b\) by returning the product. However, returns is costly to the firm because the product returned close to or after the season essentially becomes the leftover inventory which has zero salvage value. Even if the product is returned during the selling period, reselling it requires both recovery time and cost. For simplicity, we assume that the returned product has zero value to the firm. Since there are two types of customers, with a finite inventory, the firm offers two returns policies in sequel, each of which is intended for one type of customers. In other words, the firm offers return policy \((p_i, b_i), i = 1, 2\), over a two-period selling horizon. The returns policies are preannounced and credibly committed by the firm.

The customers behave strategically; they compare their expected surpluses of purchasing the product in different periods and choose the one that has a higher expected surplus. A customer of type \(\theta\) arriving in the first period, should she find the product available, decides whether to purchase now or wait for the second period. If she purchases in the first period under policy \((p_1, b_1)\), her expected surplus is \(E_{v_\theta} \max\{v_\theta, b_1\} - p_1\). If she waits to purchase in the second period, she may not be able to obtain a product due to limited supply. The probability that she can obtain the product in the second period, denoted by \(F\). Hence, the customer earns an expected surplus of \(F(E_{v_\theta} \max\{v_\theta, b_2\} - p_2)\) when she waits to buy in the second period. Several expressions have been used in literature to determine \(F\). For example, under the uniform allocation rule, \(F\) is the ratio of expected sales and expected demand in the second period; under the priority rule, \(F\) is equal to the complementary probability of the event of stockouts at the beginning of second period. We note that our qualitative results hold for all these commonly used forms of \(F\). Nevertheless, for expository convenience, we
adopt the uniform allocation rule with

$$F = E_{N_\theta, N_\tilde{\theta}} \min(N_\theta, (Q - N_\theta)^+) / EN_\tilde{\theta}$$

when the firm serves type-\( \theta \) customers in the first period and type-\( \tilde{\theta} \) customers in the second period.

4 Optimal Dynamic Pricing without Returns

As a benchmark, we characterize the firm’s optimal pricing decisions without returns. The firm faces two options. One is to induce the brand loyal customers to purchase in the first period and the regular shoppers to purchase in the second period. The other option is to reverse the sequence. Let \( \theta \) (\( \tilde{\theta} \)) be the type of customers who purchase in the first (second) period. The firm’s expected profit in the first period is \( p_1 E_{N_\theta} \min(N_\theta, Q) \) and its expected profit in the second period is \( p_2 E_{N_\theta, N_\tilde{\theta}} \min(N_\tilde{\theta}, (Q - N_\theta)^+) \), where \((Q - N_\theta)^+\) is the remaining inventory available for the second period. Therefore, the firm’s optimal pricing problem can be formulated as follows, denoted by \((P^w)\):

$$\max_{p_1 \geq 0, p_2 \geq 0, \theta, \tilde{\theta} \in \{L, S\}, \theta \neq \tilde{\theta}} \{ p_1 E_{N_\theta} \min(N_\theta, Q) + p_2 E_{N_\theta, N_\tilde{\theta}} \min(N_\tilde{\theta}, (Q - N_\theta)^+) \}$$

s.t. \( Ev_\theta - p_1 \geq F (Ev_\theta - p_2) \), \hspace{1cm} (IC1)

\( F (Ev_{\tilde{\theta}} - p_2) \geq Ev_{\tilde{\theta}} - p_1 \), \hspace{1cm} (IC2)

\( Ev_\theta - p_1 \geq 0 \), \hspace{1cm} (IR1)

\( Ev_{\tilde{\theta}} - p_2 \geq 0 \). \hspace{1cm} (IR2)

Constraint (IC1) implies that a customer of type \( \theta \) earns a larger expected surplus if she buys in the first period than that if she delays her purchase to the second period. Thus, (IC1) ensures that it is in the best interest of the type-\( \theta \) customer to purchase in the first period. Similarly, (IC2) ensures that the type-\( \tilde{\theta} \) customer is better off by purchasing in the
second period. Constraints (IR1) and (IR2) ensure that every customer earns an expected surplus no less than her reservation level which is normalized to zero.

**Proposition 1** Without returns, the optimal price in the first period, denoted by $p_{nr}^1$, is $p_{nr}^1 = Ev_L - F(Ev_L - Ev_S)$; and the optimal price in the second period, denoted by $p_{nr}^2$, is $p_{nr}^2 = Ev_S$. Under the optimal pricing, the brand loyal customers purchase in the first period and the regular shoppers purchase in the second period.

Under the optimal pricing, the firm offers a higher price targeting the brand loyal customers in the first period and marks it down to a lower price that is intended for the regular shoppers in the second period. The firm achieves price discrimination for the two customer segments via the shortage risk associated with the delayed purchase due to the limited inventory and uncertain demand. Under the optimal pricing, the firm is able to fully extract the expected value from the regular shoppers in the second period. However, the price offered in the first period is lower than the expected value of the brand loyal customers (i.e., $p_{nr}^1 \leq Ev_L$). The profit loss, i.e., $Ev_L - p_{nr}^1$, increases in both the fill rate $F$ and the customer heterogeneity measured by $Ev_L - Ev_S$. This is a consequence of the firm’s lowering the first period selling price to cope with the brand loyal customers’ strategic delay purchase behavior, which is strengthened as the product availability improves and as the gap of product valuations of the two segments widens.

**5 Optimal Dynamic Pricing with Returns**

When customers face valuation uncertainty, returns can be used to encourage purchases because returns protects customers against the risk of product misfit. But returns is costly for the firm as it pays a refund for each returned product. Will the firm benefit from offering returns, faced with strategic customers who have heterogeneous and uncertain valuations on the product? To answer this question, we first characterize the firm’s optimal returns policies.
Under a returns policy \((p_1, b_1)\) in the first period and \((p_2, b_2)\) in the second period, a type-\(\theta\) customer obtains the expected surplus of \(-p_1 + E_{v_\theta \max(v_\theta, b_1)}\) if she purchases in the first period. If she delays purchase, she may not be able to obtain the product due to limited inventory, and the probability of obtaining it is \(F\). Therefore, the type-\(\theta\) customer chooses to purchase in the first period if and only if

\[-p_1 + E_{v_\theta \max(v_\theta, b_1)} \geq F(-p_2 + E_{v_\theta \max(v_\theta, b_2)}), \quad \theta \in \{L, S\}.
\]

The firm’s problem is to determine the optimal returns policies \((p_1, b_1)\) and \((p_2, b_2)\) targeting one type of customers in each period so as to maximize its expected profit. Denote the firm’s problem by \((P^*)\), which is formulated as follows:

\[
\begin{align*}
\max_{(p_1, b_1) \geq 0, (p_2, b_2) \geq 0, \theta, \tilde{\theta} \in \{L, S\}, \theta \neq \tilde{\theta}} & \{(p_1 - b_1 F_\theta(b_1)) E_{N_\theta \min(N_\theta, Q)} \\
& + (p_2 - b_2 F_{\tilde{\theta}}(b_2)) E_{N_{\tilde{\theta}} \min(N_{\tilde{\theta}}, (Q - N_\theta)^+)}\} \\
\text{s.t.} & \quad -p_1 + E_{v_\theta \max(v_\theta, b_1)} \geq F(-p_2 + E_{v_\theta \max(v_\theta, b_2)}), \quad \text{(IC1)} \\
& \quad F(-p_2 + E_{v_{\tilde{\theta}} \max(v_{\tilde{\theta}}, b_2)}) \geq p_1 + E_{v_{\tilde{\theta}} \max(v_{\tilde{\theta}}, b_1)}, \quad \text{(IC2)} \\
& \quad -p_1 + E_{v_\theta \max(v_\theta, b_1)} \geq 0, \quad \text{(IR1)} \\
& \quad -p_2 + E_{v_{\tilde{\theta}} \max(v_{\tilde{\theta}}, b_2)} \geq 0. \quad \text{(IR2)}
\end{align*}
\]

The firm’s expected profit consists the profit earned from selling to type \(\theta\) in the first period and that from type \(\tilde{\theta}\) in the second period. Clearly, a type-\(\theta\) (type-\(\tilde{\theta}\)) customers with realized valuations greater than \(b_1\) (\(b_2\)) will keep the product while those with realized valuations below \(b_1\) (\(b_2\)) will return it. Therefore, the probability that a type-\(\theta\) (type-\(\tilde{\theta}\)) customer returns the product is \(F_{\theta}(b_1)\) (\(F_{\tilde{\theta}}(b_2)\)), and consequently, the profit margin earned from selling to a type-\(\theta\) (type-\(\tilde{\theta}\)) customer is \(p_1 - b_1 F_\theta(b_1)\) \((p_2 - b_2 F_{\tilde{\theta}}(b_2))\). Constraints (IC1) and (IR1) ensure that the type-\(\theta\) customers purchase in the first period; Constraints (IC2) and (IR2) ensure that the type-\(\tilde{\theta}\) customers purchase in the second period.
Proposition 2 The optimal returns policies, denoted by \((p^*_1, b^*_1)\) in the first period and \((p^*_2, b^*_2)\) in the second period, are

\[
p^*_1 = Ev_L - F [Ev_L \max(v_L, b^*_2) - Ev_S \max(v_S, b^*_2)],
\]
\[
b^*_1 = 0,
\]
\[
p^*_2 = Ev_S \max(v_S, b^*_2),
\]
\[
b^*_2 = \arg \max_{b_2 \geq 0} \{ [Ev_S \max(v_S, b_2) - b_2 F_S(b_2)] EN_{L,N_S} \min(N_S, (Q - N_L)^+) - F [Ev_L \max(v_L, b_2) - Ev_S \max(v_S, b_2)] EN_L \min(N_L, Q) \},
\]

where \(F = EN_{L,N_S} \min(N_S, (Q - N_L)^+)/EN_S\). Under the optimal returns policies, the brand loyal customers purchase in the first period and the regular shoppers purchase in the second period.

Consistent with the result in the benchmark, it is still in the firm’s best interest to serve the brand loyal customers in the first period and then the regular shoppers in the second period. The intuition is that the firm can make more profits from selling to the brand loyal customers because they have higher (in the first-order dominance sense) valuations than the regular shoppers, and hence the firm should target the brand loyal customers first. This is true regardless of whether or not returns is offered.

Under the optimal returns policy, the firm should not offer any returns refund to the brand loyal customers, i.e., \(b^*_1 = 0\). This result is consistent with Su (2008). It is because offering a positive refund allows the firm to extract positive value from the customer only when her realized valuation is no less than the refund, thereby forgoing the opportunity to extract value from the customer when her realized valuation is lower than the refund but still positive. This is the negative effect of offering returns, which we call the effect of surplus reduction.

Interestingly, the firm should offer positive returns refund to the regular shoppers, despite the negative effect of surplus reduction. This is because a positive effect of offering returns emerges with the presence of strategic delay purchase incentives. Relative to zero refund, a
positive refund in the second period allows the firm to increase the associated selling price to an extent such that the whole sales package is still acceptable to the regular shoppers but less attractive for the loyal customers, because the regular shoppers value more on the returns than the loyal customers. This implies that including a positive refund into the sales package in the second period weakens the loyal customers’ incentive to delay purchase. We call it the effect of delay mitigation. The optimal refund size is thus determined by balancing the tradeoff between the negative effect of surplus reduction and the positive effect of delay mitigation.

Next we turn to the impact of returns on the firm’s pricing strategy. In contrast to the benchmark where the markdown pricing is always optimal, the following proposition shows that both markdown and markup pricing policy can be optimal, depending on the firm’s inventory level.

**Proposition 3** There exist \( Q \) and \( \overline{Q} \) such that \( p_1^r > p_2^r \) for \( Q \leq \overline{Q} \) and \( p_1^r < p_2^r \) for \( Q \geq \overline{Q} \).

When the firm has sufficiently low inventory, the shortage risk is prominent and the loyal customers’ strategic delay incentives are weak, implying that the positive effect of delay mitigation is insignificant and thus the firm should offer a stingy refund in the second period due to the negative effect of surplus reduction. In such a scenario, the stingy refund has little impact on the selling prices, and thus the optimal pricing policy remains to be markdown. In contrast, when the firm has sufficiently high inventory, there is little shortage risk and the loyal customers’ strategic delay incentives are strong. Hence, the positive effect of delay mitigation becomes significant, implying that there is a strong need for the firm to offer a generous refund (driving up the selling price in the second period) to counteract the loyal customers’ delay incentives. The refund can be so generous that it reverses the order of the two selling prices, resulting in the markup pricing being optimal.

An implication of the above proposition is that when the firm has ample starting inventory so that the shortage risk is small, the firm should opt for the markup policy, where a price discount without returns is offered earlier and a regular price with returns is provided later. Such a selling strategy is in contrast with the conventional markdown pricing strategy where
Figure 1: Relationship of optimal pricing policy and inventory level.

A price discount is offered in the later stage of the selling season. The markup pricing is consistent with the practice of early promotion as a strategy to induce early purchases. Our result suggests that with a high starting inventory level, the firm offer price promotion early on, instead of waiting till the later stage of the season to do clearance sales.

The above proposition is silent when the inventory level is intermediate. As a remedy, a numerical study suggests that as the inventory level increases, the firm’s optimal pricing takes the form of markdown and then switches to markup. Figure 1 depicts how the price difference changes in the inventory level for a representative example with the following parameter values: the market size of each type of customers follows a Gamma distribution with mean 100 for both types, and standard deviation 50 and 25 for the loyal customers and regular shoppers respectively. The valuations of the loyal customers and regular shoppers are uniformly distributed over [5, 10] and [3, 8], respectively.
6 Drivers of Benefits of Returns

We have shown that returns is beneficial for the firm because it serves as a strategic tool to more effectively price discriminate customers who have heterogeneous and uncertain valuations on the product. Concerning that there are administrative costs related to returns, it is meaningful to understand to what extent returns policy outperforms no returns. To this end, we identify two drivers and examine how each influences the benefits of returns.

6.1 Inventory Level

Let $\Pi^\text{nr}_\theta$ and $\Pi^\text{r}_\theta$ denote the optimal profit gained from the type-$\theta$ customers without returns and with returns, respectively; $\theta \in \{L, S\}$. Then

$$
\Pi^\text{nr}_L = p^\text{nr}_1 E_{N_L} \min(N_L, Q), \\
\Pi^\text{nr}_S = p^\text{nr}_2 E_{N_L,N_S} \min(N_S,(Q - N_L)^+), \\
\Pi^\text{r}_L = p^\text{r}_1 E_{N_L} \min(N_L, Q), \\
\Pi^\text{r}_S = (p^\text{r}_2 - b^\text{r}_2 F_S(b^\text{r}_2)) E_{N_L,N_S} \min(N_S,(Q - N_L)^+)
$$

where the optimal prices and the optimal refunds are characterized in Proposition 1 and Proposition 2. Further, let $\Pi^\text{nr}$ and $\Pi^\text{r}$ be the firm’s optimal profit gained from both types of customers without returns and with returns, respectively; then $\Pi^\text{nr} = \Pi^\text{nr}_L + \Pi^\text{nr}_S$ and $\Pi^\text{r} = \Pi^\text{r}_L + \Pi^\text{r}_S$.

**Proposition 4** The firm earns lower profits from the regular shoppers with returns than without returns. The opposite is true for the firm’s profits from the brand loyal customers. That is, $\Pi^\text{r}_S \leq \Pi^\text{nr}_S$ and $\Pi^\text{r}_L \geq \Pi^\text{nr}_L$.

The above proposition implies that offering returns to the regular shoppers reduces the profits that the firm can earn from them. This result is due to the negative effect of surplus reduction. However, returns improves the profits that the firm can earn from the brand loyal
customers, because of the positive effect of delay mitigation. Because the firm can adjust the returns refund size to strike a balance of the tradeoff between the two effects, returns improves the firm’s total profits, i.e., $\Pi^r \geq \Pi^{nr}$.

Intuitively, the value of returns for the firm depends on the extent of the positive effect of delay mitigation. As the firm’s starting inventory level $Q$ increases, the shortage risk decreases and the brand loyal customers’ incentive to delay purchase increases, resulting in a stronger need to use returns as a strategic tool to deter delay purchase. Therefore, a higher starting inventory level enhances the positive effect of delay mitigation and improves the value of returns for the firm. This intuition is confirmed by the following analytical result.

**Proposition 5** The firm’s gain in expected profits with returns relative to without returns, i.e., $\Pi^r - \Pi^{nr}$, increases in the inventory level $Q$.

The literature has proposed the inventory rationing (i.e., deliberately installing a low starting inventory level) as a strategic tool to deter strategic customers’ delay purchase behavior. However, for products with lucrative profit margins, inventory rationing can be very costly because of the high cost of demand loss. The above proposition implies that in such situations, offering returns can be a good substitute of costly inventory rationing for the firm to price discriminate customers and extract more values.

### 6.2 Customer Valuation Uncertainty

A key feature in our model is that the customer’s product valuation is uncertain prior to purchase. Such uncertainty might arise due to the customer’s lack of experience and unfamiliarity of some unknown features of the product, which might turn out to be well appreciated or disliked by the customer after purchase. In such scenarios, the firm can potentially reduce customer valuation uncertainty by providing the customer with better access to product information such as in-store sales assistance, on-line peer reviews, free samples, product demonstrations, etc. Therefore, an important question is whether or not the firm can benefit from such valuation uncertainty reduction initiatives. Obviously, under the
no-returns policy, the customer valuation uncertainty has no impact on the firm’s expected profit because the firm’s optimal prices and customers’ purchase decisions depend on the customer product valuation only via the expected value (see Proposition 1). In contrast, under the returns policy, the customer valuation uncertainty should have impacts on the firm’s expected profit because it influences not only the customer’s purchase decision but also the probability of product returns. Intuitively, the smaller valuation uncertainty decreases the chance of customer returning the product, and thus leads to the firm’s lower cost of returns. Therefore, one may conjecture that under the returns policy, the firm should benefit from the reduction in customer valuation uncertainty. However, we show in this subsection that the opposite is true.

Define \( v_{\theta}^{(1)} = \mu_{\theta} + k_1 \varepsilon \) and \( v_{\theta}^{(2)} = \mu_{\theta} + k_2 \varepsilon \) for \( \theta = L, S \), where \( \mu_L > \mu_S, k_1 \leq k_2 \), and \( \varepsilon \) is a random variable with zero mean and a finite support such that \( v_{\theta}^{(1)} \) and \( v_{\theta}^{(2)} \) are nonnegative. Clearly, the customers with valuation \( v_{L}^{(1)} \) (for the brand loyal customer) and \( v_{S}^{(1)} \) (for the regular shopper) have lower valuation uncertainty than those with \( v_{L}^{(2)} \) and \( v_{S}^{(2)} \). Let the firm’s expected profit under the optimal returns policy be \( \Pi^r(k_1) \) (\( \Pi^r(k_2) \)) when the customer valuation uncertainty is low (high).

**Proposition 6** The firm’s expected profit under the optimal returns policy is lower when the customer valuation uncertainty is smaller. That is, \( \Pi^r(k_1) \leq \Pi^r(k_2) \).

Proposition 6 provides a sharp result that the firm is hurt by a reduction in the customer valuation uncertainty. The intuition is provided as follows. When the regular shoppers are more uncertain about the product value and thus value more on returns, the firm can increase the selling price in the second period so that the whole sales package is still attractive to the regular shoppers. In doing so, it can make the sales package in the second period even less appealing to the brand loyal customers because the incremental value they place on the returns due to the increase of valuation uncertainty is less than that of the regular shoppers. This implies that the increase in valuation uncertainty for both the loyal customers and the regular shoppers strengthens the returns’ positive effect of delay mitigation, and hence benefits the firm.
The above finding has an implication that the firm is actually worse off by providing information or assistance to better inform its customers about the product features so as to reduce valuation uncertainty, even if such information provision is costless to the firm. However, this does not mean that information provision is useless. It is straightforward to show that if such information provision is targeted only to the loyal customers, then the firm is better off. Therefore, our finding suggests that the firm should provide product information to a particular customer segment, not to every segment.

7 Conclusion

Our work reveals the behavioral role that returns plays in mitigating the negative consequences of strategic customer behavior. We identify two effects of returns in dealing with heterogeneous customers who are strategic and uncertain about the product value prior to purchase: the negative effect of surplus reduction and the positive effect of delay mitigation. Driven by the tradeoff of these two effects, the firm should offer a positive refund to regular shoppers in the second period but no refund to brand loyal customers in the first period. We find that under returns, the markup pricing policy can be optimal, implying that the firm should offer early promotion instead of waiting to do clearance sales later on when the firm has a high starting inventory level. We also identify two drivers of benefits of using returns: the inventory level and the customer valuation uncertainty. We find that returns is more beneficial when the inventory level is higher. While a reduction in the customer valuation uncertainty has no impact on the firm’s profit under no returns, it hurts the firm with returns.

Our work can be extended in several directions. The current model assumes that the returned product cannot be resold to customers. When product resale is allowed, specifically, the products sold and returned during the first period can be put into sales in the second period, Su (2009) suggests that the optimal refund be equal to the resell value of the returned product when customers are homogeneous. However, with heterogeneous customers, the optimal refund in the first period should be lower than the resell value. The reason is that
the returned products are added up to the firm’s remaining inventory and thus increase the product availability in a later period, strengthening the incentive for loyal customers to delay purchases. Therefore, to counteract the strategic delay behavior, the firm should lower the refund in the first period. Another extension would allow the inventory to be determined endogenously. Because of its delay mitigation effect, returns can mitigate the firm’s downward distortion in inventory ordering and thus improves the supply/demand mismatch.

References


**Appendix**

**Proof of Proposition 1.** We solve the firm’s problem \((P_{nr})\) in two cases. Case 1. The firm serves the loyal customers in the first period and the regular shoppers in the second period, i.e., \(\theta = L\) and \(\hat{\theta} = S\). Case 2. The firm serves the regular shoppers in the first period and the loyal customers in the second period, i.e., \(\theta = S\) and \(\hat{\theta} = L\). We first derive the closed form expressions for the optimal prices in case 1, and then show that the firm earns higher expected profits in case 1 than in case 2.

Consider the firm’s problem \((P_{nr})\) with \(\theta = L\) and \(\hat{\theta} = S\). Let the optimal prices be \(p_{1nr}^{nr1}\) and \(p_{2nr}^{nr1}\), and the firm’s optimal expected profit be \(\Pi_{nr1}\). We solve the firm’s problem in the following three steps. First, it follows from the model assumption \(Ev_L \geq Ev_S\) that (IC1) and (IR2) are sufficient to infer (IR1), implying that we can remove (IR1) from the constraints without loss of optimality. Second, (IR2) must be binding at the optimal solution, because otherwise the firm’s objective value can be improved by increasing \(p_1\) and \(p_2\) by a sufficiently small amount without violating the constraints (IC1), (IC2), and (IR2). The binding (IR2) constraint leads to \(p_{2nr}^{nr1} = Ev_S\). Third, (IC1) must be binding at the optimal solution, because otherwise the objective value can be improved by increasing \(p_1\) by a sufficiently small amount without violating the constraints (IC1), (IC2), and (IR2). The binding constraint
(IC1) leads to \( p_{1}^{nr} = Ev_{L} - F (Ev_{L} - Ev_{S}) \). Under the optimal prices \( p_{1}^{nr} \) and \( p_{2}^{nr} \), the firm’s expected profit is

\[
\Pi^{nr} = p_{1}^{nr} E_{N_{L}} \min(N_{L}, Q) + p_{2}^{nr} E_{N_{L}, N_{S}} \min(N_{S}, (Q - N_{L})^{+}). \tag{1}
\]

Next we turn the firm’s problem (Pnr) with \( \theta = S \) and \( \tilde{\theta} = L \). Let the optimal prices be \( p_{1}^{nr} \) and \( p_{2}^{nr} \), and the firm’s optimal expected profit be \( \Pi^{nr} \). We will derive an upper bound on \( \Pi^{nr} \), and show that this upper bound is no larger than \( \Pi^{nr} \). It follows from (IR1) that \( p_{1}^{nr} \leq Ev_{S} = p_{2}^{nr} \). This, together with (IC2), implies that \( p_{2}^{nr} \leq Ev_{L} - (Ev_{L} - Ev_{S}) / F \). Because \( F \leq 1 \), we have \( p_{2}^{nr} \leq Ev_{L} - F (Ev_{L} - Ev_{S}) = p_{1}^{nr} \). Hence, we have the following upper bound on \( \Pi^{nr} \):

\[
\Pi^{nr} = p_{1}^{nr} E_{N_{S}} \min(N_{S}, Q) + p_{2}^{nr} E_{N_{L}, N_{S}} \min(N_{L}, (Q - N_{S})^{+}), \tag{2}
\]

By (1) and (2), we have

\[
\Pi^{nr} - \Pi^{nr} \geq p_{1}^{nr} [E_{N_{L}} \min(N_{L}, Q) - E_{N_{L}, N_{S}} \min(N_{L}, (Q - N_{S})^{+})]
- p_{2}^{nr} [E_{N_{S}} \min(N_{S}, Q) - E_{N_{L}, N_{S}} \min(N_{S}, (Q - N_{L})^{+})]
\geq p_{2}^{nr} [E_{N_{L}} \min(N_{L}, Q) - E_{N_{L}, N_{S}} \min(N_{L}, (Q - N_{S})^{+})]
- p_{2}^{nr} [E_{N_{S}} \min(N_{S}, Q) - E_{N_{L}, N_{S}} \min(N_{S}, (Q - N_{L})^{+})]
= 0
\]

where the second inequality follows from the fact that \( p_{1}^{nr} \geq p_{2}^{nr} \) and \( E_{N_{L}} \min(N_{L}, Q) - E_{N_{L}, N_{S}} \min(N_{L}, (Q - N_{S})^{+}) \geq 0 \), and the last equality holds because

\[
E_{N_{L}} \min(N_{L}, Q) + E_{N_{L}, N_{S}} \min(N_{S}, (Q - N_{L})^{+}) = E_{N_{L}, N_{S}} \min(N_{L} + N_{S}, Q)
\]

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\[ E_{N_S} \min(N_S, Q) + E_{N_L, N_S} \min(N_L, (Q - N_S)^+) = E_{N_L, N_S} \min(N_L + N_S, Q). \]

Therefore, the firm is better off by serving the loyal customers first, under which the optimal prices are \( p_{1r}^{nr} = p_{1r}^{nr1} = Ev_L - F(Ev_L - Ev_S) \) and \( p_{2r}^{nr} = p_{2r}^{nr1} = Ev_S \). This completes the proof.

**Proof of Proposition 2.** We solve the firm’s problem \((P^r)\) in two cases. Case 1. The firm serves the loyal customers in the first period and the regular shoppers in the second period, i.e., \( \theta = L \) and \( \hat{\theta} = S \). Case 2. The firm serves the regular shoppers in the first period and the loyal customers in the second period, i.e., \( \theta = S \) and \( \hat{\theta} = L \). We first derive the closed form expressions for the optimal prices in case 1, and then show that the firm earns higher expected profits in case 1 than in case 2.

Consider the firm’s problem \((P^r)\) with \( \theta = L \) and \( \hat{\theta} = S \). Let the optimal returns policies be \((p_1^{r1}, b_1^{r1})\) and \((p_2^{r1}, b_2^{r1})\), and the firm’s optimal expected profit be \( \Pi^{r1} \). Using the same arguments for the case 1 in the proof of Proposition 1, we can show that both the constraints (IC1) and (IR2) are binding. This leads to

\[ p_2 = Ev_S \max(v_S, b_2) \]  
(3)

and

\[ p_1 = Ev_L \max(v_L, b_1) - F \max(Ev_L, b_2) - Ev_S \max(v_S, b_2). \]  
(4)

Substituting \( p_1 \) and \( p_2 \) by the right hand side of (3) and (4), respectively, we can rewrite the firm’s objective function as

\[
\begin{align*}
\max_{b_1 \geq 0, b_2 \geq 0} & \left\{ \left[ Ev_L \max(v_L, b_1) - b_1 F_L(b_1) - F \max(Ev_L, b_2) ight. \\
& \left. - Ev_S \max(v_S, b_2) \right] \right\} E_{N_L} \min(N_L, Q) \\
& + \left[ Ev_S \max(v_S, b_2) - b_2 F_S(b_2) \right] E_{N_L, N_S} \min(N_S, (Q - N_L)^+),
\end{align*}
\]
implying that the optimal returns refunds are $b_{r1} = 0$ and

$$b_{r2} = \arg \max_{b_2 \geq 0} \{ [E_{v_s} \max(v_S, b_2) - b_2 F_S(b_2)] E_{N_L, N_S} \min(N_S, (Q - N_L)^+) - F [E_{v_L} \max(v_L, b_2) - E_{v_s} \max(v_S, b_2)] E_{N_L} \min(N_L, Q) \}.$$  

This, together with (3) and (4), determines $p_{r1}^2$ and $p_{r2}^2$. It is verifiable that the solution \{(p_{r1}^2, b_{r1}^1), (p_{r2}^2, b_{r2}^1)\} satisfies the constraint (IC2), and thus is the optimal solution to the problem \(P^r\) with $\theta = L$ and $\tilde{\theta} = S$.

Next we turn the problem \(P^r\) with $\theta = S$ and $\tilde{\theta} = L$. Let the optimal returns policies be \((p_{r1}^r, b_{r1}^r)\) and \((p_{r2}^r, b_{r2}^r)\), and the firm’s optimal expected profit be $\Pi^r$. Following the same arguments of case 2 in the proof of Proposition 1, we have

$$p_1 \leq E_{v_s} \max(v_S, b_1)$$

and

$$p_2 \leq E_{v_L} \max(v_L, b_2) - F [E_{v_L} \max(v_L, b_1) - E_{v_s} \max(v_S, b_1)].$$

Hence, we have the following upper bound on $\Pi^r$:

$$\Pi^r \leq \max_{b_1 \geq 0, b_2 \geq 0} \{ [E_{v_s} \max(v_S, b_1) - b_1 F_S(b_1)] E_{N_S} \min(N_S, Q) + [E_{v_L} \max(v_L, b_2) - b_2 F_L(b_2)$$

$$- F [E_{v_L} \max(v_L, b_1) - E_{v_s} \max(v_S, b_1)] E_{N_L, N_S} \min(N_L, (Q - N_S)^+)\}$$

$$\leq \max_{b_1 \geq 0} \{ [E_{v_s} \max(v_S, b_1) - b_1 F_S(b_1)] E_{N_S} \min(N_S, Q)$$

$$+ [E_{v_L} - F [E_{v_L} \max(v_L, b_1) - E_{v_s} \max(v_S, b_1)] E_{N_L, N_S} \min(N_L, (Q - N_S)^+)\}$$

$$= [E_{v_s} \max(v_S, b_{r1}^2) - b_{r1}^2 F_S(b_{r1}^2)] E_{N_S} \min(N_S, Q)$$

$$+ [E_{v_L} - F [E_{v_L} \max(v_L, b_{r1}^2) - E_{v_s} \max(v_S, b_{r1}^2)] E_{N_L, N_S} \min(N_L, (Q - N_S)^+)\}.$$
Recall that

\[ \Pi^1 = \max_{b_2 \geq 0} \{ [Ev_L - F [Ev_L \max(v_L, b_2) - Ev_s \max(v_S, b_2)]]E_{N_L} \min(N_L, Q) + [Ev_s \max(v_S, b_2) - b_2F_S(b_2)]E_{N_L,N_S} \min(N_S, (Q - N_L)^+) \} \]

This implies that there exists \( v \) such that \( Ev \max(v_S, b_1^2) - b_1^2F_S(b_1^2)]E_{N_L,N_S} \min(N_S, (Q - N_L)^+) \)

Hence,

\[ \Pi^1 - \Pi^2 \geq [Ev_L - F [Ev_L \max(v_L, b_1^2) - Ev_s \max(v_S, b_1^2)]][E_{N_L} \min(N_L, Q) - E_{N_L,N_S} \min(N_S, (Q - N_L)^+)] \]

where the second inequality is due to

\[ Ev_L - [Ev_s \max(v_S, b_1^2) - b_1^2F_S(b_1^2)] \geq Ev_L \max(v_L, b_1^2) - b_1^2F_L(b_1^2) - [Ev_s \max(v_S, b_1^2) - b_1^2F_S(b_1^2)] \]

This completes the proof.

**Proof of Proposition 3.** Recall from Proposition 2 that \( p^*_1 = Ev_L - F [Ev_L \max(v_L, b_2^r) - Ev_s \max(v_S, b_2^s)] \) and \( p^*_2 = Ev_s \max(v_S, b_2^s) \). As \( Q \) goes to zero, \( F \) goes to zero. By the definition of \( b_2^r \) in Proposition 2, \( b_2^s \) also goes to zero, implying that \( p^*_1 - p^*_2 \) goes to \( Ev_L - Ev_S > 0 \).

This implies that there exists \( Q \) such that \( p^*_1 > p^*_2 \) for \( Q \leq Q \). As \( Q \) goes to infinity, \( F \) goes
to 1, and thus $p_1^r - p_2^r$ goes to $E_{v_L} - E_{v_S} \max(v_L, b_2^r) < 0$, implying that there exists $\overline{Q}$ such that $p_1^r > p_2^r$ for $Q \geq \overline{Q}$.

**Proof of Proposition 4.** The proposition follows from the definition of $\Pi_S^r$, $\Pi_S^nr$, $\Pi_L^nr$, and $\Pi_L^r$:

$$\Pi_S^r = (p_2^{nr} - b_2^r F_S(b_2^r))E_{N_L,N_S}\min(N_S, (Q - N_L)^+)$$

$$= (E_{v_S} \max(v_S, b_2^r) - b_2^r F_S(b_2^r))E_{N_L,N_S}\min(N_S, (Q - N_L)^+)$$

$$\leq E_{v_S}E_{N_L,N_S}\min(N_S, (Q - N_L)^+)$$

$$= p_2^{nr}E_{N_L,N_S}\min(N_S, (Q - N_L)^+)$$

$$= \Pi_S^{nr},$$

and

$$\Pi_L^{nr} = p_1^{nr}E_{N_L}\min(N_L, Q)$$

$$= [E_{v_L} - F(E_{v_L} - E_{v_S})]E_{N_L}\min(N_L, Q)$$

$$\leq [E_{v_L} - F(E_{v_L} \max(v_L, b_2^r) - E_{v_S} \max(v_S, b_2^r))]E_{N_L}\min(N_L, Q)$$

$$= \Pi_L^{r}. $$
Proof of Proposition 5. By definition of $\Pi^r$ and $\Pi^{ar}$, we have

$$\Pi^r - \Pi^{ar} = [Ev_L - F [Ev_L \max(v_L, b_2^r) - Ev_S \max(v_S, b_2^s)]]E_{N_L \min(N_L, Q)}$$

$$+ [Ev_S \max(v_S, b_2^s) - b_2^s F_S(b_2^s)]E_{N_L, N_S \min(N_S, (Q - N_L)^+)}$$

$$- [Ev_L - F [Ev_L - Ev_S]]]E_{N_L \min(N_L, Q)} - Ev_S E_{N_L, N_S \min(N_S, (Q - N_L)^+)}$$

$$= F E_{N_L \min(N_L, Q)}[Ev_L - Ev_S - [Ev_L \max(v_L, b_2^r) - Ev_S \max(v_S, b_2^s)]]$$

$$- [Ev_S - [Ev_S \max(v_S, b_2^s) - b_2^s F_S(b_2^s)]]E_{N_L, N_S \min(N_S, (Q - N_L)^+)}$$

$$= E_{N_L, N_S \min(N_S, (Q - N_L)^+)}((E_{N_L \min(N_L, Q)} / E_{N_S})[Ev_L - Ev_S]$$

$$- [Ev_L \max(v_L, b_2^r) - Ev_S \max(v_S, b_2^s)] - [Ev_S - [Ev_S \max(v_S, b_2^s) - b_2^s F_S(b_2^s)]])$$

which increases in $Q$ because both $E_{N_L, N_S \min(N_S, (Q - N_L)^+)}$ and $E_{N_L, N_S \min(N_S, (Q - N_L)^+)}$ increase in $Q$.

Proof of Proposition 6. Let the firm’s expected profit under the optimal returns policy be $\Pi^r(k)$ when the product valuations of the loyal customers and of the regular shoppers are given by $v_L = \mu_L + k\varepsilon$ and $v_S = \mu_S + k\varepsilon$. Let $g$ and $G$ be the density and distribution function of $\varepsilon$. Let $b(k)$ be the optimal returns refund in the second period. It follows from the definition of $\Pi^r(k)$ that

$$\Pi^r(k) = [\mu_L - F [E\varepsilon \max(\mu_L + k\varepsilon, b(k)) - E\varepsilon \max(\mu_S + k\varepsilon, b(k))]]E_{N_L \min(N_L, Q)}$$

$$+ [E\varepsilon \max(\mu_S + k\varepsilon, b(k)) - b(k)G(\frac{b(k) - \mu_S}{k})]E_{N_L, N_S \min(N_S, (Q - N_L)^+)}.$$
Taking the derivative of $\Pi^\ast(k)$ with respect to $k$, we have

$$
\frac{\partial \Pi^\ast(k)}{\partial k} = \left[ \int_{b(k) - \mu_S}^{+\infty} \frac{\varepsilon g(\varepsilon)}{k} d\varepsilon - \int_{b(k) - \mu_L}^{+\infty} \frac{\varepsilon g(\varepsilon)}{k} d\varepsilon \right] F E_{NL \min(N_L, Q)} \\
+ \left[ \frac{b(k) - \mu_S}{k} b(k) g\left(\frac{b(k) - \mu_S}{k}\right) + \int_{b(k) - \mu_S}^{+\infty} \frac{\varepsilon g(\varepsilon)}{k} d\varepsilon \right] E_{NL, N_S \min(N_S, (Q - N_L)^+)} \\
\geq 0.
$$

It suffices to prove that $\frac{\partial \Pi^\ast(k)}{\partial k} \geq 0$. Because $b(k)$ is the maximizer over $[0, +\infty)$, it follows from the first-order necessary condition that

$$
\left[ G\left(\frac{b(k) - \mu_S}{k}\right) - G\left(\frac{b(k) - \mu_L}{k}\right) \right] F E_{NL \min(N_L, Q)} \\
\leq \frac{b(k) - \mu_S}{k} b(k) g\left(\frac{b(k) - \mu_S}{k}\right) E_{NL, N_S \min(N_S, (Q - N_L)^+)}.
$$

By (5) and (6), we have

$$
\frac{\partial \Pi^\ast(k)}{\partial k} \\
\geq \left[ - \int_{b(k) - \mu_L}^{b(k) - \mu_S} \frac{\varepsilon g(\varepsilon)}{k} d\varepsilon + \frac{b(k) - \mu_S}{k} \left[ G\left(\frac{b(k) - \mu_S}{k}\right) - G\left(\frac{b(k) - \mu_L}{k}\right) \right] \right] F E_{NL \min(N_L, Q)} \\
+ \int_{b(k) - \mu_S}^{+\infty} \frac{\varepsilon g(\varepsilon)}{k} d\varepsilon \] E_{NL, N_S \min(N_S, (Q - N_L)^+)} \\
= \left[ \int_{b(k) - \mu_S}^{b(k) - \mu_L} \left[ \frac{b(k) - \mu_S}{k} - \varepsilon \right] g(\varepsilon) d\varepsilon \right] F E_{NL \min(N_L, Q)} \\
+ \int_{b(k) - \mu_S}^{+\infty} \frac{\varepsilon g(\varepsilon)}{k} d\varepsilon \] E_{NL, N_S \min(N_S, (Q - N_L)^+)} \\
\geq 0.
$$

where the last inequality holds because $\int_{b(k) - \mu_L}^{b(k) - \mu_S} \left[ \frac{b(k) - \mu_S}{k} - \varepsilon \right] g(\varepsilon) d\varepsilon \geq 0$ and $\int_{b(k) - \mu_S}^{+\infty} \frac{\varepsilon g(\varepsilon)}{k} d\varepsilon \geq 0$ (since $E \varepsilon = 0$).