Estimation Error in the Assessment of Financial Risk Exposure

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Value at Risk and similar measures of financial risk exposure require predicting the tail of an asset returns distribution. Assuming a specific form, such as the normal, for the distribution, the standard deviation (and possibly other parameters) are estimated from recent historical data and the tail cutoff value is computed. But this standard procedure ignores estimation error, which we find to be substantial even under the best of conditions. In practice, a "tail event" may represent a truly rare occurrence, or it may simply be a not-so-rare occurrence at a time when the predicted volatility underestimates the true volatility, due to sampling error. This problem gets worse the further in the tail one is trying to predict.

Using a simulation of 10,000 years of daily returns, we first examine estimation risk when volatility is an unknown constant parameter. We then consider the more realistic, but more problematical, case of volatility that drifts stochastically over time. This substantially increases estimation error, although strong mean reversion in the variance tends to dampen the effect. Non-normal fat-tailed return shocks makes overall risk assessment much worse, especially in the extreme tails, but estimation error per se does not add much beyond the effect of tail fatness. Using an exponentially weighted moving average to downweight older data hurts accuracy if volatility is constant or only slowly changing. But with more volatile variance, an optimal decay rate emerges, with better performance for the most extreme tails being achieved using a relatively greater rate of downweighting.

We first simulate non-overlapping independent samples, but in practical risk management, risk exposure is estimated day by day on a rolling basis. This produces strong autocorrelation in the estimation errors, and bunching of apparently extreme events. We find that with stochastic volatility, estimation error can increase the probabilities of multi-day events, like three 1% tail events in a row, by several orders of magnitude. Finally, we report empirical results using 40 years of daily S&P 500 returns which confirm that the issues we have examined in simulations are also present in the real world.

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