Duration: Formulas and Calculations

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1. Definition

\[
D = \frac{\sum_{t=1}^{n} \frac{C_t}{(1 + r)^t}}{\sum_{t=1}^{n} C_t}
\]

2. Explicit Sample Calculations

(a) For an 8% coupon (annual pay) four-year bond with a yield to maturity of 10%, we have:

\[
D = \frac{80 \frac{1}{1.10} (1) + 80 \frac{1}{(1.10)^2} (2) + 80 \frac{1}{(1.10)^3} (3) + 1080 \frac{1}{(1.10)^4} (4)}{80 \frac{1}{1.10} + 80 \frac{1}{(1.10)^2} + 80 \frac{1}{(1.10)^3} + 1080 \frac{1}{(1.10)^4}}
\]

\[
D = 3.56
\]

(b) If the coupon were 4% rather than 8%, the formula would be:

\[
D = \frac{40 \frac{1}{1.10} (1) + 40 \frac{1}{(1.10)^2} (2) + 40 \frac{1}{(1.10)^3} (3) + 1040 \frac{1}{(1.10)^4} (4)}{40 \frac{1}{1.10} + 40 \frac{1}{(1.10)^2} + 40 \frac{1}{(1.10)^3} + 1040 \frac{1}{(1.10)^4}}
\]

\[
D = 3.75
\]
(c) Finally, for a zero coupon bond with four years to maturity we have:

\[
D = \frac{1080}{(1.10)^4} = 4
\]

3. Duration Table for an 11.75% Coupon Bond

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(3a)</th>
<th>(4)</th>
<th>(4a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupon</td>
<td>MAT</td>
<td>YTM</td>
<td>DUR</td>
<td>YTM</td>
<td>DUR</td>
</tr>
<tr>
<td>11.75</td>
<td>3YR</td>
<td>11.75</td>
<td>2.70</td>
<td>6.75</td>
<td>2.71</td>
</tr>
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<td>7</td>
<td>11.75</td>
<td>5.14</td>
<td>6.75</td>
<td>5.36</td>
</tr>
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<td>10</td>
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<td>6.38</td>
<td>6.75</td>
<td>6.90</td>
</tr>
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<td>11.75</td>
<td>8.48</td>
<td>6.75</td>
<td>10.43</td>
</tr>
<tr>
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<td>30</td>
<td>11.75</td>
<td>9.17</td>
<td>6.75</td>
<td>12.54</td>
</tr>
</tbody>
</table>

Notes:
(1) Column 3a shows duration increasing with maturity, but less than proportionately

(2) Column 4a compared with 3a shows that a decline in yield to maturity (from 11.75% to 6.75%) increases duration, especially for the longer maturities.