

# Statistics and Data Analysis

## B01.1305

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### More Sample Probability Problems

**PROBLEM:** Suppose that  $X$  is a binomial random variable based on  $n$  trials, and that the probability of success on any single trial is  $p$ . Find the probability of at least one success.

**SOLUTION:** This is  $P[ X \geq 1 ] = 1 - P[ X = 0 ] = 1 - (1 - p)^n$ . This can also appear in other forms.

**PROBLEM:** Fred has decided to play fifty rounds at roulette, each time betting on a single number. What is the probability that Fred will have at least one winner?

**SOLUTION:** Observe that  $n = 50$  and  $p = 1/38 \approx 0.026316$ . Let  $X$  be the number of winners.

Then  $P[ X \geq 1 ] = 1 - P[ X = 0 ] = 1 - \left(\frac{37}{38}\right)^{50} = 1 - 0.973684^{50} = 0.736424 \approx 74\%$ .

**PROBLEM:** In the previous situation, what is the probability that Fred will leave with more money than he started with?

**SOLUTION:** The cost of each play is irrelevant to this question, but let's make it definite by saying that each play costs \$1. It will cost Fred \$50 to play the fifty rounds. Each time he wins, he gets back \$36. This would be a net win of +\$35, but we can still think of this as (pay \$1, get back \$36). Fred needs to win two or more times in order to finish with more money than he started with. Here's the easiest way to organize this arithmetic:

$$\begin{aligned}
 P[ X \geq 2 ] &= 1 - \{ P[ X = 0 ] + P[ X = 1 ] \} \\
 &= 1 - \left\{ \left(\frac{37}{38}\right)^{50} + \binom{50}{1} \left(\frac{37}{38}\right)^{49} \left(\frac{1}{38}\right) \right\} = 1 - \left\{ \left(\frac{37}{38}\right)^{50} + 50 \left(\frac{37}{38}\right)^{49} \left(\frac{1}{38}\right) \right\} = 1 - \left(\frac{37}{38}\right)^{49} \left\{ \left(\frac{37}{38}\right) + 50 \left(\frac{1}{38}\right) \right\} \\
 &\approx 1 - 0.270700 \{ 2.289474 \} \approx 0.380239 \approx 38\%
 \end{aligned}$$

PROBLEM: If  $X$  is a normal random variable with mean 120 and standard deviation 36, what is the probability that  $X$  exceeds 129?

**SOLUTION:** This can be done with routine standardizing.

$$\begin{aligned} P[ X > 129 ] &= P\left[ \frac{X-120}{36} > \frac{129-120}{36} \right] \\ &= P[ Z > 0.25 ] \\ &= 0.50 - P[ 0 \leq Z \leq 0.25 ] = 0.50 - 0.0987 = 0.4013 \approx 40\% \end{aligned}$$

This uses the fact that  $[(X-120)/36]$  is normal with mean 0 and standard deviation 1; this is a standardized normal, and it's common to identify it as symbol  $Z$ .

PROBLEM: If  $X$  is a normal random variable with mean 0.0024 and standard deviation 0.0019, what is the probability that  $X$  will lie between 0.0020 and 0.0030?

**SOLUTION:** Again, routine standardizing does the job.

$$\begin{aligned} P[ 0.0020 < X < 0.0030 ] &= P\left[ \frac{0.0020-0.0024}{0.0019} < \frac{X-0.0024}{0.0019} < \frac{0.0030-0.0024}{0.0019} \right] \\ &\approx P[ -0.21 < Z < 0.32 ] \\ &= P[ -0.21 < Z \leq 0 ] + P[ 0 < Z < 0.32 ] \\ &= P[ 0 < Z \leq 0.21 ] + P[ 0 < Z < 0.32 ] \\ &= 0.0832 + 0.1255 \\ &= 0.2087 \approx 21\%. \end{aligned}$$

Suppose that  $Z$  is a standard normal random variable.

PROBLEM: Find  $P[ Z > 0.35 ]$ .

**SOLUTION:** Convert this to a "0 to  $z$ " form.

$$P[ Z > 0.35 ] = 0.50 - P[ 0 \leq Z \leq 0.35 ] = 0.50 - 0.1368 = 0.3632$$

PROBLEM: Find  $P[ 0.25 \leq Z \leq 0.80 ]$ .

**SOLUTION:**

$$P[ 0.25 \leq Z \leq 0.80 ] = P[ 0 \leq Z \leq 0.80 ] - P[ 0 \leq Z \leq 0.25 ] = 0.2882 - 0.0987 = 0.1895$$

PROBLEM: Find  $P[ -0.25 \leq Z \leq 0.80 ]$ .

**SOLUTION:**

$$P[ -0.25 \leq Z \leq 0.80 ] = P[ -0.25 \leq Z \leq 0 ] + P[ 0 \leq Z \leq 0.80 ] = 0.0987 + 0.2882 = 0.3869$$

PROBLEM: Find  $P[ | Z | \geq 1.75 ]$ .

**SOLUTION:**

$$\begin{aligned} P[ | Z | \geq 1.75 ] &= P[ Z \leq -1.75 ] + P[ Z \geq 1.75 ] = 2 \times P[ Z \geq 1.75 ] \\ &= 2 \times \{ 0.50 - P[ 0 \leq Z \leq 1.75 ] \} = 1 - 2 \times P[ 0 \leq Z \leq 1.75 ] \end{aligned}$$

$$= 1 - 2 \times 0.4599 = 1 - 0.9198 = 0.0802$$