Estimating the Implied Risk Neutral Density for the U.S. Market Portfolio

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Abstract

The market's risk neutral probability distribution for the value of an asset on a future date can be extracted from the prices of a set of options that mature on that date, but two key technical problems arise. In order to obtain a full well-behaved density, the option market prices must be smoothed and interpolated, and some way must be found to complete the tails beyond the range spanned by the available options. This paper develops an approach that solves both problems, with a combination of smoothing techniques from the literature modified to take account of the market's bid-ask spread, and a new method of completing the density with tails drawn from a Generalized Extreme Value distribution. We extract twelve years of daily risk neutral densities from S&P 500 index options and find that they are quite different from the lognormal densities assumed in the Black-Scholes framework, and that their shapes change in a regular way as the underlying index moves. Our approach is quite general and has the potential to reveal valuable insights about how information and risk preferences are incorporated into prices in many financial markets.

Keywords: Risk neutral density; pricing kernel; implied probability

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