Equity Valuation Formulas

William L. Silber and Jessica Wachter

I. The Dividend Discount Model

Suppose a stock with price $P_0$ pays dividend $D_1$ one year from now, $D_2$ two years from now, and so on, for the rest of time. $P_0$ is then equal to the discounted value of the future dividends:

\[ P_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \cdots \]

The discount factor, $k$, is the firm's cost of equity capital and is given by the CAPM's required rate of return for holding the stock:

\[ k = R_f + \beta (R_M - R_f) \]

$k$ is sometimes called the firm’s capitalization rate.

II. Some Simplifications and Extensions

We can simplify equation (1) by assuming that the company pays the same expected dividend forever. For some companies, this is not a bad approximation. Then:

\[ P_0 = \frac{D}{1+k} + \frac{D}{(1+k)^2} + \frac{D}{(1+k)^3} + \cdots \]

$P_0$ is simply a perpetuity with cash payment $D$ and discount rate $k$. Using the formula for perpetuities:

\[ P_0 = \frac{D}{k} \]
This formula can be related to the price-earnings ratio because dividends are paid out of earnings. Let \( b \) denote the plowback ratio, i.e., the fraction of earnings that are “plowed back” into the company. The rest are paid out as dividends. Then:

\[
D = (1 - b)E.
\]

Substituting this expression into equation (2) above, we have:

\[
P_0 = \frac{E(1 - b)}{k}
\]

which implies:

\[
(3) \quad \frac{P_0}{E} = \frac{1 - b}{k}.
\]

This simple model implies that the price-earnings ratio is inversely related to the firm’s cost of equity capital, \( k \). The lower is \( k \) the higher is the firm’s price-earnings ratio. Note that when \( b = 0 \) the price-earnings ratio becomes \( 1/k \). More on this special case below.

III. Dividend Growth Model

The simplified dividend discount model does not capture a feature that is important for many companies: dividends are expected to grow over time. We need to modify the model to account for dividend growth. A simple assumption is that dividends are expected to grow at a constant rate, \( g \), forever. This means:

\[
D_1 = (1 + g)D_0
\]

\[
D_2 = (1 + g)^2 D_0
\]

\[
D_3 = (1 + g)^3 D_0
\]

and so forth. When we substitute these into equation (1) above we get:

\[
P_0 = \frac{D_0(1 + g)}{1 + k} + \frac{D_0(1 + g)^2}{(1 + k)^2} + \frac{D_0(1 + g)^3}{(1 + k)^3} + \ldots
\]

Factoring out \( D_0(1 + g) \), produces:
\[
P_0 = D_0 (1 + g) \left[ \frac{1 + g}{1 + k} + \frac{1 + g}{(1 + k)^2} + \frac{(1 + g)^2}{(1 + k)^3} + \ldots \right]
\]

It turns out that when \( k > g \) the term in brackets has a finite sum equal to \( 1/(k - g) \). If \( k < g \) the bracketed sum goes to infinity. Thus, as long as \( k > g \), we have:

\[
(4) \quad P_0 = \frac{D_0 (1 + g)}{k - g} = \frac{D_1}{k - g}
\]

Equation (4) gives the value of a stock according to the dividend growth model.

IV. Where Does Growth Come From?

The dividend growth model works better than the model with constant expected dividends, but it does require an estimate for \( g \), the growth rate. A simple numerical example shows that if ROE is the return on the firm’s equity, then:

\[ g = \text{ROE} \cdot b \]

In words, the growth rate equals the return on equity times the plowback ratio, or growth is determined by how much of earnings is put back into the firm, and how profitable those earnings are. Therefore, substituting \( g = \text{ROE} \cdot b \) into equation (4), produces:

\[
(5) \quad P_0 = \frac{D_1}{k - \text{ROE} \cdot b}
\]

V. Determinants of the Price-Earnings Ratio

We can go one step further and derive the implications of the dividend growth model for a firm’s price-earnings ratio. Because \( D_i = E_i (1 - b) \) we can rewrite (5) as:
\[ P_0 = \frac{E_i(1 - b)}{k - \text{ROE} \cdot b} \]

This gives us:

\[
\frac{P_0}{E_i} = \frac{1 - b}{k - \text{ROE} \cdot b}
\]

Thus, the price-earnings ratio is determined by the market capitalization rate \( k \), the plowback ratio \( b \), and the return on equity \( \text{ROE} \).

When \( \text{ROE} = k \), something interesting happens:

\[
\frac{P_0}{E_i} = \frac{1}{k}
\]

It doesn’t matter how much is plowed back into the firm in this case. Why does this happen? When \( \text{ROE} = k \), the earnings that are kept in the firm provide the same return as earnings that are paid out. So the price of the firm is the same whether those earnings are left in the firm or returned to shareholders.

When \( \text{ROE} \neq k \) the price-earnings ratio does depend on the plowback ratio but the direction of the effect depends on whether \( \text{ROE} > k \) or \( \text{ROE} < k \). In particular, the derivative of \( P/E \) in expression (6) with respect to \( b \) is, as follows:

\[
\frac{\partial \left( \frac{P_0}{E_i} \right)}{\partial b} = \frac{(k - \text{ROE} \cdot b)(-1) - (1 - b)(- \text{ROE})}{(k - \text{ROE} \cdot b)^2}
\]

This expression can be simplified to:
The sign of (7) is positive if $ROE > k$ and the sign is negative if $ROE < k$. This is a comforting result because only if a company's $ROE$ is greater than its cost of capital, $k$, will an increase in the fraction plowed back into the firm be rewarded with a high price-earnings ratio. Companies that have a low $ROE$ relative to $k$ will be punished with a low price-earnings ratio when they plow back a larger fraction of earnings.