Dynamic Programming and Optimal Control

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Course Description
Dynamic Programming (DP) provides a set of general methods for making sequential, interrelated decisions under uncertainty. This course brings a new dimension to static models studied in a standard optimization course, by investigating dynamic systems and their optimization over time. The focus of the course is on modelling and deriving structural properties for discrete-time and continuous-time stochastic problems. The techniques are illustrated through concrete applications from Operations, Marketing, Decision Sciences, Economics and Finance.

Prerequisites: An introductory course in Optimization and Probability.

Required and Recommended Textbooks

Required Material:
• Lecture Notes “Dynamic Programming with Applications” prepared by the instructor to be distributed before the beginning of the class.

Recommended Textbooks:
Description of other readings and case material, which will be distributed in class
The course also includes some additional readings, mostly research papers that we will use to complement the material and discussion covered in class. Some of these papers described important applications of dynamic programming in Operations Management and other fields.

Schedule
Classroom: KMC 8-150 (OM-Stats conference room).
Time: Thursdays 10:00am-1:00pm

TOPICS
The following is the list of sessions and topics that we will cover in this class. These topics serve as an introduction to Dynamic Programming. The coverage of the discipline is very selective: We concentrate on a small number of powerful themes that have emerged as the central building blocks in the theory of sequential optimization under uncertainty.

In preparation to class, students should read the REQUIRED READINGS indicated below under each session (including the chapters in Bertsekas’s textbook and the lecture notes provided). Due to time limitations, we will not be able to review all the material covered in these readings during the lectures. If you have specific questions about concepts that are not discussed in class, please contact the instructor to schedule additional office hours.

Session 1 & 2: Introduction to Dynamic Programming and Optimal Control
We will first introduce some general ideas of optimizations in vector spaces most notoriously the ideas of extremals and admissible variations. These concepts will lead us to formulation of the classical Calculus of Variations and Euler’s equation. We will proceed to formulate a “general” optimal deterministic control problem and derive a set of necessary conditions (Pontryagin Minimum principle) that characterize an optimal solution. Finally, we will discuss an alternative way of characterizing an optimal solution using the important idea of the “Principle of Optimality” pioneered by Richard Bellman. This approach will lead us to derive the so-called Hamilton-Jacobi-Bellman (HJB) sufficient optimality conditions. We will complement the discussion reviewing a paper on optimal price trajectory in a retail environment by Smith and Achabal (1998).

REQUIRED READINGS:
• Chapter 3 in Bertsekas.
• Chapter 1 in the Lecture Notes.

Session 3 & 4: Discrete Dynamic Programming
In these sessions, we review the classical model of dynamic programming (DP) in discrete time and finite time horizon. First, we discuss deterministic DP models and interpret it as a shortest path problem in an appropriate network. Different algorithms to find the shortest past are discussed. We then extend the DP framework to include uncertainty (both in the payoffs and the evolution of the system) and connect it to the theory on Markov Decision process. We review some basic properties of the value function and numerical methods to compute it.

REQUIRED READINGS:
• Chapters 1 & 2 in Bertsekas.
• Chapter 2, sections 2.1-2.4 in Lecture Notes.
Session 5: Extensions to the Basic Dynamic Programming Model
In this session we discuss some fundamental properties and extensions of the classical DP model discussed in the previous lecture. We discuss in detail a particular but important special case, the so-called Linear-Quadratic problem. We also discuss the connection between DP and supermodularity. Finally, we discuss some extensions of the DP model regarding state-space augmentation and the value of information.

REQUIRED READINGS:
• Chapter 2 in Bertsekas.
• Chapter 2, sections 2.5-2.7 in Lecture Notes.

Session 6: Applications of Dynamic Programming
This session is dedicated to review three important applications of DP. The first application that we discussed is on the optimality of (S,s) policies in a multi-period inventory control setting. We then review the single-leg multiclass revenue management problem. We conclude studying the application of DP to optimal stopping problems.

REQUIRED READINGS:
• Chapter 4 in Bertsekas.
• Chapter 3 in Lecture Notes.

Session 7: Dynamic Programming with Imperfect State Information
In this section we extend the basic DP framework to the case in which the controller has only imperfect (noisy) information about the state of the system at any given time. This is a common situation in many practical applications (e.g., firms do not know the exact type of a customer; a repairman does not know the status of a machine, etc.). We will discuss an efficient formulation of this problem and find conditions under which a sufficient set of statistics can be used to describe the available information. We will also revisit the LQ problem and review the Kalman filtering theory.

REQUIRED READINGS:
• Chapter 5 in Bertsekas.
• Chapter 4 in Lecture Notes.

Session 8: Infinite Horizon and Semi-Markov Decisions Models
In this section we extend the models discussed in the previous sessions to the case in which the planning horizon is infinite. We review alternative formulations of the problem (e.g., discounted versus average objective criteria) and derive the associated Bellman equation for these formulations. We also discuss the connection between DP and semi-Markov decision theory.

REQUIRED READINGS:
• Chapter 7 in Bertsekas.
• Chapter 5 in Lecture Notes.
Session 9: Point Process Control
In this section we consider the problem of how to optimally control the intensity of a Poisson process. This problem (and some of its variations) has become an important building block in many applications including dynamic pricing models. We will review the basic theory and some concrete applications in revenue management and marketing.

REQUIRED READINGS:
- Chapter 6 in Lecture Notes

Session 10: Review of Stochastic Processes and Itô Calculus
In preparation for the study of the optimal control of diffusion processes, we review some classical results of stochastic processes, Brownian motion, martingale theory and Itô calculus. We discuss the notions of infinitesimal generator of a Markov process, Dynkin’s formula and their connection to Hamilton-Jacobi-Bellman equations.

REQUIRED READINGS:
- Chapter 5 in W. Fleming and R. Rishel.
- Chapter 7 in Lecture Notes.

Session 11 and 12: Diffusion Control
In these sessions we discuss the fundamental results in the optimal control of diffusion processes. We characterize the HJB optimality conditions for finite and infinite horizon problems and discussed conditions for the existence of an optimal solution. We will discuss variations in the form of bang-bang, singular, and impulse controls. We also review some numerical methods for solving these optimization problems.

REQUIRED READINGS:
- Chapter 6 in W. Fleming and R. Rishel
- Chapter 8 in Lecture Notes.

The Grading Scheme
1. There are five individual assignments that will be assigned. Students have a week to prepare their solutions, which will be collected at the beginning of the following session. Homework should be considered as a take-home exam and must be done individually. In the same spirit, students are not supposed to consult solutions from previous year. Presentation is part of the grading of these assignments. Assignments must be submitted on time; late submissions will not be accepted.
2. There is an in-class midterm. There is also a take-home exam to be distributed the last day of class. Students will have two weeks to prepare and submit their solutions. The exam will be cumulative and will include the implementation of a computational algorithm.

Final Score
- 40% Individual Homework
- 20% Midterm
- 40% Final Exam