

## Yield Calculations for Treasury Bills

William L. Silber

### Question

Suppose you could buy a 91-day T-bill at an asked price of \$98 per \$100 face value and you could sell to the dealer at a bid price of \$97.95 per \$100 face value. What are the quotation conventions on this bill and how is the yield calculated? What is the best measure of the yield on a T-bill?

### Answer

1) This T-bill would be listed in a table as follows:

Days to Maturity	Bid	Ask	Ask Yield
91	8.11	7.91	8.186

2) The ask yield in the last column is the bond yield equivalent (b.y.e) of this T-bill. This is the yield (assuming simple interest) if you bought the bill at the ask price of 98 per 100 face value and held it for the full 91 days.

$$B.Y.E. = \left( \frac{F - P}{P} \right) / t$$

F = Face Value (=\$100)

P = Price Paid

t = Fraction of a year

In our case,

$$\begin{aligned} B.Y.E. &= \frac{100 - 98}{98} / \left( \frac{91}{365} \right) \\ &= .08186 = 8.186\% \end{aligned}$$

NB: This formula is simple interest because it comes from:

$$P(1 + rt) = F$$

which can be solved for r as

$$r = \left( \frac{F - P}{P} \right) / t$$

3. The 7.91 under the word Ask in the table comes from the discount rate calculated on the bill. The discount rate is defined as:

$$\text{discount rate} = \left( \frac{F - P}{F} \right) / \left( \frac{x}{360} \right)$$

where X = days to maturity

In our case (using the ask price):

$$\begin{aligned} (\text{ask}) \text{ discount rate} &= \left( \frac{100 - 98}{100} \right) / \left( \frac{91}{360} \right) \\ &= .0791 = 7.91\% \end{aligned}$$

4) The 8.11 in the table under the word bid uses the same discount rate calculation as above except it uses the bid price (=97.95) in the formula

$$\begin{aligned} (\text{bid}) \text{ discount rate} &= \left( \frac{100 - 97.95}{100} \right) / \left( \frac{91}{360} \right) \\ &= .0811 = 8.11\% \end{aligned}$$

5) Note that the (ask) discount rate will always be lower than the ask yield based on the b.y.e. formula because F appears in the denominator of the discount rate formula while P is in the denominator of the b.y.e. formula (and F>P as long as yields are positive). In addition, 360<365 in the year part of the formulas and those numbers wind up in the numerator.

6) Of these calculations, the best measure of yield earned when buying a T-bill is the b.y.e. since it uses P as the base rather than F (and because 365 is correct). Why does the discount rate calculation exist? Because it is a shorthand calculation that was easier before hand calculators existed. It allowed people to translate price into yield quickly. In fact, this tradition is perpetuated by dealers who quote T-bills in discount

rates rather than prices. Thus our T-bill in the table is quoted as 8.11 bid, offered at 7.91.

7) The effective annual rate on this bill would annualize the b.y.e. of 8.18% (which uses simple interest) using the familiar formula:

$$EAR = \left( 1 + \frac{\text{quoted rate}}{n} \right)^n - 1$$

where n = number of compounding periods per year.

$$\begin{aligned} EAR &= \left( 1 + \frac{.08186}{365/91} \right)^{365/91} - 1 \\ &= .0844 = 8.44\% \end{aligned}$$