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Foundations of Finance (COR1-GB.2311)

Law of One Price Arbitrage Examples

1. CATs vs. TIGRs

Data:
(a) One YR CAT is priced at $94.34 per $100 (YTM = 6%)
(b) One YR TIGR is priced at $95.238 per $100 (YTM = 5%)
(c) Fee charged (earned) for borrowing (lending) TIGRs for 1YR is $.05 per $100 (due at the end of the year). CATS have no borrowing or lending fee

Questions:
(1) Can you set up a transaction that will yield a riskless profit, assuming the cost of transacting is zero (or included in the prices above) and that default risk by market participants is zero (everyone does what they promise to do)?
(2) Will this arbitrage push down the price of TIGRs and push up the price of CATs until the yields on the two securities are the same?

Suggestion: Keep track of the cash flows.

Set up the following portfolio today:

	Cash Flows
(1) Buy 1 CAT	-$94.34
(2) Sell short 1 TIGR	+$95.238
(3) Borrow 1 TIGR from Chase and deliver to the person you sold the TIGR to (you must deposit the CAT as collateral with Chase to keep them happy)	0
Net cash flow today	+$.898

At the end of one year:

(1) Retrieve CAT from Chase	0
(2) Redeem CAT (at face value) from US Treas	+$100.00
(3) Deliver face value of TIGR to Chase	- $100.00
(4) Pay fee to Chase	- $ .05
Net cash flow at end of year	-$ .05

Net profit: $.898 - .05 = $.848 (this ignores interest for one year earned on $.898)

Recommendation: Do this as often as you can because it is riskless (no matter what happens to prices of CATs and TIGRs during the year you will earn $.848 per $100).

Outcome: Buying of CAT drives up the price of CAT and selling of TIGR drives down TIGR's price until the price of TIGR is $.05 above CAT's prices to reflect he $.05 per $100 borrowing fee attached to the short sale of TIGR.

Lesson: Securities with identical cash flows must sell for the same price. In this case the cash flows are identical except for the $.05 borrowing fee for TIGR, thus the two yields will be nearly the same.

What Can Go Wrong: Unless you arrange to borrow the TIGR for one year, Chase might demand that you return the TIGR before the end of the year. You will then have to sell the CAT and buy a TIGR in the market and the prices you transact at might not yield a profit.

Extensions: What would happen if both CATs and TIGRs had a $.05 borrowing and lending fee? In this case, after buying the CAT you would lend it out. This would generate an additional $.05 cash flow. Thus, the profit would be $.898 plus and minus $.05 so the arbitrage would go on until the prices of CATs and TIGRs were identical. The same would be true, of course, if the borrowing and lending fees were zero for both.

2. U.S. Treasuries (denominated in US$) vs. Japanese Treasuries (denominated in Yen)

Data:
(a) One year U.S. Treasuries yield 5% in US$
(b) One year Japanese Treasuries yield 1% in Yen
(c) The current exchange rate is 110 Yen per $
(d) There are no transactions costs and no fees for borrowing or lending securities and no default risk in any transaction

Questions:
(1) Can you set up a riskless transaction to capture the 4% yield differential between U.S. and Japanese bonds?
(2) Will this arbitrage push down U.S. yields and push up Japanese yields until the two yields are equal?
(3) If the answer to (2) is no, can these yields be arbitrarily far apart?

Proposed transaction today:

		Cash Flows
1.	Sell short Japanese Treasuries with a face value of 1,000,000 Yen (carrying a 1% coupon payable in one year)	+1,000,000 Yen
2.	Sell 1,000,000 Yen in spot foreign exchange market (at a rate of 110 Yen per dollar), producing $9090.9	-1,000,000 Yen +$9090.9
3.	Buy U.S. Treasuries with face value of $9090.9 (carrying a 5% coupon payable in one year). Think of this as buying a little more than 9 one thousand dollar face value bonds	-$9090.9
	Net cash flow	0

Expected transaction in one year:

		Cash Flows
1.	Receive face value of U.S. bond [=$9090.9 x (1.05)]	+$9545.45
2.	Sell $9545.45 and buy Yen on spot foreign exchange market at *assumed* 110 Yen per US$ [=$9545.45 x 110 Yen/$]	-$9545.45 +1,049,999 Yen
3.	Buy back Japanese security (including 1% interest payment) for face value (1,000,000 Yen) x 1.01	-1,010,000 Yen
	Net cash flow	+39,999 Yen

Answers to Questions:

This transaction earned 39,999 Yen on a 1,000,000 Yen transaction, equal to the 4% yield differential between Japanese and U.S. interest rates. (Converting 39,999 Yen into $363.63 at the 110 Yen per dollar exchange rate produces the same 4% on $9090.9.) The problem is that this transaction is not riskless. In particular, in this so-called “uncovered interest arbitrage” transaction, there is no guarantee that in one year the exchange rate will be 110 Yen per US$, as we assumed in step 2 of “expected transactions.” If the exchange rate were 103 Yen per $ then step 2 produces only 983,181.4 Yen, leaving a net loss of 26,818.6 Yen instead of a 39,999 Yen profit. Thus, although the initial short sale of Japanese bonds drives down price and drives yields up and the purchase of U.S. bonds drives up price and drives down yields, these transactions will not be done in unlimited amounts. Therefore the U.S. and Japanese yields will not be driven together. On the other hand, this “risky transaction” yields an expected profit, so U.S. and Japanese yields cannot be arbitrarily far apart. In particular, potential risk taking “arbitrageurs” will decide to undertake this transaction depending upon the expected exchange rate in one year and what the “breakeven” rate is in the proposed transactions.

<u>Breakeven Calculation</u>: What is the “breakeven rate” for Yen per $ in one year that leaves no profit or loss?

You will breakeven when you can convert the proceeds of the U.S. bond ($9,090.9 x 1.05) into Yen so that you can repay what you owe on the short sale (1,000,000 Yen x 1.01) and nothing is left over.

Therefore if Z is the Yen per $ exchange rate in one year, we have:

$$[\$9{,}009.9(1.05)] \cdot Z = 1{,}000{,}000 \text{ Yen}(1.01)$$

$$Z = \frac{1{,}000{,}000 \text{ Yen } (1.01)}{\$9{,}090.9\ (1.05)}$$

$$Z = 105.81 \frac{\text{Yen}}{\$}$$

Note: This result makes sense because if the Yen appreciates to 105.81 (slightly less than 4 percent from 110), then the gains from the “arbitrage” to capture the 4% yield differential will be wiped out. (You gain the 4% yield differential with a one year delay, so it too is slightly less than 4%.)

<u>Breakeven Calculation and Forward Rates</u>: We can transform this result into a general breakeven formula for Yen per $ in the Future given Spot Yen per $:

From above we have,

$$Z = \frac{1{,}000{,}000 \text{ Yen}}{\$9{,}090.9} \cdot \frac{(1.01)}{(1.05)}$$

Since Z is the exchange rate in the future (F) and 1,000,000 Yen/$9,090.9 is the spot exchange rate (S), we have,

$$F = S \cdot \left[\frac{1 + \text{Yen rate}}{1 + \text{US rate}} \right]$$

An Aside:

For currencies quoted in dollars per foreign currency, like the British Pound and the Euro (rather than as foreign currency per dollar, like the Yen) the formula becomes:

$$F = S \cdot \left[\frac{1 + \text{US rate}}{1 + \text{foreign rate}} \right]$$

Why the Breakeven Rate is the *Equilibrium* Forward Rate

The breakeven rate in this calculation is the equilibrium forward rate quoted at the beginning of the year because otherwise there will be a riskless and profitable arbitrage available by "covering" your foreign exchange risk. The so-called "covered interest arbitrage" transaction goes like this. Suppose the forward rate at the beginning of the year were 110 Yen per $ (identical to the spot rate). Then you would add a step to "proposed transactions today" to contract at that forward rate, thereby guaranteeing that you could exchange $9,545.45 for 1,049,999 Yen at the end of the year. This would make the "expected transactions" in one year a certainty and would produce a riskless profit. In this case, arbitrageurs would generate an unlimited demand to buy Yen and sell dollars at the forward rate of 110 Yen per dollar because they need to transform the proceeds of their U.S. Treasuries into Yen to pay off their short position in Japanese Treasuries to complete the arbitrage. Dealers would be forced to raise the value of the Yen for delivery in one year. Only when dealers quoted a forward rate equal to 105.81 Yen per $ will the riskless profit from the "covered interest arbitrage" disappear. That is why the breakeven rate is the equilibrium forward rate.

Final Note: See the next page for an example of forward and spot relationships from the *Wall Street Journal.*