Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance (Xavier Gabaix, 2009)

Presented by: Oliver Randall

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Disaster = cumulative decline in real per capita personal consumer expenditure (or GDP) of at least 10%.

Risk premia result from the possibility of rare, but severe, disasters (e.g. economic depressions or wars).

During a disaster an asset’s fundamental value falls by a time-varying amount.

Generates time-varying risk premia, and thus volatile asset prices and return predictability.
10 Puzzles

Aggregate stock market puzzles:
1. Equity premium puzzle
2. Risk-free rate puzzle
3. Excess volatility puzzle
4. Aggregate return predictability

Cross-sectional stock market puzzles:
5. Value/growth puzzle
6. Characteristics vs covariances puzzle

Nominal bond puzzles:
7. Yield curve slope puzzle
8. Long-term bond predictability puzzles
9. Credit spread puzzle

Option puzzles:
10. Option puzzles

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Variable Rare Disasters: An Exactly Solved Framework for Ten
Macro Model

- Representative agent with CRRA $\gamma$.
- Receives consumption endowment $C_t$.
- At $t + 1$ a disaster occurs with probability $p_t$.
- Consumption growth:

$$\frac{C_{t+1}}{C_t} = e^{gc} \times \begin{cases} 1 & \text{if no disaster at } t + 1 \\ B_{t+1} & \text{if disaster at } t + 1 \end{cases}$$  \hspace{1cm} (1)

(In calibration $\bar{B} = 0.66$)

- It follows that the SDF evolves as:

$$\frac{M_{t+1}}{M_t} = e^{-(\delta + \gamma gc)} \times \begin{cases} 1 & \text{if no disaster at } t + 1 \\ B_{t+1}^{-\gamma} & \text{if disaster at } t + 1 \end{cases}$$  \hspace{1cm} (2)
• Stock $i$ pays stream of dividends $\{D_{i,t}\}_{t \geq 0}$:

$$\frac{D_{i,t+1}}{D_{i,t}} = e^{g_{i,D}} \left( 1 + \varepsilon_{i,t+1}^D \right) \times \begin{cases} 1 & \text{if no disaster at } t + 1 \\ F_{i,t+1} & \text{if disaster at } t + 1 \end{cases}$$

(In calibration: $F_t \in [0, 1], F_* = \bar{B}$.)

• Expected resilience:

$$H_{i,t} = p_t E_t [B_{t+1}^{-\gamma} F_{i,t+1} - 1 | \text{disaster at } t = 1]$$

$$= H_* + \hat{H}_t$$

• Dynamics of $\hat{H}_t$ specified as LG process, so prices are linear in factors and independent of functional form of noise:

$$\hat{H}_{t+1} = \left( \frac{1 + H_*}{1 + H_t} \right) e^{-\phi} \hat{H}_t + \varepsilon_{t+1}^H$$

• $\hat{H}_t$ mean-reverts to 0, as ”twisted” AR(1).
Bonds

- Growth in real value of money:
  \[
  \frac{Q_{t+1}}{Q_t} = (1 - i_t + \varepsilon_{t+1}^Q) \times \begin{cases} 
    1 & \text{if no disaster at } t + 1 \\
    F_{t+1}^S & \text{if disaster at } t + 1
  \end{cases}
  \tag{7}
  \]
  (In calibration \( F^S \equiv 1 \).)

- Inflation:
  \[
  i_t = i^* + \hat{i}_t
  \tag{8}
  \]

  \[
  \hat{i}_{t+1} = \left( \frac{1 - i^*}{1 - i_t} \right) \left( e^{-\phi_i \hat{i}_t} + 1 \{ \text{Disaster at } t + 1 \} \left( j^* + \hat{j}_t \right) \right) + \varepsilon_{t+1}^i
  \tag{9}
  \]

  \[
  \hat{j}_{t+1} = \left( \frac{1 - j^*}{1 - j_t} \right) e^{-\phi_j \hat{j}_t} + \varepsilon_{t+1}^j
  \tag{10}
  \]

  Jump in inflation makes long-term bonds particularly risky.

- Variable part of bond risk premium (analogous to \(-\hat{H}_t\)):
  \[
  \pi_t \equiv \frac{p_t E_t[B_{t+1}^{-\gamma} F_{t+1}^S] \hat{j}_t}{1 + H^S}
  \tag{11}
  \]
1. Equity premium puzzle

- Define $P_{t+1}^\# = P_{t+1} + D_{t+1}$ the payoff conditional on a disaster at $t + 1$. Expected excess return:

$$r_t^e = \frac{1}{1 - p_t} \left( e^R - p_t E_t \left[ B_{t+1}^{-\gamma} \frac{P_{t+1}^\#}{P_t} \right] \right) - 1 \quad (12)$$

⇒ only behavior in disasters creates an equity premium.

- $\gamma = 4$

- Equity premium conditional on no disasters is 6.5%.

- Unconditional equity premium is 5.3%.

⇒ Excess returns of stocks mostly represent a risk premium, not a peso problem.
3. Excess volatility puzzle

- Puzzle: Stock prices are more volatile than warranted by a model with a constant discount rate.
- Disaster reduces fundamental value of stock by time-varying amount.
- Time-varying risk premium generates a time-varying price-dividend ratio.
- Price-dividend ratios are volatile in a way potentially independent of innovations to dividends.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean $P/D$</td>
<td>23</td>
<td>18.2</td>
</tr>
<tr>
<td>Stdev $\ln P/D$</td>
<td>0.33</td>
<td>0.30</td>
</tr>
<tr>
<td>Stdev of stock returns</td>
<td>0.18</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Explanation: Stock market moments. The data are Campbell (2003, Table 1 and 10)’s calculation for the USA 1891–1997.
4. Aggregate return predictability

- \( P/D \) mean-reverts:

\[
\frac{P_t}{D_t} = \frac{1}{r_i} \left( 1 + \frac{\hat{H}_t}{r_i + \phi_H} \right)
\]  

(13)

where \( r_i = R - G_d - h_* \).

- Expected return on stock \( i \), conditional on no disasters:

\[
 r_{i,t}^{e} = R - H_t
\]  

(14)

- When \( \hat{H}_t \) is high:
  - \( P/D \) high
  - risk premium low
and vice versa.

- Since \( P/D \) mean-reverts: when \( P/D \) is lower than normal, expect it to increase, and expect subsequent returns to be higher.
4. Aggregate return predictability (cont’d)

- Predictive regression:

\[ r_{t \rightarrow t+T} = \alpha_T + \beta_T \ln(D/P)_t + \text{noise} \]  

(15)

where \( \beta_T = (r_i + \phi_H) T \)

- \( \beta_T \) proportional to \( T \), since returns over horizon \( T \) are proportional to \( T \).

- When \( P/D \) is 1% below baseline, returns increase:
  - dividend yield increases by \( r_i \%) \).
  - mean-reversion of \( P/D \) creates capital gains of \( \phi_H \% \).

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Data</th>
<th>Model</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Slope</td>
<td>s.e.</td>
</tr>
<tr>
<td></td>
<td>0.11</td>
<td>(0.053)</td>
</tr>
<tr>
<td>4</td>
<td>0.42</td>
<td>(0.18)</td>
</tr>
<tr>
<td>8</td>
<td>0.85</td>
<td>(0.20)</td>
</tr>
</tbody>
</table>

Explanation: Predictive regression \( E_t [r_{t \rightarrow t+T}] = \alpha_T + \beta_T \ln(D/P)_t \), at horizon \( T \) (annual frequency). The data are Campbell (2003, Table 10 and 11B)’s calculation for the US 1891–1997. The model’s predictions are derived analytically in Proposition 3.
5. Value/growth puzzle

- Puzzle: Stocks with a high $P/D$ ratio have lower future returns, even after controlling for their covariance with the aggregate stock market, and vice versa.

- Stocks with high $P/D$ ratios are more resilient:

$$\frac{P_t}{D_t} = \frac{1}{r_i} \left(1 + \frac{\hat{H}_t}{r_i + \phi_H}\right)\quad (16)$$

- Expected return, conditional on no disaster:

$$r_{i,t}^e = R - H_{i,t}\quad (17)$$

- Stocks which are more resilient have a lower ex-ante risk premium.

$\Rightarrow$ Stocks with high $P/D$ ratios have lower expected returns.
Puzzle: Stock characteristics (e.g. the $P/D$ ratio) often predict future returns at least as well as covariances with risk factors.

In a sample without disasters, covariances between stocks are due to covariances between cash flows in non-disaster times.

But risk premia are only due to behavior in disasters.

Hence non-disaster betas have no relation to risk premia.

Characteristics like $P/D$ will predict returns better than covariances.
7. Yield curve slope puzzle

- **Puzzle**: The yield curve slopes up on average. The premium of long-term yields over short-term yields is too high to be explained by a traditional RBC model.

- **Bond yield decreases in bond price**:
  \[
  y_t(T) = -\frac{\ln Z_t(T)}{T} \tag{18}
  \]

- **Bond prices decrease with inflation**:
  \[
  Z_t(T) = e^{-(R - H^\$ + i_{**})T} \left( 1 - \left( \frac{1 - e^{-\psi_i T}}{\psi_i} \right) (i_t - i_{**}) - \left( \frac{1 - e^{-\psi_i T}}{\psi_i} - \frac{1 - e^{-\psi_j T}}{\psi_j} \right) \left( \frac{1}{\psi_j - \psi_i} \right) \pi_t \right) \geq 0, \text{increasing in } T \tag{19}
  \]

  where \( i_{**} \equiv i_* + \kappa, \psi_i \equiv \phi_i - 2\kappa, \psi_j \equiv \phi_j - \kappa \).

  Here \( \kappa \) parameterizes the permanent risk of a jump in inflation.

- **Disaster leads to temporary jump in inflation**: greater detrimental impact on long-term bonds, so higher risk premium than short term bonds.
8. Long-term bond predictability puzzle

- Puzzle: Risk premium on long-term bonds increases with the difference in the long-term rate minus the short-term rate.
- Size of expected jump in inflation varies: slope of yield curve will vary and predict excess bond returns. A high slope will mean-revert and thus predicts a fall in the long rate and high returns on long-term bonds.
- Fama-Bliss (1987), Campbell-Shiller (1991), Cochrane-Piazzesi (2005) show that a high slope of the yield curve predicts high excess returns on long term bonds, and that long term interest rates will fall.
Fama-Bliss (1987) regression

\[ r_{x_{t+1}}(T) = \alpha + \beta_T(f_t(T) - r_t) + \text{noise} \]  

(20)

If changes in slope of forward curve come from changes in bond risk premium, rather than changes in drift of short rate:

\[ \beta_T = 1 + \frac{\psi_i}{2} T + O(T^2) \]  

(21)

<table>
<thead>
<tr>
<th>Maturity T</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta ) (s.e.)</td>
<td>( R^2 )</td>
</tr>
<tr>
<td>2</td>
<td>0.99 (0.33)</td>
<td>0.16</td>
</tr>
<tr>
<td>3</td>
<td>1.35 (0.41)</td>
<td>0.17</td>
</tr>
<tr>
<td>4</td>
<td>1.61 (0.48)</td>
<td>0.18</td>
</tr>
<tr>
<td>5</td>
<td>1.27 (0.64)</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Explanation: The regressions are the excess returns on a zero-coupon bond of maturity \( T \), regressed on the spread between the \( T \) forward rate and the short term rate: \( r_{x_{t+1}}(T) = \alpha + \beta (f_t(T) - f_t(1)) + \varepsilon_{t+1}(T) \). The unit of time is one year. The empirical results are from Cochrane and Piazzesi (2005, Table 2). The expectation hypothesis implies \( \beta = 0 \). The model’s predictions are derived analytically in Proposition 5.
Campbell-Shiller (1991) regression:

\[ y_{t+1}(T - 1) - y_t(T) = \alpha + \frac{\beta_T}{T - 1}(y_t(T) - y_t(1)) + \text{noise} \quad (22) \]

EH implies \( \beta_T = 1 \).

Campbell-Shiller find that a high slope in the yield curve predicts that future long-term rates will fall.

\[ \beta_T = -\left(1 + \frac{2\psi\pi - \psi_i}{3} T\right) + o(T) \quad (23) \]

<table>
<thead>
<tr>
<th>Maturity T</th>
<th>Data (s.e.)</th>
<th>Model</th>
<th>Data (s.e.)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-0.15 (0.28)</td>
<td>-1.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.83 (0.44)</td>
<td>-1.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>-1.43 (0.60)</td>
<td>-1.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>-1.45 (1.00)</td>
<td>-1.92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>-2.27 (1.46)</td>
<td>-2.83</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Explanation: The regressions are the change in bond yield on the slope of the yield curve:

\[ y_{t+1}(T - 1) - y_t(T) = \alpha + \frac{\beta_T}{T - 1}(y_t(T) - y_t(1)) + \varepsilon_{t+1}(T) \]

The time unit is one month. The empirical results are from Campbell, Lo, MacKinlay (1997, Table 10.3). The expectation hypothesis implies \( \beta = 1 \). The model’s predictions are derived analytically in Proposition 6.
Cochrane and Piazzesi establish:

- Bond premia can be captured by a single factor model ($\pi_t$).
- (Zero-coupon) bond premia are proportional to bond maturity.
- This risk premium is well proxied by a tent-shaped linear combination of forward rates.

\[
\begin{align*}
 f_t(T) &= F(T) + e^{-\psi_i T} i_t + G(T) \pi_t \\
\text{where } G(T) &= \frac{e^{-\psi T} - e^{-\psi_{\pi} T}}{\psi_{\pi} - \psi_i} 
\end{align*}
\]  

is tent-shaped.
Puzzle: Corporate bond spreads are too high compared to historical default rates (Huang and Huang, 2003).

It is only during disasters that very safe corporations will default.

Hence the risk premia on default risk will be very high.

Risk premia cannot be accounted for by their behavior during normal times.
10. Options puzzles

- Puzzle: deep OTM puts have high prices.
- \( V_t = V_t^{ND} + V_t^D \)
  
  \[
  V_t^{ND} = e^{-R+\mu}(1-p_t)V_{BS}^{put}(Ke^{-\mu}, \sigma)
  \]
  \[
  V_t^D = e^{-R+\mu}p_tE_t[B_{t+1}^{-\gamma}(Ke^{-\mu} - F_{t+1})^+] \]

- Model generates volatility smirk: high put prices for deep OTM puts.

![Implied Volatility vs. Strike](image)

Figure 1: This Figure shows the Black-Scholes annualized implied volatility of a 1-month put on the stock market. The solid line is from the model’s calibration. The dots are the empirical average (January 2001 - February 2006) for the options on the S&P 500 index, calculated as in Figlewski (2008). The initial value of the market is normalized to 1. The implied volatility on deep out-of-the-money puts is higher than the implied volatility on at-the-money puts, which reflects the probability of rare disasters.
Ambitious to try to solve so many puzzles at once, but to be admired.

"To keep things parsimonious, the probability and conditional severity of macroeconomic disasters are constant."

Multi-period disasters.

In options-pricing literature, models are calibrated with frequent, moderately sized jumps; in rare-disasters literature they occur every few decades and are larger.

\[ p = 3.63\% \] calibrated to OECD + non-OECD countries: overstated?

Barro and Ursua (2008)…