Frontiers of Real Estate Research

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Real Estate is a Topic Area

- Research on real estate uses draws on tools and ideas from many areas in economics and finance
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Broadly speaking there are three large sub-areas, but the lines can be a bit blurry even between these sub-areas:

1. Real Estate Finance
2. Urban Economics
3. Real Estate Policy
Life-cycle/portfolio theory with housing: how to rationalize observed housing and mortgage choices (alongside rest of portfolio)
Real Estate Finance

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- House price determination: booms-busts, regional variation, return predictability, REITS
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- Foreclosures: role of strategic default, role of equity extraction, effect on house prices, spatial misallocation of labor, loan modifications
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- Contracting: optimal (reverse) mortgage contract design
Urban Economics

- Agglomeration effects: economies of scale and productivity
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- Spatial equilibrium models: free mobility, city-size distribution, types of cities
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- Congestion externalities
- Housing supply regulation and impact on house prices, impact of rent control
Effect of government involvement in housing markets on house prices, availability of credit, loan performance, in normal times and in crises?
Real Estate Policy

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- How to reform the government-sponsored enterprises?
Real Estate Policy

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- How to reform the government-sponsored enterprises?
- What would happen to house prices, housing stock, if welfare mortgage interest rate deductibility was abolished?
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How to reform the government-sponsored enterprises?

What would happen to house prices, housing stock, if welfare mortgage interest rate deductibility was abolished?

How to mitigate foreclosures ex-post? Effect of regulation on banks’ recourse and on loan-to-value limits ex-ante?
Housing and Portfolio Choice

- Managing long-term housing risk important for lifetime wealth
  - Housing is the largest asset class in household portfolio
  - Homeowners work and live in the same market, tying their fortunes to the fate of the local economy
  - Large moving costs impede frequent reoptimization: house price dynamics important for location choice
  - Difficult to manage risk with standard products/instruments
Aggregate Household Portfolio Shares

Flow of Funds; left panel is 2006.Q2, right panel is 2010.Q4
• Individual cities are volatile over long horizons
• Much of the house price risk is local (idiosyncratic) and not aggregate
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Hizmo & Van Nieuwerburgh  Real Estate
Individual cities are volatile over long horizons
Much of the house price risk is local (idiosyncratic) and not aggregate
Within City Heterogeneity

San Francisco: HPI by Price Quartile minus Median HPI

Source: Dataquick
Within City Heterogeneity

Chicago: HPI by Price Quartile minus Median HPI

Source: Dataquick
Within City Heterogeneity

Phoenix: HPI by Price Quartile minus Median HPI

Source: Dataquick
Within City Heterogeneity

Miami: HPI by Price Quartile minus Median HPI

Source: Dataquick
Within City Heterogeneity

Cleveland: HPI by Price Quartile minus Median HPI

Source: Dataquick
Questions

Theoretical:
- How does local nondiversifiable risk affect:
  - Individual location and portfolio decision
  - Equilibrium house prices (asset prices?)

Empirical:
- How is risk capitalized into house prices?
  - Do homes in more volatile cities sell at a discount?

Policy:
- What are the costs of market incompleteness?
  - Risk held by local residents + misallocation of human capital
  - Benefits of financial instruments that allow for hedging
### Literature Overview

<table>
<thead>
<tr>
<th>Urban/Spatial Models</th>
<th>Portfolio Choice with Income and/or Housing</th>
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- Add risk aversion and portfolio choice to urban literature
- Add heterogeneity and space to models in finance

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<tr>
<th>Benefits of Hedging/Insuring Risks</th>
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Hizmo (2011) - General Setting

- System of \( L \) cities with fixed supply of \( n^l \) homes and countryside
- Overlapping generations: agents live for \( T \) periods
- Timing
  1. Agents are born and learn their types
  2. They decide where to live and buy one house once
  3. Receive wages and invest in financial market
  4. Consume accumulated wealth at end of life
Economy driven by $m$ independent Brownian motions

$$B_t = \left( B_t^1 ... B_t^m \right)'$$
Uncertainty and Financial Assets

- Economy driven by $m$ independent Brownian motions
  \[ B_t = \left( B_t^1 ... B_t^m \right) '\]
- One risk-less asset with return $r$
- $n$ risky securities with returns:
  \[ \frac{dP^i_t}{P^i_t} = \mu^i dt + \sigma_{i1} dB^1_t + \sigma_{i2} dB^2_t + ... + \sigma_{im} dB^m_t \]
  where $\mu$ is to be determined in equilibrium and $n \leq m$.  

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Economy driven by $m$ independent Brownian motions

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where $\mu$ is to be determined in equilibrium and $n \leq m$.

In matrix form:

$$D_{P_t}^{-1} dP_t = \mu dt + \Sigma_{P} dB_t$$
Wages are set to marginal product:

$$w_{it}^l = y_t^l + \zeta_i + \epsilon_i^l$$

where $\zeta_i$, $\epsilon_i^l$ individual specific.
Wages and Productivity

- Wages are set to marginal product:

\[ w_{it} = y_t^l + \zeta_i + \epsilon_i^l \]

where \( \zeta_i, \epsilon_i^l \) individual specific.

- Productivity of a city \( y_t^l \) follows:

\[ dy_t^l = s_t^l dt \]
\[ ds_t^l = \phi^l \left( m^l - s_t^l \right) dt + \Sigma_s^l dB_t \]

where \( s_t \) is instantaneous change in income and
\( B_t = (B_t^1 ... B_t^m)' \) underlying factors that drive economy.
The Households’ Problem

At birth households solve:

\[ V(X_t, w_t) = \sup_{\theta_t, l} \left[ -E_t e^{-\gamma_i \left[ X_{t+T} + p_t^l T + p_t^l e^{r(T-t)} + M^l \beta^1 \frac{1}{r} (e^{r(T-t)} - 1) \right] } \right] \]

The wealth \( X_t \) evolves according to:

\[ dX_t = \sum_{j=1}^{n} \theta_{jt} \frac{dP_j^j}{P_j^j} + r (X_t - \theta_t) dt + w_it dt \]

where \( P_t \) = stock price, \( p_t^l \) = house price, \( \theta_t \) = investment in stocks, \( M^l \) = amenities
Equilibrium Summary

- Individuals maximize utility
  1. Given a location, solve portfolio choice problem
  2. Given optimal portfolio rules, choose best location

- Asset markets clear
  - Aggregate demand to get equilibrium returns

- Housing markets clear
  - Find equilibrium prices by setting demand=supply
Focus on stationary equilibria linear in the states:

\[ p_t^l = A^l y_t^l + B^l s_t^l + C^l \]

**Proposition.** Suppose prices are given as above and agent decides to live in city \( l \). Then his demand for assets is:

\[ \theta_{ilt} = (\Sigma_{PP})^{-1} \left[ \frac{(\mu - 1r)}{\gamma_i e^{r(T-t)}} - \frac{\Sigma_l P_s}{r(r + \phi^l)} \right] \]  

(1)

Setting aggregate demand equal to supply \( S \):

\[ \mu = \frac{r \Sigma_{PP} S + \sum_{l=1}^L \left( \frac{n^l \Sigma_l P_s}{r + \phi^l} \right)}{\left( 1 - e^{-rT} \right) \int_{\gamma_i} \frac{1}{\gamma_i} d\Gamma(\gamma_i)} + 1r \]  

(2)
Portfolio Choice and Asset Returns

- Focus on stationary equilibria linear in the states:

\[ p^l_t = A^l y^l_t + B^l s^l_t + C^l \]

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\]

- Tools: HJB transformations (Henderson 2005) + Feynman-Kac to get CARA-normal setting.
Proposition 3. Given optimal portfolio rules for location $l$, the log value function is:

$$
\max_l V^l = -r \pi^l - \sum_{j=1}^{n} \lambda_j \sigma_j^l c_1^l - \gamma_i \sum_{j=n+1}^{m} \left( \sigma_j^l \right)^2 c_2^l + M^l \beta_i + \epsilon_i^l
$$

and $\lambda = \frac{\mu - r}{\sigma_p}$ market prices of risk, $\sigma_j^l$ = city’s exposure to $B_t^j$

If $\epsilon_i^l$ is drawn from a continuous distribution there exists a unique vector $\pi^l$ that solves the sorting problem.

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- Complete markets:

$$
V^l = -r\pi^l - \sum_{j=1}^{n} \lambda_j \sigma_j^l c_1^l + M^l \beta_i + \epsilon_i^l
$$

(3)
Interpretation: No heterogeneity

\[ p_t^l = E_t \left[ \int_t \exp(-r(u-t)) D_u^l du \right] + \pi^l \]

\[ = \frac{1}{r} y_t^l + \frac{1}{r (r + \phi^l)} s_t^l + \frac{\phi^l m_t^l}{r^2 (r + \phi^l)} + \frac{1}{r} \beta M_t^l + \pi^l \]

Income \quad \text{Instant. Growth} \quad \text{Expected Growth} \quad \text{Amenities} \quad \text{Risk Premium}

where

\[ \pi^l = - \sum_{i=1}^{n} \lambda_i \sigma_i^l c_1^l - \gamma \sum_{i=n+1}^{m} \left( \sigma_i^l \right)^2 c_2^l \]

Traded Risk \quad Nontraded Risk

and \( \lambda = \frac{\mu - r}{\sigma_p} \) market prices of risk, \( \sigma_j^l = \text{city’s exposure to } B_t^j \)
Empirical Results Overview

- Estimate model using house price, wage, and amenities data for 216 cities since 1985
  - Find three common risk factors that drive house prices
  - Only the national housing market factor correlated with stocks
  - Use a city’s exposure to these factors to compute risk premia
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- Estimate city-specific housing risk premia = portion of prices due to volatility
  - Premia imply average prices are $20000 or 10% cheaper than if there was no volatility
  - Homes over $100000 cheaper for volatile coastal markets
  - On average 10% of growth rate due to risk premia
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  - Completing markets ⇒ wages increase by 14%
  - No gains if agents homogeneous
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  - On average homeowners willing to pay $10000-$20000 for access to new instruments
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- Welfare gains = Risk sharing + Higher wages
  - On average homeowners willing to pay $10000-$20000 for access to new instruments
- House prices increase by 20% due to falling risk premia
Fact 1: Broad Observations

- Residential real estate large and volatile component of household wealth (Houses accounted for 25% of net worth in 2000 but 37% end of 2006)
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- Residential investment is volatile and highly pro-cyclical

- Large swings in house prices relative to housing fundamentals.

- Real estate is prime collateral for households (and small business) for borrowing
  - House price fluctuations affect risk-sharing opportunities in economy
  - When house prices fall, collateral values shrink, less borrowing is possible, compensation for risk goes up in all asset markets (e.g., equity risk premium)
Fact 2: National House Price-Rent Ratio

Price–Rent Ratio (Core Logic and BLS)

Price: Core Logic (SA) repeat-sales house price index,
Rent: BLS shelter index

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Fact 3: Collapse in Aggregate Home Equity

Flow of Funds, 1952Q4-2010Q4

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Fact 4: Financial Market Liberalization in Mortgage Lending

- Housing boom 2000-2006:
  - Widespread relaxation borrowing constraints: (rising LTV ratios, prime and sub-prime, end of 2006 100% financing routine).
  - Declining transactions costs: Sharp drop in cost of mortgage refinance, home equity withdrawal, fees/charges (McCarthy and Stiendel ’07; Mian and Sufi ’09).
  - Sustained depression interest rates: real rate on 10-yr T-bond 3.6% ↓ 0.93%, 2000-2006.

- Credit crisis 2007-2009:
  - Back to more normal minimum combined (1st and 2nd mortgage) LTV ratio ≤ 75-80% of appraised value.
  - Latest evidence: transactions costs moving back up.
Fact 5: Large Capital Inflows into U.S. Safe Assets

- Foreign ownership U.S. Treasuries: **13.5% to 61%**
  1984-2008

U.S. Department of Treasury (1974-2010)
Fact 5: Large Capital Inflows into U.S. Safe Assets

- Foreign holdings U.S. Treasuries/Agency Debt range **15-30% GDP** by end of 2008

U.S. Department of Treasury (1974-2010)
Fact 5: Large Capital Inflows into U.S. Safe Assets

- Assume these holdings are perfectly inelastic (regulatory motive)

U.S. Department of Treasury (1974-2010)
Research Question: Role of Housing in the Macroeconomy?

To what extent is national house price appreciation attributable to:

- A liberalization in housing finance?
- An infusion of foreign governmental capital into U.S. bond markets?
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- How do movements in house prices affect expectations of future housing fundamentals and future home price appreciation?

- How do housing wealth and housing finance affect
  1. Risk premia in housing and equity markets,
  2. Cross-sectional risk-sharing,
  3. Life-cycle wealth/savings patterns,
  4. Housing “wealth effects” on consumer spending?
Incomplete markets and equity pricing

Aiyagari and Gertler ’91; Telmer ’93; Lucas ’94, Heaton and Lucas ’96; Basak and Cocco ’98; Luttmer ’99; Lustig and Van Nieuwerburgh ’05; Storesletten et. al ’07; Gomes and Michaelides ’08; Favilukis ’08.
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Incomplete markets and housing

- Fernandez-Villaverde and Krueger ’05; Rios-Rull and Marcos ‘06; Peterson ‘06; Ortalo-Magne and Rady ’06; Lustig and Van Nieuwerburgh ’07 ’08; Piazzesi and Schneider ’08; Kiyotaki et. al., ’08; Corbae and Quintin ’09.
Related Literature

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- Complete markets/rep agent and housing
  - Davis and Heathcoate ‘05; Kahn ‘08; Piazzesi, Schneider, and Tuzel ‘07.
Goal: provide theoretical answers using a model that accounts for the endogenous interactions among

1. financial and housing wealth,
2. output, investment, and consumption,
3. rates of return and risk premia (housing and equity markets),
4. consumption and wealth inequality.
Favilukis, Ludvigson, and Van Nieuwerburgh’s Model

Goal: provide theoretical answers using a model that accounts for the endogenous interactions among

- Use model to study consequences of three systemic changes in housing finance.
  1. Collateralized borrowing constraints
  2. Housing transactions costs
  3. Secular decline in interest rates driven by influx of foreign governmental capital
Favilukis, Ludvigson, and Van Nieuwerburgh’s Model

- Goal: provide theoretical answers using a model that accounts for the endogenous interactions among

- Use model to study consequences of three systemic changes in housing finance.

- All three changed markedly during or preceding period of rapid home price appreciation 2000-2006.
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Use model to study consequences of three systemic changes in housing finance.

All three changed markedly during or preceding period of rapid home price appreciation 2000-2006.

Some changes have been reversed in aftermath of credit crisis of 2007/2008.
Model: Production Side

- Time is discrete, a period = one year.
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- Two sectors: **consumption** and **housing** output:

  \[
  Y_{C,t} = Z_{C,t}K_{C,t}^\alpha N_{C,t}^{1-\alpha}, \quad Y_{H,t} = Z_{H,t}K_{H,t}^\nu N_{H,t}^{1-\nu}
  \]
Model: Production Side

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  \[ Y_{C,t} = Z_{C,t} K_{C,t}^{\alpha} N_{C,t}^{1-\alpha}, \quad Y_{H,t} = Z_{H,t} K_{H,t}^{v} N_{H,t}^{1-v} \]

- Capital accumulates

  \[ K_{C,t+1} = (1 - \delta) K_{C,t} + I_{C,t} \quad K_{H,t+1} = (1 - \delta) K_{H,t} + I_{H,t} \]
Model: Production Side

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  \[ K_{C,t+1} = (1 - \delta) K_{C,t} + I_{C,t} \quad K_{H,t+1} = (1 - \delta) K_{H,t} + I_{H,t} \]

- \( Y_{H,t} \) residential investment \( \Rightarrow \) housing stock evolves \( H_t \)
  \[ H_{t+1} = (1 - \delta_H) H_t + Y_{H,t}. \]
Model: Production Side

- Adj costs capital, $\phi_i(\cdot)$, deduction from earnings.
Model: Production Side

- Adj costs capital, \( \phi_i(\cdot) \), deduction from earnings.
- No new shares issued. Dividends = earnings:

\[
D_{C,t} = Y_{C,t} - w_t N_{C,t} - I_{C,t} - \phi_C \left( \frac{I_{C,t}}{K_{C,t}} \right) K_{C,t}.
\]

\[
D_{H,t} = p^H_t Y_{H,t} - w_t N_{H,t} - I_{H,t} - \phi_H \left( \frac{I_{H,t}}{K_{H,t}} \right) K_{H,t}.
\]

\( p^H \) relative price housing
Model: Production Side

- Adj costs capital, \( \phi_i(\cdot) \), deduction from earnings.
- No new shares issued. Dividends = earnings:

\[
D_{C,t} = \gamma_{C,t} - w_t N_{C,t} - I_{C,t} - \phi_C \left( \frac{I_{C,t}}{K_{C,t}} \right) K_{C,t}.
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D_{H,t} = p^H_t \gamma_{H,t} - w_t N_{H,t} - I_{H,t} - \phi_H \left( \frac{I_{H,t}}{K_{H,t}} \right) K_{H,t}.
\]

\( p^H \) relative price housing

- Firms maximize value; SDF = \( \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} \):

\[
V_{C,t} = \max_{N_{C,t}, I_{C,t}} E_t \sum_{k=0}^{\beta^k \Lambda_{t+k}} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} D_{C,t+k}
\]

\[
V_{H,t} = \max_{N_{H,t}, I_{H,t}} E_t \sum_{k=0}^{\beta^k \Lambda_{t+k}} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} D_{H,t+k}
\]
Model: Asset Returns

- **Cum dividend returns**

\[
R_{YH,t+1} = \frac{V_{H,t+1}}{(V_{H,t} - D_{H,t})} \quad R_{YC,t+1} = \frac{V_{C,t+1}}{(V_{C,t} - D_{C,t})}.
\]

- Value-weighted portfolio of equity in C and H sectors:

\[
R_{K,t} = \frac{V_{H,t}}{V_{H,t} + V_{C,t}} R_{YH,t} + \frac{V_{C,t}}{V_{H,t} + V_{C,t}} R_{YC,t}.
\]

- 1-period risk-free bond, \(B_{t+1}\), with price \(q_t\) and gross return

\[
R_{f,t+1} = \frac{1}{q_t}.
\]
Model: Consumer Side

- A overlapping generations, indexed by $a = 1, \ldots, A$.
- 2 life stages: working (21-65) & retirement (65-100). Retired workers die with age-dependent probability.
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- CRRA utility function, Cobb-Douglas over non-housing and housing consumption
Model: Consumer Side

- A overlapping generations, indexed by $a = 1, \ldots, A$.
- 2 life stages: working (21-65) & retirement (65-100). Retired workers die with age-dependent probability.
- CRRA utility function, Cobb-Douglas over non-housing and housing consumption.
- Heterogeneous labor income: $Y_{a,t}^i = w_t L_{a,t}^i,$

\[-\frac{L_{a,t}^i}{G_a Z_{a,t}^i} = \frac{G_a Z_{a,t}^i}{L_{a,t}^i} \]

\[-\log \left( Z_{a,t}^i \right) = \log \left( Z_{a-1,t-1}^i \right) + \epsilon_{a,t}^i, \quad \epsilon_{a,t}^i \sim i.i.d. \left( 0, \sigma_t^2 \right), \]

- Two-state volatility (Storesletten, Telmer, Yaron ’04)

\[-\sigma_t^2 = \begin{cases} \sigma_E^2 & \text{if } Z_{C,t} \geq E \left( Z_{C,t} \right) \\ \sigma_R^2 & \text{if } Z_{C,t} < E \left( Z_{C,t} \right) \end{cases}, \quad \sigma_R^2 > \sigma_E^2 \]
Model: Consumer Side

- Invests in one-period bond \((B^i)\), in a mutual fund of firms \((\theta^i)\), and in housing \((H^i)\)
Model: Consumer Side

- Invests in one-period bond ($B^i$), in a mutual fund of firms ($\theta^i$), and in housing ($H^i$)
- Changing housing incurs transaction costs $F$

$$F_{H,a,t}^i \begin{cases} 
0 & \text{if } H_{a+1,t+1}^i = (1 - \delta_H) H_{a,t}^i \\
\psi_0 + \psi_1 p_t^H H_{a,t}^i & \text{if } H_{a+1,t+1}^i \neq (1 - \delta_H) H_{a,t}^i 
\end{cases}$$
Model: Consumer Side

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- Changing housing incurs transaction costs $F$

- Budget constraint, working age

$$C_{a,t} + B_{a+1,t+1}^i q_t + \theta_{a+1,t+1}^i (V_{C,t} + V_{H,t}^e) \leq W_{a,t}^i + (1 - \tau) w_t L_{a,t}^i + p_t^H \left( (1 - \delta_H) H_{a,t}^i - H_{a+1,t+1}^i \right) - F_{a,t}^i$$

$$W_{a+1,t+1}^i \geq - (1 - \omega) p_t^H H_{a,t+1}^i, \quad \forall a, t$$

- Maximum combined (mortgage and home equity) LTV ratio $= (1 - \omega)$. 
Model: Consumer Side

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\[
C^i_{a,t} + B^i_{a+1,t+1}q_t + \theta^i_{a+1,t+1} (V^e_{C,t} + V^e_{H,t}) \leq W^i_{a,t} + (1 - \tau) w_t L^i_{a,t} + p^H_t \left( (1 - \delta_H)H^i_{a,t} - H^i_{a+1,t+1} \right) - F^i_{a,t}
\]

\[
W^i_{a+1,t+1} \geq -(1 - \omega) p^H_t H^i_{a,t+1}, \quad \forall a, t
\]

- Maximum combined (mortgage and home equity) LTV ratio = $(1 - \omega)$. 
- Retired workers earn a pension paid for by social security taxes on labor income, $\tau$; replaces wage income.
Model: Equilibrium

Set $q_t = q(\mu_t, Z_t)$, $p^H_t = p^H(\mu_t, Z_t)$, $w_t = w(\mu_t, Z_t)$, $R_{K,t} = R_K(\mu_t, Z_t)$, set of cohort-specific functions $i$, \( \left\{ V_a, H^i_{a+1,t+1}, \theta^i_{a+1,t+1} B^i_{a+1,t+1} \right\}_{a=1}^A \), and law of motion $\mu$, $\mu_{t+1} = \Gamma(\mu_t, Z_t, Z_{t+1})$ such that:

1. Households optimize (dynamic problem), subject to constraints.
2. Firm’s maximize value.
3. Wages $w_t = w(\mu_t, Z_t)$ given by marginal product of labor in each sector.
4. Housing market clears: $p^H_t = p^H(\mu_t, Z_t)$ is such that
   \[
   Y_{H,t} = \int_S \left( H^i_{a,t+1} - H^i_{a,t} (1 - \delta_H) \right) d\mu.
   \]
5. Bond market clears: $q_t = q(\mu_t, Z_t)$ is such that
   \[
   \int_S B^i_{a,t} d\mu + B^F_t = 0 \quad B^F_t \geq 0
   \]
6. Risky asset market clears:
   \[
   1 = \int_S \theta^i_{a,t} d\mu.
   \]
7. The labor market clears:
   \[
   N_t \equiv N_{C,t} + N_{H,t} = \int_S L^i_{a,t} d\mu.
   \]
8. Total taxes equal total pension benefits.
9. Presumed law of motion $\mu_{t+1} = \Gamma(\mu_t, Z_t, Z_{t+1})$ is consistent with individual beliefs.
Keeping Track of Wealth Distribution

\[ Z_t \equiv (Z_{C,t}, Z_{H,t})'. \text{ State of economy is } (Z, \mu), \mu \text{ is a measure defined over } S = (A \times Z \times \mathcal{W} \times \mathcal{H}). \]

- \( A = \{1, 2, \ldots A\} \) is the set of ages
- \( Z \) set of all possible idiosyncratic shocks
- \( \mathcal{W} \) set of all possible BOP financial wealth realizations
- \( \mathcal{H} \) set of all possible BOP housing wealth realizations
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- Law of motion, \( \Gamma \), for \( \mu \),
  \[
  \mu_{t+1} = \Gamma (\mu_t, Z_t, Z_{t+1}).
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- Law of motion, \( \Gamma \), for \( \mu \),

\[
\mu_{t+1} = \Gamma (\mu_t, Z_t, Z_{t+1})
\]

- Approximate infinite dimensional \( \mu \) with finite dimensional object - use version of Krusell-Smith algorithm (1998). Notes:
Stochastic Discount Factor

- SDF = weighted avg of shareholders’ intertemporal MRS (IMRS), weighted by prop. ownership:

\[
\frac{\beta \Lambda_{t+1}}{\Lambda_t} \equiv \int_S \theta_{a+1,t+1} \frac{\beta \partial U / \partial C_{i,a+1,t+1}}{\partial U / \partial C_{i,a,t}} d\mu
\]

- Only IMRS of those long in risky asset receive weight.

- One paper without adj. costs: (Carceles & Coen-Pirani ’09), equilibrium:
  1. same as identical economy with “static” firms that rent capital.
  2. invariant to choice of SDF among IMRS that satisfy Euler equation for risky asset.

- Our adj costs small; check robustness of our equilibrium to various SDFs.
Transition Dynamics: National Price-Rent Ratio

- Price/Rent rises 41% 2000-2006, boosted by economic growth, FML, and lower interest rates (foreign capital).

![Graph showing price and price-rent ratio trends over years from 2000 to 2008.](image)
Price/Rent declines 17% 2007-2009, driven by economic contraction and a reversal of the FML.
Stijn’s other work in RE

- Housing collateral ratio predicts future stock returns and can account for cross-sectional variation in value-growth returns (JF 2005)
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- Optimal mortgage choice depends on bond risk premium (JFE 2009)
How to hedge house price and income risk?
- Short the local stock market
- Construct local stock index using data on location of businesses

How does availability of credit affect house prices?
- Study how increases in FHA limits affected house prices in 2008
- Dataquick transaction level data + Multiple Listings Service data
Some Useful Datasets

- **House price data**
  - Dataquick: transaction level, extremely detailed
  - Case-Shiller: reliable for up to zip code level
  - FHFA: down to MSA level, only for conforming loan homes
  - Zillow: partly hedonic
  - Corelogic HPI
  - MLS: house listings data

- **Mortgages / Credit**
  - Equifax: individual level, debt and credit score, location
  - McDash: mortgage performance data
  - HMDA: loan applications information

- **General Micro Level Datasets**
  - Decennial Census
  - American Housing Survey
  - PSID: detailed panel of individuals
  - BEA REIS, QCEW: detailed local wage/productivity data
References


