Can Rare Events Explain the Equity Premium Puzzle?

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Presented by Jason Levine for NYU Asset Pricing Seminar, Fall 2009
Introduction

• Over last century excess return on U.S. stocks over 1-month treasuries is 7% on average
• Representative agent CRRA calibrated to times series properties of consumption and asset returns generates a risk premium of <1%
• Paper examines if the Rare Events Hypothesis can rationalize the equity premium puzzle.
  – Investors may appear irrational since in a small sample-size world, the realization may not accurately represent the true probabilities
Contribution 1

• Using an information-theoretic alternative to GMM, estimate the consumption Euler equation for the risk premium explicitly allowing probabilities to differ from their sample frequencies
  – C-CAPM still rejected by the data
  – Requires a very high risk aversion
  – Holds for a variety of sources, including back to 1890
  – Methodologies belong to the Generalized Likelihood family
    • More robust (by construction) to a rare events problem
    • Better small sample and asymptotic properties than standard GMM
    • Permits the used of Bayesian posterior inference not relying on asymptotic properties less likely to be met, in finite sample, in the presence of rare events
  – Shows that the information-theoretic estimation approaches can also be used to identify (nonparametrically) the rare events distribution needed to rationalize the equity premium puzzle
Contribution 2

• Using the estimated rare events distribution, generate counterfactual histories to determine the probability of observing an equity premium puzzle of the same size as historical samples.

• Finding: if data were generated by the rare events distribution needed to rationalize the equity premium puzzle, with low risk aversion, the puzzle would be unlikely to arise.
Contribution 3

- Can rare events rationalize the poor performance of the C-CAPM in pricing the cross-section of asset returns?
- Finding: it worsens the ability to explain the cross-section
  - Correlation jumps up during market crashes, reducing dispersion.
  - Crashes are emphasized more, reducing the cross-sectional dispersion of consumption risk across assets
Outline

• Related Literature
• Theoretical Underpinnings of the Estimation and Testing Approaches
• Data Description
• Results of Estimation and Testing
• Section 6
  – Rare events distribution of data needed to rationalize equity premium puzzle with low level of risk aversion
  – What would be the likelihood of observing an equity premium puzzle in samples of the same size as historical ones, if the rare events hypothesis were the true explanation of the puzzle?
  – Implications of the rare events hypothesis for the ability of C-CAPM to price the cross-section of asset returns
• Comparison with Barro (2006)
• Conclusion
Related Literature

– Mandelbrot (1963, 1967) pointed out that tails are fatter than Gaussian.

– Applications of Extreme Value Theory (Beirlant, Schoutens, and Segers (2004))

– Tail Events cause the Equity Premium Puzzle (Rietz (1988))
  • Uses standard C-CAPM
    \[
    E \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} R_t^e \right] = 0
    \]
  • Rietz adds a low probability depression-like state
    – <2% probability is sufficient to generate historical equity premium
Related Literature (2)

• Barro (2006) constructs a model that extends Rietz (1988) and calibrates disasters from 20th century global history.
• Gabaix (2007a) argues that rare events can explain this and 9 other macro-finance puzzles
• Veronesi (2004) – peso problem hypothesis implies most stylized facts about stock returns
• Gourio (2008a, 2008b) – rare events model has counterfactual implications
Econometric Methodology

- Methodology belongs to the Generalized Empirical Likelihood family
  - Use 4 estimation and testing approaches
  - ET – Exponential Tilting (Kitamura and Stutzer (1997))
  - BEL – Bayesian Empirical Likelihood (Lazar (2003))
  - BETEL – Bayesian Exponentially Tilted Empirical Likelihood (Schennach (2005))
Basic Estimators

  – Simplest estimator
  – Permits the underlying distribution to be a parameter to estimate instead of determined by the data
  – Enables inference that does not require distributional assumptions
  – Minimizes the Kullback-Leibler Information Criterion (KLIC)

• Exponential Tilting (ET)
  – ET compensates for the asymmetry of the KLIC in EL
EL & ET

- Are asymptotically equivalent to the optimal GMM estimator
- Have smaller second order bias than the GMM estimator
- Bias-corrected EL is third-order efficient
- Local Analysis - Kunitomo and Matsushita (2003)
  - Find that EL is more centered and concentrated around the true parameter value
  - Asymptotic normal approximation appears more appropriate for EL than for GMM
- Global Analysis
  - EL Ratio – Uniformly most powerful – Kitamura (2001)
  - Important since the distributions are unknown
- Based on minimization of relative entropy between estimated and unknown probability measure
Bayesian Estimators

• EL and ET can be used in the likelihood part of Bayes theorem
• Bayesian Empirical Likelihood (BEL) ← Empirical Likelihood
• Bayesian Exponentially Tilted Empirical Likelihood (BETEL) ← Exponential Tilting
• Using Bayesian estimation leads to the conditional Euler equation

\[ E_{t-1} \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} R_t^e \right] = 0 \]

– Assumes no autocorrelation, but requires no more than “weak dependency”
Data

• Range
  – Annual: 1929 – 2006
  – Annual 1890 – 1995: from Campbell (2003) for robustness

• Asset data
  – Market Return: CRSP Value-Weighted Index of all stocks on the NYSE, AMEX, and NASDAQ
  – Risk-free rate: 1-month t-bill
  – Quarterly and annual returns computed by compounding monthly returns
  – Converted to real using the personal consumption deflator

• Consumption: per capita real consumption expenditures on nondurable goods from the National Income and Product Accounts
  – Assume consumption takes places at end of quarter
Data (continued)

- Cross-sectional analysis
  - Quarterly returns on the 25 Fama and French (1992) portfolios
    - Intersection of 5 portfolios formed on size and 5 portfolios formed on book-to-market
    - Denote by size rank then book to market rank (portfolio 5.1 is biggest size, lowest book-to-market)
- Covers (from the 20th century) annual (quarterly)
  - 11 (7) of 15 stock market crashes
  - 13 (10) of 22 NBER Recessions
  - 52.3% (32.4%) of months of economic contraction
  - 4 (3) of all (10) major U.S. wars – 91.7% (22.3%) of total cost
  - 85% (59%) of major hurricanes and 39% (18%) of related deaths
  - 87% (68%) of deadly earthquakes and 20% (12%) of related deaths
  - Both annual samples include 2 of 65 major rare economic disasters – Great Depression and World War 2 aftermath
Estimation Results

• Table 1 – Estimation results for quarterly and annual data
  – EL and ET
    • Very high risk aversion (and large standard errors) which is significantly above 10 in Panels A & B
    • Panel C cannot reject risk-aversions below 10
  – BEL and BETEL
    • 95% confidence intervals do not include 10
  – Second row – joint hypothesis of risk aversion as small as 10 and the identify restriction given by the consumption Euler equation
    • both tests reject $\gamma \leq 10$
  – Third row – probabilities of $\gamma \leq 10$, never $>1$
  – C-CAPM rejected – requires a very high risk aversion to rationalize equity risk premium
    • Holds up to robustness checks in the appendix
Table 1: Euler Equation Estimation

<table>
<thead>
<tr>
<th></th>
<th>EL</th>
<th>ET</th>
<th>BEL</th>
<th>BETEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{\gamma})</td>
<td>102</td>
<td>146</td>
<td>102</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>(48.0)</td>
<td>(32.3)</td>
<td>[24.8, 263.1]</td>
<td>[19.5, 164.9]</td>
</tr>
<tr>
<td>(\chi^2)_{(1)}</td>
<td>9.87</td>
<td>10.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.002)</td>
<td>(.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Pr(\gamma \leq 10</td>
<td>\text{data}))</td>
<td>.64%</td>
<td>.92%</td>
<td></td>
</tr>
<tr>
<td>Panel B. Annual Data: 1929-2006</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\hat{\gamma})</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>(10.5)</td>
<td>(10.5)</td>
<td>[13.4, 64.1]</td>
<td>[13.8, 57.1]</td>
</tr>
<tr>
<td>(\chi^2)_{(1)}</td>
<td>5.26</td>
<td>5.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.022)</td>
<td>(.015)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Pr(\gamma \leq 10</td>
<td>\text{data}))</td>
<td>1.00%</td>
<td>.84%</td>
<td></td>
</tr>
<tr>
<td>(\hat{\gamma})</td>
<td>49</td>
<td>49</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>(39.5)</td>
<td>(39.6)</td>
<td>[24.5, 244.7]</td>
<td>[20.8, 225.4]</td>
</tr>
<tr>
<td>(\chi^2)_{(1)}</td>
<td>7.08</td>
<td>8.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.008)</td>
<td>(.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Pr(\gamma \leq 10</td>
<td>\text{data}))</td>
<td>.08%</td>
<td>.08%</td>
<td></td>
</tr>
</tbody>
</table>

Note: EL, ET, BEL and BETEL estimation results for the consumption Euler equation (1). The first row of each panel reports the EL and ET point estimates (with s.e. underneath), and the BEL and BETEL posterior modes (with 95% confidence regions underneath), of the relative risk aversion coefficient \(\gamma\). The second row of each panel reports the EL and ET Likelihood Ratio tests (with \(p\)-values underneath) for the joint hypothesis of a \(\gamma\) as small as 10 and for the identifying restriction given by the consumption Euler equation (1). The third row of each panel reports the BEL and BETEL posterior probabilities of \(\gamma\) being smaller than, or equal to, 10.
Prior beliefs needed to not reject the model

- Using BEL and BETEL, how strong a prior is necessary to keep risk aversion at 10 or lower?
- Paper assumes a lognormal prior centered at $\gamma = 4$ (value from Barro (2006))
- The following figures will suggest that “only an economist endowed with an almost dogmatic belief in favor of $\gamma \leq 10$ would still believe in the Consumption-CAPM with CRRA utility and low risk aversion, after having observed the data.”
Table 2: Beliefs Necessary *not* to Reject the Model

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BEL</td>
<td>BETEL</td>
<td>BEL</td>
</tr>
<tr>
<td>Panel A: Prior necessary for having a posterior peak at $\gamma = 10$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prior $Pr(\gamma \leq 10)$</td>
<td>92.32%</td>
<td>93.18%</td>
<td>94.01%</td>
</tr>
<tr>
<td>Prior $Pr(\gamma \geq \hat{\gamma})$</td>
<td>0.06%</td>
<td>0.04%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Posterior $Pr(\gamma \leq 10)$</td>
<td>29.88%</td>
<td>28.89%</td>
<td>29.99%</td>
</tr>
<tr>
<td>Posterior $Pr(\gamma \geq \hat{\gamma})$</td>
<td>4.97%</td>
<td>3.92%</td>
<td>0.47%</td>
</tr>
<tr>
<td>Panel B: Prior necessary for having a posterior $Pr(\gamma \leq 10) = 5%$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prior $Pr(\gamma \leq 10)$</td>
<td>80.50%</td>
<td>82.10%</td>
<td>86.09%</td>
</tr>
<tr>
<td>Prior $Pr(\gamma \geq \hat{\gamma})$</td>
<td>2.55%</td>
<td>1.85%</td>
<td>0.15%</td>
</tr>
<tr>
<td>Posterior $Pr(\gamma \geq \hat{\gamma})$</td>
<td>31.96%</td>
<td>28.89%</td>
<td>12.90%</td>
</tr>
</tbody>
</table>
Counterfactual Analysis

- Set $\gamma$ to a “reasonable” value and use EL and ET estimations to identify distribution of data that can solve the Equity Premium Puzzle.
- Question 1: assuming the low risk aversion and a matching distribution, what is the probability of observing an equity premium puzzle in a sample of the same size as the historical sample?
- Question 2: Assuming rare events were the cause of the equity premium puzzle, would this account for the poor performance of the C-CAPM in pricing the cross-section of asset returns?
A World without the Equity Premium Puzzle

\[
\begin{align*}
E^F \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} R_t^e \right] & = \frac{E^F \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} \right] \cdot \text{Cov}^F \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma}, R_t^e \right]}{E^F \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} \right]} \\
& = epp^F(\gamma)
\end{align*}
\]

Where \( F \) is the true probability distribution of the data

- EL and ET estimated nonparametrically the unknown distribution \( F \) with probability weights

\[
\{ \hat{\beta}_t^j(\gamma) \}_{t=1}^T \ (\text{where } j \in \{ EL, ET \})
\]
A World without the Equity Premium Puzzle (2)

\[
\sum_{t=1}^{T} \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} R_t^e \hat{p}_t^j (\gamma) = 0 \ \forall \gamma
\]

\[
E^{\hat{p}_j}(\gamma) \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} R_t^e \right] = 0 \Rightarrow epp^j (\gamma) = 0
\]

- Assuming the SDF and excess return have finite first and second moments
- EL and ET minimize the Kullback-Leibler divergence between calibrated distribution and unknown data generating process
Do the implied probabilities make sense?

- EL and ET are similar (correlation > 0.93)
- Both assign higher probability weight to recessions
  - Actual frequency of recession: 20% (35%)
  - Estimated probabilities of being in recession: 21% (39%)
- Increases in probabilities of observing a recession largely driven by assign higher probabilities to recessions that occur with market crashes
- EL and ET assign high probabilities to most market crash periods
  - Actual: 6.6% (21%)
  - Estimated: 10% (28%)
- Estimated put highest weights on periods with both crash and recession
  - Equivalent to states with which consumption risk of stock market is extremely high
    - Actual 0.4% (1.3%)
    - EL: 1.1% (3.5%)
    - ET: 0.9% (2.9%)
    - For 1890-1995: 0.9% versus 3.5% and 2.5%
- EL and ET have thicker negative tails – leftward shift in distributions
- Panels D, E, F similar
- Level curves of Epanechnikov kernel estimates of joint distributions
- EL and ET skew towards lower left – high stocks market consumption risk states (In line with the rare events hypothesis)
How likely is the Equity Premium Puzzle?

- **Process**
  - Use the estimated distributions to generate counterfactual samples of data (same size as historical samples)
  - bootstrap from observed data
  - 100,000 counterfactual samples (quarterly and annually)
  - Compute realized equity premium puzzle as:

\[
\text{epp}_i^T (\gamma) = E^T \left[ R_{i,t}^e \right] + \frac{\text{Cov}^T \left[ \left( \frac{C_{i,t}}{C_{i,t-1}} \right)^{-\gamma}, R_{i,t}^e \right]}{E^T \left[ \left( \frac{C_{i,t}}{C_{i,t-1}} \right)^{-\gamma} \right]}
\]
### Table 3: Counterfactual Equity Premium Puzzle

<table>
<thead>
<tr>
<th></th>
<th>$epp^T$</th>
<th>$epp^T_i$</th>
<th>$\Pr(epp^T_i \geq epp^T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Quarterly Data: 1947:Q1 2003:Q3</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{P}^{EL} (\gamma = 5)$</td>
<td>7.4%</td>
<td>0.0%</td>
<td>0.10%</td>
</tr>
<tr>
<td>$\hat{P}^{EL} (\gamma = 10)$</td>
<td>7.3%</td>
<td>0.0%</td>
<td>0.12%</td>
</tr>
<tr>
<td>$\hat{P}^{ET} (\gamma = 5)$</td>
<td>7.4%</td>
<td>0.0%</td>
<td>0.10%</td>
</tr>
<tr>
<td>$\hat{P}^{ET} (\gamma = 10)$</td>
<td>7.3%</td>
<td>0.0%</td>
<td>0.13%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$epp^T$</th>
<th>$epp^T_i$</th>
<th>$\Pr(epp^T_i \geq epp^T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel B. Annual Data: 1929-2006</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{P}^{EL} (\gamma = 5)$</td>
<td>7.2%</td>
<td>0.0%</td>
<td>0.37%</td>
</tr>
<tr>
<td>$\hat{P}^{EL} (\gamma = 10)$</td>
<td>6.5%</td>
<td>0.0%</td>
<td>1.22%</td>
</tr>
<tr>
<td>$\hat{P}^{ET} (\gamma = 5)$</td>
<td>7.2%</td>
<td>0.0%</td>
<td>0.33%</td>
</tr>
<tr>
<td>$\hat{P}^{ET} (\gamma = 10)$</td>
<td>6.5%</td>
<td>0.0%</td>
<td>0.98%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$epp^T$</th>
<th>$epp^T_i$</th>
<th>$\Pr(epp^T_i \geq epp^T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{P}^{EL} (\gamma = 5)$</td>
<td>6.8%</td>
<td>−0.4%</td>
<td>0.00%</td>
</tr>
<tr>
<td>$\hat{P}^{EL} (\gamma = 10)$</td>
<td>6.1%</td>
<td>−1.4%</td>
<td>0.50%</td>
</tr>
<tr>
<td>$\hat{P}^{ET} (\gamma = 5)$</td>
<td>6.8%</td>
<td>1.2%</td>
<td>0.65%</td>
</tr>
<tr>
<td>$\hat{P}^{ET} (\gamma = 10)$</td>
<td>6.1%</td>
<td>−0.6%</td>
<td>0.85%</td>
</tr>
</tbody>
</table>

Columns: historical puzzle, counterfactual puzzle, probability of equity puzzle arising
Historical equity premium puzzle unlikely to arise: probability <2%
Assessing the Power of the Estimation Approach

- Computed the probability of obtaining in the counterfactual samples estimates of the relative risk aversion larger than the historical samples
  - Reduce computing time
    - only use the 2 annual samples
    - 50,000 counterfactual samples for each
  - Small values imply that rare events are unlikely to deliver the large estimates of risk aversion from historical sample
### Table 4: Counterfactual Risk Aversion Estimates

<table>
<thead>
<tr>
<th></th>
<th>EL/BEL</th>
<th>ET</th>
<th>BETEL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Annual Data: 1929-2006</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{P}^{EL} (\gamma = 5)$</td>
<td>2.06%</td>
<td>2.06%</td>
<td>2.05%</td>
</tr>
<tr>
<td>$\hat{P}^{EL} (\gamma = 10)$</td>
<td>3.03%</td>
<td>3.03%</td>
<td>3.03%</td>
</tr>
<tr>
<td>$\hat{P}^{ET} (\gamma = 5)$</td>
<td>1.98%</td>
<td>1.99%</td>
<td>1.99%</td>
</tr>
<tr>
<td>$\hat{P}^{ET} (\gamma = 10)$</td>
<td>3.13%</td>
<td>3.13%</td>
<td>3.14%</td>
</tr>
<tr>
<td>$\hat{P}^{EL} (\gamma = 5)$</td>
<td>0.93%</td>
<td>0.96%</td>
<td>0.91%</td>
</tr>
<tr>
<td>$\hat{P}^{EL} (\gamma = 10)$</td>
<td>2.32%</td>
<td>2.36%</td>
<td>2.34%</td>
</tr>
<tr>
<td>$\hat{P}^{ET} (\gamma = 5)$</td>
<td>1.15%</td>
<td>1.18%</td>
<td>1.13%</td>
</tr>
<tr>
<td>$\hat{P}^{ET} (\gamma = 10)$</td>
<td>2.32%</td>
<td>2.36%</td>
<td>2.34%</td>
</tr>
</tbody>
</table>
Rare events and the cross-section of asset returns

\[ E^F [R_{m,t}^e] = \alpha - \frac{Cov^F \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma}, R_{m,t}^e \right]}{E^F \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} \right]} \lambda \]

= : \beta_m

• Should explain all excess returns with \( \alpha=0 \) and \( \lambda=1 \)
  - \( \beta_m \) is a measure of consumption risk of investing in asset \( m \)
• Log linearization
  \[ E^F [R_{m,t}^e] = \alpha + Cov^F \left( \ln \frac{C_t}{C_{t-1}}, R_{m,t}^e \right) \lambda \]

= : \beta_m

- with \( \alpha=0 \) and \( \lambda>0 \)
• If Rare Events make the C-CAPM work, then these should hold with the parameters from the EL and ET distributions
Estimation of parameters for Cross-Section

- Assume $\gamma = 10$
- Estimate $\alpha$ and $\lambda$
- Quarterly cross-section of 25 Fama-French portfolios
  - Excess over 3-month t-bill rate (this is not the rate described in the data section)
- Two-step Fama-MacBeth (1973) cross-sectional procedure
\[ \rho_m := corr \left( \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma}, R_{m,t}^e \right) \]

- Log linearization:

\[ \rho_m := corr \left( \ln \frac{C_t}{C_{t-1}}, R_{m,t}^e \right) \]
Table 5: Counterfactual Cross-Sectional Regressions

<table>
<thead>
<tr>
<th>Moments:</th>
<th>$R^2$</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\lambda}$</th>
<th>$\Delta \frac{\text{Var}(\beta_m)}{\text{Var}(\mathbb{E}[R_{m,t+1}])}$</th>
<th>$\Delta \text{Var}(\rho_m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>0.11</td>
<td>0.017</td>
<td>6.28</td>
<td>(5.04)</td>
<td></td>
</tr>
<tr>
<td>$\hat{P}^{EL}(\gamma)$</td>
<td>0.00</td>
<td>0.007</td>
<td>-1.15</td>
<td>-35.4%</td>
<td>-18.4%</td>
</tr>
<tr>
<td>(0.006)</td>
<td></td>
<td>(5.09)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{P}^{ET}(\gamma)$</td>
<td>0.00</td>
<td>0.006</td>
<td>-0.78</td>
<td>-38.2%</td>
<td>-12.9%</td>
</tr>
<tr>
<td>(0.006)</td>
<td></td>
<td>(5.09)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Panel A:** C-CAPM, $\gamma = 10$

| Sample   | 0.12  | 0.017          | 63.35           | (49.89)                         |                     |
| $\hat{P}^{EL}(\gamma)$ | 0.00  | 0.007          | -12.18          | -34.9%                         | -19.4%             |
| (0.006)  |       | (50.31)        |                 |                                 |                     |
| $\hat{P}^{ET}(\gamma)$ | 0.00  | 0.006          | -8.49           | -37.8%                         | -13.7%             |
| (0.006)  |       | (50.37)        |                 |                                 |                     |

**Panel B:** linearized C-CAPM

Note: Fama and MacBeth (1973) standard errors in parentheses under the estimated coefficients.

- First column: percentage of cross-sectional variation explained
- Fourth column: percentage change in the ratio of the cross-sectional variance of consumption risk measures to the cross-sectional variance of average excess returns caused by using the estimated probability weights
- Fifth column: change is cross-sectional variance of the correlations between the pricing kernel and excess returns in the expected excess return equations
Estimation of parameters for Cross-Section (cont)

• Sample data
  – Positive $\alpha$ is the equity premium puzzle
  – $\lambda$ much larger than the C-CAPM result

• EL & ET
  – Smaller $\alpha$ – smaller mispricing
  – $\lambda$ has opposite sign than theory predicts
  – Other than reduction in $\alpha$, these perform worse under the EL and ET measures

• Can rationalize the equity premium puzzle, but fail to explain the cross-section

• Panel B is similar to panel A
Top Half: If C-CAPM worked, all portfolios would be on the 45 degree line. EL & ET fit better. Portfolios on both sides of 45 degree line.

Bottom half: mean excess returns and covariance between consumption growth and excess returns. EL & ET enable C-CAPM to better fit the average risk premium of the 25 portfolios. Do NOT observe the increase in cross-sectional variation necessary to fit the cross-sectional.
Table 4: Estimation and Counterfactual EPP with Calibrated Disaster

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$\hat{\gamma}$</td>
<td>21 (5.3) 21 (5.3) 21 [12.6, 39.4] 21 [12.9, 38.9]</td>
<td>11 (2.7) 11 (2.7) 11 [6.3, 19.8] 11 [6.4, 19.7]</td>
</tr>
<tr>
<td>$\chi^2_{(1)}$</td>
<td>6.95 (.008) 8.47 (.003)</td>
<td>0.07 (.792) 0.07 (.784)</td>
</tr>
<tr>
<td>$Pr(\gamma \leq 10</td>
<td>\text{data})$</td>
<td>3.41% 2.32%</td>
</tr>
<tr>
<td>$Pr(\epsilon_{pp_t}(\gamma) \geq \epsilon_{pp}^T(\gamma))$</td>
<td>0.95% 1.34%</td>
<td>43.60% 43.30%</td>
</tr>
</tbody>
</table>

- Estimates of $\gamma$ are smaller than with the full sample
- Table 4 shows that if Barro (2006) calibrated an annual model with a cumulative multi-year contraction during disasters and replace the consumption drop with a GDP drop, both papers would reach the same conclusions
Conclusion

• Even with allowing probabilities to differ from their sample frequencies, C-CAPM with CRRA preferences is rejected.
• If the rare events explanation is correct, one should believe that the equity premium puzzle is a rare event.
• Failure to price the cross-section of returns.
• Rejection of Barro (2006) because Barro overstates the consumption risk.