Asset Pricing with Heterogeneous Consumers

Constantinides & Duffie, JPE 1996
Presented by: Rustom Irani, NYU Stern

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Outline

1. Introduction
   - Motivation
   - Contribution

2. Model
   - Assumptions
   - Equilibrium

3. Results
   - Mechanism
   - Empirical Implications of Idiosyncratic Risk

4. Conclusion

5. Discussion
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Representative Agent Consumption-Based Asset Pricing

- Breeden-Lucas consumption-CAPM model:
  1. Complete markets;
  2. No trading frictions;
  3. Time-additive utility.

- Aggregate consumption growth only priced factor:
  - Fluctuations too small in data!

- Euler equation estimation (Hansen & Singleton, 1982):
  - Unreasonably high RRA required to deliver empirical average excess returns;
  - “Equity-premium puzzle.”
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...Could relaxing the complete markets assumption help?
What does Market Incompleteness mean?

- Agents are unable to insure against risks they face:
  - Earnings, health, investments, etc.;
  - Asymmetric information or limited enforcement of contracts...
- Complete markets require all state-contingent contracts may be exchanged;
- However, observe very few insurance mechanisms;
- Models we consider have *exogenously* incomplete markets:
  - Model markets and institutions as we see them (e.g., stocks & bonds, social security, health insurance, etc.);
  - Don’t attempt to micro-founded.
Relaxing full consumption insurance promising:
- Individual consumption growth is much more volatile than aggregate consumption growth;
- Can this be exploited to get asset pricing right?

**Key insights:**
1. If individual and aggregate consumption risk vary systematically, then individual risk impacts equity premium;
2. Persistence and heteroscedasticity of shocks matters;

**This paper:** Uninsurable earnings risk might matter.
Main Contribution

- Closed-form solutions in presence of uninsurable earnings risk:
  - X-sectional distribution of consumption growth matters!
  - Misspecification of Euler equation;
  - Implications for risk-return relationship;
- Particular earnings process **constructed** s.t. equilibrium pricing kernel depends on x-sectional distn. of consumption;
  - “Back-solving” (Sims, JBES, 1990);
- Conversely, a class of models is identified in which market incompleteness is irrelevant:
  - If x-sectional variance of consumption growth is orthogonal to returns then incompleteness irrelevant and Breeden-Lucas CCAPM can be used for asset pricing;
  - Krueger & Lustig (JET, 2009) extends this class.
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Endowment economy;
2 Continuum of ex-ante identical, infinitely-lived consumers;
3 Finite set of securities available for trade:
   - Market incompleteness arises because cannot insure individual earnings risk using this set of securities!
Market Arrangement

At every time $t$:

1. $n$ securities:
   - Net dividend $d_{jt}$, ex-dividend price $P_{jt}$;
   - Each security in positive net supply;
   - Consumer $i$ has holding $\theta_{ijt}$.

2. $T$ bonds:
   - Default-free discount bond, paying one unit of consumption;
   - Each bond in zero net supply;
   - Consumer $i$ has holding $b_{ijt}$. 
Consumer’s Problem

\[ V_{i0} = \max_{\{\theta_{it}, b_{it}, C_{it}\}_{i,t}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} e^{-\rho t} \frac{C_{it}^{1-\alpha}}{1-\alpha} \right] \]

s.t. \quad C_{it} + \theta_{it} P_t + b_{it} B_t^T = I_{it} + \theta_{i,t-1} (P_t + d_t) + b_{i,t-1} B_{t-1}^T

- \( I_{it} \) is consumer \( i \)'s labor income endowment;
- Consumption + Savings = Nonfinancial + Financial Income.
Add up consumption, dividends and labor income:

1. \[ C_t = \int_{i \in I} C_{it}; \]
2. \[ D_t = \sum_{j=1}^{n} d_{jt}; \]
3. \[ l_t = \int_{i \in I} l_{it} = C_t - D_t. \]
Income Process

- Let $M_t > 0$ be an SDF implied by no-arbitrage;
- Assume $i$’s labor endowment is as follows:

\[
I_{it} = \delta_{it} C_t - D_t
\]

s.t. \[\delta_{it} = \exp \left[ \sum_{s=1}^{t} (\eta_{is} y_s - y_s^2 / 2) \right] \]

\[
y_t = \left( \frac{2}{\alpha^2 + \alpha} \left[ \Delta m_t + \rho + \alpha \Delta c_t \right] \right)^{1/2}
\]

- IID multiplicative unit-root earnings shock $\eta_{it} \sim N(0,1)$. 
An equilibrium is a value function, decision rules for the investor, and pricing functions \( \{V^*, C^*, \theta^*, b^*, P^*, B^*\} \) s.t.:

1. **Optimality:** Given \((P^*, B^*)\),
   - \((C^*, \theta^*, b^*)\) maximizes utility and \(V^*\) is associated value;

2. **Market Clearing:** \(\forall j, t\)
   - \(\int_{i \in I} \theta^*_{ijt} = 1\);
   - \(\int_{i \in I} b^*_{ijt} = 0\).
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An Equilibrium with Autarky

Under the maintained assumptions, if:

1. $E[M_t] \to 0$, as $t \to \infty$;
2. $M_{t+1}/M_t \geq e^{-\rho} (C_{t+1}/C_t)^{-\alpha}$;

then there exists an equilibrium with no trade.
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- Given any \((I_t, P_t, B_t)\) there exists \((I_{it})\) consistent with equilibrium concept!
- “No trade” means that agent consumes labor earnings each period and does not trade in financial markets:
  - This is unrealistic, but facilitates subsequent analysis;
  - Follows from (unusual) choice of earnings process.  

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- “No trade” means that agent consumes labor earnings each period and does not trade in financial markets:
  - This is unrealistic, but facilitates subsequent analysis;
  - Follows from (unusual) choice of earnings process.
In equilibrium, $C_{it} = \delta_{it} C_t$, hence:

$$\ln C^i_t = \ln C_t + \epsilon^i_t \quad \text{s.t.} \quad \epsilon^i_t = \sum_{s=1}^{t} (\eta^i_t y_s - \frac{y_s^2}{2})$$

Hence $y^2_{t+1}$ corresponds to cross-sectional variance of consumption growth (conditional on the aggregate state):

$$\Var^{xs} \left[ \ln \left( \frac{C^i_{t+1} / C^i_t}{C^i_{t+1} / C^i_t} \right) \mid C_{t+1}, y_{t+1} \right] = y^2_{t+1}$$
Use equilibrium consumption and EE to extract SDF:

\[ 1 = E_t \left[ e^{-\rho} \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} R_{t+1}^j \right] \]

\[ \Rightarrow 1 = E_t \left[ e^{-\rho + \frac{1}{2} \alpha (1+\alpha) y_{t+1}^2} \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} R_{t+1}^j \right] \]

\[ \Rightarrow M_{t+1} = e^{-\rho + \frac{1}{2} \alpha (1+\alpha) y_{t+1}^2} \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} \]

Notice that \( M_{t+1} \) is a function of the x-sectional distribution!
Euler Equation Estimation

\[ M_{t+1} = e^{-\rho + \frac{1}{2} \alpha (1+\alpha) y_{t+1}^2} \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} \]

- In the presence of uninsurable idiosyncratic risk \( y_{t+1}^2 \neq 0 \), \( M_{t+1} = g\left( \frac{C_{t+1}}{C_t}, y_{t+1}^2 \right) \);
- Standard Euler equation \( (M_{t+1} = f\left( \frac{C_{t+1}}{C_t} \right)) \) misspecified and estimates of the coefficient of RRA will be biased:
Risk-Return Relation

\[ \mathbb{E}_t [R_{jt+1} - R_f] = \frac{-\text{Cov}_t [R_{jt+1}, M_{t+1}]}{\text{Var}_t [M_{t+1}]} \cdot \frac{\text{Var}_t [M_{t+1}]}{\mathbb{E}_t [M_{t+1}]} \]

- SDF suggests relationship between x-sectional distribution of nonfinancial income risk and asset risk/return:
  1. Time-series properties of \( y^2_{t+1} \) will affect market price of risk:
     - In particular, \( \uparrow y^2_t \) in bad times implies counter-cyclical MPR;
     - Counter-cyclical idiosyncratic labor income risk confirmed in STY (JPE, 2004);
  2. Returns that covary negatively with \( y^2_{t+1} \) will have higher \( \beta \) and risk premium.
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1. Asset pricing in an incomplete market endowment economy:
   - Very special example, but results generalize;

2. Modified Euler equation now depends on properties of idiosyncratic risk process:
   - Strong assumptions on income process;
   - No trade in financial markets in equilibrium;
   - Tractability, but at what cost?

3. Asset pricing implications for:
   - Euler equation estimation of preference parameters;
   - Risk/return relationship.
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   • Empirical Evidence: BCG (JPE, 2002)
Euler Equation Errors: Set Up

- Brav, Constantinides & Geczy (JPE, 2002) investigate EE errors using an incomplete markets SDF:

  1. CRRA SDF: \( M_{t+1}(g_{t+1}^i) = \beta(g_{t+1}^i)^{-\gamma} \), where \( g_{t+1}^i \equiv \frac{c_{t+1}^i}{c_t^i} \)

  2. Assume standard EE holds for every household and asset:

\[
1 = E_t \left[ \beta(g_{t+1}^i)^{-\gamma} R_{t+1}^j \right] \quad \forall \ i, j
\]
Euler Equation Errors: Approach

- Under complete markets, HH’s fully insure and equalize their MRS state-by-state:
  - Consumption growth rates are equalized across HHs;
  - CCAPM holds, i.e., $M_{t+1} = \beta (g_{t+1})^{-\gamma}$ is a valid SDF;
  - We know this doesn’t work!

- With incomplete markets:
  - We do not have full insurance;
  - HHs do not equate MRS/consumption growth rates;

\[ E \text{ Euler Equation Errors: Approach} \]
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- With incomplete markets:
  - We do not have full insurance;
  - HHs do not equate MRS/consumption growth rates;

- However, if EE holds $\forall i$, any linear combination of individual SDFs should be valid;

- BCG test if equally-weighted sum of HH’s MRS is valid SDF:

  $$M_{t+1} = \beta I^{-1} \sum_{i=1}^{l} (g_{t+1}^i)^{-\gamma}$$
Euler Equation Errors: Approach

- Interested in EE errors of the form:
  \[
  u_{t+1} = I^{-1} \left( \beta \sum_{i=1}^{I} (g_{t+1}^{i})^{-\gamma} \right) \left( R_{t+1} - R_{t+1}^{f} \right)
  \]

- Conduct standard tests of: \( \frac{1}{T} \sum_{t=1}^{T} u_{t+1} = 0; \)
- They find that Euler equation is satisfied for reasonable \( \gamma \).
**Euler Equation Errors: Result 1**

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<th>Unexplained Premium</th>
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**A. Value-Weighted Equity Premium**

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**B. Equally Weighted Equity Premium**

**Constantinides & Duffie**

**Asset Pricing with Heterogeneous Consumers**
They also perform the same test for the Constantinides & Duffie (JPE, 1996) SDF:

\[ M_{t+1} = \beta \left( \frac{\sum I_{i=1}^I c_{t+1}^i}{\sum I_{i=1}^I c_t^i} \right)^{-\alpha} \exp \left\{ \frac{\alpha(\alpha+1)}{2} I^{-1} \sum I_{i=1}^I \left[ \log(g_{t+1}^i) - \log(g_{t+1}^i)^2 \right] \right\} \]

This is equivalent to testing if most of the x-sectional variation of the consumption growth rate is captured by idiosyncratic income shocks that are:

1. Multiplicative;
2. i.i.d. lognormal.
Euler Equation Errors: Result 2

- Euler equation errors \textit{increase} with RRA;
- However, error is statistically insignificant for RRA > 1;
- Paper argues that this highlights the importance of the \textbf{x-sectional skewness} of the HH’s consumption growth rate, combined with the first two moments, which is a major contribution.
Zero Euler equation errors is one dimension along which to test an asset pricing model;

There are other dimensions too:

1. Return predictability;
2. Implied wealth-consumption ratio;
3. Persistence of SDF;

Can incomplete markets models explain some of these facts?