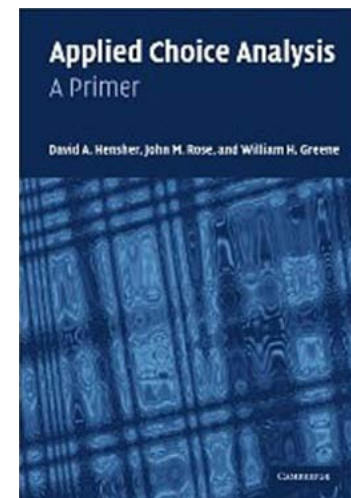
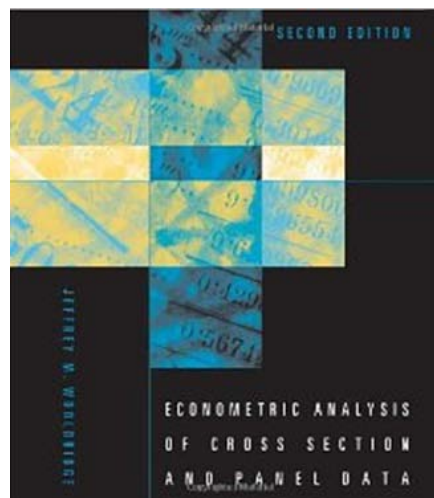
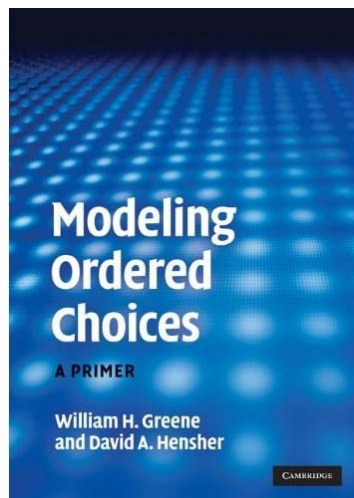


Part 3: Basic Linear Panel Data Models





Benefits of Panel Data

- Time and individual variation in behavior unobservable in cross sections or aggregate time series
- Observable and unobservable individual heterogeneity
- Rich hierarchical structures
- More complicated models
- Features that cannot be modeled with only cross section or aggregate time series data alone
- Dynamics in economic behavior



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British Household Panel Survey

BHPS

The British Household Panel Survey began in 1991 and is a multi-purpose study whose unique value resides in the fact that:




- it follows the same representative sample of individuals – the panel – over a period of years;
- it is household-based, interviewing every adult member of sampled households;
- it contains sufficient cases for meaningful analysis of certain groups such as the elderly or lone parent families.

The wave 1 panel consists of some 5,500 households and 10,300 individuals drawn from 250 areas of Great Britain. Additional samples of 1,500 households in each of Scotland and Wales were added to the main sample in 1999, and in 2001 a sample of 2,000 households was added in Northern Ireland, making the panel suitable for UK-wide research.


- [BHPS wave 18 data and documentation](#) are available from the UK Data Archive.



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Short Description

The German Socio-Economic Panel Study (SOEP) is a wide-ranging representative longitudinal study of private households, located at the German Institute for Economic Research, DIW Berlin. Every year, there were nearly 11,000 households, and more than 20,000 persons sampled by the fieldwork organization TNS Infratest Sozialforschung.

The data provide information on all household members, consisting of Germans living in the Old and New German States, Foreigners, and recent Immigrants to Germany. The Panel was started in 1984.

Some of the many topics include household composition, occupational biographies, employment, earnings, health and satisfaction indicators.




PSID

A national study of socioeconomics and health
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The Panel Study of Income Dynamics - PSID - is the longest running longitudinal household survey in the world.

The study began in 1968 with a nationally representative sample of over 18,000 individuals living in 5,000 families in the United States. Information on these individuals and their descendants has been collected continuously, including data covering employment, income, wealth, expenditures, health, marriage, childbearing, child development, philanthropy, education, and numerous other topics. The PSID is directed by faculty at the University of Michigan, and the data are available on this website without cost to researchers and analysts.

The data are used by researchers, policy analysts, and teachers around the globe. Over 3,000 peer-reviewed publications have been based on the PSID. Recognizing the importance of the data, numerous countries have created their own PSID-like studies that now facilitate cross-national comparative research. The National Science Foundation recognized the PSID as one of the **60 most significant advances funded by NSF** in its 60 year history.



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Survey of Income and Program Participation (SIPP)

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(Formerly, DEWS)

URL: <http://www.census.gov/sipp/>


*Source: U.S. Census Bureau, Demographics Survey Division,
Survey of Income and Program Participation branch*

Created: February 14, 2002

Last revised: June 6, 2012

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Access to microdata	European Community Household Panel (EHP)	See Also
<p>Introduction</p> <p>European Community Household Panel</p> <p style="padding-left: 20px;">Publications</p> <p>European Union Labour Force Survey</p> <p>Community Innovation Statistics</p> <p style="padding-left: 20px;">Publications</p> <p>European Union Statistics on Income and Living Conditions</p> <p style="padding-left: 20px;">Publications</p> <p>Structure of Earnings Survey</p> <p style="padding-left: 20px;">Publications</p> <p>Adult Education Survey</p> <p style="padding-left: 20px;">Publications</p>	<p> EHP microdata for scientific purposes: how to obtain them?</p> <p>Description of dataset</p> <p>The European Community Household Panel (EHP) is a panel survey in which a sample of households and persons have been interviewed year after year.</p> <p>These interviews cover a wide range of topics concerning living conditions. They include detailed income information, financial situation in a wider sense, working life, housing situation, social relations, health and biographical information of the interviewed.</p> <p>The total duration of the EHP was 8 years, running from 1994-2001 (8 waves).</p> <p>EHP based data in the database</p> <p>99% of the "income and living conditions" domain under theme "Population and social conditions" is derived from EHP. This includes many indicators of relative monetary poverty and of income inequality, analysed in different ways (eg. different cut-off thresholds, by age, gender, activity status, tenure status...).</p> <p>It also includes a selection of indicators of social exclusion and non-monetary deprivation derived from EHP, notably on housing.</p> <p>Of these, 4 have been chosen as structural indicators, namely the at-risk-of-poverty rate before cash social transfers, the persistent at-risk-of-poverty rate and the s80/s20 income quintile share ratio. The at-risk-of-poverty rate after social transfers is a headline indicator.</p> <p>A selection of indicators in the "health status" and "health care" collections of the "public health" domain also under the above-mentioned same theme are derived from EHP as well.</p>	<p>Additional information on EHP</p> <p>Income, Social Inclusion and Living Conditions</p>



National Longitudinal Surveys

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The **National Longitudinal Surveys (NLS)** are a set of surveys designed to gather information at multiple points in time on the labor market activities and other significant life events of several groups of men and women. For more than 4 decades, NLS data have served as an important tool for economists, sociologists, and other researchers.

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NLS General Overviews

- [National Longitudinal Survey of Youth 1997 \(NLSY97\)](#)-- Survey of young men and women born in the years 1980-84; respondents were ages 12-17 when first interviewed in 1997.
- [National Longitudinal Survey of Youth 1979 \(NLSY79\)](#)-- Survey of men and women born in the years 1957-64; respondents were ages 14-22 when first interviewed in 1979.
- [NLSY79 Children and Young Adults](#)-- Survey of the biological children of women in the NLSY79.
- [National Longitudinal Surveys of Young Women and Mature Women \(NLSW\)](#)-- The Young Women's survey includes women who were ages 14-24 when first interviewed in 1968. The Mature Women's survey includes women who were ages 30-44 when first interviewed in 1967. These surveys were discontinued in 2003.
- [National Longitudinal Surveys of Young Men and Older Men](#)-- The Young Men's survey, which was discontinued in 1981, includes men who were ages 14-24 when first interviewed in 1966. The Older Men's survey, which was discontinued in 1990, includes men who were ages 45-59 when first interviewed in 1966.



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The Household, Income and Labour Dynamics
in Australia (HILDA) Survey

HILDA Survey

The Household, Income and Labour Dynamics in Australia (HILDA) Survey is a household-based panel study which began in 2001. It has the following key features:

- It collects information about economic and subjective well-being, labour market dynamics and family dynamics.
- Special questionnaire modules are included each wave.
- The wave 1 panel consisted of 7,682 households and 19,914 individuals. In wave 11 this was topped up with an additional 2,153 households and 5,477 individuals.
- Interviews are conducted annually with all adult members of each household.
- The panel members are followed over time.
- The funding has been guaranteed for sixteen waves, though the survey is designed to continue for longer than this.
- Academic and other researchers can apply to use the General Release datasets for their research.

HILDA Survey Research Conference 2013

The call for submissions has now been released.

Further information can be obtained by clicking on the following link.

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The annual Agricultural Resource Management Survey (ARMS) is USDA's primary source of information on the financial condition, production practices, and resource use of America's farm businesses and the economic well-being of America's farm households. ARMS data are essential to USDA, congressional, administration, and industry decision makers when weighing alternative policies and programs that touch the farm sector or affect farm families.

Sponsored jointly by ERS and the National Agricultural Statistics Service (NASS), ARMS is the only national survey that provides observations of field-level farm practices, the economics of the farm businesses operating the field (or dairy herd, green house, nursery, poultry house, etc.), and the characteristics of farm operators and their households (age, education, occupation, farm and off-farm work, types of employment, family living expenses, etc.)—all collected in a representative sample. Information about crop production, farm production, business, and households includes data for selected surveyed States where available. [See more background on ARMS....](#)



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The Medical Expenditure Panel Survey (MEPS) is a set of large-scale surveys of families and individuals, their medical providers, and employers across the United States. MEPS is the most complete source of data on the cost and use of health care and health insurance coverage. [Learn more about MEPS.](#)

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Cornwell and Rupert Data

Cornwell and Rupert Returns to Schooling Data, 595 Individuals, 7 Years

Variables in the file are

EXP = work experience
WKS = weeks worked
OCC = occupation, 1 if blue collar,
IND = 1 if manufacturing industry
SOUTH = 1 if resides in south
SMSA = 1 if resides in a city (SMSA)
MS = 1 if married
FEM = 1 if female
UNION = 1 if wage set by union contract
ED = years of education
BLK = 1 if individual is black

LWAGE = log of wage = dependent variable in regressions

These data were analyzed in Cornwell, C. and Rupert, P., "Efficient Estimation with Panel Data: An Empirical Comparison of Instrumental Variable Estimators," *Journal of Applied Econometrics*, 3, 1988, pp. 149-155. See Baltagi, page 122 for further analysis. The data were downloaded from the website for Baltagi's text.



Data Editor			
28/900 Vars; 11111 Rows: 4165 Obs Cell: 0			
	LOGWAGE	EDUC	
1 »	5.56068	9	
2 »	5.72031	9	
3 »	5.99645	9	
4 »	5.99645	9	
5 »	6.06146	9	
6 »	6.17379	9	
7 »	6.24417	9	
8 »	6.16331	11	
9 »	6.21461	11	
10 »	6.2634	11	
11 »	6.54391	11	
12 »	6.69703	11	
13 »	6.79122	11	
14 »	6.81564	11	
15 »	5.65249	12	
16 »	6.43615	12	
17 »	6.54822	12	
18 »	6.60259	12	
19 »	6.6958	12	
20 »	6.77878	12	
21 »	6.86066	12	



Balanced and Unbalanced Panels

- Distinction: Balanced vs. Unbalanced Panels
- A notation to help with mechanics

$$z_{i,t}, i = 1, \dots, N; \boxed{t = 1, \dots, T_i}$$

- The role of the assumption
 - Mathematical and notational convenience:
 - Balanced, $n=NT$
 - Unbalanced: $\boxed{n = \sum_{i=1}^N T_i}$
 - Is the fixed T_i assumption ever necessary? Almost never.
- Is unbalancedness due to **nonrandom** attrition from an otherwise balanced panel? This would require special considerations.



Application: Health Care Usage

German Health Care Usage Data, 7,293 Individuals, Varying Numbers of Periods

This is an unbalanced panel with 7,293 individuals. There are altogether 27,326 observations. The number of observations ranges from 1 to 7.

(Frequencies are: 1=1525, 2=2158, 3=825, 4=926, 5=1051, 6=1000, 7=987).

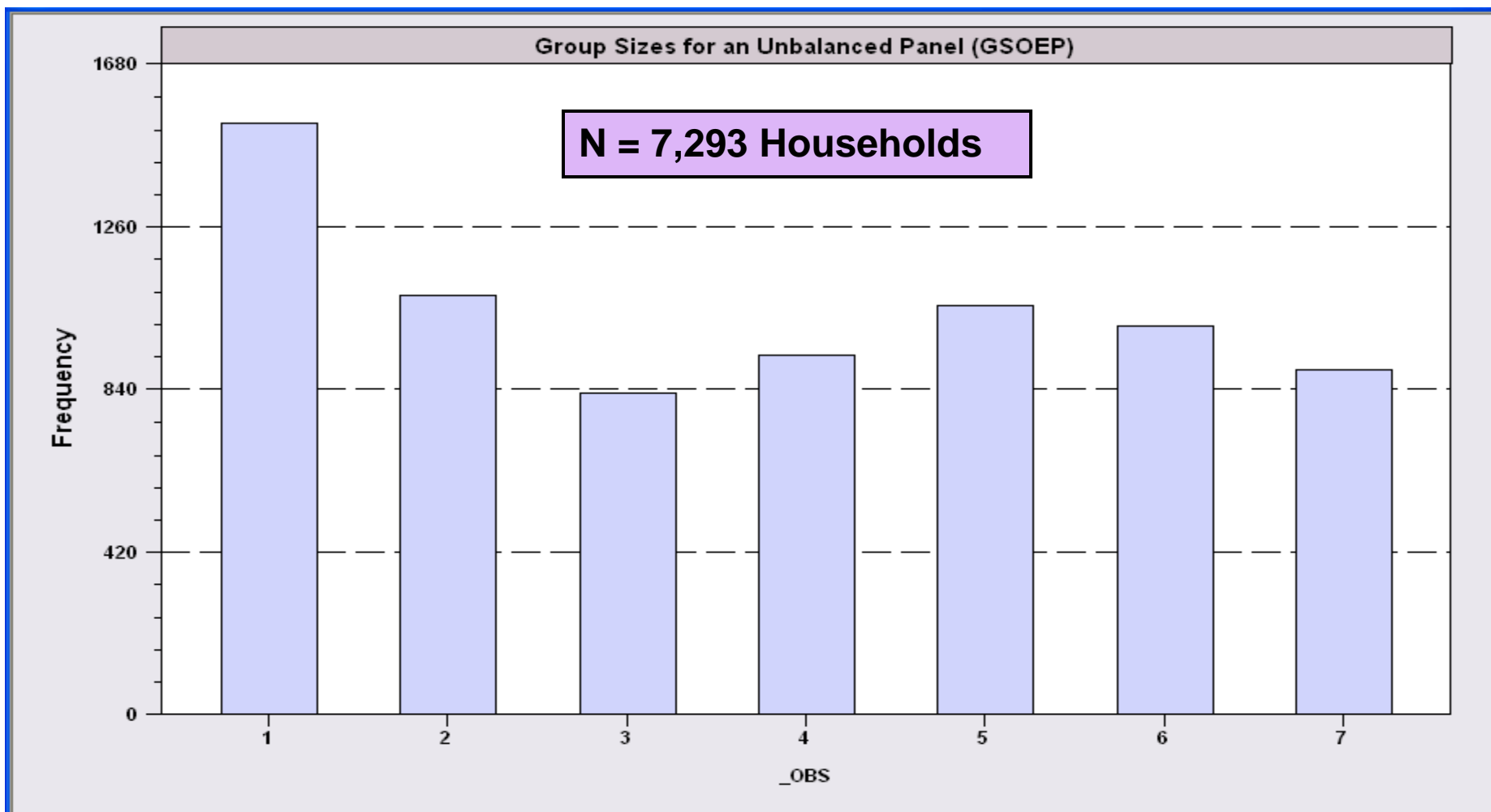
(Downloaded from the JAE Archive)

Variables in the file are

- DOCTOR** = 1(Number of doctor visits > 0)
- HOSPITAL** = 1(Number of hospital visits > 0)
- HSAT** = health satisfaction, coded 0 (low) - 10 (high)
- DOCVIS** = number of doctor visits in last three months
- HOSPVIS** = number of hospital visits in last calendar year
- PUBLIC** = insured in public health insurance = 1; otherwise = 0
- ADDON** = insured by add-on insurance = 1; otherwise = 0
- HHNINC** = household nominal monthly net income in German marks / 10000.
(4 observations with income=0 were dropped)
- HHKIDS** = children under age 16 in the household = 1; otherwise = 0
- EDUC** = years of schooling
- AGE** = age in years
- MARRIED** = marital status



An Unbalanced Panel: RWM's GSOEP Data on Health Care





Fixed and Random Effects

- Unobserved individual effects in regression: $E[y_{it} \mid \mathbf{x}_{it}, c_i]$

Notation: $y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + c_i + \varepsilon_{it}$

$$\mathbf{X}_i = \begin{bmatrix} \mathbf{x}_{i1}' \\ \mathbf{x}_{i2}' \\ \vdots \\ \mathbf{x}_{iT_i}' \end{bmatrix} \quad T_i \text{ rows, } K \text{ columns}$$

- Linear specification:

Fixed Effects: $E[c_i \mid \mathbf{X}_i] = g(\mathbf{X}_i)$. $\text{Cov}[\mathbf{x}_{it}, c_i] \neq 0$
effects are correlated with included variables.

Random Effects: $E[c_i \mid \mathbf{X}_i] = 0$. $\text{Cov}[\mathbf{x}_{it}, c_i] = 0$



Convenient Notation

- Fixed Effects – the ‘dummy variable model’

$$y_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + \varepsilon_{it}$$

Individual specific constant terms.

- Random Effects – the ‘error components model’

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta} + \varepsilon_{it} + u_i$$

Compound (“composed”) disturbance



Estimating β

- β is the partial effect of interest
- Can it be estimated (consistently) in the presence of (unmeasured) c_i ?
 - Does pooled least squares “work?”
 - Strategies for “controlling for c_i ” using the sample data



The Pooled Regression

- Presence of omitted effects

$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + c_i + \varepsilon_{it}$, observation for person i at time t

$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{c}_i + \boldsymbol{\varepsilon}_i$, T_i observations in group i

$= \mathbf{X}_i\boldsymbol{\beta} + \mathbf{c}_i + \boldsymbol{\varepsilon}_i$, note $\mathbf{c}_i = (c_i, c_i, \dots, c_i)'$

$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{c} + \boldsymbol{\varepsilon}$, $\sum_{i=1}^N T_i$ observations in the sample

- Potential bias/inconsistency of OLS – depends on 'fixed' or 'random'



OLS with Individual Effects

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{y}$$

$$\boldsymbol{\beta} + \left[\frac{1}{N} \sum_{i=1}^N \mathbf{x}_i \mathbf{x}_i' \right]^{-1} \left[\frac{1}{N} \sum_{i=1}^N \mathbf{x}_i c_i \right] \quad (\text{part due to the omitted } c_i)$$

$$+ \left[\frac{1}{N} \sum_{i=1}^N \mathbf{x}_i \mathbf{x}_i' \right]^{-1} \left[\frac{1}{N} \sum_{i=1}^N \mathbf{x}_i' \right] (c_i \text{ variance of } \text{ and will} = 0)$$

The third term vanishes asymptotically by assumption

$$\text{plim } \mathbf{b}\boldsymbol{\beta} = \text{plim} \left[\frac{1}{N} \sum_{i=1}^N \mathbf{x}_i \mathbf{x}_i' \right]^{-1} \left[\frac{1}{N} \sum_{i=1}^N \mathbf{x}_i' c_i \right] \quad (\text{left out variable for mula})$$

So, what becomes of $\sum_{i=1}^N \mathbf{x}_i' c_i$?

$$\text{plim } \mathbf{b}\boldsymbol{\beta} = \text{plim} \left[\frac{1}{N} \sum_{i=1}^N \mathbf{x}_i \mathbf{x}_i' \right]^{-1} \left[\frac{1}{N} \sum_{i=1}^N \mathbf{x}_i' c_i \right] \quad \text{if the covariance of } \mathbf{x}_i \text{ and } c_i \text{ converges to zero.}$$



Estimating the Sampling Variance of b

- $s^2(\mathbf{X}'\mathbf{X})^{-1}$? Inappropriate because
 - Correlation across observations
 - (Possibly) Heteroscedasticity
- A 'robust' covariance matrix
 - Robust estimation (in general)
 - The White estimator
 - A Robust estimator for OLS.



Cluster Estimator

Robust variance estimator for $\text{Var}[\mathbf{b}]$

Est. $\text{Var}[\mathbf{b}]$

$$= (\mathbf{X}'\mathbf{X})^{-1} \left[\sum_{i=1}^N \left(\sum_{t=1}^{T_i} \mathbf{x}_{it} \hat{v}_{it} \right) \left(\sum_{t=1}^{T_i} \mathbf{x}_{it}' \hat{v}_{it} \right) \right] (\mathbf{X}'\mathbf{X})^{-1}$$

$$= (\mathbf{X}'\mathbf{X})^{-1} \left[\sum_{i=1}^N \left(\sum_{t=1}^{T_i} \sum_{s=1}^{T_i} \hat{v}_{it} \hat{v}_{is} \mathbf{x}_{it} \mathbf{x}_{is}' \right) \right] (\mathbf{X}'\mathbf{X})^{-1}$$

\hat{v}_{it} = a least squares residual = $\widehat{\varepsilon_{it} + c_i}$

(If $T_i = 1$, this is the White estimator.)



Application: Cornwell and Rupert

Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
Constant	5.66098218	.04685914	120.808	.0000	
OCC	-.11220205	.01464317	-7.662	.0000	.51116447
SMSA	.15504405	.01233744	12.567	.0000	.65378151
MS	.09569050	.02133490	4.485	.0000	.81440576
FEM	-.39478212	.02603413	-15.164	.0000	.11260504
ED	.05688005	.00267743	21.244	.0000	12.8453782
EXP	.01043785	.00054206	19.256	.0000	19.8537815
Covariance matrix for the model is adjusted for data clustering.					
Sample of 4165 observations contained 595 clusters defined by					
7 observations (fixed number) in each cluster.					
Sample of 4165 observations contained 1 strata defined by					
4165 observations (fixed number) in each stratum.					
Constant	5.66098218	.10026368	56.461	.0000	
OCC	-.11220205	.02653437	-4.229	.0000	.51116447
SMSA	.15504405	.02540156	6.104	.0000	.65378151
MS	.09569050	.04656766	2.055	.0399	.81440576
FEM	-.39478212	.05319458	-7.421	.0000	.11260504
ED	.05688005	.00568214	10.010	.0000	12.8453782
EXP	.01043785	.00131647	7.929	.0000	19.8537815



Data Editor

28/900 Vars; 11111 Rows; 4165 Obs Cell: 0

	LOGWAGE	EDUC
1 »	5.56068	9
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17 »	6.54822	12
18 »	6.60259	12
19 »	6.6958	12
20 »	6.77878	12
21 »	6.86066	12

Bootstrap variance for a panel data estimator

- **Panel Bootstrap = Block Bootstrap**
- Data set is N groups of size T_i
- Bootstrap sample is N groups of size T_i drawn with replacement.



LWAGE	Coefficient	Standard Error	z	Prob. z >Z*	95% Confidence Interval		
Constant	5.66098***	.04686	120.81	.0000	5.56914	5.75282	OLS
OCC	-.11220***	.01464	-7.66	.0000	-.14090	-.08350	
SMSA	.15504***	.01234	12.57	.0000	.13086	.17922	
MS	.09569***	.02133	4.49	.0000	.05387	.13751	
FEM	-.39478***	.02603	-15.16	.0000	-.44581	-.34376	
ED	.05688***	.00268	21.24	.0000	.05163	.06213	
EXP	.01044***	.00054	19.26	.0000	.00938	.01150	
							Bootstrap
B001	5.66098***	.04683	120.89	.0000	5.56920	5.75276	
B002	-.11220***	.01326	-8.46	.0000	-.13820	-.08620	Assumes no
B003	.15504***	.01205	12.87	.0000	.13143	.17866	correlation
B004	.09569***	.01953	4.90	.0000	.05742	.13396	within groups
B005	-.39478***	.01863	-21.19	.0000	-.43129	-.35827	
B006	.05688***	.00325	17.52	.0000	.05052	.06324	
B007	.01044***	.00053	19.67	.0000	.00940	.01148	
							Cluster
Constant	5.66098***	.10026	56.46	.0000	5.46447	5.85750	
OCC	-.11220***	.02653	-4.23	.0000	-.16421	-.06020	Accounts for
SMSA	.15504***	.02540	6.10	.0000	.10526	.20483	within group
MS	.09569**	.04657	2.05	.0399	.00442	.18696	correlation
FEM	-.39478***	.05319	-7.42	.0000	-.49904	-.29052	
ED	.05688***	.00568	10.01	.0000	.04574	.06802	
EXP	.01044***	.00132	7.93	.0000	.00786	.01302	
							Block Bootstrap
B001	5.66098***	.09497	59.61	.0000	5.47484	5.84712	
B002	-.11220***	.02617	-4.29	.0000	-.16349	-.06092	Mimics results
B003	.15504***	.02351	6.60	.0000	.10897	.20112	of panel
B004	.09569***	.03542	2.70	.0069	.02627	.16511	correction
B005	-.39478***	.04287	-9.21	.0000	-.47880	-.31077	
B006	.05688***	.00536	10.61	.0000	.04637	.06739	
B007	.01044***	.00138	7.57	.0000	.00774	.01314	



The Fixed Effects Model

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{d}_i\mathbf{a}_i + \boldsymbol{\varepsilon}_i, \text{ for each individual } i$$

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{pmatrix} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{d}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{X}_2 & \mathbf{0} & \mathbf{d}_2 & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_N & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{d}_N \end{bmatrix} \begin{pmatrix} \boldsymbol{\beta} \\ \mathbf{a} \end{pmatrix} + \boldsymbol{\varepsilon}$$

$$= [\mathbf{X}, \mathbf{D}] \begin{pmatrix} \boldsymbol{\beta} \\ \mathbf{a} \end{pmatrix} +$$

$$\boldsymbol{\varepsilon}$$

$E[\mathbf{c}_i | \mathbf{X}_i] = g(\mathbf{X}_i)$; Effects are correlated with included variables.

$$\text{Cov}[\mathbf{x}_{it}, \mathbf{c}_i] \neq \mathbf{0}$$



Estimating the Fixed Effects Model

- The FEM is a plain vanilla regression model but with many independent variables
- Least squares is unbiased, consistent, efficient, but inconvenient if N is large.

$$\begin{pmatrix} \mathbf{b} \\ \mathbf{a} \end{pmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{D} \\ \mathbf{D}'\mathbf{X} & \mathbf{D}'\mathbf{D} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{D}'\mathbf{y} \end{bmatrix}$$

Using the Frisch-Waugh theorem

$$\mathbf{b} = [\mathbf{X}'\mathbf{M}_D\mathbf{X}]^{-1} [\mathbf{X}'\mathbf{M}_D\mathbf{y}]$$



The Within Groups Transformation Removes the Effects

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + C_i + \varepsilon_{it}$$

$$\bar{y}_i = \mathbf{x}_i'\boldsymbol{\beta} + C_i + \bar{\varepsilon}_i$$

$$y_{it} - \bar{y}_i = (\mathbf{x}_{it} - \mathbf{x}_i)' \boldsymbol{\beta} + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

Use least squares to estimate $\boldsymbol{\beta}$.



Least Squares Dummy Variable Estimator

- **b** is obtained by '**within**' groups least squares (group mean deviations)
- **a** is estimated using the normal equations:
$$\mathbf{D}'\mathbf{X}\mathbf{b} + \mathbf{D}'\mathbf{D}\mathbf{a} = \mathbf{D}'\mathbf{y}$$

$$\mathbf{a} = (\mathbf{D}'\mathbf{D})^{-1}\mathbf{D}'(\mathbf{y} - \mathbf{X}\mathbf{b})$$

$$a_i = (1/T_i) \sum_{t=1}^{T_i} (y_{it} - \mathbf{x}'_{it}\mathbf{b}) = \bar{e}_i$$



Application Cornwell and Rupert

+-----+-----+			
Panel Data Analysis of LWAGE		[ONE way]	
Unconditional ANOVA (No regressors)			
Source	Variation	Deg. Free.	Mean Square
Between	646.254	594.	1.08797
Residual	240.651	3570.	.674093E-01
Total	886.905	4164.	.212994
+-----+-----+			

OLS Without Group Dummy Variables			
LHS=LWAGE	Mean	=	6.676346
	Standard deviation	=	.4615122
Model size	Parameters	=	5
	Degrees of freedom	=	4160
Residuals	Sum of squares	=	651.7870
	Standard error of e	=	.3958277
Fit	R-squared	=	.2650993
	Adjusted R-squared	=	.2643927
Model test	F[4, 4160] (prob)	=	375.16 (.0000)
+-----+-----+			

+-----+-----+-----+-----+-----+-----+					
Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
+-----+-----+-----+-----+-----+-----+					
OCC	-.29227536	.01259221	-23.211	.0000	.51116447
SMSA	.17712491	.01327104	13.347	.0000	.65378151
MS	.35695474	.01610229	22.168	.0000	.81440576
EXP	.00746892	.00057035	13.095	.0000	19.8537815
Constant	6.27095389	.02041864	307.119	.0000	



LSDV Results

Least Squares with Group Dummy Variables				
LHS=LWAGE	Mean	=	6.676346	
	Standard deviation	=	.4615122	
Model size	Parameters	=	599	
	Degrees of freedom	=	3566	
Residuals	Sum of squares	=	83.88505	
	Standard error of e	=	.1533740	
Fit	R-squared	=	.9054182	
	Adjusted R-squared	=	.8895573	
Model test	F[598, 3566] (prob)	=	57.08 (.0000)	

Panel:Groups	Empty	0,	Valid data	595
	Smallest	7,	Largest	7
	Average group size			7.00

Note huge changes in the coefficients. SMSA and MS change signs. Significance changes completely!

-----+-----+-----+-----+-----+-----+-----						Pooled OLS	
Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X		
OCC	-.02021384	.01374007	-1.471	.1412	.51116447	-.29227536	.01259221
SMSA	-.04250645	.01950085	-2.180	.0293	.65378151	.17712491	.01327104
MS	-.02946444	.01913652	-1.540	.1236	.81440576	.35695474	.01610229
EXP	.09665711	.00119162	81.114	.0000	19.8537815	.00746892	.00057035

The Effect of the Effects

Test Statistics for the Classical Model								
Model		Log-Likelihood		Sum of Squares			R-squared	
(1)	Constant term only	-2688.80597		.8869049390D+03			.0000000	
(2)	Group effects only	27.58464		.2406511943D+03			.7286618	
(3)	X - variables only	-2047.35445		.6517870323D+03			.2650993	
(4)	X and group effects	2222.33376		.8388505089D+02			.9054182	
Hypothesis Tests								
Likelihood Ratio Test					F Tests			
	Chi-squared	d.f.	Prob.	F	num.	denom.	Prob	value
(2) vs (1)	5432.781	594	.00000	16.140	594	3570	.00000	
(3) vs (1)	1282.903	4	.00000	375.157	4	4160	.00000	
(4) vs (1)	9822.279	598	.00000	57.085	598	3566	.00000	
(4) vs (2)	4389.498	4	.00000	1666.054	4	3566	.00000	
(4) vs (3)	8539.376	594	.00000	40.643	594	3566	.00000	



A Caution About Stata and R^2

$$R^2 = 1 - \frac{\text{Residual Sum of Squares}}{\text{Total Sum of Squares}}$$

Or is it? What is the total sum of squares?

$$\text{Conventional: Total Sum of Squares} = \sum_{i=1}^N \sum_{t=1}^{T_i} (y_{it} - \bar{y})^2$$

$$\text{"Within Sum of Squares"} = \sum_{i=1}^N \sum_{t=1}^{T_i} (y_{it} - \bar{y}_i)^2$$

For the FE model above,

$$R^2 = 0.90542$$

$$R^2 = 0.65142$$

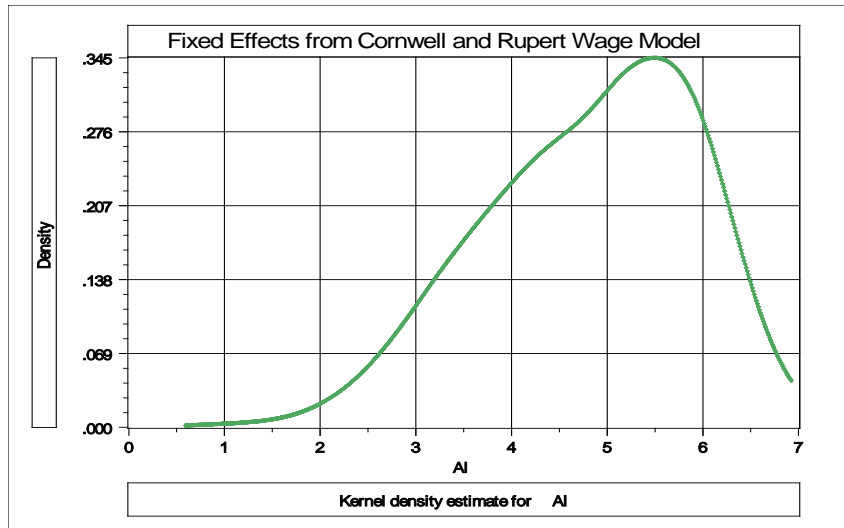
Which should appear in the denominator of R^2

The coefficient estimates and standard errors are the same. The calculation of the R^2 is different. In the **areg** procedure, you are estimating coefficients for each of your covariates plus each dummy variable for your groups. In the **xtreg, fe** procedure the R^2 reported is obtained by only fitting a mean deviated model where the effects of the groups (all of the dummy variables) are assumed to be fixed quantities. So, all of the effects for the groups are simply subtracted out of the model and no attempt is made to quantify their overall effect on the fit of the model.

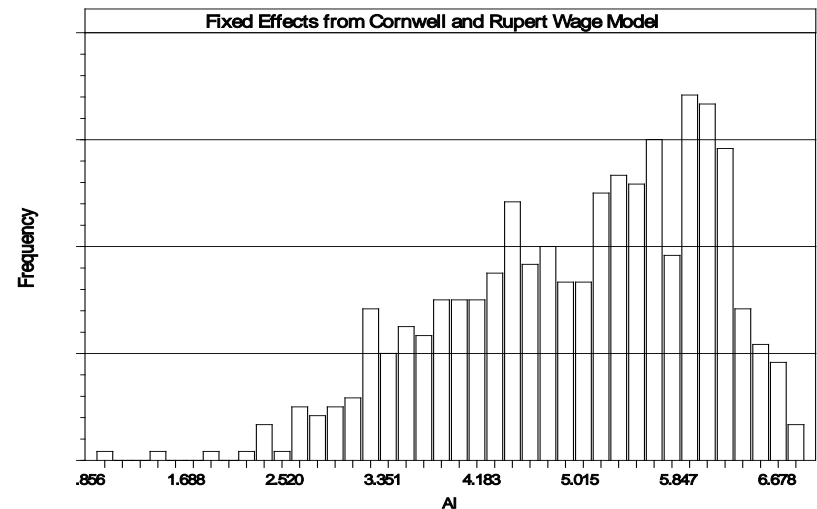
Since the SSE is the same, the $R^2 = 1 - \text{SSE}/\text{SST}$ is very different. The difference is real in that we are making different assumptions with the two approaches. In the **xtreg, fe** approach, the effects of the groups are fixed and **unestimated quantities are subtracted out of the model** before the fit is performed. In the **areg** approach, the group effects are estimated and affect the total sum of squares of the model under consideration.



Examining the Effects with a KDE



Mean = 4.819,
Standard deviation = 1.054.



Robust Covariance Matrix for LSDV Cluster Estimator for Within Estimator

Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
OCC	-.02021	.01374007	-1.471	.1412	.5111645
SMSA	-.04251**	.01950085	-2.180	.0293	.6537815
MS	-.02946	.01913652	-1.540	.1236	.8144058
EXP	.09666***	.00119162	81.114	.0000	19.853782

Covariance matrix for the model is adjusted for data clustering.
Sample of 4165 observations contained 595 clusters defined by
7 observations (fixed number) in each cluster.

Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
DOCC	-.02021	.01982162	-1.020	.3078	.00000
DSMSA	-.04251	.03091685	-1.375	.1692	.00000
DMS	-.02946	.02635035	-1.118	.2635	.00000
DEXP	.09666***	.00176599	54.732	.0000	.00000



Time Invariant Regressors

- Time invariant \mathbf{x}_{it} is defined as invariant for all i . E.g., sex dummy variable, FEM and ED (education in the Cornwell/Rupert data).
- If $\mathbf{x}_{it,k}$ is invariant for all t , then the group mean deviations are all 0.



FE With Time Invariant Variables

+-----+					
There are 3 vars. with no within group variation.					
FEM ED BLK					
+-----+					
+-----+-----+-----+-----+-----+-----+					
Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
+-----+-----+-----+-----+-----+-----+					
EXP	.09671227	.00119137	81.177	.0000	19.8537815
WKS	.00118483	.00060357	1.963	.0496	46.8115246
OCC	-.02145609	.01375327	-1.560	.1187	.51116447
SMSA	-.04454343	.01946544	-2.288	.0221	.65378151
FEM	.000000(Fixed Parameter).....			
ED	.000000(Fixed Parameter).....			
BLK	.000000(Fixed Parameter).....			
+-----+					
Test Statistics for the Classical Model					
+-----+					
Model Log-Likelihood Sum of Squares R-squared					
(1)	Constant term only	-2688.80597	886.90494	.00000	
(2)	Group effects only	27.58464	240.65119	.72866	
(3)	X - variables only	-1688.12010	548.51596	.38154	
(4)	X and group effects	2223.20087	83.85013	.90546	
+-----+					



Drop The Time Invariant Variables Same Results

Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
EXP	.09671227	.00119087	81.211	.0000	19.8537815
WKS	.00118483	.00060332	1.964	.0495	46.8115246
OCC	-.02145609	.01374749	-1.561	.1186	.51116447
SMSA	-.04454343	.01945725	-2.289	.0221	.65378151

Test Statistics for the Classical Model				
Model	Log-Likelihood	Sum of Squares	R-squared	
(1) Constant term only	-2688.80597	886.90494	.00000	
(2) Group effects only	27.58464	240.65119	.72866	
(3) X - variables only	-1688.12010	548.51596	.38154	
(4) X and group effects	2223.20087	83.85013	.90546	

No change in the sum of squared residuals



Fixed Effects Vector Decomposition

Efficient Estimation of Time
Invariant and Rarely Changing
Variables in Finite Sample Panel
Analyses with Unit Fixed Effects

Thomas Plümper and Vera Troeger
Political Analysis, 2007



Introduction

[T]he FE model ... does not allow the estimation of time invariant variables. A second drawback of the FE model ... results from its inefficiency in estimating the effect of variables that have very little within variance.

This article discusses a remedy to the related problems of estimating time invariant and rarely changing variables in FE models with unit effects



The Model

$$y_{it} = \alpha_i + \sum_{k=1}^K \beta_k x_{kit} + \sum_{m=1}^M \gamma_m z_{mit} + \varepsilon_{it}$$

where α_i denote the N unit effects.



Fixed Effects Vector Decomposition

Step 1: Compute the fixed effects regression to get the “estimated unit effects.” “We run this FE model with the sole intention to obtain estimates of the unit effects, α_i .”

$$\hat{\alpha}_i = \bar{y}_i - \sum_{k=1}^K b_k^{FE} \bar{x}_{ki}$$



Step 2

Regress a_i on \mathbf{z}_i and compute residuals

$$a_i = \sum_{m=1}^M \beta_m z_{im} + h_i$$

h_i is orthogonal to \mathbf{z}_i (since it is a residual)

Vector \mathbf{h}_i is expanded so each element

h_i is replicated T_i times - \mathbf{h} is the length of the full sample.



Step 3

Regress y_{it} on a constant, \mathbf{X} , \mathbf{Z} and \mathbf{h} using ordinary least squares to estimate α , $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, δ .

$$y_{it} = \alpha + \sum_{k=1}^K \beta_k x_{kit} + \sum_{m=1}^M \gamma_m z_{mit} + \delta h_i + \varepsilon_{it}$$

Notice that α_i in the original model has become $\alpha + \delta h_i$ in the revised model.



Step 1 (Based on full sample)

These 3 variables have no within group variation.

FEM ED BLK

F.E. estimates are based on a generalized inverse.

LWAGE	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
EXP	.09663***	.00119	81.13	.0000	19.8538
WKS	.00114*	.00060	1.88	.0600	46.8115
OCC	-.02496*	.01390	-1.80	.0724	.51116
IND	.02042	.01558	1.31	.1899	.39544
SOUTH	-.00091	.03457	-.03	.9791	.29028
SMSA	-.04581**	.01955	-2.34	.0191	.65378
UNION	.03411**	.01505	2.27	.0234	.36399
FEM	.000(Fixed Parameter).....			.11261
ED	.000(Fixed Parameter).....			12.8454
BLK	.000(Fixed Parameter).....			.07227



Step 2 (Based on 595 observations)

		Standard		Prob.	Mean
UHI	Coefficient	Error	z	z> Z	of x
Constant	2.88090***	.07172	40.17	.0000	
FEM	-.09963**	.04842	-2.06	.0396	.11261
ED	.14616***	.00541	27.02	.0000	12.8454
BLK	-.27615***	.05954	-4.64	.0000	.07227



Step 3!

LWAGE	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
Constant	2.88090***	.03282	87.78	.0000	
EXP	.09663***	.00061	157.53	.0000	19.8538
WKS	.00114***	.00044	2.58	.0098	46.8115
OCC	-.02496***	.00601	-4.16	.0000	.51116
IND	.02042***	.00479	4.26	.0000	.39544
SOUTH	-.00091	.00510	-.18	.8590	.29028
SMSA	-.04581***	.00506	-9.06	.0000	.65378
UNION	.03411***	.00521	6.55	.0000	.36399
FEM	-.09963***	.00767	-13.00	.0000	.11261
ED	.14616***	.00122	120.19	.0000	12.8454
BLK	-.27615***	.00894	-30.90	.0000	.07227
HI	1.00000***	.00670	149.26	.0000	-.103D-13



The Magic

Step 1

	Coefficient	Standard Error
LWAGE		

EXP	.09663***	.00119
WKS	.00114*	.00060
OCC	-.02496*	.01390
IND	.02042	.01558
SOUTH	-.00091	.03457
SMSA	-.04581**	.01955
UNION	.03411**	.01505

Step 2

	Coefficient	Standard Error
UHI		
Constant	2.88090***	.07172
FEM	-.09963**	.04842
ED	.14616***	.00541
BLK	-.27615***	.05954

Step 3

	Coefficient	Standard Error
	2.88090***	.03282
	.09663***	.00061
	.00114***	.00044
	-.02496***	.00601
	.02042***	.00479
	-.00091	.00510
	-.04581***	.00506
	.03411***	.00521
	-.09963***	.00767
	.14616***	.00122
	-.27615***	.00894
	1.00000***	.00670



What happened here?

$$y_{it} = \alpha_i + \sum_{k=1}^K \beta_k x_{kit} + \sum_{m=1}^M \gamma_m z_{mit} + \varepsilon_{it}$$

where α_i denote the N unit effects.

An assumption is added along the way

$\text{Cov}(\alpha_i, Z_i) = \mathbf{0}$. This is exactly the number of orthogonality assumptions needed to identify γ . It is not part of the original model.



The Random Effects Model

- The random effects model

$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + c_i + \varepsilon_{it}$, observation for person i at time t

$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{c}_i + \boldsymbol{\varepsilon}_i$, T_i observations in group i

$= \mathbf{X}_i\boldsymbol{\beta} + \mathbf{c}_i + \boldsymbol{\varepsilon}_i$, note $\mathbf{c}_i = (c_i, c_i, \dots, c_i)'$

$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{c} + \boldsymbol{\varepsilon}$, $\sum_{i=1}^N T_i$ observations in the sample

$\mathbf{c} = (\mathbf{c}'_1, \mathbf{c}'_2, \dots, \mathbf{c}'_N)'$, $\sum_{i=1}^N T_i$ by 1 vector

- c_i is uncorrelated with \mathbf{x}_{it} for all t ;

$$E[c_i | \mathbf{X}_i] = 0$$

$$E[\varepsilon_{it} | \mathbf{X}_i, c_i] = 0$$



Error Components Model

A Generalized Regression Model

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + \varepsilon_{it} + u_i$$

$$E[\varepsilon_{it} | \mathbf{X}_i] = 0$$

$$E[\varepsilon_{it}^2 | \mathbf{X}_i] = \sigma_\varepsilon^2$$

$$E[u_i | \mathbf{X}_i] = 0$$

$$E[u_i^2 | \mathbf{X}_i] = \sigma_u^2$$

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \boldsymbol{\varepsilon}_i + \mathbf{1}_i u_i \text{ for } T_i \text{ observations}$$

$$\text{Var}[\boldsymbol{\varepsilon}_i + u_i \mathbf{1}_i] = \begin{bmatrix} \sigma_\varepsilon^2 + \sigma_u^2 & \sigma_u^2 & \dots & \sigma_u^2 \\ \sigma_u^2 & \sigma_u^2 + \sigma_\varepsilon^2 & \dots & \sigma_u^2 \\ \dots & \dots & \ddots & \dots \\ \sigma_u^2 & \sigma_u^2 & \dots & \sigma_u^2 + \sigma_\varepsilon^2 \end{bmatrix} = \boldsymbol{\Omega}_i$$



Random vs. Fixed Effects

- Random Effects
 - Small number of parameters
 - Efficient estimation
 - Objectionable orthogonality assumption ($c_i \perp \mathbf{X}_i$)
- Fixed Effects
 - Robust – generally consistent
 - Large number of parameters



Ordinary Least Squares

- Standard results for OLS in a GR model
 - Consistent
 - Unbiased
 - Inefficient
- True variance of the least squares estimator

$$\text{Var}[\mathbf{b} \mid \mathbf{X}] = \frac{1}{\sum_{i=1}^N T_i} \left[\frac{\mathbf{X} \mathbf{Q} \mathbf{X}}{\sum_{i=1}^N T_i} \right]^{-1} \frac{\mathbf{X} \mathbf{J} \mathbf{X}}{\sum_{i=1}^N T_i} \left[\frac{\mathbf{J}}{\sum_{i=1}^N T_i} \right]^{-1}$$

$$\rightarrow \mathbf{0} \times \rightarrow \mathbf{Q}^{-1} \times \rightarrow \mathbf{Q}^* \times \rightarrow \mathbf{Q}^{-1}$$

$$\rightarrow \mathbf{0} \text{ as } N \rightarrow \infty$$



Estimating the Variance for OLS

$$\text{Var}[\mathbf{b} \mid \mathbf{X}] = \frac{1}{\sum_{i=1}^N T_i} \left[\frac{\mathbf{X}\hat{\Omega}\mathbf{X}'}{\sum_{i=1}^N T_i} \right]^{-1} \left(\frac{\mathbf{X}\hat{\Omega}\mathbf{X}'}{\sum_{i=1}^N T_i} \right) \left[\frac{\mathbf{X}\hat{\Omega}\mathbf{X}'}{\sum_{i=1}^N T_i} \right]^{-1}$$

In the spirit of the White estimator, use

$$\frac{\mathbf{X}\hat{\Omega}\mathbf{X}'}{\sum_{i=1}^N T_i} = \sum_{i=1}^N f_i \frac{\mathbf{X}_i' \hat{\mathbf{w}}_i \hat{\mathbf{w}}_i' \mathbf{X}_i}{T_i}, \quad \hat{\mathbf{w}}_i = \mathbf{y}_i - \mathbf{X}_i \mathbf{b}, \quad f_i = \frac{T_i}{\sum_{i=1}^N T_i}$$

Hypothesis tests are then based on Wald statistics.

THIS IS THE 'CLUSTER' ESTIMATOR



OLS Results for Cornwell and Rupert

Residuals	Sum of squares	=	522.2008
	Standard error of e	=	.3544712
Fit	R-squared	=	.4112099
	Adjusted R-squared	=	.4100766

Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
Constant	5.40159723	.04838934	111.628	.0000	
EXP	.04084968	.00218534	18.693	.0000	19.8537815
EXPSQ	-.00068788	.480428D-04	-14.318	.0000	514.405042
OCC	-.13830480	.01480107	-9.344	.0000	.51116447
SMSA	.14856267	.01206772	12.311	.0000	.65378151
MS	.06798358	.02074599	3.277	.0010	.81440576
FEM	-.40020215	.02526118	-15.843	.0000	.11260504
UNION	.09409925	.01253203	7.509	.0000	.36398559
ED	.05812166	.00260039	22.351	.0000	12.8453782



Alternative Variance Estimators

Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]
Constant	5.40159723	.04838934	111.628	.0000
EXP	.04084968	.00218534	18.693	.0000
EXPSQ	-.00068788	.480428D-04	-14.318	.0000
OCC	-.13830480	.01480107	-9.344	.0000
SMSA	.14856267	.01206772	12.311	.0000
MS	.06798358	.02074599	3.277	.0010
FEM	-.40020215	.02526118	-15.843	.0000
UNION	.09409925	.01253203	7.509	.0000
ED	.05812166	.00260039	22.351	.0000

Robust - Cluster

Constant	5.40159723	.10156038	53.186	.0000
EXP	.04084968	.00432272	9.450	.0000
EXPSQ	-.00068788	.983981D-04	-6.991	.0000
OCC	-.13830480	.02772631	-4.988	.0000
SMSA	.14856267	.02423668	6.130	.0000
MS	.06798358	.04382220	1.551	.1208
FEM	-.40020215	.04961926	-8.065	.0000
UNION	.09409925	.02422669	3.884	.0001
ED	.05812166	.00555697	10.459	.0000



Generalized Least Squares

GLS is equivalent to OLS regression of

$$y_{it}^* = y_{it} - \theta_i \bar{y}_i \text{ on } \mathbf{x}_{it}^* = \mathbf{x}_{it} - \theta_i \bar{\mathbf{x}}_i,$$

$$\text{where } \theta_i = 1 - \frac{\sigma_\varepsilon}{\sqrt{\sigma_\varepsilon^2 + T_i \sigma_u^2}}$$

$$\text{Asy.Var}[\hat{\boldsymbol{\beta}}] = [\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X}]^{-1} = \sigma_\varepsilon^2 [\mathbf{X}' * \mathbf{X}^*]^{-1}$$



Estimators for the Variances

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + \varepsilon_{it} + u_i$$

Using the OLS estimator of $\boldsymbol{\beta}$, \mathbf{b}_{OLS} ,

$$\frac{\sum_{i=1}^N \sum_{t=1}^{T_i} (y_{it} - a - \mathbf{x}_{it}'\mathbf{b})^2}{\left(\sum_{i=1}^N T_i\right) - 1 - K} \text{ estimates } \sigma_{\varepsilon}^2 + \sigma_u^2$$

With the LSDV estimates, a_i and \mathbf{b}_{LSDV} ,

$$\frac{\sum_{i=1}^N \sum_{t=1}^{T_i} (y_{it} - a_i - \mathbf{x}_{it}'\mathbf{b})^2}{\left(\sum_{i=1}^N T_i\right) - N - K} \text{ estimates } \sigma_{\varepsilon}^2$$

Using the difference of the two,

$$\left[\frac{\sum_{i=1}^N \sum_{t=1}^{T_i} (y_{it} - a - \mathbf{x}_{it}'\mathbf{b})^2}{\left(\sum_{i=1}^N T_i\right) - 1 - K} \right] - \left[\frac{\sum_{i=1}^N \sum_{t=1}^{T_i} (y_{it} - a_i - \mathbf{x}_{it}'\mathbf{b})^2}{\left(\sum_{i=1}^N T_i\right) - N - K} \right] \text{ estimates } \sigma_u^2$$



Practical Problems with FGLS

- The preceding regularly produce negative estimates of σ_u^2 .
- Estimation is made very complicated in unbalanced panels.

A bulletproof solution (originally used in TSP, now NLOGIT and others).

From the robust LSDV estimator:
$$\hat{\sigma}_\varepsilon^2 = \frac{\sum_{i=1}^N \sum_{t=1}^{T_i} (y_{it} - a_i - \mathbf{x}'_{it} \mathbf{b}_{\text{LSDV}})^2}{\sum_{i=1}^N T_i}$$

From the pooled OLS estimator:
$$\text{Est}(\sigma_\varepsilon^2 + \sigma_u^2) = \frac{\sum_{i=1}^N \sum_{t=1}^{T_i} (y_{it} - a_{\text{OLS}} - \mathbf{x}'_{it} \mathbf{b}_{\text{OLS}})^2}{\sum_{i=1}^N T_i} \geq \hat{\sigma}_\varepsilon^2$$

$$\hat{\sigma}_u^2 = \frac{\sum_{i=1}^N \sum_{t=1}^{T_i} (y_{it} - a_{\text{OLS}} - \mathbf{x}'_{it} \mathbf{b}_{\text{OLS}})^2 - \sum_{i=1}^N \sum_{t=1}^{T_i} (y_{it} - a_i - \mathbf{x}'_{it} \mathbf{b}_{\text{LSDV}})^2}{\sum_{i=1}^N T_i} \geq 0$$



Stata Variance Estimators

$$\hat{\sigma}_{\varepsilon}^2 = \frac{\sum_{i=1}^N \sum_{t=1}^{T_i} (y_{it} - a_i - \mathbf{x}'_{it} \mathbf{b}_{\text{LSDV}})^2}{\sum_{i=1}^N T_i - K - N} > 0 \text{ based on FE estimates}$$

$$\hat{\sigma}_u^2 = \text{Max} \left[0, \frac{\text{SSE}(\text{group means})}{N - A} - \frac{(N - K) \hat{\sigma}_{\varepsilon}^2}{(N - A) \bar{T}} \right] \geq 0$$

where $A = K$ or if $\hat{\sigma}_u^2$ is negative,

A = trace of a matrix that somewhat resembles \mathbf{I}_K .

Many other adjustments exist. None guaranteed to be positive. No optimality properties or even guaranteed consistency.



Application

```
+-----+
| Random Effects Model: v(i,t) = e(i,t) + u(i) |
| Estimates:  Var[e]           = .231188D-01 |
|              Var[u]           = .102531D+00 |
|              Corr[v(i,t),v(i,s)] = .816006 |
| Variance estimators are based on OLS residuals. |
+-----+
```

**No problems arise
in this sample.**

Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
EXP	.08819204	.00224823	39.227	.0000	19.8537815
EXPSQ	-.00076604	.496074D-04	-15.442	.0000	514.405042
OCC	-.04243576	.01298466	-3.268	.0011	.51116447
SMSA	-.03404260	.01620508	-2.101	.0357	.65378151
MS	-.06708159	.01794516	-3.738	.0002	.81440576
FEM	-.34346104	.04536453	-7.571	.0000	.11260504
UNION	.05752770	.01350031	4.261	.0000	.36398559
ED	.11028379	.00510008	21.624	.0000	12.8453782
Constant	4.01913257	.07724830	52.029	.0000	



Testing for Effects: An LM Test

Breusch and Pagan Lagrange Multiplier statistic

$$y_{it} = \beta' x_{it} + u_i + \varepsilon_{it}, \quad u_i \text{ and } \varepsilon_{it} \sim \text{Normal} \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{pmatrix} \right]$$

$$H_0 : \sigma_u^2 = 0$$

$$LM = \frac{(\sum_{i=1}^N T_i)^2}{2 \sum_{i=1}^N T_i (T_i - 1)} \left[\frac{\sum_{i=1}^N (T_i \bar{e}_i)^2}{\sum_{i=1}^N \sum_{t=1}^T e_{it}^2} - 1 \right]^2 \longrightarrow \chi^2[1]$$

Application: Cornwell-Rupert

+-----+-----+-----+-----+-----+-----+					
Ordinary least squares regression					
LHS=LWAGE	Mean	=	6.676346		
	Standard deviation	=	.4615122		
Model size	Parameters	=	7		
	Degrees of freedom	=	4158		
Residuals	Sum of squares	=	556.3030		
	Standard error of e	=	.3657745		
Fit	R-squared	=	.3727592		
	Adjusted R-squared	=	.3718541		
+-----+-----+-----+-----+-----+-----+					
+-----+-----+-----+-----+-----+-----+					
Variable	Coefficient		Standard Error	b/St. Er.	P[Z >z] Mean of X
+-----+-----+-----+-----+-----+-----+					
Constant	5.66098218		.04685914	120.808	.0000
FEM	-.39478212		.02603413	-15.164	.0000 .11260504
ED	.05688005		.00267743	21.244	.0000 12.8453782
OCC	-.11220205		.01464317	-7.662	.0000 .51116447
SMSA	.15504405		.01233744	12.567	.0000 .65378151
MS	.09569050		.02133490	4.485	.0000 .81440576
EXP	.01043785		.00054206	19.256	.0000 19.8537815
+-----+-----+-----+-----+-----+-----+					
Random Effects Model: $v(i,t) = e(i,t) + u(i)$					
Estimates: Var[e]	= .235368D+01				
Var[u]	= .110254D+00				
Corr[v(i,t),v(i,s)]	= .824078				
Lagrange Multiplier Test vs. Model (3)	= 3797.07				
(1 df, prob value = .000000)					
(High values of LM favor FEM/REM over CR model.)					
+-----+-----+-----+-----+-----+-----+					
Constant	4.24669585		.07763394	54.702	.0000
FEM	-.34715010		.04681514	-7.415	.0000 .11260504
ED	.11120152		.00525209	21.173	.0000 12.8453782
OCC	-.03908144		.01298962	-3.009	.0026 .51116447
SMSA	-.03881553		.01645862	-2.358	.0184 .65378151
MS	-.06557030		.01815465	-3.612	.0003 .81440576
EXP	.05737298		.00088467	64.852	.0000 19.8537815





Hausman Test for FE vs. RE

Estimator	Random Effects $E[c_i \mathbf{X}_i] = 0$	Fixed Effects $E[c_i \mathbf{X}_i] \neq 0$
FGLS (Random Effects)	Consistent and Efficient	Inconsistent
LSDV (Fixed Effects)	Consistent Inefficient	Consistent Possibly Efficient



Computing the Hausman Statistic

$$\text{Est.Var}[\hat{\boldsymbol{\beta}}_{\text{FE}}] = \hat{\sigma}_{\varepsilon}^2 \left[\sum_{i=1}^N \mathbf{X}_i' \left(\mathbf{I} - \frac{1}{T_i} \mathbf{ii}' \right) \mathbf{X}_i \right]^{-1}$$

$$\text{Est.Var}[\hat{\boldsymbol{\beta}}_{\text{RE}}] = \hat{\sigma}_{\varepsilon}^2 \left[\sum_{i=1}^N \mathbf{X}_i' \left(\mathbf{I} - \frac{\hat{\gamma}_i}{T_i} \mathbf{ii}' \right) \mathbf{X}_i \right]^{-1}, \quad 0 \leq \hat{\gamma}_i = \frac{T_i \hat{\sigma}_u^2}{\hat{\sigma}_{\varepsilon}^2 + T_i \hat{\sigma}_u^2} \leq 1$$

As long as $\hat{\sigma}_{\varepsilon}^2$ and $\hat{\sigma}_u^2$ are consistent, as $N \rightarrow \infty$, $\text{Est.Var}[\hat{\boldsymbol{\beta}}_{\text{FE}}] - \text{Est.Var}[\hat{\boldsymbol{\beta}}_{\text{RE}}]$ will be nonnegative definite. In a finite sample, to ensure this, both must be computed using the same estimate of $\hat{\sigma}_{\varepsilon}^2$. The one based on LSDV will generally be the better choice.

Note that columns of zeros will appear in $\text{Est.Var}[\hat{\boldsymbol{\beta}}_{\text{FE}}]$ if there are time invariant variables in \mathbf{X} .

$\boldsymbol{\beta}$ does not contain the constant term in the preceding.



Hausman Test

```
+-----+
| Random Effects Model:  $v(i,t) = e(i,t) + u(i)$  |
| Estimates:  Var[e]                = .235368D-01 |
|              Var[u]                = .110254D+00 |
|              Corr[v(i,t),v(i,s)] = .824078      |
| Lagrange Multiplier Test vs. Model (3) = 3797.07 |
| ( 1 df, prob value = .000000)                  |
| (High values of LM favor FEM/REM over CR model.) |
+-----+
| Fixed vs. Random Effects (Hausman)      = 2632.34 |
| ( 4 df, prob value = .000000)           |
| (High (low) values of H favor FEM (REM).) |
+-----+
```



Variable Addition

A Fixed Effects Model

$$y_{it} = \alpha_i + \beta' \mathbf{x}_{it} + \varepsilon_{it}$$

LSDV estimator - Deviations from group means:

To estimate β , regress $(y_{it} - \bar{y}_i)$ on $(\mathbf{x}_{it} - \bar{\mathbf{x}}_i)$

Algebraic equivalent: OLS regress y_{it} on $(\mathbf{x}_{it}, \bar{\mathbf{x}}_i)$

Mundlak interpretation: $\alpha_i = \alpha + \delta' \bar{\mathbf{x}}_i + u_i$

Model becomes $y_{it} = \alpha + \delta' \bar{\mathbf{x}}_i + u_i + \beta' \mathbf{x}_{it} + \varepsilon_{it}$

$$= \alpha + \delta' \bar{\mathbf{x}}_i + \beta' \mathbf{x}_{it} + \varepsilon_{it} + u_i$$

= a random effects model with the group means.

Estimate by FGLS.



A Variable Addition Test

- Asymptotic equivalent to Hausman
- Also equivalent to Mundlak formulation
- In the random effects model, using FGLS
 - Only applies to time varying variables
 - Add expanded group means to the regression (i.e., observation i,t gets same group means for all t .)
 - Use Wald test to test for coefficients on means equal to 0. Large chi-squared weighs against random effects specification.



Fixed Effects

```

+-----+
| Panel:Groups   Empty      0,   Valid data   595 |
|               Smallest   7,   Largest      7   |
|               Average group size           7.00 |
| There are 3 vars. with no within group variation. |
| ED           BLK           FEM                 |
| Look for huge standard errors and fixed parameters. |
| F.E. results are based on a generalized inverse.   |
| They will be highly erratic. (Problematic model.) |
| Unable to compute std.errors for dummy var. coeffs. |
+-----+
  
```

```

+-----+-----+-----+-----+-----+-----+
|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|
+-----+-----+-----+-----+-----+-----+
|WKS     | .00083      | .00060003      | 1.381   |.1672   | 46.811525|
|OCC     | -.02157     | .01379216      | -1.564  |.1178   | .5111645|
|IND     | .01888      | .01545450      | 1.221   |.2219   | .3954382|
|SOUTH   | .00039      | .03429053      | .011    |.9909   | .2902761|
|SMSA    | -.04451**   | .01939659      | -2.295  |.0217   | .6537815|
|UNION   | .03274**    | .01493217      | 2.192   |.0283   | .3639856|
|EXP     | .11327***   | .00247221      | 45.819  |.0000   | 19.853782|
|EXPSQ   | -.00042***  | .546283D-04    | -7.664  |.0000   | 514.40504|
|ED       | .000        | .....(Fixed Parameter)..... |
|BLK      | .000        | .....(Fixed Parameter)..... |
|FEM      | .000        | .....(Fixed Parameter)..... |
+-----+-----+-----+-----+-----+-----+
  
```




Random Effects

```

+-----+
| Random Effects Model: v(i,t) = e(i,t) + u(i) |
| Estimates:  Var[e]                = .235368D-01 |
|              Var[u]                = .110254D+00 |
|              Corr[v(i,t),v(i,s)] = .824078      |
| Lagrange Multiplier Test vs. Model (3) = 3797.07 |
| ( 1 df, prob value = .000000)                  |
| (High values of LM favor FEM/REM over CR model.) |
+-----+
  
```

Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
WKS	.00094	.00059308	1.586	.1128	46.811525
OCC	-.04367***	.01299206	-3.361	.0008	.5111645
IND	.00271	.01373256	.197	.8434	.3954382
SOUTH	-.00664	.02246416	-.295	.7677	.2902761
SMSA	-.03117*	.01615455	-1.930	.0536	.6537815
UNION	.05802***	.01349982	4.298	.0000	.3639856
EXP	.08744***	.00224705	38.913	.0000	19.853782
EXPSQ	-.00076***	.495876D-04	-15.411	.0000	514.40504
ED	.10724***	.00511463	20.967	.0000	12.845378
BLK	-.21178***	.05252013	-4.032	.0001	.0722689
FEM	-.24786***	.04283536	-5.786	.0000	.1126050
Constant	3.97756***	.08178139	48.637	.0000	



The Hausman Test, by Hand

```
--> matrix; br=b(1:8) ; vr=varb(1:8,1:8)$  
--> matrix ; db = bf - br ; dv = vf - vr $  
--> matrix ; list ; h =db'<dv>db$
```

Matrix H has 1 rows and 1 columns.
1

```
+-----  
1 | 2523.64910
```

```
--> calc,list;ctb(.95,8)$
```

```
+-----+  
| Listed Calculator Results |  
+-----+  
Result = 15.507313
```



Means Added to REM - Mundlak

Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
WKS	.00083	.00060070	1.380	.1677	46.811525
OCC	-.02157	.01380769	-1.562	.1182	.5111645
IND	.01888	.01547189	1.220	.2224	.3954382
SOUTH	.00039	.03432914	.011	.9909	.2902761
SMSA	-.04451**	.01941842	-2.292	.0219	.6537815
UNION	.03274**	.01494898	2.190	.0285	.3639856
EXP	.11327***	.00247500	45.768	.0000	19.853782
EXPSQ	-.00042***	.546898D-04	-7.655	.0000	514.40504
ED	.05199***	.00552893	9.404	.0000	12.845378
BLK	-.16983***	.04456572	-3.811	.0001	.0722689
FEM	-.41306***	.03732204	-11.067	.0000	.1126050
WKSB	.00863**	.00363907	2.371	.0177	46.811525
OCCB	-.14656***	.03640885	-4.025	.0001	.5111645
INDB	.04142	.02976363	1.392	.1640	.3954382
SOUTHB	-.05551	.04297816	-1.292	.1965	.2902761
SMSAB	.21607***	.03213205	6.724	.0000	.6537815
UNIONB	.08152**	.03266438	2.496	.0126	.3639856
EXPB	-.08005***	.00533603	-15.002	.0000	19.853782
EXPSQB	-.00017	.00011763	-1.416	.1567	514.40504
Constant	5.19036***	.20147201	25.762	.0000	



Wu (Variable Addition) Test

```
--> matrix ; bm=b(12:19);vm=varb(12:19,12:19)$  
--> matrix ; list ; wu = bm'<vm>bm $
```

Matrix WU has 1 rows and 1 columns.

```
      1  
+-----  
1 | 3004.38076
```



A Hierarchical Linear Model

Interpretation of the FE Model

$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + \alpha_i + \varepsilon_{it}$ (does not contain a constant)

$$E[\varepsilon_{it} | \mathbf{X}_i, c_i] = 0, \text{Var}[\varepsilon_{it} | \mathbf{X}_i, c_i] = \sigma_\varepsilon^2$$

$$c_i = \alpha + \mathbf{z}_i\boldsymbol{\delta} + u_i,$$

$$E[u_i | \mathbf{z}_i'] = 0, \text{Var}[u_i | \mathbf{z}_i'] = \sigma_u^2$$

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + [\alpha + \mathbf{z}_i\boldsymbol{\delta} + u_i] + \varepsilon_{it}$$



Hierarchical Linear Model as REM

```

+-----+
| Random Effects Model: v(i,t) = e(i,t) + u(i) |
| Estimates:  Var[e]           = .235368D-01 |
|              Var[u]          = .110254D+00 |
|              Corr[v(i,t),v(i,s)] = .824078 |
|              Sigma(u)         = 0.3303     |
+-----+
  
```

Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
OCC	-.03908144	.01298962	-3.009	.0026	.51116447
SMSA	-.03881553	.01645862	-2.358	.0184	.65378151
MS	-.06557030	.01815465	-3.612	.0003	.81440576
EXP	.05737298	.00088467	64.852	.0000	19.8537815
FEM	-.34715010	.04681514	-7.415	.0000	.11260504
ED	.11120152	.00525209	21.173	.0000	12.8453782
Constant	4.24669585	.07763394	54.702	.0000	



Evolution: Correlated Random Effects

Unknown parameters

$$y_{it} = \alpha_i + \beta' \mathbf{x}_{it} + \varepsilon_{it}, \quad \Theta = [\alpha_1, \alpha_2, \dots, \alpha_N, \beta, \sigma_\varepsilon^2]$$

Standard estimation based on LS (dummy variables)

Ambiguous definition of the distribution of y_{it}

Effects model, nonorthogonality, heterogeneity

$$y_{it} = \alpha_i + \beta' \mathbf{x}_{it} + \varepsilon_{it}, \quad E[\alpha_i | \mathbf{X}_i] = g(\mathbf{X}_i) \neq 0$$

Contrast to random effects $E[\alpha_i | \mathbf{X}_i] = \alpha$

Standard estimation (still) based on LS (dummy variables)

Correlated random effects, more detailed model

$$y_{it} = \alpha_i + \beta' \mathbf{x}_{it} + \varepsilon_{it}, \quad P[\alpha_i | \mathbf{X}_i] = g(\mathbf{X}_i) \neq 0$$

Linear projection? $\alpha_i = \boldsymbol{\theta}' \mathbf{x}_i + u_i \quad \text{Cor}(u_i, \mathbf{x}_i) = 0$



Mundlak's Estimator

Mundlak, Y., "On the Pooling of Time Series and Cross Section Data, *Econometrica*, 46, 1978, pp. 69-85.

Write $c_i = \bar{\mathbf{x}}_i \boldsymbol{\delta} + u_i$, $E[c_i | \mathbf{x}_{i1}, \mathbf{x}_{i2}, \dots, \mathbf{x}_{iT_i}] = \bar{\mathbf{x}}_i \boldsymbol{\delta}$

Assume c_i contains all time invariant information

$$\begin{aligned} \mathbf{y}_i &= \boldsymbol{\beta} \mathbf{x}_i + c_i \boldsymbol{\epsilon}_i + \mathbf{u}_i, \quad T_i \text{ observations in group } i \\ &= \mathbf{x}_i \boldsymbol{\beta} + \bar{\mathbf{x}}_i \boldsymbol{\delta} + \boldsymbol{\epsilon}_i + \mathbf{u}_i \end{aligned}$$

Looks like random effects.

$$\text{Var}[\boldsymbol{\epsilon}_i + \mathbf{u}_i] = \boldsymbol{\Omega}_i + \sigma_u^2 \mathbf{ii}'$$

This is the model we used for the Wu test.



Correlated Random Effects

Mundlak

$$c_i = \bar{\mathbf{x}}_i \boldsymbol{\delta} + u_i, \quad E[c_i | \mathbf{x}_{i1}, \mathbf{x}_{i2}, \dots, \mathbf{x}_{iT_i}] = \bar{\mathbf{x}}_i \boldsymbol{\delta}$$

Assume c_i contains all time invariant information

$$\begin{aligned} \mathbf{y}_i &= \mathbf{X}_i \boldsymbol{\beta} + c_i \mathbf{i} + \boldsymbol{\varepsilon}_i, \quad T_i \text{ observations in group } i \\ &= \mathbf{X}_i \boldsymbol{\beta} + \mathbf{i} \bar{\mathbf{x}}_i \boldsymbol{\delta} + \boldsymbol{\varepsilon}_i + u_i \end{aligned}$$

Chamberlain / Wooldridge

$$c_i = \mathbf{x}_{i1} \boldsymbol{\delta}_1 + \mathbf{x}_{i2} \boldsymbol{\delta}_2 + \dots + \mathbf{x}_{iT_i} \boldsymbol{\delta}_{T_i} + u_i$$

$$\begin{aligned} \mathbf{y}_i &= \boldsymbol{\beta}_i \mathbf{i} + \mathbf{x}_{i1} \boldsymbol{\delta}'_1 + \mathbf{x}_{i2} \boldsymbol{\delta}'_2 + \dots + \mathbf{x}_{iT_i} \boldsymbol{\delta}'_{T_i} + \boldsymbol{\varepsilon}_i + u_i \\ &\quad \underbrace{T \times K}_{} + \underbrace{T \times K}_{} + \underbrace{T \times K}_{} + \underbrace{T \times K}_{} \text{ etc.} \end{aligned}$$

Problems: Requires balanced panels

Modern panels have large T ; models have large K





Mundlak's Approach for an FE Model with Time Invariant Variables

$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + \mathbf{z}_i\boldsymbol{\delta} + \varepsilon_{it}$ (does not contain a constant)

$$E[\varepsilon_{it} | \mathbf{X}_i, c_i] = 0, \text{Var}[\varepsilon_{it} | \mathbf{X}_i, c_i] = \sigma_\varepsilon^2$$

$$c_i = \alpha + \bar{\mathbf{x}}_i\boldsymbol{\theta} + w_i,$$

$$E[w_i | \mathbf{X}_i, \mathbf{z}_i] = 0, \text{Var}[w_i | \mathbf{X}_i, \mathbf{z}_i] = \sigma_w^2$$

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + \mathbf{z}_i\boldsymbol{\delta} + \alpha + \bar{\mathbf{x}}_i\boldsymbol{\theta} + w_i + \varepsilon_{it}$$

= random effects model including group means of time varying variables.



Mundlak Form of FE Model

Variable	Coefficient	Standard Error	b/St.Er.	P[Z >z]	Mean of X
x(i,t)=====					
OCC	-.02021384	.01375165	-1.470	.1416	.51116447
SMSA	-.04250645	.01951727	-2.178	.0294	.65378151
MS	-.02946444	.01915264	-1.538	.1240	.81440576
EXP	.09665711	.00119262	81.046	.0000	19.8537815
z(i)=====					
FEM	-.34322129	.05725632	-5.994	.0000	.11260504
ED	.05099781	.00575551	8.861	.0000	12.8453782
Means of x(i,t) and constant=====					
Constant	5.72655261	.10300460	55.595	.0000	
OCCB	-.10850252	.03635921	-2.984	.0028	.51116447
SMSAB	.22934020	.03282197	6.987	.0000	.65378151
MSB	.20453332	.05329948	3.837	.0001	.81440576
EXPB	-.08988632	.00165025	-54.468	.0000	19.8537815
Variance Estimates=====					
Var[e]	.0235632				
Var[u]	.0773825				



Panel Data Extensions

- Dynamic models: lagged effects of the dependent variable
- Endogenous RHS variables
- Cross country comparisons– large T
- More general parameter heterogeneity – not only the constant term
- Nonlinear models such as binary choice



The Hausman and Taylor Model

$$y_{it} = \mathbf{x1}_i \boldsymbol{\beta}_1 + \mathbf{x2}_{it} \boldsymbol{\beta}_2 + \mathbf{z1}_i \boldsymbol{\alpha}_1 + \mathbf{z2}_{it} \boldsymbol{\alpha}_2 + \varepsilon_{it} + u_i$$

Model: $\mathbf{x2}$ and $\mathbf{z2}$ are correlated with u .

Deviations from group means removes all time invariant variables

$$y_{it} - \bar{y}_i = (\mathbf{x1}_{it} - \bar{\mathbf{x1}}_i)' \boldsymbol{\beta}_1 + (\mathbf{x2}_{it} - \bar{\mathbf{x2}}_i)' \boldsymbol{\beta}_2 + \varepsilon_{it}$$

Implication: $\boldsymbol{\beta}_1, \boldsymbol{\beta}_2$ are consistently estimated by LSDV.

$(\mathbf{x1}_{it} - \bar{\mathbf{x1}}_i) = K_1$ instrumental variables

$(\mathbf{x2}_{it} - \bar{\mathbf{x2}}_i) = K_2$ instrumental variables

$\mathbf{z1}_i = L_1$ instrumental variables (uncorrelated with u)

? = L_2 instrumental variables (where do we get them?)

H&T: $\bar{\mathbf{x1}}_i = K_1$ additional instrumental variables. Needs $K_1 \geq L_2$.



H&T's 4 Step FGLS Estimator

(1) LSDV estimates of $\beta_1, \beta_2, \sigma_\varepsilon^2$

(2) $(\mathbf{e}^*)' = (\bar{e}_1, \bar{e}_1, \dots, \bar{e}_1), (\bar{e}_2, \bar{e}_2, \dots, \bar{e}_2), \dots, (\bar{e}_N, \bar{e}_N, \dots, \bar{e}_N)$

IV regression of \mathbf{e}^* on \mathbf{Z}^* with instruments

\mathbf{W}_i consistently estimates β_1 and β_2 .

(3) With fixed T, residual variance in (2) estimates $\sigma_u^2 + \sigma_\varepsilon^2 / T$

With unbalanced panel, it estimates $\sigma_u^2 + \sigma_\varepsilon^2 \overline{(1/T)}$ or something resembling this. (1) provided an estimate of σ_ε^2 so use the two to obtain estimates of σ_u^2 and σ_ε^2 . For each group, compute

$$\hat{\theta}_i = 1 - \sqrt{\hat{\sigma}_\varepsilon^2 / (\hat{\sigma}_\varepsilon^2 + T_i \hat{\sigma}_u^2)}$$

(4) Transform $[\mathbf{x}_{it1}, \mathbf{x}_{it2}, \mathbf{z}_{i1}, \mathbf{z}_{i2}]$ to

$$\mathbf{W}_i^* = [\mathbf{x}_{it1}, \mathbf{x}_{it2}, \mathbf{z}_{i1}, \mathbf{z}_{i2}] - \hat{\theta}_i [\bar{\mathbf{x}}_{i1}, \bar{\mathbf{x}}_{i2}, \mathbf{z}_{i1}, \mathbf{z}_{i2}]$$

and y_{it} to $y_{it}^* = y_{it} - \hat{\theta}_i \bar{y}_i$.



H&T's 4 STEP IV Estimator

Instrumental Variables $\mathbf{V}_i =$

$(\mathbf{x1}_{it} - \overline{\mathbf{x1}_i}) = K_1$ instrumental variables

$(\mathbf{x2}_{it} - \overline{\mathbf{x2}_i}) = K_2$ instrumental variables

$\mathbf{z1}_i = L_1$ instrumental variables (uncorrelated with u)

$\overline{\mathbf{x1}_i} = K_1$ additional instrumental variables.

Now do 2SLS of \mathbf{y}^* on \mathbf{W}^* with instruments \mathbf{V} to estimate all parameters. I.e.,

$$[\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2] = (\hat{\mathbf{W}}^{*'} \hat{\mathbf{W}}^*)^{-1} \hat{\mathbf{W}}^{*'} \mathbf{y}^*.$$



TABLE 13.3 Estimated Log Wage Equations

<i>Variables</i>		<i>OLS</i>	<i>GLS/RE</i>	<i>LSDV</i>	<i>HT/IV-GLS</i>	<i>HT/IV-GLS</i>
x_1	Experience	0.0132 (0.0011) ^a	0.0133 (0.0017)	0.0241 (0.0042)	0.0217 (0.0031)	
	Bad health	−0.0843 (0.0412)	−0.0300 (0.0363)	−0.0388 (0.0460)	−0.0278 (0.0307)	−0.0388 (0.0348)
	Unemployed Last Year	−0.0015 (0.0267)	−0.0402 (0.0207)	−0.0560 (0.0295)	−0.0559 (0.0246)	
	Time	NR ^b	NR	NR	NR	NR
	Experience					0.0241 (0.0045)
x_2	Unemployed					−0.0560 (0.0279)
	Race	−0.0853 (0.0328)	−0.0878 (0.0518)		−0.0278 (0.0752)	−0.0175 (0.0764)
	Union	0.0450 (0.0191)	0.0374 (0.0296)		0.1227 (0.0473)	0.2240 (0.2863)
	Schooling	0.0669 (0.0033)	0.0676 (0.0052)			
	Constant	NR	NR	NR	NR	NR
z_2	Schooling				0.1246 (0.0434)	0.2169 (0.0979)
	σ_ε	0.321	0.192	0.160	0.190	0.629
	$\rho = \sqrt{\sigma_u^2 / (\sigma_u^2 + \sigma_\varepsilon^2)}$		0.632		0.661	0.817
	Spec. Test [3]		20.2		2.24	0.00

^aEstimated asymptotic standard errors are given in parentheses.

^bNR indicates that the coefficient estimate was not reported in the study.



Arellano/Bond/Bover's Formulation Builds on Hausman and Taylor

$$y_{it} = \mathbf{x1}_{it}\boldsymbol{\beta}_1 + \mathbf{x2}_{it}\boldsymbol{\beta}_2 + \mathbf{z1}_{it}\boldsymbol{\alpha}_1 + \mathbf{z2}_{it}\boldsymbol{\alpha}_2 + \varepsilon_{it} + u_i$$

Instrumental variables for period t

$(\mathbf{x1}_{it} - \overline{\mathbf{x1}_i}) = K_1$ instrumental variables

$(\mathbf{x2}_{it} - \overline{\mathbf{x2}_i}) = K_2$ instrumental variables

$\mathbf{z1}_i = L_1$ instrumental variables (uncorrelated with u)

$\overline{\mathbf{x1}_i} = K_1$ additional instrumental variables. $K_1 \geq L_2$.

Let $v_{it} = \varepsilon_{it} + u_i$

Let $\mathbf{z}'_{it} = [(\mathbf{x1}_{it} - \overline{\mathbf{x1}_i})', (\mathbf{x2}_{it} - \overline{\mathbf{x2}_i})', \mathbf{z1}'_i, \overline{\mathbf{x1}_i}']$

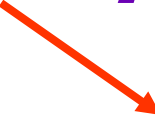
Then $E[\mathbf{z}_{it}v_{it}] = \mathbf{0}$

We formulate this for the T_i observations in group i.



Arellano/Bond/Bover's Formulation

Adds a Lagged DV to H&T



$$y_{it} = \delta y_{i,t-1} + \mathbf{x1}_i' \boldsymbol{\beta}_1 + \mathbf{x2}_i' \boldsymbol{\beta}_2 + \mathbf{z1}_i' \boldsymbol{\alpha}_1 + \mathbf{z2}_i' \boldsymbol{\alpha}_2 + \varepsilon_{it} + u_i$$

Parameters $\boldsymbol{\theta} = [\boldsymbol{\beta}_1' \boldsymbol{\beta}_2' \boldsymbol{\alpha}_1' \boldsymbol{\alpha}_2']$

The data

$$\mathbf{y}_i = \begin{bmatrix} y_{i,2} \\ y_{i,3} \\ \vdots \\ y_{i,T_i} \end{bmatrix}, \quad \mathbf{X}_i = \begin{bmatrix} y_{i,1} & \mathbf{x1}_{i2}' & \mathbf{x2}_{i2}' & \mathbf{z1}_i' & \mathbf{z2}_i' \\ y_{i,2} & \mathbf{x1}_{i3}' & \mathbf{x2}_{i3}' & \mathbf{z1}_i' & \mathbf{z2}_i' \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{i,T-1} & \mathbf{x1}_{iT-1}' & \mathbf{x2}_{iT-1}' & \mathbf{z1}_i' & \mathbf{z2}_i' \end{bmatrix}, \quad T_i - 1 \text{ rows}$$

1 K1 K2 L1 L2 columns

This formulation is the same as H&T with $y_{i,t-1}$ contained in $\mathbf{x2}_{it}$.



Dynamic (Linear) Panel Data (DPD) Models

- Application
- Bias in Conventional Estimation
- Development of Consistent Estimators
- Efficient GMM Estimators



Dynamic Linear Model

Balestra-Nerlove (1966), 36 States, 11 Years

Demand for Natural Gas

Structure

New Demand: $G_{i,t}^* = G_{i,t} - (1 - \delta)G_{i,t-1}$

Demand Function $G_{i,t}^* = \beta_1 + \beta_2 P_{i,t} + \beta_3 \Delta N_{i,t} + \beta_4 N_{i,t} + \beta_5 \Delta Y_{i,t} + \beta_6 Y_{i,t} + \varepsilon_{i,t}$

G=gas demand

N = population

P = price

Y = per capita income

Reduced Form

$G_{i,t} = \beta_1 + \beta_2 P_{i,t} + \beta_3 \Delta N_{i,t} + \beta_4 N_{i,t} + \beta_5 \Delta Y_{i,t} + \beta_6 Y_{i,t} + \beta_7 G_{i,t-1} + \alpha_i + \varepsilon_{i,t}$



A General DPD model

$$y_{i,t} = \mathbf{x}_{i,t} \boldsymbol{\beta} + \boxed{\delta y_{i,t-1}} + \boxed{c_i} + \varepsilon_{i,t}$$

$$E[\varepsilon_{i,t} \mid \mathbf{X}_i, c_i] = 0$$

$$E[\varepsilon_{i,t}^2 \mid \mathbf{X}_i, c_i] = \sigma_\varepsilon^2, \quad E[\varepsilon_{i,t} \varepsilon_{i,s} \mid \mathbf{X}_i, c_i] = 0 \text{ if } t \neq s.$$

$$E[c_i \mid \mathbf{X}_i] = g(\mathbf{X}_i)$$

No correlation across individuals

OLS and GLS are both inconsistent.



Arellano and Bond Estimator

Base on first differences

$$y_{i,t} - y_{i,t-1} = (\mathbf{x}_{i,t} \boldsymbol{\beta} \mathbf{x}_{i,t-1})' + \delta(y_{i,t-1} - y_{i,t-2}) + (\varepsilon_{i,t} - \varepsilon_{i,t-1})$$

Instrumental variables

$$y_{i,3} - y_{i,2} = (\mathbf{x}_{i,3} \boldsymbol{\beta} \mathbf{x}_{i,2})' + \delta(y_{i,2} - y_{i,1}) + (\varepsilon_{i,3} - \varepsilon_{i,2})$$

Can use y_{i1}

$$y_{i,4} - y_{i,3} = (\mathbf{x}_{i,4} \boldsymbol{\beta} \mathbf{x}_{i,3})' + \delta(y_{i,3} - y_{i,2}) + (\varepsilon_{i,4} - \varepsilon_{i,3})$$

Can use $y_{i,1}$ and y_{i2}

$$y_{i,5} - y_{i,4} = (\mathbf{x}_{i,5} \boldsymbol{\beta} \mathbf{x}_{i,4})' + \delta(y_{i,4} - y_{i,3}) + (\varepsilon_{i,5} - \varepsilon_{i,4})$$

Can use $y_{i,1}$ and y_{i2} and $y_{i,3}$



Arellano and Bond Estimator

More instrumental variables - Predetermined X

$$y_{i,3} - y_{i,2} = (\mathbf{x}_{i,3} \boldsymbol{\beta} \mathbf{x}_{i,2})' + \delta(y_{i,2} - y_{i,1}) + (\varepsilon_{i,3} - \varepsilon_{i,2})$$

Can use y_{i1} and $\mathbf{x}_{i,1}, \mathbf{x}_{i,2}$

$$y_{i,4} - y_{i,3} = (\mathbf{x}_{i,4} \boldsymbol{\beta} \mathbf{x}_{i,3})' + \delta(y_{i,3} - y_{i,2}) + (\varepsilon_{i,4} - \varepsilon_{i,3})$$

Can use $y_{i,1}, y_{i,2}, \mathbf{x}_{i,1}, \mathbf{x}_{i,2}, \mathbf{x}_{i,3}$

$$y_{i,5} - y_{i,4} = (\mathbf{x}_{i,5} \boldsymbol{\beta} \mathbf{x}_{i,4})' + \delta(y_{i,4} - y_{i,3}) + (\varepsilon_{i,5} - \varepsilon_{i,4})$$

Can use $y_{i,1}, y_{i,2}, y_{i,3}, \mathbf{x}_{i,1}, \mathbf{x}_{i,2}, \mathbf{x}_{i,3}, \mathbf{x}_{i,4}$



Arellano and Bond Estimator

Even more instrumental variables - Strictly exogenous X

$$y_{i,3} - y_{i,2} = (\mathbf{x}_{i,3} \boldsymbol{\beta} \mathbf{x}_{i,2})' + \delta(y_{i,2} - y_{i,1}) + (\varepsilon_{i,3} - \varepsilon_{i,2})$$

Can use y_{i1} and $\mathbf{x}_{i,1}, \mathbf{x}_{i,2}, \dots, \mathbf{x}_{i,T}$ (all periods)

$$y_{i,4} - y_{i,3} = (\mathbf{x}_{i,4} \boldsymbol{\beta} \mathbf{x}_{i,3})' + \delta(y_{i,3} - y_{i,2}) + (\varepsilon_{i,4} - \varepsilon_{i,3})$$

Can use $y_{i,1}, y_{i,2}, \mathbf{x}_{i,1}, \mathbf{x}_{i,2}, \dots, \mathbf{x}_{i,T}$

$$y_{i,5} - y_{i,4} = (\mathbf{x}_{i,5} \boldsymbol{\beta} \mathbf{x}_{i,4})' + \delta(y_{i,4} - y_{i,3}) + (\varepsilon_{i,5} - \varepsilon_{i,4})$$

Can use $y_{i,1}, y_{i,2}, y_{i,3}, \mathbf{x}_{i,1}, \mathbf{x}_{i,2}, \dots, \mathbf{x}_{i,T}$

The number of potential instruments is huge.

These define the rows of \mathbf{Z}_i . These can be used for simple instrumental variable estimation.



Application: Maquiladora

The U.S. and Mexico: Are We Still Connected?
Federal Reserve Bank of Dallas, El Paso Branch
Network of Border Economics (Red de la Economía Fronteriza)
Centro de Investigación y Docencia Económicas A.C.
Houston, Texas. November 18, 2005

Maquila: volatility and Mexico-US economic integration

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Maquiladora

Model: Labor Demand in Maquila Industry

Dynamic Panel Data:

$$Ltrab_{it} = \alpha_0 + \alpha_1 Ltrab_{i(t-1)} + \alpha_2 Ltrab_{i(t-2)} + \beta_1 Lrppd_{it} + \beta_2 Lpibusa_{it} + v_i + u_{it}$$

t= 1990.1 – 2005.3 quarterly

i = The Following 13 States where maquila mainly operates: Baja California, Sonora, Chihuahua, Coahuila, Nuevo León, Tamaulipas, Durango, Aguascalientes, Jalisco, Guanajuato, Mexico-DF, Puebla y Yucatán.

Variables:

Ltrab= log of maquila employment

Lrppd = wage per worker in dollars

Lpibusa = log of: USA GDP (2000 prices) over distance

Estimates

Model: Labor Demand in Maquila Industry

```

Arellano-Bond dynamic panel-data estimation      Number of obs      =      695
Group variable (i): estado                       Number of groups   =      13
                                                Wald chi2(4)       =  18500.45
Time variable (t): trim                         Obs per group: min =      35
                                                avg =  53.46154
                                                max =      59
  
```

One-step results

D.ltrab		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
ltrab						
	LD	1.220175	.0362107	33.70	0.000	1.149204 1.291147
	L2D	-.262198	.0355168	-7.38	0.000	-.3318095 -.1925864
lrppd						
	D1	-.0804483	.0115187	-6.98	0.000	-.1030246 -.0578721
lpibusa						
	D1	.4801248	.1643802	2.92	0.003	.1579454 .8023041
_cons		-.0023032	.0012531	-1.84	0.066	-.0047592 .0001528

Sargan test of over-identifying restrictions:

chi2(1827) = 695.25 Prob > chi2 = 1.0000

Arellano-Bond test that average autocovariance in residuals of order 1 is 0:

H0: no autocorrelation z = -13.42 Pr > z = 0.0000

Arellano-Bond test that average autocovariance in residuals of order 2 is 0:

H0: no autocorrelation z = -1.30 Pr > z = 0.1927