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Communication

The Optimal Cash Balance Proposition: Maurice Allais' Priority

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Maurice Allais' well-deserved Nobel Prize fortuitously brought to our attention an injustice inadvertently done him, to which we were unknowing accessories. For years the literature has ascribed to us the parentage of the transactions-cost model of optimal cash balances, with its notorious square-root formula derived from inventory theory.[1] *Recently, we found that its essence is contained in Allais' 1947* Economie et Intérêt *(pp. 238–41). As Jacob Viner used to say, no matter to what source the origin of an economic proposition is ascribed, someone is sure to come up with an earlier one.*

In any event, here is a translation of the pertinent passages. Allais describes the model in footnotes (11) and (12) to the following text (pp. 238–41):

> One may note that the value of the benefits obtained from an additional [money] balance of 100 francs when the average balance is already 10,000 francs is much less than when that balance is zero. If, for example, a consumer carelessly invests all his funds he will almost certainly have trouble meeting his basic needs, of which some will perhaps be essential and for which he will not even have the time to have recourse to credit. Holding the mere sum of 100 francs will by itself avoid for him a thousand petty inconveniences by permitting him to have lunch, go home on the bus, buy a newspaper, give a tip, etc., and maintenance of this balance could provide him a yield of much higher value, say 500 francs, for example, representing a liquidity premium of 500 percent. In contrast, when his average balance is already high, say 10,000 francs, for example, an additional 100 francs would evidently have a very low value, of the order tenths or hundredths. (footnote 11)
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> It is then easy to see how the amount of the average balance held in equilibrium by an economic agent is determined. In effect, to the extent that the marginal liquidity premium ℓ_m of the average balance is higher than the pure interest rate, it in the agent's interest to keep his capital in the form of a money balance. This balance is only superficially "sterile," because in reality it brings, each time period, advantages equal in value to the liquidity premium. When, on the other hand, the liquidity premium becomes smaller than the pure interest rate, investment of disposable capital becomes more advantageous than keeping it in the form of money balance. *The optimal situation in equilibrium obtains when the size of the balance*

[1] The inventory-theoretic derivation goes back to 1925–27 when it was independently contributed by some half-dozen authors. This was described in Baumol (1952) on the basis of information obtained from Thomson Whitin who had first called the literature to Baumol's attention. On the early literature see Whitin (1953, chapter 2, esp. p. 32. footnote 4).

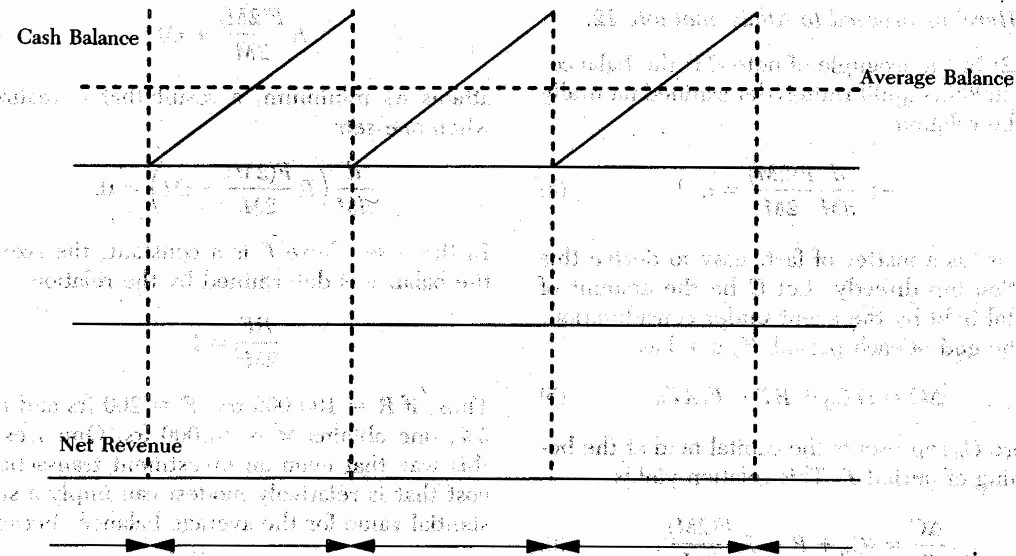

Figure 1

is such that its marginal liquidity premium is equal to the pure rate of interest [emphasis in original]. (footnote 12)

Thus, one sees that the service of money is advantageous but that no price is paid for it directly. Its cost comes *indirectly*, from gain forgone because of the loss of interest that would arise from investment of the money balance. Equilibrium occurs when the marginal value of liquidity, the service rendered by money, is precisely equal to the interest lost, and the money balance is increased or diminished as the marginal liquidity premium is higher or lower than the market interest rate.

The footnotes follow: We begin with Allais' footnote 11.

(11) It is in fact easy to make precise via a particular example these intuitive, but obviously rather fuzzy, ideas.

Consider, for example, an economic agent who has a continuous inflow of net receipts equal to R per unit of time (Figure 1) and suppose that he only invests the money received in this way at intervals of time of length T, when his balance reaches some certain value. In these circumstances the curve representing his balance as a function of time will be made up of a series of linear segments and will oscillate between zero and RT, so that its mean value will be

$$M = \frac{RT}{2}. \tag{1}$$

If one then used $F(V)$ to represent the [transactions] cost of investing an amount equal to V in value, the transactions cost per unit of time will be equal to $F(RT)/T$, that is, to $RF(2M)/2M$. If the average balance increases by ΔM, the cost saving per unit of time, which is equal to the marginal liquidity premium of the average balance, will equal the derivative of the quantity, so that one will have

$$\ell_m = -R \frac{d}{dM} \frac{F(2M)}{2M}. \tag{2}$$

If F is constant one then obtains

$$\ell_m = \frac{RF}{2M^2}, \tag{3}$$

a function decreasing in the average balance, M.

This extremely simple example has the advantage of making it easy to understand how things work out and of showing how a rigorous and general theory of liquidity can be constructed.

Here we proceed to Allais' footnote 12.

(12) In the example of note (11) the balance, M_E, held in equilibrium is determined naturally by the relation

$$-R\frac{d}{dM}\frac{F(2M)}{2M} = i. \qquad (4)$$

It is, as a matter of fact, easy to derive this relationship directly. Let C be the amount of capital held by the agent under consideration. At the end of each period, T, one has

$$\Delta C = iTC_0 + RT - F(RT), \qquad (5)$$

where C_0 represents the capital held at the beginning of period T. This relation yields

$$\frac{\Delta C}{T} = iC_0 + R - R\frac{F(2M)}{2M}. \qquad (6)$$

One may be tempted to use the rate $\Delta C/T$ as a first approximation to the derivative dC/dT, but such an approximation amounts to the assumption that the income R is invested in a continuous manner, whereas in fact it is invested only discontinuously. This assumption would attribute to dC too high a figure for the amount of interest earned on the balance during each period, T, an excess approximately equal to the product, iM, of its average value, M, and the market rate of interest, i. In fact, it is clear that one should write

$$\frac{dC}{dt} = iC + R - R\frac{F(2M)}{2M} - iM. \qquad (7)$$

The optimal balance is then attained when capital accumulates most rapidly, that is to say, when the sum

$$R\frac{F(2M)}{2M} + iM \qquad (8)$$

attains its minimum, a result that is realized when one sets

$$\frac{d}{dM}\left(R\frac{F(2M)}{2M} + iM\right) = 0. \qquad (9)$$

In the case where F is a constant, the size of the balance is determined by the relation

$$\frac{RF}{2M^2} = i.$$

Thus, if R = 100,000 frs, F = 200 frs and i = 5%, one obtains M = 14,000 frs. One sees in this way that even an investment transactions cost that is relatively modest can imply a substantial value for the average balance. Because

$$\frac{M}{R} = \sqrt{\frac{F}{2iR}}$$

one sees that in the case considered the relationship of the average balance to annual income is weaker the higher is this income. This result continues to hold if F increases with income R, but less rapidly than the latter. This is a state of affairs that seems to prevail quite generally.

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