The Cross-Section and Time-Series of Stock and Bond Returns

Ralph S.J. Koijen, Hanno Lustig, and Stijn Van Nieuwerburgh

University of Chicago, UCLA & NBER, and NYU, NBER & CEPR

UC Berkeley, September 10, 2009
Unified Stochastic Discount Factor Model

- Models that price bonds do not explain expected stock returns nor stock return predictability
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- Develop a parsimonious SDF model that:
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  2. Captures the predictability of bond returns, dynamics of bond yields, and stock return predictability
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  3. Improves our understanding of connection between stock and bond returns
Motivation

Model Estimation Extensions Conclusions Extras

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**Ultimate goal:** understanding of (macroeconomic) sources of risk agents demand compensation for when holding financial assets
Our SDF model contains three priced risk factors
Developing a Unified Stochastic Discount Factor

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- Motivated by a temporal decomposition of risk
  Alvarez and Jermann (2005) and Hansen and Scheinkman (2008)

  - Shocks to the dividend yield – *permanent shocks to SDF*
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- Shocks to the dividend yield – *permanent shocks to SDF*
- Shocks to the level factor of the yield curve – *transitory shocks to SDF*
- Shocks to the Cochrane-Piazzesi (2005) factor – *relative importance of the two components*
Interpreting the Value Premium

Differential exposure to $CP$ shocks enables the model to account for the spread on average returns between value and growth stocks.

Value stocks have a positive exposure to $CP$ shocks, while growth stocks have a small, negative exposure.
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- $CP$ turns out to be a strong forecaster of future economic activity.

  $\Rightarrow$ Value stocks pay off when economic activity is expected to increase.
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- \( CP \) turns out to be a strong forecaster of future economic activity.

  \[ \Rightarrow \text{Value stocks pay off when economic activity is expected to increase} \]

- This explains why value stocks are riskier and why the price of \( CP \) risk is expected to be positive.
Large, separately developed literatures on pricing bonds and on pricing stocks
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Recent term structure models are able to:

- Explain the cross-section of expected bond returns across maturities
- Capture the predictability of bond returns across maturities
Large, separately developed literatures on pricing bonds and on pricing stocks

Recent equity valuation models are able to:

- Explain the cross-section of expected stock returns
  *E.g.*, *Fama and French* (1992)

- Capture the predictability of returns on market and on cross-section of stocks
Large, separately developed literatures on pricing bonds and on pricing stocks

Empirical work on joint properties of stocks and bonds
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Empirical work on joint properties of stocks and bonds

Theoretical work on joint pricing of stocks and bonds
  - Habit and long-run risk models
Outline

- Affine valuation model for stocks and bonds
  - Decomposition of SDF into permanent and transitory component
- Estimation from cross-section and time series of expected stock and bond returns
- What is CP and why does it help account for value premium
- Extensions
Affine Valuation Models

- We show how to decompose the SDF in any affine valuation model.

The SDF is given by:

\[
SDF_{t+1} = \frac{M_{t+1}}{M_t} = \exp\left(-y_t - \frac{1}{2} \Lambda'_t \Sigma \Lambda_t - \Lambda'_t \epsilon_{t+1}\right)
\]

- Short rate, risk prices, and state dynamics:

\[
\begin{align*}
y_t &= \delta_0 + \delta'_1 X_t, \\
\Lambda_t &= \Lambda_0 + \Lambda_1 X_t, \\
X_{t+1} &= \gamma_0 + \Gamma X_t + \epsilon_{t+1}
\end{align*}
\]
The model implies:

\[ P_t(\tau) = \exp \left( A_\tau + B'_\tau X_t \right), \]

\[ A(\tau) \] and \[ B(\tau) \] follow from the recursions:

\[ A(\tau) = -\delta_0 + A(\tau - 1) + B'(\tau - 1) \gamma_0 - \Lambda'_0 \Sigma B(\tau - 1) + \frac{1}{2} B'(\tau - 1) \Sigma B(\tau - 1), \]

\[ B(\tau) = -\delta_1 + (\Gamma - \Sigma \Lambda_1)' B(\tau - 1), \]

initiated at \[ A(0) = 0 \] and \[ B(0) = 0_{1 \times N}. \]
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\[ B(\tau) = -(I - \Theta^\tau)(I - \Theta)^{-1}\delta_1. \]

\[ B_\infty \equiv \lim_{\tau \to \infty} B(\tau) = -(I - \Theta)^{-1}\delta_1. \]
No-Arbitrage Condition for Risky Assets

- For each risky asset $j$:

$$\log E_t \left[ SDF_{t+1} R^j_{t+1} \right] = 0,$$
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For each risky asset $j$:

$$\log E_t \left[ SDF_{t+1} R_{t+1}^j \right] = 0,$$

Let $r_{t+1}^j = E_t [r_{t+1}^j] + \eta_{t+1}^j$, and $\Sigma_X^j = Cov(\varepsilon, \eta^j)$.
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Define $rx^j_{t+1} \equiv r^j_{t+1} - y_t + \frac{1}{2} \text{Var} \left[ \eta^j_{t+1} \right]$. 

Koijen (Chicago Booth), Lustig (UCLA), and Van Nieuwerburgh (Stern)
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$$E \left[ rx^j_{t+1} \right] = \Sigma X_j (\Lambda_0 + \Lambda_1 E [X_t]) = \Sigma X_j \hat{\Lambda}_0.$$
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- Estimate $\hat{\Lambda}_0$ to match average excess returns on test assets
Following Alvearez and Jermann (2005), we decompose the pricing kernel as $M_t = M_t^P M_t^T$, with:

$$M_t^T = \lim_{\tau \to \infty} \beta^{t+\tau} / P_t(\tau),$$

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Decomposing Affine Valuation Models

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**Proposition**

The SDF of any affine model can be decomposed into:

$$\frac{M_{t+1}^T}{M_t^T} = \beta \exp \left( -B'_\infty \gamma_0 + B'_\infty (I - \Gamma) X_t - B'_\infty \epsilon_{t+1} \right),$$

$$\frac{M_{t+1}^P}{M_t^P} = \beta^{-1} \exp \left( -\delta_0 + B'_\infty \gamma_0 - [B'_\infty (I - \Gamma) + \delta'_1] X_t - \frac{1}{2} \Lambda'_t \Sigma \Lambda_t - (\Lambda'_t - B'_\infty) \epsilon_{t+1} \right),$$

with constant $\beta$ given by

$$\beta = \exp \left( -\delta_0 + (\gamma'_0 - \Lambda'_0 \Sigma) B_\infty + \frac{1}{2} B'_\infty \Sigma B_\infty \right).$$
Conditional Variance Ratio

- AJ use metric $L_t (SDF_{t+1}) = \log E_t [SDF_{t+1}] - E_t [\log SDF_{t+1}]$

- In Gaussian models: $L_t (SDF_{t+1}) = \frac{1}{2} V_t [sdf_{t+1}]$

- The conditional variance of the SDF that comes from the permanent component

\[
\omega_t = \frac{L_t (M_{t+1}^P / M_t^P)}{L_t (SDF_{t+1})}
\]

\[
= 1 - \frac{B'_\infty \Sigma \Lambda_t - \frac{1}{2} B'_\infty \Sigma B_\infty}{\frac{1}{2} \Lambda'_t \Sigma \Lambda_t},
\]

\[
= 1 - \frac{E_t[r_{t+1}^b (\infty) - y_t]}{\max_j E_t[r_{t+1}^j - y_t]}.
\]
Variation in the transitory component of SDF linked to bond market

\[ \frac{M_{t+1}^T}{M_t^T} = (R_{t+1}(\infty))^{-1} \]

Without transitory component, yield curve is constant and bond risk premia are zero (e.g., C-CAPM)

⇒ **Shocks to Level factor** capture shocks to the **transitory component**
Link with Stocks and Bonds

- Variation in the transitory component of SDF linked to bond market

- Variation in the permanent component of SDF linked to stock market
  - Without permanent component, highest risk premium is that on long bond
  - AJ use stocks and bonds to find $E[\omega] \approx 1$
    - $\Rightarrow$ Shocks to DP factor capture shocks to the permanent component
Link with Stocks and Bonds

- Variation in the transitory component of SDF linked to bond market

- Variation in the permanent component of SDF linked to stock market

- Return predictability shows $\omega_t$ cannot be constant
  - Bond risk premia move over time, with $CP$, generating time variation in conditional variance of transitory component
  - Stock risk premia move over time, with $DP$, generating time variation in conditional variance of permanent component
  \[\Rightarrow \text{Shocks to CP & DP factor capture shocks to the conditional variance ratio } \omega_t\]
Example of Structural Model

- Preferences are CRRA
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Aggregate consumption has two components: $C_{t+1} = C_{t+1}^P C_{t+1}^T$

- Transitory component follows AR(1) in logs
- Permanent component is random walk in levels
- Variance of shock to permanent component, $s_t^2$, varies over time (time-varying economic uncertainty)
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- Conditional variance ratio $\omega_t$ varies positively with $s_t^2$
Estimation Strategy

Estimate risk-neutral companion matrix $\Theta$ and short-rate parameters in $\delta_1$ to match dynamics of forward rates (20); estimate $\delta_0$ to match average level of interest rate (1), as in Cochrane and Piazzesi (2008)
Estimation Strategy

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- Estimate average prices of Level, $DP$, and $CP$ risk in $\hat{\Lambda}_0$ to match unconditional expected returns on stock and bond portfolios (3)
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- Estimate average prices of Level, DP, and CP risk in $\hat{\Lambda}_0$ to match unconditional expected returns on stock and bond portfolios (3).

- Estimate the time-variation in risk prices in $\Lambda_1$ to match conditional expected returns on stock and bond portfolios (3).
The state vector $X_t$ contains:

- CP factor (as in Cochrane and Piazzesi 2005)
- Level factor (first principal comp. of Fama-Bliss yields)
- Slope factor (unpriced)
- Curvature factor (unpriced)
- DP factor (log dividend yield on stock market)
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Dividend yield is not a priced risk factor in the bond market by setting $\Gamma_{(1:4,5)} = 0_{4 \times 1}$. 
Main set of 16 test assets

- CRSP value-weighted market portfolio (AMEX/NASDAQ/NYSE)
- 10 Fama-French portfolios sorted along book-to-market (BM)
- 5 CRSP bond portfolios with maturities 1y, 2y, 5y, 7y, and 10y
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Data

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- Data at monthly frequency
- Sample period June 1952 - December 2008
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- Sample period June 1952 - December 2008
- Robustness analysis: corporate bonds, other equity portfolios
# 1. Cross-Section of Unconditional Expected Returns

## Panel A: Pricing Errors (in % per year)

<table>
<thead>
<tr>
<th></th>
<th>RN</th>
<th>Our Model</th>
<th>Level</th>
<th>Level-only bonds</th>
<th>DP</th>
<th>Level + DP</th>
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</thead>
<tbody>
<tr>
<td>1-yr</td>
<td>1.11</td>
<td>-0.43</td>
<td>-0.41</td>
<td>0.69</td>
<td>1.00</td>
<td>0.68</td>
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<tr>
<td>2-yr</td>
<td>1.31</td>
<td>-0.55</td>
<td>-1.62</td>
<td>0.50</td>
<td>1.15</td>
<td>0.53</td>
</tr>
<tr>
<td>5-yr</td>
<td>1.69</td>
<td>-0.19</td>
<td>-4.10</td>
<td>0.09</td>
<td>1.43</td>
<td>0.19</td>
</tr>
<tr>
<td>7-yr</td>
<td>1.99</td>
<td>0.38</td>
<td>-4.82</td>
<td>0.11</td>
<td>1.61</td>
<td>0.15</td>
</tr>
<tr>
<td>10-yr</td>
<td>1.62</td>
<td>0.17</td>
<td>-6.02</td>
<td>-0.49</td>
<td>1.09</td>
<td>-0.54</td>
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<tr>
<td>Mkt</td>
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<td>-0.74</td>
<td>4.22</td>
<td>5.51</td>
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<td>-0.06</td>
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<tr>
<td>BM1</td>
<td>4.97</td>
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<td>3.38</td>
<td>4.53</td>
<td>-3.28</td>
<td>-3.15</td>
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<tr>
<td>BM2</td>
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<td>-0.36</td>
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<td>-0.86</td>
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<td>-0.99</td>
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<td>BM5</td>
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<tr>
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<td>MAPE</td>
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<td>0.40</td>
<td>4.83</td>
<td>4.81</td>
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<td>1.23</td>
</tr>
</tbody>
</table>

## Panel B: Prices of Risk Estimates

<table>
<thead>
<tr>
<th></th>
<th>CP</th>
<th>Level</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>88.06</td>
<td>-23.98</td>
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<tr>
<td>0</td>
<td>0</td>
<td>-42.22</td>
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<tr>
<td>0</td>
<td>0</td>
<td>-11.69</td>
<td>-3.46</td>
</tr>
</tbody>
</table>

Koijen (Chicago Booth), Lustig (UCLA), and Van Nieuwerburgh (Stern) Stock and Bond Returns
Decomposing Risk Premia

Decomposition of the market and bond risk premia

Decomposition of risk premia on value and growth

Koijen (Chicago Booth), Lustig (UCLA), and Van Nieuwerburgh (Stern)
2. Time Series of Conditional Expected Returns

- Recall $r^j_{t+1} = E_t[r^j_{t+1}] + \eta^j_{t+1}$
- Return predictability governed by $\Lambda_1$ matrix
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- Recall $r^j_{t+1} = E_t[r^j_{t+1}] + \eta^j_{t+1}$
- Return predictability governed by $\Lambda_1$ matrix
- Bond return predictability:
  - Price of level risk depends on $CP$: $\Lambda_1(2,1) \neq 0$
  - Chosen to exactly match predictive coefficient of equally-weighted portfolio of CRSP bonds on lagged $CP$
2. Time Series of Conditional Expected Returns

- Recall \( r^j_{t+1} = E_t[r^j_{t+1}] + \eta^j_{t+1} \)

- Return predictability governed by \( \Lambda_1 \) matrix

- Bond return predictability:
  - Price of level risk depends on \( CP \): \( \Lambda_1(2,1) \neq 0 \)
  - Chosen to exactly match predictive coefficient of equally-weighted portfolio of CRSP bonds on lagged \( CP \)
  - Implies predictability of annual 2-yr through 5-yr returns, constructed from Fama-Bliss yields, on lagged \( CP \)
2. Time Series of Conditional Expected Returns

- Recall $r_{t+1}^j = E_t[r_{t+1}^j] + \eta_{t+1}^j$

- Return predictability governed by $\Lambda_1$ matrix

- Bond return predictability:
  - Stock return predictability
    - Price of $DP$ risk depends on $DP$: $\Lambda_{1(5,5)} \neq 0$
    - Chosen to exactly match predictive coefficient of the aggregate stock market on lagged $DP$
    - $\Lambda_{1(5,1)} \neq 0$ to make sure $CP$ does not predict aggregate stock return
2. Time Series of Conditional Expected Returns

- Recall $r^j_{t+1} = E_t[r^j_{t+1}] + \eta^j_{t+1}$
- Return predictability governed by $\Lambda_1$ matrix
- Bond return predictability:
  - Stock return predictability
    - Price of $DP$ risk depends on $DP$: $\Lambda_{1(5,5)} \neq 0$
    - Chosen to exactly match predictive coefficient of the aggregate stock market on lagged $DP$
    - $\Lambda_{1(5,1)} \neq 0$ to make sure $CP$ does not predict aggregate stock return
  - Implies predictive coefficients for 10 book-to-market portfolios on lagged $DP$
Rerun the CP (2005) bond return predictability regressions using simulations to form yields, forward rates, and bond returns.
Conditional Risk Pricing: Stock Return Predictability

Regress 10 BM portfolio excess returns on lagged DP

Predictability of value and growth portfolios

Koijen (Chicago Booth), Lustig (UCLA), and Van Nieuwerburgh (Stern)
Two frequencies in risk premia

equity risk premium has 83 mo. half-life, bond risk premium only 3 mo.
3. Term Structure of Interest Rates

- **Short Rate:** $y_t(1) = \delta_0 + \delta'_1 X_t$.

- **Recall:** Bond yields of maturity $\tau$ affine in $X$:
  \[
  y_t(\tau) = -\frac{A(\tau)}{\tau} - \frac{B'(\tau)}{\tau} X_t
  \]

- **Forward rates also affine**

- **Estimate** $\Theta$ and $\delta_1$ to minimize pricing errors on 1-yr through 5-yr Fama-Bliss demeaned forward rates

- **Estimate** $\delta_0$ to match average yield level of Fama-Bliss bonds
Implied Fit for Bond Yields

Volatility of yield pricing errors: 12bp, 7bp, 6bp, 9bp, 3bp per year

Koijen (Chicago Booth), Lustig (UCLA), and Van Nieuwerburgh (Stern)
Decomposing the SDF suggests three priced factors:

1. *CP* innovations to explain the cross-section of expected stock returns

2. *Level* innovations to explain the cross-section of expected bond returns, price of *Level* risk varies with *CP*

3. *DP* innovations to capture the level of expected stock returns, price of *DP* risk varies with *DP*
Decomposing the SDF suggests three priced factors:

1. *CP* innovations to explain the cross-section of expected stock returns

2. *Level* innovations to explain the cross-section of expected bond returns, price of *Level* risk varies with *CP*

3. *DP* innovations to capture the level of expected stock returns, price of *DP* risk varies with *DP*

- What is *CP* factor?
- Why are value stocks riskier than growth stocks?
- Why is the price of *CP* risk positive?
Revisiting Conditional Variance Ratio

- Most the of the variation in $\omega$ is due to $CP$: correlation is -99.2%
- When $CP$ is high, importance of the transitory shocks is high, and persistence of pricing kernel is low
- This happens when economic activity is expected to increase
Interpreting the $CP$ Factor

- When $CP$ is high, economic activity is expected to increase.

- Predicting economic activity $k = 1, \ldots, 36$ months ahead with $CP$:

$$CFNAI_{t+k} = c_k + \beta_k CP_t + \varepsilon_{t+k},$$

- Where $CFNAI$ is the Chicago FED National Activity Indicator.

- Shocks to $CP$ are good news for the economy, hence positive price of risk.
Interpreting the $CP$ Factor

Koijen (Chicago Booth), Lustig (UCLA), and Van Nieuwerburgh (Stern) Stock and Bond Returns
Interpreting the \textit{CP} Factor and the Value Premium

- High values of \textit{CP} forecast an increase in future economic activity

- Returns on value stocks are positively correlated with \textit{CP} shocks (while growth stocks have zero or even negative correlation)

$\Rightarrow$ Value stocks are riskier because they pay off exactly when economic activity is expected to increase
Stocks Also Predicted by CP

\[ r_{x_t}^j = r_{x_0} + \zeta_{s1}^j DP_t + \zeta_{s2}^j CP_t + \eta_{t+1}^j \]
## Comparison to Fama-French Three-Factor Model

### Panel A: Pricing Errors (in % per year)

<table>
<thead>
<tr>
<th></th>
<th>RN</th>
<th>Our Model</th>
<th>FF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-yr</td>
<td>1.11</td>
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<td>0.91</td>
</tr>
<tr>
<td>2-yr</td>
<td>1.31</td>
<td>-0.55</td>
<td>0.97</td>
</tr>
<tr>
<td>5-yr</td>
<td>1.69</td>
<td>-0.19</td>
<td>1.08</td>
</tr>
<tr>
<td>7-yr</td>
<td>1.99</td>
<td>0.38</td>
<td>1.22</td>
</tr>
<tr>
<td>10-yr</td>
<td>1.62</td>
<td>0.17</td>
<td>0.56</td>
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<tr>
<td>Mkt</td>
<td>6.00</td>
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<td>-0.06</td>
</tr>
<tr>
<td>BM1</td>
<td>4.97</td>
<td>0.02</td>
<td>0.53</td>
</tr>
<tr>
<td>BM2</td>
<td>5.86</td>
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</tr>
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<td>BM3</td>
<td>6.65</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>BM4</td>
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<td>-0.84</td>
</tr>
<tr>
<td>BM5</td>
<td>7.30</td>
<td>0.79</td>
<td>-0.17</td>
</tr>
<tr>
<td>BM6</td>
<td>7.35</td>
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<tr>
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<td>7.32</td>
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<td>0.05</td>
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<tr>
<td><strong>MAPE</strong></td>
<td><strong>5.53</strong></td>
<td><strong>0.40</strong></td>
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### Panel B: Prices of Risk Estimates

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<th>CP</th>
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<th>DP</th>
<th>MKT</th>
<th>SMB</th>
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<td>-23.98</td>
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<tr>
<td>DP</td>
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<td>6.93</td>
<td>5.08</td>
<td>-4.76</td>
<td>-1.98</td>
</tr>
</tbody>
</table>

Koijen (Chicago Booth), Lustig (UCLA), and Van Nieuwerburgh (Stern)
### Pricing Corporate Bonds

<table>
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<tr>
<th></th>
<th>RN SDF</th>
<th>Our SDF not re-estimated</th>
<th>Our SDF re-estimated</th>
<th>FF SDF</th>
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<tr>
<td>1-yr</td>
<td>1.62</td>
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<td>3-yr</td>
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<td>0.39</td>
<td>0.96</td>
<td>0.66</td>
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<td>5-yr</td>
<td>1.69</td>
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<td>0.35</td>
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<tr>
<td>7-yr</td>
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<td>Market</td>
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<td>-0.81</td>
<td>0.18</td>
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<tr>
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<td>0.79</td>
</tr>
<tr>
<td>BM2</td>
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<td>-0.34</td>
<td>-0.46</td>
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<tr>
<td>BM3</td>
<td>6.65</td>
<td>0.04</td>
<td>-0.01</td>
<td>-0.15</td>
</tr>
<tr>
<td>BM4</td>
<td>6.37</td>
<td>-0.05</td>
<td>-0.12</td>
<td>-1.10</td>
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<tr>
<td>BM5</td>
<td>7.30</td>
<td>0.78</td>
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</tr>
<tr>
<td>BM6</td>
<td>7.35</td>
<td>0.26</td>
<td>0.32</td>
<td>-0.65</td>
</tr>
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<td>-0.93</td>
<td>-1.63</td>
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<tr>
<td>BM8</td>
<td>9.10</td>
<td>0.28</td>
<td>0.51</td>
<td>-0.32</td>
</tr>
<tr>
<td>BM9</td>
<td>9.69</td>
<td>0.69</td>
<td>0.91</td>
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</tr>
<tr>
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<tr>
<td>Credit2</td>
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<td>Credit3</td>
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<td>1.01</td>
</tr>
<tr>
<td>Credit4</td>
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<td>-1.53</td>
<td>-0.71</td>
<td>0.90</td>
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<td>MAPE</td>
<td>5.12</td>
<td>0.58</td>
<td>0.49</td>
<td>0.77</td>
</tr>
</tbody>
</table>

**Stock and Bond Returns**

- CP/Market: 88.16, 79.61, 6.36
- Level/SMB: -24.03, -18.84, -10.88
- DP/HML: -1.96, -2.15, 8.08
## Pricing Other Test Assets

<table>
<thead>
<tr>
<th></th>
<th>10 FF Size</th>
<th></th>
<th>10 FF E-P</th>
<th></th>
<th>25 FF Size/Value</th>
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<tr>
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<td>RN</td>
<td>KLN</td>
<td>FF</td>
<td>RN</td>
<td>KLN</td>
<td>FF</td>
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<tr>
<td><strong>MAPE</strong></td>
<td>5.84</td>
<td>0.52</td>
<td>0.55</td>
<td>3.55</td>
<td>0.78</td>
<td>0.91</td>
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<tr>
<td><strong>Market Prices of Risk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CP/Market</td>
<td>48.21</td>
<td>6.22</td>
<td>81.22</td>
<td>2.19</td>
<td>134.73</td>
<td>4.04</td>
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<td>Level/SMB</td>
<td>-15.59</td>
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<td>-22.84</td>
<td>4.19</td>
<td>-29.83</td>
<td>2.16</td>
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<tr>
<td>Market/HML</td>
<td>-2.56</td>
<td>17.48</td>
<td>-0.83</td>
<td>5.37</td>
<td>-1.45</td>
<td>8.82</td>
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### Different Sample Periods

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<td>KLN kernel</td>
<td>FF kernel</td>
<td>RN kernel</td>
<td>KLN kernel</td>
<td>FF kernel</td>
</tr>
<tr>
<td><strong>MAPE</strong></td>
<td>4.88</td>
<td>0.40</td>
<td>0.72</td>
<td>5.78</td>
<td>0.59</td>
<td>1.06</td>
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</table>

**Panel B: Market Prices of Risk**

<p>| | | | | | | |</p>
<table>
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<tr>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>CP/Market</td>
<td>61.39</td>
<td>4.19</td>
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<td>41.12</td>
<td>5.40</td>
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<tr>
<td>DP/HML</td>
<td>-0.83</td>
<td>6.73</td>
<td></td>
<td>-1.35</td>
<td>4.40</td>
<td></td>
</tr>
</tbody>
</table>
Maximum Sharpe Ratio

- Affine term structure models commonly imply maximum Sharpe ratios that are very high.
- Maximum SR achieved through large long-short positions in bonds.
- Impose constraints on positions: \(-\alpha \leq w_i \leq 1\), with \(\alpha = 0, .5, 1\).

<table>
<thead>
<tr>
<th></th>
<th>Model mean</th>
<th>Model st.dev.</th>
<th>Data mean</th>
<th>Data st.dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Annual Sharpe ratios on individual bonds</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-yr</td>
<td>0.81</td>
<td>(0.28)</td>
<td>0.71</td>
<td>(0.28)</td>
</tr>
<tr>
<td>2-yr</td>
<td>0.71</td>
<td>(0.31)</td>
<td>0.48</td>
<td>(0.32)</td>
</tr>
<tr>
<td>5-yr</td>
<td>0.45</td>
<td>(0.33)</td>
<td>0.33</td>
<td>(0.30)</td>
</tr>
<tr>
<td>7-yr</td>
<td>0.33</td>
<td>(0.33)</td>
<td>0.33</td>
<td>(0.29)</td>
</tr>
<tr>
<td>10-yr</td>
<td>0.24</td>
<td>(0.32)</td>
<td>0.22</td>
<td>(0.30)</td>
</tr>
</tbody>
</table>

|                |            |               |           |              |
| Panel B: Annual Sharpe ratios on bond portfolios |            |               |           |              |
| unconstr.      | 2.62       | (0.07)        |           |              |
| \(\alpha = 0.0\) | 0.93       | (0.30)        |           |              |
| \(\alpha = 0.5\) | 0.93       | (0.30)        |           |              |
| \(\alpha = 1.0\) | 0.93       | (0.30)        |           |              |
Conclusions

Main contribution: develop a parsimonious SDF model which

1. Explains the cross-section of expected stock and bond returns
2. Captures the predictability of bond and stock returns
3. Captures the dynamics of bond yields
Conclusions

- Main contribution: develop a parsimonious SDF model which

- Requires only three priced factors:
  1. $CP$ innovations to explain the cross-section of expected stock returns
  2. Level innovations to explain the cross-section of expected bond returns
  3. $DP$ innovations to capture the level of expected stock returns
Conclusions

- Main contribution: develop a parsimonious SDF model which
  - Requires only three priced factors:

- Main economic insights:
  - $CP$ factor captures the variation in importance of transitory component of pricing kernel
  - This transitory component becomes more important 1-2 years before economic activity peaks
Conclusions

- Main contribution: develop a parsimonious SDF model which
  - Requires only three priced factors:

- Main economic insights:
  - *CP* factor captures the variation in importance of transitory component of pricing kernel
  - This transitory component becomes more important 1-2 years before economic activity peaks
  - Values stocks have cash-flows that are more sensitive to this cyclical component, hence the value premium
Model with Temporary and Permanent Component in Consumption

- **Endowment:**  
  \[ C_{t+1} = C_{t+1}^P C_{t+1}^T \]  
  with  
  \[
  \begin{align*}
  c_{t+1}^T &= \mu_c + \rho c_t^T + \sigma \epsilon_{t+1}, \\
  c_{t+1}^P &= c_t^P - \frac{1}{2}s_t^2 + s_t\eta_{t+1}, \\
  s_{t+1}^2 - \bar{s}^2 &= \nu(s_t^2 - \bar{s}^2) + \sigma_w w_{t+1},
  \end{align*}
  \]

- **Preferences are CRRA:**
  \[
  sdf_{t+1} = \log(\beta) - \gamma \Delta c_{t+1},
  \]
  \[
  = \log(\beta) - \gamma \mu_c + \gamma (1 - \rho) c_t^T + \frac{\gamma}{2}s_t^2 - \gamma \sigma \epsilon_{t+1} - \gamma s_t\eta_{t+1},
  \]

---

Koijen (Chicago Booth), Lustig (UCLA), and Van Nieuwerburgh (Stern)

Stock and Bond Returns
Model with Temporary and Permanent Component in Consumption

- Term structure is affine:

\[ P_t(\tau) = \exp \left\{ A(\tau) + B(\tau)c_t^T + C(\tau) \left( s_t^2 - \bar{s}^2 \right) \right\}. \]

- Decomposition \( sdf_{t+1} = sdf_{t+1}^T + sdf_{t+1}^P \) follows general model

- Conditional variance ratio

\[ \omega_t = \frac{\frac{1}{2} V_t \left[ sdf_{t+1}^P \right]}{\frac{1}{2} V_t \left[ sdf_{t+1} \right]} = 1 - \frac{B_\infty \gamma \sigma^2 - \frac{1}{2} B_\infty^2 \sigma^2}{\frac{1}{2} \left( \gamma^2 \sigma^2 + \gamma^2 s_t^2 \right)} \]

\[ = 1 - \text{bond risk premium} / \text{maximum risk premium} \]

- When economic uncertainty \( s_t^2 \) decreases, transitory component of SDF becomes more important
The \textit{CP} Factor and NBER Recessions

Motivation Model Estimation Extensions Conclusions Extras

Koijen (Chicago Booth), Lustig (UCLA), and Van Nieuwerburgh (Stern) Stock and Bond Returns
Consistent Risk Pricing Across Stocks and Bonds

- GMM estimation where we allow for $\neq$ risk price on $CP$ and $Level$ for stocks and bonds

- Point estimates in $\Lambda_0$: $91.98/88.16$, $-14.19/-24.03$, $-2.12/-1.96$

- Incremental risk prices for bonds (GMM s.e.): $-42.92$ (38.57) and $-3.78$ (37.77)

- Cannot reject null hypothesis that risk prices are same for stocks and bonds
Internal consistency with $CP$ Factor

- Consistency between first VAR element and implied $CP$?
  - Simulate SDF model for 100,000 months
  - Calculate model-implied nominal yields, forwards, and bond returns from simulation
  - Re-estimate $CP$ factor on model-implied bond returns and yields
  - Does $\hat{CP}$ equal the first element of the state vector?
- Their correlation is 80%
Does the Slope Factor Also Work?

- No: Innovations of slope factor have correlation with CP innovations of only 17%

- Estimate model where slope is priced instead of CP: \( \Lambda_{0(1)} = 0 \) and \( \Lambda_{0(3)} \neq 0 \)

- and price of level risk depends on slope instead of CP: \( \Lambda_{1(2,1)} = 0 \) and \( \Lambda_{1(2,3)} \neq 0 \).

- MAPE deteriorates substantially from 40bp to 105bp per year

- Pricing errors contain a value spread, and a spread between short- and long-horizon bonds

- Slope factor plays no meaningful role in pricing either stock or bond portfolios
Why Are Value Stocks Riskier? Role of Cash-Flow News

Decompose returns

\[ r_{t+1} - E_t[r_{t+1}] = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta r_{t+1+j} \]

\[ \text{Cov}(\varepsilon_{t+1}^R, \varepsilon_{t+1}^{CP}) = \text{Cov}(\varepsilon_{t+1}^{NCFDG}, \varepsilon_{t+1}^{CP}) - \text{Cov}(\varepsilon_{t+1}^{NFR}, \varepsilon_{t+1}^{CP}) \]

CP ↑ ⇒ future bond returns high ⇒ NCFDG on value stocks is good