Pricing and Liquidity
in the US Corporate Bond Market *

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January 16th, 2013

Abstract

I show, both theoretically and empirically, that if dealers are risk averse, transaction prices, liquidity provision, and dealers’ inventory positions depend on their inventory holding costs in over-the-counter, search markets. Using the solution to a dynamic equilibrium model, I can rationalize the following stylized facts in the US corporate bond market: (i) a reduction in dealers’ inventories in the financial crisis of 2007-09 and sovereign debt crisis of 2011-12; (ii) a reduction in average trade size since the onset of the financial crisis and the subsequently tighter regulatory environment; (iii) the dependence of prices and liquidity on dealers’ holding costs, with asymmetric effects for buys versus sells; (iv) a generally negative relationship between transaction costs and trade size. Using a richer transactions data-set than in previous studies, my model correctly predicts that liquidity is worse when dealers become effectively more risk averse relative to customers. Also consistent with the model, (a) this effect is stronger for bonds with lower credit ratings and for customers with lower bargaining power; and (b) conditioning on customer bargaining power, the effect is more pronounced for larger trades.

* The author would like to thank Joel Hasbrouck, Anthony Lynch, Lasse Pedersen, Emiliano Pagnotta, Marti Subrahmanyam, and seminar participants at NYU Stern for helpful comments and suggestions, and the Salomon Center for the Study of Financial Institutions and a NASDAQ grant for providing funding for data. All errors are of course my own.

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1 Introduction

As of June 2012 there was $8.5 trillion of US corporate debt outstanding\(^1\). But, relative to equity markets, the microstructure of the US corporate bond market has been under-researched. Dealers in the bond market have reduced their inventories substantially since October 2007\(^2\). This coincided with the start of large write-downs on the balance sheets of several banks (Citi, Merrill Lynch, UBS), whose dealer businesses had substantial share in liquidity provision for the trading of US corporate bonds. In the financial press\(^3\) this reduction in inventory has been attributed to a reduction in dealer risk appetite during the financial crisis of 2007-09. Bonds which suffered credit rating downgrades not only represented greater inventory risk to the dealing arm of banks, but they also contributed less to the banks’ Fed-mandated capital requirements on a risk-weighted basis than they did before. Reducing dealer inventories would not only have ameliorated inventory risk, but would also have freed up capital for the banks, at a time of anticipated tightening of capital requirements and greater demand for liquid balance sheets. Anecdotal evidence from practitioners is that this reduction in dealer inventory led to an overall deterioration in liquidity in the secondary market for US corporate bonds\(^4\). Figure 3 confirms that average liquidity, measured by the bid-ask spread, did worsen in the financial crisis, but there was an asymmetric effect for customers buying from dealers, versus those selling to them. Average liquidity actually improved for customers buying from dealers, but it deteriorated for customers selling to them.

I use the solution to a dynamic model of over-the-counter (OTC) trading to establish equilibrium relationships, showing how liquidity and dealers’ optimal inventory positions are related to their inventory holding costs and customer bargaining power, if dealers are “effectively risk averse”\(^5\). My theory emphasizes that it is the time-varying cost of holding inventory, rather than inventory per se, that drives these relationships in dealer markets. Using a richer data-set than previous studies, I show that the predictions of these relationships are correct: (1) Dealers reduced their inventory positions of non-investment grade bonds more when they become effectively more risk averse relative to customers, and increased their inventory of investment grade bonds, suggesting a flight to quality; (2) Liquidity was worse when dealers become effectively more risk averse relative to customers. (3) This effect was generally stronger for non-investment grades bonds than investment grades bonds, consistent with the theory that both ag-

\(^1\)Source: http://www.sifma.org/research/statistics.aspx
\(^2\)See Figure 2.
\(^3\)e.g. ”Slimmer bond inventories as dealers reduce risk” November 8th, 2011 (FT.com)
\(^4\)e.g. ”Dealers Slash Bond Holdings as Conviction in Rally Wanes: Credit Markets” July 15th, 2010 (Bloomberg.com)
\(^5\)See Etula (2009) and Adrian, Etula, Shin (2010)
aggregate and bond-specific risk drives the dealers’ implicit costs of holding inventory. (4) This effect was also stronger for customers with lower bargaining power, with a strikingly monotonic relationship in bond-level regression results. (5) Conditioning on customer bargaining power, the effect was more pronounced for larger trades. To the best of my knowledge, my model is the first dynamic trading model of an OTC search market with both dealers and customers, where the dealers are averse to inventory risk and trade size is non-trivial. All these features are necessary to match the stylized facts in the US corporate bond market in the last several years, and also allow me to make new, nuanced predictions, which hold true in the data.

In the theory section of my paper, I solve a dynamic equilibrium model of trading of a risky asset in a hybrid market, consisting of a competitive inter-dealer market, an OTC search market where customers and dealers bargain over price and quantity, and less frequent trade between dealers and the issuer of the asset. I measure liquidity as the dealers’ “markup”, which is defined as the difference between the average price per unit at which they trade with a customer, and the price at which a hypothetical inter-dealer trade would have occurred, had there been one at that time. I derive closed-form expressions for customer-dealer prices, dealer markups, and dealers’ optimal inventory. The model gives intuition for the relationship between liquidity and trade size in an OTC setting. It also rationalizes why dealers reduced their inventories of US corporate bonds in the financial crisis of 2007-09 and subsequent sovereign debt crisis, given an increase in their “effective risk aversion”, as modeled in other markets by Etula (2009) and Adrian, Etula, Shin (2010). Unlike the previous literature of dynamic trade in OTC markets, the structural model allows me to demonstrate how the dependence of liquidity on dealer inventory holding costs, amongst other factors, can be tested empirically in the US corporate bond market.

In the seminal models of dynamic, OTC trading of Duffie, Garleanu, Pedersen (2005, 2007), dealer inventory is restricted to 0 or 1 unit. However, in order to understand the relationship between trade size and liquidity, trade size must be non-trivial. Lagos and Rocheteau (2009) weaken this restriction to allow dealers to hold positive inventory of any amount. Weill (2007) and Lagos, Rocheteau and Weill (2011) show that when dealers are risk-neutral, if customers become more risk averse in a crisis, dealers will “lean against the wind” and stock-pile risky assets to sell them back to customers at a profit, once customers' risk tolerance returns to pre-crisis levels. Though that model may be applicable to other crises or other markets, it is not applicable to the US corporate bond market during the financial crisis of 2007-09, nor to the sovereign debt crisis of 2010-11, when dealers reduced their aggregate inventories, rather than increasing them. In fact, the data suggests that it is the relative level of constraint under which dealers and customers operate which determines which set of agents builds
or reduces inventory. In that sense, customers took the role of leaning against the wind these crises. These empirical facts can be rationalized in my model when the effective risk aversion of dealers, relative to risk-neutral customers, increases in a crisis. Afonso and Lagos (2012) model inter-dealer trading in the overnight federal funds market. The model allows dealers to have more flexible utility functions, but any aversion to holding inventory is constant over time, and trade size is restricted. Empirically, dealers’ corporate bond holdings suggest that time-varying risk aversion is an important feature of this market, and unrestricted trade size allow me to better understand the relationship between liquidity and trade size. I employ a friction of “trading blackouts” - that dealers can only trade with other dealers, customers, and the issuer of the asset at known, discrete points in time - as in Longstaff (2001, 2009) and Garleanu (2008), to emphasize the periods of time at which dealers bear inventory risk. All these features are necessary to match the stylized facts in the US corporate bond market in the last several years.

The effect of dealer inventory on liquidity has been shown empirically in equity (Comerton-Forde et al, 2010), foreign exchange (Lyons, 1997) and credit default swap (Shachar, 2012) markets. It is likely to be more pronounced in markets where counterparties are less readily available, where trading is at a lower frequency, and subsequently dealers are exposed to holding inventory for longer periods. The over-the-counter, low frequency nature of trading in the US corporate bond market make it a prime candidate to empirically observe the effect of dealer inventory holding costs on prices and liquidity. It is also the only OTC market where a comprehensive database of secondary market transactions is publicly available; it offers not only a broad cross-section of traded bonds, but also a large heterogeneity in liquidity, especially compared to the equity market.

In the empirical section of the paper I analyze corporate bond transaction data which is richer than most previous academic studies. Since November 2008, the database of secondary market trades, TRACE, has included information in real time on whether a trade was between a customer and dealer or two dealers, and the direction of trade between a dealer and customer. One can purchase this information from FINRA to a much earlier date - to the start of TRACE in July 2002. This increased transparency in transaction data allows me to be the first to test the strength of the relationship between liquidity and dealer inventory holding costs in this market. Using proxies for search like bonds’ notional outstanding, time since issuance, and time to maturity, the previous empirical literature, e.g. Jankowitsch, Nashikkar, Subrahmanyam (2010) and Zitzewitz (2010), found worse liquidity for bonds with higher search costs. The intuition for this is that the bond issues are in fixed supply, and get absorbed in long-term investors’ portfolios, making searching for a particular issue slow and costly.
Dealers pass at least some of that cost onto their customers. Dealer inventory is the other side of the coin to search: search costs are less relevant when dealers hold an inventory, and inventory risk is less relevant when search costs are low. Bond issues with higher credit ratings have also been shown to have better secondary market liquidity than lower ones. But the structural relationship my model produces allows me to test the relationship between liquidity and inventory holding costs in a much more careful way, resulting in more nuanced predictions, which are empirically accurate.

The dealer markup is the theoretical analogue to the empirical “unique round trip” measures of Dick-Nielsen, Feldhutter, Lando (2012), Feldhutter (2012), and Zitzewitz (2010), which were proposed to quantify liquidity in this market. These previous studies focus on “paired” customer-dealer trades: ones that are accompanied by an inter-dealer trade within a few minutes, often for the exact same quantity, as the pairing dealer immediately lays off the customer trade in the inter-dealer market. The unique round-trip measure is the difference between the two prices in a paired trade. This allows straightforward computation of the markup, at least if trade direction is known, but ignores the majority of the customer-dealer trades. In Zitzewitz (2010) only about one third of customer-dealer trades are paired with an inter-dealer trade. As he notes, this represents a selection bias, with a disproportionate number of small trades being paired. As a contribution to the empirical methodology in this field, I show how to interpolate inter-dealer prices at the time of customer-trades, to extend the analysis to unpaired trades. The markups for the paired trades are likely due more to bargaining power rather than dealer inventory holding costs, since effectively the pairing dealer is taking on little or no inventory risk, only matchmaking a customer with another dealer. I therefore focus my analysis on the unpaired trades to identify my mechanism more clearly.

The dealers’ markup is related to the bid-ask spread, but is an improvement on that measure of liquidity in two ways. Firstly, I show that it captures the asymmetry between customer buys and sells, which the bid-ask spread does not. When dealers have excessive inventory they want to sell, they charge lower transaction costs when selling to customers and higher when buying from them, as in Amihud and Mendelson (1980). Secondly, my equilibrium relationship between the markup and trade size shows how customer-dealer transaction prices vary with quantity, whereas bid and ask quotes are offered for a fixed size. The model gives theoretical intuition for the empirical fact that, without conditioning on customer bargaining power, larger trades in the US corporate bond market are generally associated with smaller markups. This is the

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6 Data limitations meant that most studies in this market did not have this information. As Feldhutter (2010) notes, without it the unique round-trip measure is a noisy mix of the full bid-ask spread and the half-spread.

7 See Figure 3.
reverse of the relationship between trade size and liquidity for US equities which trade in limit order markets. In a limit order market, market orders which exceed the depth of the best quote walk up or down the limit order book, and so larger trades, unless split up over time, are mechanically associated with worse volume-weighted average prices. The equity limit order markets are anonymous, but the OTC corporate bond market is not. A possible explanation for the disparity in the liquidity-size relationship between the two markets is found in the empirical literature on US corporate bonds (e.g. Zitzewitz, 2010). It is a market where dealers know the identity of customers with whom they are trading, and larger trades are associated with larger customers, who have higher bargaining power. For small and medium-sized trades this bargaining power effect dominates the inventory holding cost effect, which is why for these size trades there is a generally negative relationship between markup and trade size. In my model too, all else equal, higher bargaining power leads to smaller markups. But it also produces the novel prediction that, controlling for customer bargaining power, larger trades are associated with larger markups when holding costs are high. This is borne out in transaction-level regressions, with the effect generally larger for high yield bonds versus investment grade bonds. I also correctly predict that this effect is more pronounced in smaller trades, where customer bargaining power is likely to be lower. There is a strikingly monotonic relationship between the effect and bargaining power group, for both high yield and investment grade bonds.

Dick-Nielsen, Feldhutter and Lando (2012) show the empirical significance of the illiquidity discount in bond prices, particularly during the financial crisis. He and Milbradt (2011) show theoretically how worsening secondary market liquidity for corporate bonds reduces the amount of capital that can be raised through issuance in the primary market. An increase in the cost of capital, or reduction in available financing, could lead to under-investment in projects which have positive net present value. Thus a deterioration in liquidity in the secondary market can have negative consequences for the real economy. Understanding the determinants of liquidity in the secondary market, in particular the extent to which increased dealer inventory holding costs could inhibit liquidity provision, and the subsequent spillover to the cost of financing to businesses, helps inform policy decisions about the trading behavior and capital requirements of banks who also serve a role as liquidity-provider with their dealer businesses.

Section 2 describes the model. Section 3 describes the data. Section 4 describes the empirical methodology. Section 5 describes testable hypotheses from the model. Section 6 describes the results. Section 7 addresses alternative theories. Section 8 concludes. Proofs are relegated to the Appendix.
2 The model

In this section I solve a dynamic equilibrium model of trading in a hybrid market, where there is secondary trading in a competitive inter-dealer market, between customers and dealers in an OTC search market, and, less frequently, between dealers and the issuer of the asset. I derive expressions for the prices and quantities at which customers and dealers trade, dealers’ optimal inventory, and the dealers’ “markup” which is defined as the difference between the average price per unit at which they trade with customers, and the price at which dealers would trade in a hypothetical inter-dealer market had it occurred.

2.1 The Economy

Time is discrete, and runs forever. There is a risk-free asset paying gross interest $R$ each period. At any point in time there is a single risky asset, with a known finite maturity. Upon maturity, a new risky asset is issued. There are a continuum of customers and dealers. Customers are risk neutral, trade only once at birth, and consume all their wealth one period later. Dealers are risk averse, and infinitely-lived. Each period they receive interest on their cash holdings, pay a holding cost on their risky asset position, and may get to trade with a customer or another dealer, but not both, or may not trade.

At regular, discrete points in time there are rounds of trading. With probability $\tilde{\pi}_i$, there is inter-dealer trading, but only amongst a proportion $\frac{\tilde{\pi}_i}{\tilde{\pi}_i}$ of the dealers, and no customer-dealer trading. The dealers who trade are drawn uniformly and without replacement from the population of dealers. Each dealer therefore trades in the inter-dealer market with probability $\pi_i$. With probability $\tilde{\pi}_c$, there is customer-dealer trading, but only amongst a proportion $\frac{\tilde{\pi}_c}{\tilde{\pi}_c}$ of the dealers, and no inter-dealer trading. The dealers and customers who trade are drawn uniformly and without replacement from the population of dealers and customers, respectively, independent of each other. Each dealer therefore trades in the customer-dealer market with probability $\pi_c$. With probability $1 - \tilde{\pi}_i - \tilde{\pi}_c$ there is no trade by any dealer at time $t$. With probability $1 - \pi_i - \pi_c$ there is no trade by dealer $k$ at time $t$. Draws are independent across rounds.

I denote the current state at time $t$ as $s_t$, whose value changes just after each round of trading according to a Markov process. I denote the history of states since issuance of the risky asset as $\mathcal{S}_t$. The time-line is displayed in figure 2.1.
possible trade

$\pi_j^c$

trade with customer $j$

inter-dealer trade

$\pi_i$

$1 - \pi_i - \sum_j \pi_j^c$

no trade

Figure 1: **Dealer time-line.** $\pi_j^c$ is the exogenous probability that each dealer trades with a customer of type $j$ at time $t$, who has relative bargaining power $\eta_j$ and reservation value $V_i^c$. $\pi_i$ is the exogenous probability that each dealer trades in the interdealer market at time $t$. $1 - \pi_i - \sum_j \pi_j^c$ is the exogenous probability that no dealer trades that period. The probabilities of trade are independent across time. This cycle repeats until the asset matures, at which point asset holders receive a payoff, and a new asset is issued.

### 2.2 The Dealers

There are a continuum of dealers of mass 1 with holdings of (cash, risky asset units) $= (s^d_t, a^d_t)$ when entering time $t$. $s^d_t$ includes interest for the period $(t - 1, t]$, which accumulates at rate gross rate $R$ per period, and is paid at time $t$. Each dealer is risk-neutral over her cash-flows, which include trading revenues, interest on cash, and inventory holding costs. But the inventory holding costs make her “effectively risk averse” as in e.g. Etula (2009).

At time $t = ..., -1, 0, 1, ...$ each dealer receives interest at rate $R$ on her cash position, and pays holding costs $f(s_t)(a^d_t)^2$ on her risky asset position, where $f(s_t)$ is strictly positive. The holding cost is quadratic in dealer inventory as in Mahanti, Nashikkar, Subrahmanyam (2008). The convexity allows bounded solutions for trade size and inventory holdings, and the quadratic form allows a linear relationship between dealers’ markup and their inventory holdings. If $f(s)$ is proportional to the variance per unit of the next price of the risky asset, then the holding cost cash-flow can be interpreted as a Value-at-Risk constraint. We interpret this holding cost as effective risk aversion as in Etula (2009) and Adrian, Etula, Shin (2010). Since customers are risk neutral, it can be thought of as relating to effective risk aversion either in an absolute sense, or relative to customers.

At time $t$ with probability $\pi_i$ the dealer trades in the inter-dealer market with a
proportion $\frac{\pi_i}{n_i}$ of the dealers, and those are the only trades that round. The dealers who trade are drawn uniformly and without replacement from the population of dealers. She buys quantity $q_i(s_t)$ units of the risky asset from the inter-dealer market at price per unit $p_i(s_t)$, so $q_i(s_t)$ is positive if the dealer buys, and negative if she sells. The inter-dealer market is competitive, and so the price per unit does not depend on the quantity she buys.

With probability $\pi_c j$ she trades instead with customer type $j$ at time $t$, and that is her only trade that round. Customer type $j$ is defined by her conditional expected valuation of the asset, $V_{c j t} \equiv E_j [V_{t+1}]$, bargaining power $\eta_j$, and interest rate $R_{c j s t}$.

The customer is drawn uniformly from the population of customers. The dealer buys quantity $q_c(s_t)$ units of the risky asset at price per unit $p_c(s_t)$, where both the price and quantity are negotiated through Nash bargaining. $q_c(s_t)$ is positive if the dealer buys, and negative if she sells. With probability $1 - \pi_i - \pi_c$ she does not trade at time $t$, where $\pi_c \equiv \sum \pi_c j$.

At known times $T_t (t = ..., -1, 0, 1, ...)$, the risky asset matures, dealers receive payoff $V_{T_t}$ for each unit they hold, and a new risky asset is issued, with dealers buying quantity $q_{\text{issuer}}(s_{T_t})$ at price per unit $p_{\text{issuer}}(s_{T_t})$. The price only depends on the state at the time of issuance: not any prior states, and not on the quantity bought or sold. $q_{\text{issuer}}(s_{T_t})$ is positive if the dealer buys, and negative if she sells. At all other times, she receives interest and pays holding costs, but there is no trading.

Henceforth, we denote $f_t, p^i_t, q^i_t, p^c_t, q^c_t, q_{\text{issuer}}^d, p_{\text{issuer}}^d$ as shorthand for $f(s_t), p^i(s_t), q^i(s_t), p^c(s_t), q^c(s_t), q_{\text{issuer}}(s_t), p_{\text{issuer}}(s_t)$ respectively.

### 2.2.1 The dealer’s problem

The dealer maximizes the present value of her cash-flows, and so her value function at time $t$ is given by:

$$V^d_t(s^d_t, a^d_t, s_t) = \max_{\{q^d_t, q^r_t, p^r_t, q^c_t, p^c_t, q_{\text{issuer}}^d, p_{\text{issuer}}^d\}} E_t^{\infty} \left[ \sum_{u=0}^{\infty} \delta^u \left( \frac{s^d_{t+u}}{R} (1 - R^{-1}) - p_{t+u} q_{t+u} - f_{t+u} (a_{t+u}^d)^2 \right) \right]$$

subject to $(p^r_t, q^r_t)$ being the solution to Nash bargaining between dealer and customer, and the budget constraints that the dealer could be facing. Here $\delta \in (0, 1)$ is the impatience parameter, and $p$ and $q$ are the price and quantity, respectively, of a trade with a dealer, customer, or issuer. Each round, the possible situations are: (1) no trade, (2) trade in the inter-dealer market, or with a customer, and (3) maturity and reissuance. The budget constraints are given by:
1. no trade at time $\tau$:
\[ s^d_{\tau+1} = R \left( s^d_{\tau} - s^d_{\tau}(1 - R^{-1}) - f_{\tau}.(a^d_{\tau})^2 \right) \]
\[ a^d_{\tau+1} = a^d_{\tau} \]

2. trade in the inter-dealer market or with a customer at time $\tau$:
\[ s^d_{\tau+1} = R \left( s^d_{\tau} - s^d_{\tau}(1 - R^{-1}) - p_{\tau}q_{\tau} - f_{\tau}.(a^d_{\tau})^2 \right) \]
\[ a^d_{\tau+1} = a^d_{\tau} + q_{\tau} \]

3. maturity and reissuance at time $\tau$:
\[ s^d_{\tau+1} = R \left( s^d_{\tau} - s^d_{\tau}(1 - R^{-1}) + a^d_{\tau}V_{\tau} - p_{\tau}q_{\tau} - f_{\tau}.(a^d_{\tau})^2 \right) \]
\[ a^d_{\tau+1} = q_{\tau} \]

where $(p, q) \in \{(p^c_{\tau}, q^c_{\tau}), (p^i_{\tau}, q^i_{\tau}), (p^\text{issuer}_{\tau}, q^\text{issuer}_{\tau})\}$. When there is no trade at time $\tau$, the dealer pays the holding cost on their inventory of risky assets, and receives interest on their cash holding. $s^d_{\tau}$ includes interest received by the dealer at $\tau$. The dealer’s pre-interest cash holding was thus $R^{-1}s^d_{\tau}$, so the interest received is $s^d_{\tau} - R^{-1}s^d_{\tau} = s^d_{\tau}(1 - R^{-1})$. After consuming the interest and paying the holding cost, the dealer is left with $s^d_{\tau} - s^d_{\tau}(1 - R^{-1}) - f_{\tau}.(a^d_{\tau})^2$, which earns gross interest at rate $R$ by $\tau+1$. When there is a trade at time $\tau$, whether with a customer, dealer, or the issuer, the dealer buys $q_{\tau}$ units of the asset at price per unit $p_{\tau}$. Thus the dealer’s cash outflow $p_{\tau}q_{\tau}$ is also deducted from the dealer’s cash holding at $\tau$. At maturity the dealer receives $V_{\tau}$ for each unit she holds, so her additional cashflow is $a^d_{\tau}V_{\tau}$.

**Theorem 2.1.** If the dealer’s impatience parameter $\delta < 1$, the dealer’s value function takes the following form at time $\tau$:
\[ V^d_{\tau}(s^d_{\tau}, a^d_{\tau}, s_{\tau}) = \gamma^d(s_{\tau})s^d_{\tau} + \gamma^a(s_{\tau})a^d_{\tau} + \gamma^{aa}(s_{\tau}),(a^d_{\tau})^2 + \gamma(s_{\tau}) \]
for some functions $\gamma^d(s_{\tau})$, $\gamma^a(s_{\tau})$, $\gamma^{aa}(s_{\tau})$, and $\gamma(s_{\tau})$, where $s_{\tau}$ is the history of states since issuance of the asset to time $\tau$.

I conjecture that the value function takes this form, since it is the same form as the dealer’s utility function. In the appendix I verify that if the value function takes this
form at some point in time, then by backwards induction on time, it takes that form at all prior periods.

2.3 The Customers

There are a continuum of risk-neutral customers, of different types. Customer type \( j \) is defined by her conditional expected valuation of the asset, \( V_{t}^{c,j} \equiv E_{t}^{j}[V_{t+1}] \), bargaining power \( \eta_{j} \), and interest rate \( R_{c}^{j}(s_{t}) \). They trade at most once, at birth, with a dealer\(^8\), where the price and quantity are determined by Nash bargaining. Because they are risk-neutral, it follows that the price and quantity they negotiate do not depend on their holdings of either the risk-free or risky asset. They consume all their wealth one period later.

2.4 Customer-Dealer Trade

When a customer and dealer meet they bargain over both the price and quantity, maximizing the Nash product:

\[
\max_{p_{t}^{c},q_{t}^{c}} \left[ \text{customer gain from trade} \right]^\eta \times \left[ \text{dealer gain from trade} \right]^{1-\eta}
\]

where \( \eta \) is the relative bargaining power of the customer, and \( 1 - \eta \) is that of the dealer.

2.4.1 Customer’s gain from trade with a dealer

Consider a customer whose conditional expected valuation of the risky asset is \( V_{t}^{c} \) per unit, and whose interest rate is \( R_{c}^{c} \). For a customer-dealer trade at time \( t \), if a dealer buys \( q_{t}^{c} \) units of the asset from that customer, at average price per unit \( p_{t}^{c} \), I show in the appendix that the customer’s gain from trade is given by:

\[
\text{Customer’s gain from trade} = \delta^{c} R_{t}^{c} q_{t}^{c} \left( p_{t}^{c} - \tilde{V}_{t}^{c} \right)
\]  

(2)

where \( \tilde{V}_{t}^{c} \equiv V_{t}^{c} / R_{t}^{c} \) is the customer’s expected valuation of the asset, discounted back to time \( t \). The gain does not depend on the customer’s pre-trade holdings in either cash or the risky asset, because he is risk-neutral. All he cares about is the difference between his discounted valuation and the price at which he can trade with the dealer: this is his marginal gain for each unit traded. Fixing the price per unit, he would want to trade an unbounded amount: buying as much as possible if his valuation is above the dealer’s quoted price, and selling otherwise. But the convexity of the dealer’s holding

\(^8\)Trade between 2 customers is extremely rare in the US corporate bond market.
cost makes this unappealing to the dealer, and bounds the agreed quantity to a finite amount.

### 2.4.2 Dealer’s gain from trade with a customer

Let $q_i^c$ denote the optimal quantity the dealer would buy if she meets a customer at time $t$. Let $q_{i+u+1}^t$ and $q_{i+u+1}^c$ denote the optimal quantity the dealer would buy if she trades in the inter-dealer market or with a customer at time $t + u + 1$, respectively, assuming her last trade was buying $q_i^c$ at time $t$.

If a dealer has inventory $a_t^d$, her gain from trade from buying $q_i^c$ units at price per unit $p_i^c(a_t^d, q_i^c)$, versus not trading, can be decomposed as follows:

1. buy $q_i^c$ today at price per unit $p_i^c$, with a cashflow of $-p_i^c q_i^c$;
2. pay per-period holding costs of $f.(a_{t+u}^d + q_i^c)^2$ instead of $f.(a_t^d)^2$ every period until the next trade;
3. buy $q_{i+u}^d$ units instead of $q_{i+u}^c + q_i^c$ units, if the next trade is at time $t + u$ and is in the inter-dealer market;
4. buy $q_{i+u}^c$ units instead of $q_{i+u}^c + q_i^c$ units, if the next trade is at time $t + u$ and is with a customer;
5. receive $V_T$ per unit on $a_t^d + q_i^c$ units instead of $a_t^d$ units at maturity time $T$, if there is no trade before time $T$.

3 and 4 follow since the dealer’s inventory position after trading does not depend on their inventory when about to trade. The total gain from trade is the sum of these 5 pieces, multiplied by the probability they occur, and discounted back to time $t$. The probability that there is no trade each period is $(1 - \pi_i - \pi_c)$. The probability the next trade after time $t$ is in the interdealer market, and is at time $t + s$ is $(1 - \pi_i - \pi_c)^s - 1 \pi_i$. The probability the next trade after time $t$ is with a customer of type $j$, and is at time $t + s$ is $(1 - \pi_i - \pi_c)^s - 1 \pi_j$. The probability that there is no trade until maturity is $(1 - \pi_i - \pi_c)^{T-t-1}$. Putting these pieces together, I show in the appendix that her gain
from trade is given by:

\[
(1 + \delta R \gamma^S_t) \left( -p^c_t q^c_t \right) \left( \left( a^d_t + q^c_t \right)^2 - (a^d_t)^2 \right) T-t-1 \delta \left( 2a^d_t + q^c_t \right) \sum_{s=0}^{T-1} \left( \delta(1 - \pi_i - \pi_c) \right)^s E_t[f_{t+u+1}] \\

\begin{align*}
&\text{(1) buy } q^c_t \text{ units from time-} t \text{ customer} \\
&\text{(2) discounted holding costs on } a^d_t + q^c_t \text{ units instead of } a^d_t, \\
&\text{until next expected trade}
\end{align*}

+ \delta \sum_{u=0}^{T-t-2} \left( \delta(1 - \pi_i - \pi_c) \right)^s \left( \pi_i E_t[p_{t+u+1}] \left( -q^c_{t+u+1} - \left( -\left( q^d_t + q^c_t \right) \right) \right) \\
+ \sum_j \pi^c_j E_t \left[ -q^c_{j+u+1} p^c_{j+u+1} \left( a^d_t + q^c_t \right) + q^c_{j+u+1} \right] - \left( -\left( q^c_t + q^c_{j+u+1} \right) - p^c_{j+u+1} \left( a^d_t + q^c_t \right) \right) \right)
\text{trade at } t \text{ and } t + u + 1 \text{ only}

+ \left( \delta(1 - \pi_i - \pi_c) \right)^{T-t-1} \delta \left( -a^d_t - \left( -\left( q^d_t + q^c_t \right) \right) \right) E_t[\text{issuer}] \left( V^d_t \right)
\text{trade at } t + u + 1 \text{ only}

\begin{align*}
\text{(3) and (4): buy } q^c_{t+u+1} \text{ units from time-} (t + u + 1) \text{ counterparty, instead of } q^c_{t+u+1} + q^c_t. \text{ The} \\
\text{counterparty is a dealer with probability } \pi_i, \text{ and customer } j \text{ with probability } \pi^c_j
\end{align*}

\begin{align*}
\text{(5) cash in } a^d_t \text{ units instead of } (a^d_t + q^c_t) \text{ at maturity time } T, \text{ if dealer} \\
\text{doesn’t meet a counterparty in the previous } T - t - 1 \text{ rounds of trading}
\end{align*}

The sum of these 5 pieces is multiplied by 1 + \delta R \gamma^S_{t+1}. In the appendix I show that \gamma^S_t is constant. The ‘1’ represents the cash-flow at time \(t\). \delta R \gamma^S_{t+1} represents the opportunity cost of not investing that one unit of cash-flow in the cash account: it would accrue interest at rate \(R\), be worth \gamma^S_{t+1} next period, but be discounted back at rate \(\delta\).

2.4.3 Customer-dealer price

The customer-dealer price can be expressed as a weighted average of the dealer’s and customer’s valuations, where the weights are given by their relative bargaining power:

\[
p^c_t = \eta V^d_t + (1 - \eta) V^c_t
\]

where \(\eta\) is the customer’s relative bargaining power, \(1 - \eta\) the dealer’s, \(V^d_t\) the dealer’s valuation of the risky asset, and \(V^c_t\) the customer’s. The customer’s valuation is exogenous. The dealer’s valuation can be characterized as the customer-dealer price such that the dealer’s gain from trade is zero. Using the expression for the gain from trade,
the customer-dealer price per unit, $p_t^c$, can thus be written recursively:

$$
\eta \left( -\delta \left( 2a_t^d + q_t^c \right) \sum_{u=0}^{T-t-1} (\delta(1 - \pi_i - \pi_c))^u E_t[f_{t+u+1}] \right) $$

holding costs on $a_t^d + q_t^c$ units instead of $a_t^d$, until next trade

$$
+ \delta \sum_{u=0}^{T-t-2} (\delta(1 - \pi_i - \pi_c))^u \left( \pi_i E_t[p_t^c | p_t^c_{t+u+1}] + \sum_j \pi_j^c E_t \left[ \frac{r_j^c_{t+u+1} \pi_j^c_{t+u+1} \left( \pi_j^c - \delta \eta \sum_{j} \pi_j^c (1 - \eta_j) \right) / q_t^c \right] \right)$$

buy $q_{t+u+1}$ units from time-$(t + u + 1)$ counterparty, instead of $q_{t+u+1} + q_t^c$. The counterparty is a dealer with probability $\pi_i$, and customer $j$ with probability $\pi_j^c$

$$
+ \delta \left( \delta(1 - \pi_i - \pi_c) \right) \sum_{u=0}^{T-t-1} \delta E_t \left[ V_{t+u+1}^{\text{issuer}} \right] $$

buy $a_t^d$ units instead of $(a_t^d + q_t^c)$ from issuer at time $T$, if dealer doesn’t meet a counterparty in the previous $T - t - 1$ rounds of trading

$$
+ (1 - \eta) V_t^c
$$

From the expression above, it can be seen that there are 2 ways that inventory holding costs affect the price at which dealers trade with customers: firstly it affects the expected holding cost until the next trade; secondly it affects the price of the next trade if it is with a customer, as it affects the dealer’s bargaining position.

In the appendix I show that the customer-dealer price, whether the dealer is buying from, or selling to, a customer, can be written as:

$$
p_t^c = V_t^c + \eta q_t^c \sum_{u=0}^{T-t-1} \left( \delta (1 - \pi_i - \sum_j \pi_j^c (1 - \eta_j)) \right) E_t[f_{t+u+1}] \quad (3)
$$

If $q_t^c$ is positive, i.e. the dealer is buying from a customer, the price per unit increases in the customer’s bargaining power, dealers’ expected holding costs, and absolute trade size. If $q_t^c$ is negative, i.e. the dealer is selling to a customer, the price per unit decreases in the customer’s bargaining power, dealers’ expected holding costs, and absolute trade size. If the customer’s bargaining power is zero, then the dealer gets all the gain from trade, the customer none, and the customer-dealer price is equal to the customer’s valuation, $V_t^c$. The higher customers’ bargaining power is, the more the price moves away from their reservation value, and the greater their gain from trade. The higher dealers’ inventory holding costs are, the more the price moves away from the customer’s reservation value, and the greater the gain the dealer gives to the customer.
2.4.4 Customer-dealer quantity

In the appendix I show that the dealer’s post-trade inventory position, after buying \( q_t^c \) units of the risky asset from a customer, is given by:

\[
a_{t+1}^{d,c} = a_t^d + q_t^c = \frac{\delta E_t \left[ \sum_{u=0}^{T-t-2} \left( \delta \left( 1 - \pi_t + \sum_j \pi_j (1 - \eta_j) \right) \right)^u \left( \mu_t p_{t+u+1} + \sum_j \pi_j (1 - \eta_j) v_{t+u+1}^j \right) + \left( \delta \left( 1 - \pi_t + \sum_j \pi_j (1 - \eta_j) \right) \right)^{T-t-1} v_{t+1}^c \right] - \tilde{v}_t^c}{2 \delta E_t \left[ \sum_{u=0}^{T-t-1} \left( \delta \left( 1 - \pi_t + \sum_j \pi_j (1 - \eta_j) \right) \right)^u f_{t+u+1} \right]} \]

If all the customers have the same interest rates, then a feature inherited from Nash bargaining is that the quantity traded, and thus also the dealer’s post-trade position, do not depend on the dealer’s pre-trade inventory. If customers with lower bargaining power suffer worse interest rates, i.e. higher when borrowing and lower when saving, then they will trade in smaller size. In this way smaller customers would trade in smaller size, and then trade size and post-trade inventory would depend on pre-trade inventory.

From the customer’s gain from trade, we know all he cares about is the difference between his discounted valuation and the price at which he can currently trade with the dealer. Fixing the price per unit, he would want to trade an unbounded amount: buying as much as possible if his valuation is above the dealer’s quoted price, and selling otherwise. But the convexity of the dealer’s holding cost bounds the agreed quantity to a finite amount.

The dealer is not a myopic market-maker who targets zero inventory to minimize her inventory holding costs. Instead, fixing holding costs, she holds a larger inventory if future expected inter-dealer prices or customer valuations are high relative to the valuation of the customer with whom she is currently trading. Since customer dealer prices are a weighted average of customer and dealer valuations, this will mean future customer-dealer prices will be relatively high, and the dealer can profit from stocking up now and selling off later. Similarly, if the current customer has a high valuation, the dealer sells to him knowing she expects to be able to profit from unwinding the position at the next trade.

The larger the expected holding cost, the smaller the dealer’s post-trade inventory position. Pre-trade inventory of one trade is post-trade inventory of the previous one. So when holding costs are higher, both pre- and post-trade inventory will be lower. Since trade size is the difference between pre- and post-trade inventory, larger holding costs will be associated with smaller trade size.

Figure 9 shows the price-quantity regions where there are gains from trade, the equilibrium total price and quantity at which the dealer and customer will transact, and how the total gain from trade is split between them, given their relative bargaining power. The solid and dashed lines represent the indifference curves for the dealer and
customer, respectively. The customer’s indifference curve is linear due to him being risk neutral. The dealer’s indifference curve is quadratic due to her inventory holding costs being quadratic in their risky asset holding position. The region below (above) the solid (dashed) line represents the region where the dealer (customer) has a gain from trade. The intersection of these regions, where there are gains from trade to both dealer and customer, are shaded gray. The quantity is independent of bargaining power, and maximizes the total surplus. The black dot is plotted at the coordinates of the optimal quantity and total price (quantity × price/unit). The length of the arrows represent the gains from trade to the customer and dealer. The price is a weighted average of the customer’s and dealer’s valuations, with the weights given by their relative bargaining power. In this example the customer’s relative bargaining power, \( \eta \), is \( \frac{1}{3} \).

### 2.5 Inter-dealer trade

In the model there is no information asymmetry between dealers, and so risk-sharing drives trading in the inter-dealer market. Reiss and Werner (1998) show that this is the primary motivation for inter-dealer trade in London equity markets. In my model dealers trade in such a way that their inventory positions after inter-dealer trading are all equal, regardless of the distribution of their inventories before. Each dealer’s inventory position after buying \( q^i_t \) units of the risky asset in the inter-dealer market is given by:

\[
q^{d,i}_{t+1} = a^d_t + q^i_t = \frac{\delta E^d_t \left[ \sum_{s=0}^{T-t-2} \left( \delta \left( 1 - \pi_i - \sum_j \pi_j (1 - \eta_j) \right) \right)^s \left( \pi_i p^i_{t+s+1} + \sum_j \pi_j (1 - \eta_j) V^{d,j}_{t+s+1} \right) + \left( \delta \left( 1 - \pi_i - \sum_j \pi_j (1 - \eta_j) \right) \right)^{T-t-1} V^{d,issuer} \right] - p^i_t}{2 \delta E^d_t \left[ \sum_{s=0}^{T-t-1} \left( \delta \left( 1 - \pi_i - \sum_j \pi_j (1 - \eta_j) \right) \right)^s f_{t+s+1} \right]}
\]

This expression is very similar to the dealer’s optimal holding after trade with a customer, but the benchmark which determines the position is the current inter-dealer price instead of the current customer valuation. As with the customer-dealer trade, the larger the expected holding cost, the smaller the dealer’s post-trade inventory position, and the smaller trade size will be. The larger the next expected price relative to the current one, the larger the position the dealer will trade to.

Market-clearing in the inter-dealer market states that net trading amongst the dealers must be zero. This gives a recursion for the inter-dealer price, \( p^i_t \), which can be solved by backwards induction on trade time, starting from the last trade before the maturity of the asset. Let \( \tilde{a}^{d,i}_{t+1} \) denote the mean inventory of dealers who trade in the inter-dealer market at time \( t \).

\[
p^i_t = \delta E^d_t \left[ \sum_{s=0}^{T-t-2} \left( \delta \left( 1 - \pi_i - \sum_j \pi_j (1 - \eta_j) \right) \right)^s \left( \pi_i p^i_{t+s+1} + \sum_j \pi_j (1 - \eta_j) V^{d,j}_{t+s+1} \right) + \left( \delta \left( 1 - \pi_i - \sum_j \pi_j (1 - \eta_j) \right) \right)^{T-t-1} V^{d,issuer} \right] \]

\[
-2 \delta \tilde{a}^{d,i}_{t+1} E^d_t \left[ \sum_{s=0}^{T-t-1} \left( \delta \left( 1 - \pi_i - \sum_j \pi_j (1 - \eta_j) \right) \right)^s f_{t+s+1} \right]
\]

(4)
Given the dynamic nature of the dealer’s optimization problem, the current inter-dealer price is increasing in expected future inter-dealer prices, expected future customer valuations, and the expected value of the asset at maturity. The larger the average inventory of dealers trading in the inter-dealer market, \( \bar{a}_{d,i} \), is, the greater the supply of the asset, and the lower the price. This is particularly pronounced when dealers’ expected inventory holding costs are high.

### 2.6 Dealer Markup

I define the dealer’s per-unit markup when trading with a customer to be the difference between the customer-dealer price and inter-dealer price, specifically:

\[
\text{markup}_t \equiv \begin{cases} 
    p_i^c - p_i & \text{if dealer buys from a customer at time } t \\
    p_i - p_i^d & \text{if dealer sells to a customer at time } t 
\end{cases}
\]

If transactions only happen at quoted prices, then the bid-ask spread would be \( p_{c,\text{ask}}^t - p_{c,\text{bid}}^t \), where \( p_{c,\text{ask}}^t \) is the ask price demanded when the customer is selling to the dealer, and \( p_{c,\text{bid}}^t \) is the bid price when the customer is buying from her. So the markup splits the bid-ask spread into 2, possibly unequal, pieces. The benchmark price is not the bid-ask midpoint, but the inter-dealer price. We can think of it as the mark-to-market profit of the dealers. It is the analytical analogue to the "unique round trip" measure from Feldhutter (2012), which focuses on paired trades, when there are customer-dealer and inter-dealer trades very close in time. In my model only one of these types of trade can occur at each point in time, i.e. I am focusing on the unpaired trades, when dealers have to bear inventory holding costs for some non-trivial time. But I can compute the theoretical inter-dealer price at the time of a customer-dealer trade. In the appendix I show that the markup when the dealer is buying from a customer is given by:

\[
p_i^c - p_i^d = \\
\delta \left( \frac{2a_{d,i}^c + q_i^c}{\text{dealer’s holding cost gain per unit}} - \frac{(a_{d,i}^d + q_i^d)}{\text{average inventory of dealers in hypothetical inter-dealer market}} \right) + (1 - \eta)q_i^c \sum_{u=0}^{T-t-1} \left( \delta \left( 1 - \pi_i - \sum_j \pi_j (1 - \eta_j) \right) \right)^u E_t[f_{t+u+1}]
\]

Whether the dealer trades with a customer or dealer at time \( t \), at time \( t + 1 \) her optimization problem look the same, though she may enter time \( t + 1 \) with different cash and risky asset positions. So there is a lot of cancelation between customer-dealer and inter-dealer prices at time \( t \), and from the expression for the markup, we see that the residual which doesn’t cancel is comprised of 3 pieces. One piece is proportional to \( a_{d,i}^d + q_i^d \), and comes from the expression for the inter-dealer price. Since net trading in the inter-dealer market is zero, all dealers trade to the same position in the inter-dealer market, \( \bar{a}_{d,i}^d \equiv a_{d,i}^d + q_i^d \). So from equation (4) we see that the inter-dealer price, and therefore the markup, is decreasing in the average inventory of the dealers trading.
The other 2 pieces come from the expression for customer-dealer price. If there’s no trade in the interval \((t, t + u + 1)\), then the difference in time-(\(t + u + 1\)) holding costs of buying \(q_t^c\) units of the risky assets from the customer versus not trading is 
\[
(2a_t^{dc} + q_t^c)q_t^c f_{t+u+1} = ((a_t^{dc} + q_t^c)^2 - (a_t^{dc})^2) f_{t+u+1}.
\]
Dividing this by \(q_t^c\) gives the per period holding cost gain, and is \((2a_t^{dc} + q_t^c)f_{t+u+1}\). The remaining piece, proportional to \((1 - \eta)q_t^c\) shows that the markup is higher when customer’s bargaining power, \(\eta\) is lower, since the customer-dealer price is then less favorable to the customer, as we see from equation (3).

Since I will test my model using corporate bond data where I do not have individual dealer inventory, I assume that the inventory of dealers trading with customers is approximately the same as the inventory of dealers trading in the inter-dealer market. If the pre-trade inventory of the dealer trading with a customer is exactly equal to the average inventory of dealers in the hypothetical inter-dealer market, i.e. \(a_t^{dc} \approx a_t^{di} + q_t^j\), the markup when the dealer is buying or selling becomes:

\[
\text{markup}_t = \delta(2 - \eta)|q_t^c| \sum_{u=0}^{T-t-1} \left( \delta \left( 1 - \pi_i - \sum_j \pi_j^c(1 - \eta_j) \right) \right)^u E_t[f_{t+u+1}] \tag{5}
\]

This is because when the dealer is selling to a customer (a) the markup is the negative of the expression for buying; and (b) the quantity traded is negative, so \(|q_t^c| = -q_t^c\).

Here \(T\) is the maturity of the bond, \(\pi_i\) the probability each dealer will trade in the inter-dealer market each round, and \(\pi_j^c\) the probability that she will trade with customer \(j\) who has bargaining power \(\eta_j\).

To gain further intuition about the expression for the markup, consider what happens as the customer’s valuation varies. At each point in time the new state is revealed. If a dealer is trading with a customer, her post-trade inventory is given in section 2.4.4. It depends on current and past states, but not the dealer’s pre-trade inventory. If the dealer’s post-trade holding is above her pre-trade holding she will buy from the customer, otherwise she will sell. The lower the customer’s discounted valuation, \(\tilde{V}_t^c\), the higher the dealer’s post-trade inventory, and the more the dealer will buy: if she’s buying she will buy more, and if selling she will sell less. Lower customer valuations are also associated with lower customer-dealer prices. Therefore, fixing the inter-dealer price, lower customer valuations are associated with higher markups when the dealer is buying, and lower markups when selling. So if the dealer is buying: lower (higher) customer valuations are associated with (a) higher (lower) absolute trade size; and (b) higher (lower) markups. If the dealer is selling: lower (higher) customer valuations are associated with (a) lower (higher) absolute trade size; and (b) lower (higher) markups. In all cases, larger absolute trade size is associated with a higher markup, as we see from equation (5).
3 Data

3.1 TRACE

Since 2001 the Trade Reporting and Compliance Engine (TRACE) has been phased in to increase transparency of the trading costs of US corporate bonds. Financial Industry Regulatory Agency (FINRA) members are now required to report all their secondary OTC corporate bond transactions within 15 minutes. The data-set now covers over 99% of total trading volume, with the remainder small retail trades on the New York Stock Exchange. TRACE includes information on: trade date, trade time to the nearest second, transaction price, trade size, type of counterparties undertaking the trade (dealer-customer or dealer-dealer), and dealer trade direction. Only the sell side of interdealer trades is reported, to avoid duplication. Most researchers have used the TRACE database disseminated through Wharton Research Data Services (WRDS). This is the information released in real time to the market, and trade size is censored above $1 million for high yield bonds, and above $5 million for investment grade bonds, so that dealers’ trades are somewhat concealed, since revealing them could compromise their positions, and discourage them from providing liquidity. In real time the counterparty types and trade directions are available only for trades since November 3rd, 2008, except in a rare few trades. Using an enhanced database purchased from FINRA\textsuperscript{9}, I have this information from the start of TRACE to December 2010, as well as uncensored trade sizes. This information is released at a lag of 18 months, by which time the dealers are less sensitive about its release because their inventory positions will mostly have been unwound. As well as clearer examination of large trades, this extra information allows me to compute the change in aggregate inventory of the dealer market at each trade. Cumulating these changes over time yields a time-series of the level of dealer inventory, up to a constant representing the dealers’ initial inventory. I follow Dick-Nielsen (2009) to remove duplicate, canceled and corrected trades. In TRACE there are obvious typos in both the price and trade size. For a few bonds, consecutive prices sometimes differ by a factor of about 10 or 100, suggesting the wrong par value has been used for one of the prices. Moreover, trade size sometimes exceeds the amount outstanding for the issue. If one transaction size exceeds the amount outstanding, or there are 2 consecutive prices different by a factor of 5 or more, I delete the whole time series for that bond. I start my sample in October 2004 which is when the trades of all bonds were disseminated through TRACE. I only include bonds with at least 50 inter-dealer trades in my sample, in order to interpolate the inter-dealer price with some accuracy. I examine customer-dealer trades which are not

\textsuperscript{9}I thank the Salomon Center for the Study of Financial Institutions, and a NASDAQ grant for their generosity in providing funding for the data.
paired in time with an inter-dealer trade, since the paired trades have previously been examined in e.g. Zitzewitz (2010) and Feldhutter (2012), and my focus is on trades where dealers are exposed to inventory risk for some non-trivial period of time. My final sample consists of 6,954,148 unpaired customer-dealer trades, across 10,580 bonds issued by 1,045 issuers. Summary statistics are provided in Table 2.

3.2 Other Data Sources: Mergent FISD, CRSP, Federal Reserve Bank of New York

Bond issue characteristics and historical S&P credit ratings are taken from Mergent FISD. The database also contains voluntary reporting of US corporate bond trades by insurance companies, with a date-stamp, but no time-stamp. This allows me to see to what extent the inventories of insurance companies has changed over my sample period. I use equity returns from CRSP to compute conditional variances of equity returns for the major US corporate bond dealers and the insurance sector. The Federal Reserve Bank of New York’s website reports weekly dealer holdings and trade volume of US corporate bonds, aggregated across all primary government bond dealers and across all bonds.

4 Empirical Methodology

My model gives me the predicted expression (5) for the markup the dealer charges the customer. I make the following simplifying assumptions, in order to test it:

1. \( \delta \approx 1 \) but still \( \delta < 1 \);
2. \( T \gg t \), i.e. there are many possible trade times until the bond matures, and the time to maturity is approximately the same for all trades;
3. The expected holding cost is linear in the current holding cost: \( E_t[ f_{t+u+1} ] \approx \beta_u + \beta_{f,u} f_t \) for all \( u \), with \( \beta_u, \beta_{f,u} > 0 \).
4. \( \left( 1 - \pi_i - \sum_j \pi_j^c (1 - \eta_j) \right)^u \approx c_u + \beta_{i,u} \pi_i + \sum_j \beta_{c,j,u} \pi_j^c \) for all \( u \), where \( \beta_{i,u}, \beta_{c,j,u} < 0 \) for all \( u \) and \( j \).

Then from equation (5) I show in the appendix that the markup per unit at time \( t \) for a dealer buying \( q_t^c \) units of the asset from a customer with bargaining power \( \eta \), for both dealer purchases and sales, is approximately:

\[
\text{markup}_t \approx (2 - \eta) |q_t^c| (\beta_0 + \beta_\pi \pi + \beta_{f,f} f_t + \beta_{\pi,f} \pi f_t)
\]

where \( \pi \equiv \pi_i + \sum_j \pi_j^c \) is the probability of meeting a customer or dealer each period, with \( \beta_0, \beta_f > 0 \) and \( \beta_\pi, \beta_{\pi,f} < 0 \).
4.1 Bargaining power

In the empirical literature, larger trades by customers are generally associated with smaller markups, except for really large trades where the pattern is reversed. The leading proposed explanation is that larger trades are done by larger customers who have greater bargaining power, and customers with greater bargaining power have lower markups. So while bargaining power is not directly observable, we can proxy it by trade size. To test this, I split the trades into \( N = 11 \) trade size buckets. I assume that average customer bargaining power is increasing as you move from the smallest trade size bucket up to the largest. Since customers with low bargaining power are unlikely to be able to fund the largest trades, the largest bucket is likely only customers with the most bargaining power. Smaller trades could be funded by any customer type, with low or high bargaining power, but the average bargaining power is likely to be lower. Within those buckets, I can test for the marginal effect of trade size on the markup. Thus the customer’s bargaining power in the trade of bond \( b \) trade at time \( t \) is written as \( \eta^b_t = \sum_{j=1}^N \eta_j 1_{t}^{b,j} \), where \( 1_{t}^{b,j} \) is a dummy variable taking the value 1 if the trade in bond \( b \) at time \( t \) is in size bucket \( j \), and zero otherwise, and \( \eta_j \) is the bargaining power of customer type \( j \).

4.2 Buys versus sells

The model predicts that the markup’s regression coefficients on trade size is the same for dealer purchases as for dealer sales. I impose this restriction.

4.3 Paired versus unpaired customer-dealer trades

In the empirical literature, paired trades have previously been examined in e.g. Zitzewitz (2010) and Feldhutter (2012). My model is focused on trades where dealers are exposed to inventory risk for some non-trivial period of time. So I test the model only for unpaired trades.

4.4 Level of dealer inventory

The recursion for the inter-dealer price in the model suggests that it depends on the level of dealer inventory. But only changes in inventory from secondary market trading can be computed using TRACE. I assume that the only changes to dealer inventory for a specific bond issue other than secondary trading are from (1) primary issuance

\[ \text{In the context of the model, a paired trade can be thought of as a random endowment of cash to the pairing dealer, but no change in their inventory, while the other dealer pays cash and gains inventory. The dealers’ cash reserve does not affect prices or quantities traded. Though the pairing dealer’s inventory is unaffected, aggregate dealer inventory changes by the size of the trade.} \]
which occurs prior to secondary trading; (2) exercised call or put options, which are embedded in the bonds; (3) maturity of the issue. Option exercise may be on all or only a fraction of the issue. Since I do not have data on which agents’ bond holdings are called or putted, I take the conservative approach that dealers’ inventory changes whenever any option is exercised. Option exercise dates can be determined by dates when there is a change in the amount outstanding, which is available from Mergent FISD. I define the change in aggregate dealer inventory since time r:

\[
\hat{a}_{t}^{D,b,r} = a_{t}^{D,b} - a_{r}^{D,b} = \sum_{u=r}^{t-1} q_{u}^{c,b}
\]

### 4.5 Aggregate versus individual dealer inventory

The model predicts that it is the inventory of the individual dealer trading that affects the markup. But currently I only have data on inventory of the aggregate dealer market\(^{11}\). So I proxy for dealer-specific inventory by the mean inventory of the aggregate dealer market at that time. This assumption is not exactly true, as for instance I am effectively assuming that between 2 consecutive customer-dealer trades the first dealer shares their extra inventory with the others without a trade being recorded. But the model predicts that all dealers would hold the same inventory after trading with each other. The more similar dealers are, and the more they specialize (i.e. each issue only traded by a few dealers) the more reasonable the assumption that dealers all hold the same inventory when trading with customers is, as a first approximation. So while there will be heterogeneity in dealer inventories, my specification assumes their inventories are at least positively correlated.

To convert the aggregate dealer inventory into the mean across all dealers, I divide it by the number of primary government bond dealers, which I assume is approximately the number of major corporate bond dealers.

### 4.6 Commission

Prices in TRACE sometimes do, and sometimes do not, have commission included. Though I do not know how large the commission is, I do know whether prices include it or not. I compute the median commission by trade size bin, for buys and sells separately. For those trades without commission in the price, I subtract the median buy commission when the dealer is buying, and add the median sell commission when she is selling, since my model makes predictions about the total price paid or received

\(^{11}\)I have approval from FINRA that I will have access to anonymous dealer ID’s shortly
by the dealer, including commission.

4.7 Dealers’ expected holding cost

The expected holding cost has bond-specific and aggregate components. I proxy this variable by the product of proxies for each. The per-calendar-period holding cost in my model is $f^b(s_t). (a_{t \mid b}^d)^2$, where $a_{t \mid b}^d$ is the dealer’s risky asset holdings of bond $b$ in units of the asset. The holding cost until the next trade is $\tau_b^f f^b(s_t). (a_{t \mid b}^d)^2$, where $\tau_b^f$ is the time until the dealer next trades bond $b$. I write $f^b(s_t) = \alpha(s_t) \hat{f}^b(s_t)\), where $\alpha(s_t)$ measures the aggregate component of the inventory holding cost, and $\hat{f}^b(s_t)$ measures the bond-specific component. The model assumes that $f(s_t)$ must be strictly positive in all states of the world.

One proxy for the bond-specific holding cost, $\hat{f}^b(s_t)$, is the bond’s credit rating. Using ratings from Standard & Poor’s available on Mergent FISD, I partition bonds into investment grade (BBB- and above) and high yield (below BBB-). To estimate the expected holding cost until dealer $d$’s next trade, I multiply this by the expected waiting time to dealer $d$’s next trade. In the following section I describe how I estimate this waiting time.

Another measure of the holding cost comes from the interpretation of it as Value-at-Risk constraint, or a penalty function for variance of the dealer’s risky asset holding. Assume $\hat{f}^b(s_t). (a_{t \mid b}^d)^2$ is the per-calendar-period dollar variance of their risky asset holdings, measured in dollars squared. Since the holding cost is $\alpha(s_t) f^b(s_t). (a_{t \mid b}^d)^2$, where $a_{t \mid b}^d$ is dealer $d$’s risky asset holdings of bond $b$ in units of the asset, $\hat{f}^b$ is the conditional variance of the next traded price of the bond in dollars per unit, estimated using all transaction prices for that bond for its whole time series. One way to forecast the variance is to use a GARCH model for the price variance. Let $Var_t[p_{t+1}^b]$ denote the conditional variance of the price of the bond next period. Since $Var_t[p_{t+1}^b] = Var_t[p_{t+1}^b - p_t^b]$, and this differenced series is likely to be better behaved, I fit a GARCH(1,1) model to price differences. I forecast variance from observed trades in trade time, rather than calendar time: this gives an estimate of $\tau_b^f f^b(s_t)$ where $\tau_b^f$ is the time to the next trade after $t$, since calendar price variance is proportional to the expected time to the next trade, at least approximately if price changes are i.i.d. Since customer-dealer prices depend on bargaining power, volatile customer-dealer prices may be due to a higher proportion of trades being with customers who have low bargaining power, which is something that dealers like rather than dislike. It is variation in fundamental value that dealers dislike. I am therefore implicitly assuming that the distribution of customer bargaining power is the same across bonds. Since some customer-dealer trades include commission, but others do not, I assume missing commissions are the median across all bonds and all time for that trade size bin, with separate estimates for the
buy commission and sell commission.

Another option is to use only inter-dealer prices when estimating price variance, which is much less contaminated by bargaining power, and commission is very rare. I multiply this variance by the fraction of trades that are inter-dealer for that bond, to convert the variance of the next interdealer price to an estimate of the variance of the next price of any type, inter-dealer or customer-dealer. I am implicitly assuming that customer-dealer and inter-dealer trades occur randomly, and that price changes are i.i.d.

In the model, \( f_b(s_t) \) captures not only the effective risk aversion of dealers in an absolute sense, but, since customers are risk neutral, also their effective risk aversion relative to customers. One proxy for the customers’ risk appetite relative to the dealers is the ratio of the variance of dealer equity returns to the variance of equity returns of their customers, denoted \( \frac{\sigma_d^2}{\sigma_c^2} \). We know the identity of the dealers from the corporate bond trading platform MarketAxess, listed in Table 1. The customers in the US corporate bond market are mostly insurance companies, pension funds, mutual funds, and hedge funds. To compute an index of equity returns for customers, I use the insurance sector. For dealers to be selling corporate bonds in large quantities from October 2007, other agents must have been buying. Aggregating the transaction data from Mergent FISD for trades by insurance companies, it can be seen that they were net buyers at that time. Figure 8 shows that my measure of insurance companies’ effective risk appetite relative to dealers was persistently high during the financial crisis.

### 4.8 Time to next trade

Figure 7 shows intra-day volume from TRACE, aggregated across all bonds. It is clear that volume is not uniform within the day. Though there is no official market open or close time, there is very little trading volume outside 7am - 6pm, Monday to Friday. Volume increases until around 11am, before a drop-off at lunch-time. There is a spike in customer-dealer volume just after 3pm, when Barclays announces the level of its corporate bond index. The shortest average waiting time is mid-morning during the week, the longest in the final hours of Friday afternoon. So for each run within a bond, I regress the log of waiting times from a customer-dealer trade to the next trade, be it inter-dealer or customer-dealer, on the hour of the week and month. By using the log, I can ensure my forecast of the wait time, exponentiated from the forecast of the log wait time, is positive, and the different orders of magnitude the wait time can take are better handled. The empirical literature on corporate bonds shows that the lower the notional outstanding of the issue, the less liquid it is, as search costs become higher. By splitting a bond’s time series into runs where the notional outstanding is constant, I account for the different trading frequencies as the notional outstanding changes.
Even though I exclude paired trades from the regression analysis, I include them when computing trade frequency, as I want a measure of the time to the next trade, paired or unpaired, as that is the when the dealer can next unwind their inventory position.

4.9 Estimating the inter-dealer price and markup

The empirical markup is the difference between an actual customer-dealer price, and the interpolated inter-dealer price at the time of that customer-dealer trade. The recursion for the inter-dealer price from the model suggests dynamics which I use to interpolate the inter-dealer price:

\[
p_{i,b}^t = c_b + \phi_b p_{i,b}^{t-1} + \beta_b a_{t}^{D,b} f_t^b + \beta_i (\text{index}_t - \text{index}_{t-1}) + \varepsilon_t^b
\]

where \(a_{t}^{D,b}\) denotes dealers’ aggregate inventory of bond \(b\) at time \(t\), \(f_t^b = Var_t[p_t^{b+1}]\) and \((\text{index}_t - \text{index}_{t-1})\) is the change in the volume-weighted daily price index. This last term helps mitigate the staleness in the last inter-dealer price. The time index is trade time. I use all inter-dealer trade prices and aggregate dealer inventory before and after time \(t\). Since I don’t know the level of aggregate dealer inventory, I estimate:

\[
p_{i,b}^t = \left( c_b + \beta_b a_{t}^{D,b} f_t^b \right) + \phi_b p_{i,b}^{t-1} + \beta_b a_{t}^{D,b,r} f_t^b + \beta_i (\text{index}_t - \text{index}_{t-1}) + \varepsilon_t^b
\]

where \(a_{t}^{D,b,r}\) is the increase in inventory since the start of the last run. This gives estimates of the interdealer price using the realized distribution of aggregate dealer inventory.

Using the estimated interdealer price, \(\tilde{p}_i^t\), I estimate the markup as \(\tilde{p}_i^t - p_c^t\) when the dealer is buying from the customer, and \(p_c^t - \tilde{p}_i^t\) when the dealer is selling to the customer, where \(p_c^t\) is the observed customer-dealer price including estimated commission. I Winsorize the markup at the 1% level. Summary statistics for the markup are provided in Table 2.

4.10 Markup regression specification

Having estimated the interdealer price at the time of customer-dealer trades, I can estimate the markup, and run the following regression specification, for buys and sells together.

\[
\text{markup}_{t,r}^{b,\eta} = (\tilde{p}_i^t - p_c^t) \times 1_t^{b,\text{buy/sell}} = |q_t^{b,\eta}| (\beta_\eta + \beta_{\eta,f} f_t^b + \beta_{\eta,\pi} \pi_t^b + \beta_{\eta,f,\pi} \pi_t^b f_t^b + \varepsilon_t^{b,\eta})
\]
for bond $b$ at time $t$ where $r$ was the time of the last change in amount outstanding or the start of the sample, whichever is more recent. $|q_t^b|$ is the par value of bond $b$ traded between the dealer and the customer at time $t$. $1_t^{b,\text{buy/sell}}$ takes the value 1 if the dealer is buying bond $b$ from a customer, and -1 if selling. $1_t^{b,\eta}$ takes the value 1 if the trade size corresponds to bargaining power $\eta$, and 0 otherwise. $f_t^b$ is the product of aggregate and bond-specific components: The aggregate component, denoted $\sigma^2_d \sigma^2_c$, is the ratio of the conditional variances from GARCH(1,1) models of the equity returns of the US dealers listed in Table 1 and the insurance sector. It is a proxy for the relative financing constraints that the dealers are suffering from versus the customers in the corporate bond market. For the bond-specific component I split the sample into investment grade and high yield, and $\tilde{f}_t^b$ represents a dummy variable for each.

5 Testable Hypotheses

The model predicts that, fixing bargaining power:

(H1) The markup is increasing in the product of trade size and inventory holding costs, for both buys and sells, for all bargaining power bins, i.e. $\beta_{t}^{\eta} > 0$ for all $\eta$.

(H2) The higher the customer’s bargaining power is, the less the markup is increasing in that product, for both buys and sells, i.e. $\beta_{t}^{\eta_2} < \beta_{t}^{\eta_1}$ for all $\eta_2 > \eta_1$.

(H3) The higher the dealers’ inventory holding costs, the less they buy and the more they sell.

In my model, trading frequency is exogenous. But if trade frequency was endogenous, a natural prediction would be that when inventory holding costs are high, dealers would more actively seek counterparties with whom to trade to reduce the time they are stuck with undesirable inventory positions.

(H4) Dealers expect their wait time to trade to be shorter when inventory holding costs are high.

6 Results

6.1 Aggregate Data

Figure 2 shows the time series of US corporate bond inventory held by primary government bond dealers, aggregated across all dealers and across all bonds, measured in market value, par value and % par of the market. By all 3 measures, there is a large decrease in inventory starting from October 2007, which coincided with a time when some dealing banks (Citi, Merrill Lynch, UBS) were experiencing large write-downs.
of their assets. A second decrease in inventory, not pictured, began in June 2011, during the sovereign debt crisis. The data is from the Federal Reserve Bank of New York’s website. Figure 3 shows the time series of the median markup, computed from TRACE, as a percentage of par, when the dealer is buying from a customer (circle) and when selling to a customer (cross). In the financial crisis of 2007-09, as dealers sought to reduce their inventory, they priced bonds low to encourage customers to buy from them, and discourage customers to sell to them. We see this effect by higher markups when dealers were buying, and lower when they were selling. Pre- and post-crisis, there is little asymmetry in the median markup. This pattern is corroborated by comparing average prices when dealers are buying from, and selling to, customers, which reveals similar asymmetry, showing that this is not a result driven by a mis-estimation of the inter-dealer price. Figure 4 shows the time series of the mean trade size, computed from TRACE, when the dealer is buying from a customer (circle), when selling to a customer (diagonal cross), and for interdealer trades (cross). During the financial crisis of 2007-09 average trade size decreased for all 3 categories of trade, and has remained lower since. This is consistent with Reiss and Werner (1998) who show that dealers used small and medium-sized customers to unwind inventory imbalances in London equity markets, as they could generate larger average spreads and commission. Once inventory levels stabilized, higher holding costs would lead to smaller inventory position deviations, and thus trade size remained low. Figure 5 shows the mean markup, computed from TRACE, as a percentage of par, split into 4 trade size buckets: retail trades, which are classified in the previous empirical literature as having a par value of less than $100,000; odd lot trades with a par value of at least $100,000 but less than $1 million; institutional trades of at least $1 million, which is a round lot in the US corporate bond market, but less than $10 million; and trades of at least $10 million which I denote as mega. The exact size of these mega trades has not been available to most researchers, as in real time trade size is censored at $5 million for investment grade bonds and $1 million for high yield bonds. Figure 6 shows the median markup at finer intervals. Figure 7 shows intraday volume, by minute, computed from TRACE, aggregated across all bonds. It is clear that volume is not uniform within the day. Though there is no official market open or close time, there is very little trading volume outside 7am - 6pm, Monday to Friday. Volume increases until around 11am, before a drop-off at lunch-time. There is a spike in customer-dealer volume just after 3pm, when Barclays announces the level of its daily corporate bond index.
6.2 Bond-level data

6.2.1 Dealer Markup: Hypotheses H1 and H2

Table 5 shows regression results testing my model’s implied relationship between the dealer’s markup and trade size. Point estimates have been multiplied by 10 million for readability, and standard errors are clustered at the issuer level. The dealer’s holding cost is proxied by the product of aggregate and bond-specific components. The aggregate component is proxied by the ratio of the conditional variance of equity returns of the major US corporate bond dealers over the conditional variance of equity returns of the insurance sector, and denoted $\sigma_d^2/\sigma_c^2$. Insurance companies constitute an important fraction of customers in this market, and on aggregate were net buyers of inventory in the financial crisis when dealers were selling. This proxy can be thought of as the financing constraint of dealers relative to customers. The bond-specific part of the holding cost is the bond’s S&P credit rating, split into investment grade and high yield.

H1 is equivalent to (a) the coefficients on the interaction of trade size and holding cost being positive for all customer bargaining power bins; and (b) those coefficients being more positive for high yield than investment grade bonds. We indeed see that when the dealers are relatively more constrained than customers (higher $\sigma_d^2/\sigma_c^2$), there is a more positive relationship between markup and trade size for every bargaining power bin, except the smallest for trades of investment grade bonds less than $50,000$ par. This anomaly could be explained by a small fixed cost of trading, or by a flight to quality: in my model I assume that the holding cost is always negative, but there is a benefit to dealers holding investment grade bonds, to satisfy mandatory capital requirements. This effect does not pervade larger trades. So it seems that outside the smaller trades, it is bargaining power, rather than a fixed cost, that causes the negative relationship between markup and trade size seen in Figure 5 for all but the largest trades. The coefficients for bins 3 - 11 for investment grade, and 2 - 11 for high yield are all strongly significant: 17 out of 19 of those coefficients are significant at the 1% level, the other 2 at the 5% level. For 8 out of 11 bargaining power bins, the magnitude of the coefficient on the interaction of trade size and holding costs is greater for high yield bonds than investment grade bonds. So there is strong empirical support for H1.

H2 is equivalent to the coefficients on the interaction of trade size and holding cost being monotonically decreasing in customer bargaining power. Other than the smallest bargaining power bin, which may be affected by a fixed cost to trading, this is strikingly true for both high yield and investment grade bonds.
6.3 Signed Trade Size: Hypothesis H3

Table 7 shows regression results testing a reduced form of the model’s implied relationship between signed trade size, \( q_c \), and dealers’ inventory holding costs. Signed trade size is defined as the positive par amount traded when the dealer is buying from a customer, and the negative of this amount when selling to a customer. I focus first on the results for the unpaired trades. The regression coefficient on \( \sigma_d^2 / \sigma_c^2 \), a proxy for the dealers’ trading constraint relative to customers, is significantly negative at the 5% level. Customers, rather than dealers, were effectively “leaning against the wind” when dealers were relatively more constrained. But the regression coefficient on the dealers’ constraint for investment grade bonds is positive, significant at the 1% level, and larger in magnitude than the negative coefficient on the dealers’ constraint on its own. So when dealers’ financing constraints relative to customers are higher, dealers sell more high yield bond inventory and sell less investment grade bond inventory. The final column shows that these results are robust when I also include the expected time to the next trade, its interaction with the dealers’ relative constraint and investment grade status, and its interaction with their product. As we might expect, these effects are not there, however, for the unpaired trades where the pairing dealer does not experience a change in their inventory holding costs. Putting the paired and unpaired trades together, the results are no longer statistically significant, but the directions are the same for the unpaired trades.

6.4 Expected Waiting Time to Next Trade: Hypothesis H4

Table 8 shows the regression of the log of the forecast of expected wait time til the dealers next get to trade on dealers’ inventory holding costs. The forecast of the expected wait time is computed in 2 steps, to reduce computation time. Firstly I regress the log of observed wait time on the hour of the day and the month, for each run of a bond separately. A run is a series of consecutive trades of a bond, without a change in the amount outstanding, such as from a partial call of the issue. We have seen how intraday volume varies systematically with the hour of the day. Volume also varies over time. Having removed the intraday and monthly effects, I regress the residual on the date to get a more precise forecast of the time to the next trade. I then regress the forecast on the aggregate and bond-specific measures for the dealers’ holding costs, and their interaction. The regression coefficient on the dealers’ relative constraint is insignificantly different from zero, but is significantly negatively if the bond is investment grade. So while it is true that dealers actively speed up their trading for investment grade bonds as their constraints bind tighter than customers’,
this is surprisingly not true for high yield bonds. This could possibly be because it is easier to find a counterparty with whom to trade investment grade bonds with, than it is to find one with whom to trade high yield bonds, in crisis times.

7 Alternative Hypotheses

7.1 Fixed costs of trading

In my model and empirical results, bargaining power rationalizes the generally negative relationship between the markup the dealer charges and trade size. An alternative explanation is that there is a fixed cost of trading for the dealer, which, when split across a large trade, is smaller per unit of par than for a small one. However Figure 6 shows that for very small trades, between $1,000 and $20,000 par, there is a positive relationship between the markup and trade size, for both buys and sells. Since the markup of these very small trades would be most affected by fixed costs of trading, it is unlikely that this is the main driver of the relationship. For trades of up to $1,000 par, there is a negative relationship between the markup and trade size, suggesting there may be a fixed cost, but that it is small.

7.2 Asymmetric Information

Figure 6 shows that for larger trades, in excess of $500,000 par, there is a positive relationship between the markup and trade size. In my model, this is rationalized by dealer’s inventory holding costs: they require compensation for a large change in their inventory holding, whether to buy or sell. An alternative explanation is that these larger trades are done by more informed customers, and the dealers require compensation for the possibility that the customer is more informed than they are, as in Glosten and Milgrom (1985) or Easley and O’Hara (1992). However, in OTC markets, it is less clear that customers have the same or better information than dealers who observe trade flow, which is the key assumption driving a positive bid-ask spread in these models. Though TRACE-eligible trades are disseminated at a delay mandated to be less than 15 minutes, only censored trade size is released that quickly. This gives dealers an informational advantage over customers. Also, since bonds are less information-sensitive, and have higher trading costs, than equities, it seems more likely that private information would be exploited in the equity market.

Though there is no asymmetric information between dealers and customers in my model, I can control for it empirically. In Table 6 I include issuer fixed effects in the markup regression. Issuers often issue multiple bonds. If there is asymmetric information it is likely to be at the issuer level: customers may have better information than
dealers about the bond issuer’s value, but are less likely to have different information about a particular bond. By adding issuer fixed effects I analyze only the variation of the markup within an issuer. As in Table 5, the higher the effective risk aversion of the dealers, the higher the markup, for all customer bargaining powers but the smallest, where a small fixed cost of trading may explain the discrepancy. This is still more pronounced for customers with less bargaining power. Conditional on bargaining power, the effect is also more pronounced for larger trades. The results are strongly statistically significant. However, the effect is no longer more pronounced for high yield bonds over investment grade ones across all customer bargaining powers, only for 3 out of 11 bargaining power bins. Given there is unlikely to be much variability in whether issues for the same issuer are investment grade or not, it may be that there is not enough power to distinguish this effect if I include issuer fixed effects.

8 Conclusion

My structural model for trading in OTC search markets allows me to show that dealers’ inventory holding costs are an important determinant of liquidity provision in the US corporate bond market. It rationalizes the most prominent stylized facts for the market, with dealers’ inventory levels, trade size, and transaction cost asymmetry (buys versus sells) all correlated with dealers’ time-varying aversion to retaining inventory for significant periods of time. In addition, it correctly predicts that, controlling for customer bargaining power, when holding costs are high enough, the generally negative relationship between dealer markups and trade size can be reversed, and this effect is stronger for both riskier bonds and trades associated with customers with lower bargaining power. Without a structural model these relationships would be harder to identify and interpret.

My model can also be tested in other over-the-counter markets, beyond that for US corporate bonds. In particular, FINRA has constructed a TRACE-like database of transactions for structured product markets: Asset-Backed Securities, Collateralized Mortgage Obligations, Mortgage-Backed Securities and To be Announced securities. The database has yet to be made public, though some research groups have had early access to it, e.g. Jankowitsch, Friewald, and Subrahmanyam (2012). This database also includes anonymous identifiers for the identity of the dealer, which is not available in TRACE currently, though will be available to me shortly. Once released, this will allow alternative and sharper tests of my model, which predicts transaction prices and liquidity depend on the inventory of the individual dealer trading.

Understanding the effect of inventory holding costs is particularly relevant in the current regulatory environment. The Volcker Rule, part of the Dodd-Frank Act, will
restrict commercial banks from proprietary trading. Many of the major corporate bond dealers are subsidiaries of banks, and will thus be affected. The Financial Stability Oversight Council has proposed that proprietary trading may be identified by the regulators as monitoring (a) the ratio of daily turnover of assets to the banks’ inventory; and (b) the ratio of customer-initiated trades to dealer-initiated trades, relative to their past trading behavior, other trading desks within the industry, and hedge funds. Banks which hold positions which are too large for too long are more likely to be cited for speculation, and would find it harder to argue these positions were purely for market-making. So going forward a component of dealers’ aversion to holding inventory is likely to be the implementation of this and other regulations. Though these may be effective in limiting opportunities for the dealing arms of banks to speculate, they also restrict their ability to make a market in infrequently traded corporate bonds and other illiquid markets. Understanding this mechanism, and given the feedback from secondary market liquidity to financing costs in the primary market, informs the policy debate.

The sum of all the markups in my sample amounts to $2.2 billion per year, which can be thought of as the mark-to-market profit of the dealers. The growth in market share of MarketAxess, an electronic auction market for US corporate bonds\textsuperscript{12}, and the development of Blackrock’s electronic bond-trading platform, highlight how the market may change in the future to circumvent these trading costs. Using a richer data-set than previous studies, I can see that total volume in the corporate bond market, released at a lag of 18 months, is 67\% higher than in the censored data, which is released publicly in real time. Dealers dislike transparency, because it reduces their informational advantage in the market, compromises their inventory positions, and reduces their profitability. Bessembinder and Maxwell (2008) document that there has been a transfer of dealer resources out of the US corporate bond market and into other asset markets, both in terms of capital and personnel. Goldstein, Hotchkiss, Sirri (2006) showed that the net effect was still that transaction costs generally fell after the introduction of TRACE. The optimal level of transparency in an OTC market, trading off better information for customers with the incentive for dealers to profit from making a market, is a topic for future research. Having access to different level of data, including censored and uncensored trade size, aggregate and individual dealer data, and trade direction, will help me to answer this.

\textsuperscript{12}See Hendershott T., and A. Madhavan (2012)
References


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[36] Lyons, R., A simultaneous trade model of the foreignexchange hot potato *Journal of International Economics* 42, 3-4, 275-298


# 9 Tables and Figures

<table>
<thead>
<tr>
<th>Primary Government Bond Dealers</th>
<th>Major MarketAxess Dealers</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNP Paribas Securities Corp.</td>
<td>BNP Paribas Securities Corp.</td>
</tr>
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<td>Barclays Capital Inc.</td>
<td>Barclays Capital Inc.</td>
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<td>CIBC World Markets</td>
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Table 1: **Identity of major bond dealers.** This table lists the primary government bond dealers, and the major dealers who trade on MarketAxess, an electronic limit order market for US corporate bonds. There is a large overlap in the two lists, so it seems reasonable that the aggregate inventory of primary government bond dealers displayed in Figure 2 is likely representative of the wider dealer market.
<table>
<thead>
<tr>
<th>S&amp;P Credit rating</th>
<th>Frequency</th>
<th>Markup to buy</th>
<th>Markup to sell</th>
<th>Amount outstanding ($ par)</th>
<th>Mean time to next trade (days)</th>
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<tr>
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<td>980,861</td>
<td>0.743</td>
<td>0.818</td>
<td>800,905,335</td>
<td>0.579</td>
</tr>
<tr>
<td>BB-</td>
<td>520,876</td>
<td>0.455</td>
<td>0.545</td>
<td>1,105,815,544</td>
<td>0.487</td>
</tr>
<tr>
<td>B+</td>
<td>535,914</td>
<td>0.478</td>
<td>0.571</td>
<td>679,178,347</td>
<td>0.515</td>
</tr>
<tr>
<td>B</td>
<td>824,165</td>
<td>0.217</td>
<td>0.860</td>
<td>830,877,631</td>
<td>0.601</td>
</tr>
<tr>
<td>B-</td>
<td>617,891</td>
<td>0.567</td>
<td>0.587</td>
<td>878,801,070</td>
<td>0.483</td>
</tr>
<tr>
<td>CCC+</td>
<td>400,248</td>
<td>0.578</td>
<td>0.633</td>
<td>1,160,108,979</td>
<td>0.457</td>
</tr>
<tr>
<td>CCC</td>
<td>363,736</td>
<td>1.128</td>
<td>0.182</td>
<td>846,822,105</td>
<td>0.570</td>
</tr>
<tr>
<td>CCC-</td>
<td>69,768</td>
<td>0.590</td>
<td>0.542</td>
<td>438,585,894</td>
<td>0.539</td>
</tr>
<tr>
<td>CC</td>
<td>93,265</td>
<td>0.443</td>
<td>0.914</td>
<td>1,212,547,318</td>
<td>0.535</td>
</tr>
<tr>
<td>C</td>
<td>14,932</td>
<td>0.564</td>
<td>0.408</td>
<td>7,526,214,886</td>
<td>0.635</td>
</tr>
<tr>
<td>D</td>
<td>41,588</td>
<td>0.486</td>
<td>0.489</td>
<td>781,118,693</td>
<td>0.270</td>
</tr>
</tbody>
</table>

Table 2: Summary statistics. This table shows summary statistics for the sample from TRACE, after applying filters for canceled and corrected trades, unreliable prices and quantities, and bonds with fewer than 50 inter-dealer trades. The period covers October 1st, 2004 - December 31st, 2010. “Markup to buy” denotes the mean markup when the dealer is buying from a customer, “markup to sell” when they are selling to a customer. “Amount outstanding” is the mean $ par amount of the notional outstanding for each issue. “Mean time to next trade” is the mean time until the next trade of any type (inter-dealer or customer-dealer), averaged across bonds. Frequency is the number of trades, including inter-dealer and customer-dealer. Customer-dealer trades include both those paired in time with inter-dealer trades, and unpaired ones.
<table>
<thead>
<tr>
<th>Trade Size</th>
<th>(Oct-Dec) 2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy 1</td>
<td>59,425</td>
<td>359,765</td>
<td>381,298</td>
<td>394,668</td>
<td>491,515</td>
<td>1,108,015</td>
<td>1,643,450</td>
<td>4,438,136</td>
</tr>
<tr>
<td>2</td>
<td>10,648</td>
<td>55,429</td>
<td>61,113</td>
<td>67,242</td>
<td>93,462</td>
<td>195,521</td>
<td>268,075</td>
<td>751,490</td>
</tr>
<tr>
<td>3</td>
<td>15,154</td>
<td>80,942</td>
<td>97,689</td>
<td>115,711</td>
<td>134,011</td>
<td>214,998</td>
<td>244,830</td>
<td>903,335</td>
</tr>
<tr>
<td>4</td>
<td>2,399</td>
<td>10,874</td>
<td>15,705</td>
<td>20,718</td>
<td>20,405</td>
<td>29,584</td>
<td>32,517</td>
<td>132,202</td>
</tr>
<tr>
<td>Total</td>
<td>87,626</td>
<td>507,010</td>
<td>555,805</td>
<td>598,339</td>
<td>739,393</td>
<td>1,548,118</td>
<td>2,188,872</td>
<td>6,225,163</td>
</tr>
<tr>
<td>Sell 1</td>
<td>64,755</td>
<td>361,619</td>
<td>455,855</td>
<td>539,799</td>
<td>1,050,178</td>
<td>2,611,822</td>
<td>3,050,045</td>
<td>8,134,073</td>
</tr>
<tr>
<td>2</td>
<td>12,274</td>
<td>59,068</td>
<td>76,291</td>
<td>84,576</td>
<td>139,081</td>
<td>307,181</td>
<td>407,110</td>
<td>1,085,581</td>
</tr>
<tr>
<td>3</td>
<td>18,147</td>
<td>87,506</td>
<td>109,723</td>
<td>118,851</td>
<td>126,089</td>
<td>200,721</td>
<td>265,252</td>
<td>926,289</td>
</tr>
<tr>
<td>4</td>
<td>1,124</td>
<td>3,551</td>
<td>6,577</td>
<td>7,834</td>
<td>5,569</td>
<td>10,177</td>
<td>11,424</td>
<td>46,256</td>
</tr>
<tr>
<td>Total</td>
<td>96,300</td>
<td>511,744</td>
<td>648,446</td>
<td>751,060</td>
<td>1,320,917</td>
<td>3,129,901</td>
<td>3,733,831</td>
<td>10,192,199</td>
</tr>
<tr>
<td>Inter-dealer 1</td>
<td>88,227</td>
<td>369,348</td>
<td>440,985</td>
<td>573,348</td>
<td>1,236,429</td>
<td>2,541,084</td>
<td>2,252,270</td>
<td>7,501,691</td>
</tr>
<tr>
<td>2</td>
<td>12,174</td>
<td>58,908</td>
<td>77,965</td>
<td>92,701</td>
<td>146,486</td>
<td>286,866</td>
<td>313,983</td>
<td>989,083</td>
</tr>
<tr>
<td>3</td>
<td>16,472</td>
<td>85,546</td>
<td>102,793</td>
<td>122,723</td>
<td>143,722</td>
<td>233,617</td>
<td>260,331</td>
<td>965,204</td>
</tr>
<tr>
<td>4</td>
<td>2,386</td>
<td>10,754</td>
<td>15,622</td>
<td>20,420</td>
<td>18,924</td>
<td>25,705</td>
<td>30,917</td>
<td>124,728</td>
</tr>
<tr>
<td>Total</td>
<td>119,259</td>
<td>524,556</td>
<td>637,365</td>
<td>809,192</td>
<td>1,545,561</td>
<td>3,087,272</td>
<td>2,857,501</td>
<td>9,580,706</td>
</tr>
</tbody>
</table>

Total 303,185 1,543,310 1,841,616 2,158,591 3,605,871 7,765,291 8,780,204 25,998,068

Table 3: **Number of trades.** This table shows the number of trades in my sample, by year and by trade size. *Buy* denotes customer-dealer trades when the dealer is buying from the customer. *Sell* denotes a sale from a dealer to a customer. '1' denotes trades less than $100,000 par; '2' denotes trades between $100,000 and $1 million par; '3' denotes trades between $1 million and $10 million par; '2' denotes trades in excess of $10 million par.
<table>
<thead>
<tr>
<th>Trade Size</th>
<th>(Oct-Dec)</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1,346,466,748</td>
<td>8,075,414,043</td>
<td>8,466,774,672</td>
<td>8,632,757,711</td>
<td>10,672,349,047</td>
<td>25,646,562,531</td>
<td>38,349,249,080</td>
<td>101,189,573,832</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3,686,554,361</td>
<td>19,324,268,274</td>
<td>21,665,577,649</td>
<td>24,107,524,773</td>
<td>33,417,552,452</td>
<td>69,420,749,249</td>
<td>93,271,567,017</td>
<td>264,893,793,775</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>48,945,557,211</td>
<td>249,212,474,625</td>
<td>311,611,062,678</td>
<td>377,925,909,700</td>
<td>411,982,461,613</td>
<td>634,119,307,687</td>
<td>724,311,713,006</td>
<td>2,758,108,486,520</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>36,888,614,000</td>
<td>159,560,301,000</td>
<td>235,819,932,707</td>
<td>312,376,067,853</td>
<td>320,884,793,286</td>
<td>495,737,041,281</td>
<td>558,285,986,511</td>
<td>2,119,552,736,638</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>90,867,192,320</td>
<td>436,172,457,942</td>
<td>577,563,347,706</td>
<td>723,042,260,037</td>
<td>776,957,156,398</td>
<td>1,224,923,660,748</td>
<td>1,414,218,515,614</td>
<td>5,243,744,590,765</td>
<td></td>
</tr>
<tr>
<td>Sell</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1,551,513,516</td>
<td>8,702,104,216</td>
<td>10,739,362,925</td>
<td>13,100,445,363</td>
<td>25,807,641,420</td>
<td>63,183,436,334</td>
<td>75,025,992,433</td>
<td>198,110,496,206</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>56,464,819,324</td>
<td>250,823,147,646</td>
<td>325,888,467,151</td>
<td>343,835,872,746</td>
<td>319,931,975,437</td>
<td>500,351,018,071</td>
<td>670,559,474,151</td>
<td>2,467,864,774,526</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15,218,042,000</td>
<td>52,420,374,000</td>
<td>90,143,130,024</td>
<td>104,887,865,384</td>
<td>82,725,042,805</td>
<td>197,592,425,700</td>
<td>180,040,077,771</td>
<td>723,026,957,684</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>77,383,573,656</td>
<td>331,783,509,669</td>
<td>452,750,128,221</td>
<td>476,194,978,902</td>
<td>486,261,138,416</td>
<td>1,057,396,873,300</td>
<td>3,747,277,440,617</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Trade volume. This table shows the volume of trades in my sample, measured in dollars of par value. Buy denotes customer-dealer trades when the dealer is buying from the customer. Sell denotes a sale from a dealer to a customer. '1' denotes trades less than $100,000 par; '2' denotes trades between $100,000 and $1 million par; '3' denotes trades between $1 million and $10 million par; '4' denotes trades in excess of $10 million par.
Table 5: Determinants of markup. This table tests my model’s expression for the markup the dealer implicitly charges the customer when they trade, given by equation (5). Results for investment grade bonds are denoted IG, and high yield HY. $|q_t^c|$ is the magnitude of trade size between a dealer and a customer at time $t$. $\pi_t = \frac{1}{E_t[time to next trade]}$ is a time-$t$ estimate of the future frequency of trading of that bond. $\frac{\sigma^2_d}{\sigma^2_c}$ is the ratio of the conditional variances from GARCH(1,1) models of the equity returns of the US dealers listed in Table 1 and the insurance sector. It is a proxy for the relative financing constraints that the dealers are suffering from versus the customers in the corporate bond market. [Min. trade size, Max. trade size) is the interval of trade sizes in $par for each bargaining power bucket, where I assume that trade size is positively correlated with customer bargaining power. $N$ denotes the number of trades in each trade size bin. Point estimates for all but the intercept have been multiplied by 10 million. Standard errors are clustered at the issuer level. $t$-statistics are in parentheses. I denote the statistical significance at a 1%, 5%, and 10% level with ***, ** and *, respectively.
### Table 6: Determinants of markup: controlling for asymmetric information.

This table tests my model’s expression for the markup the dealer implicitly charges the customer when they trade, given by equation (5). Results for investment grade bonds are denoted IG, and high yield HY. $|q_t|$ is the magnitude of trade size between a dealer and a customer at time $t$. $\pi_t = E_t[\text{time to next trade}]$ is a time-$t$ estimate of the future frequency of trading of that bond. $\sigma_d^2/\sigma_c^2$ is the ratio of the conditional variances from GARCH(1,1) models of the equity returns of the US dealers listed in Table 1 and the insurance sector. It is a proxy for the relative financing constraints that the dealers are suffering from versus the customers in the corporate bond market. [Min. trade size, Max. trade size] is the interval of trade sizes in $\text{par}$ for each bargaining power bucket, where I assume that trade size is positively correlated with customer bargaining power. $N$ denotes the number of trades in each trade size bin. Point estimates for all but the intercept have been multiplied by 10 million. $t$-statistics are in parentheses. I denote the statistical significance at a 1%, 5%, and 10% level with ***, ** and *, respectively. Issuer fixed effects are included to control for asymmetric information.

<table>
<thead>
<tr>
<th>Trade size bin</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. trade size</td>
<td>50,000</td>
<td>100,000</td>
<td>325,000</td>
<td>550,000</td>
<td>775,000</td>
<td>1,000,000</td>
<td>3,000,000</td>
<td>5,000,000</td>
<td>7,000,000</td>
<td>9,000,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. trade size</td>
<td>50,000</td>
<td>100,000</td>
<td>325,000</td>
<td>550,000</td>
<td>775,000</td>
<td>1,000,000</td>
<td>3,000,000</td>
<td>5,000,000</td>
<td>7,000,000</td>
<td>9,000,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| | IG: $|q_t| \times \pi_t$ | HY: $|q_t| \times \sigma_d^2/\sigma_c^2 \times \pi_t$ |
|---------------|-----------------|----------------------------------|
| $|q_t|$ | -38.615*** | -28.868*** |
| | (-49.36) | (-22.39) |
| $|q_t| \times \pi_t$ | -4,849.200*** | -898.900*** |
| | (-72.76) | (-14.86) |
| IG: IG: $|q_t| \times \sigma_d^2/\sigma_c^2$ | 0.665*** | 0.858*** |
| | (8.14) | (16.51) |
| HY: HY: $|q_t| \times \sigma_d^2/\sigma_c^2 \times \pi_t$ | -0.120 | 0.261*** |
| | (-0.69) | (2.37) |
| IG: IG: $|q_t| \times \sigma_d^2/\sigma_c^2 \times \pi_t$ | -226.910*** | -31.985*** |
| | (-18.66) | (-3.94) |
| HY: HY: $|q_t| \times \sigma_d^2/\sigma_c^2 \times \pi_t$ | 557.032*** | 65.927*** |
| | (26.28) | (2.88) |
| Issuer fixed effects | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y | Y |
| $N$ | 3,507,047 | 552,877 | 807,544 | 296,629 | 121,233 | 60,934 | 787,345 | 242,424 | 280,675 | 56,844 | 204,910 |
Table 7: **Signed trade size.** This table tests my model’s expression for the sign and magnitude of trades between dealers and customers: $q_t$. It shows a regression of signed trade size (trade size for a dealer purchase, negative trade size for a dealer sale) on proxies for dealer’s inventory holding costs. The “Dealers’ relative constraint” is defined as $\sigma^2_d / \sigma^2_c$, the ratio of the conditional variances from GARCH(1,1) models of the equity returns of the US dealers listed in Table 1 ($\sigma^2_d$), and the insurance sector ($\sigma^2_c$). It is a proxy for the relative financing constraints that the dealers are suffering from, versus the customers, in the corporate bond market. $\pi_t$ denotes the frequency of trading. The specification with the frequency of trading only makes sense for unpaired trades, since for paired trades the time to the next trade is zero, after accounting for a reporting lag of a few minutes. Standard errors are clustered at the issuer level. $t$-statistics are in parentheses. I denote the statistical significance at a 1%, 5%, and 10% level with ***, ** and *, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Unpaired Trades</th>
<th>Paired</th>
<th>All Trades</th>
<th>Unpaired Trades</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Investment Grade</strong></td>
<td>-4,593.6***</td>
<td>-964.4</td>
<td>-2,419.5**</td>
<td>-4,899.5**</td>
</tr>
<tr>
<td></td>
<td>(-2.23)</td>
<td>(-0.70)</td>
<td>(-2.10)</td>
<td>(-2.35)</td>
</tr>
<tr>
<td><strong>High Yield</strong></td>
<td>-12,631.2***</td>
<td>7,342.0**</td>
<td>-3,147.9</td>
<td>-13,057.5***</td>
</tr>
<tr>
<td></td>
<td>(-3.95)</td>
<td>(2.23)</td>
<td>(-1.48)</td>
<td>(-3.98)</td>
</tr>
<tr>
<td><strong>Dealers’ relative constraint</strong></td>
<td>-840.9***</td>
<td>205.8</td>
<td>-360.1</td>
<td>-868.3***</td>
</tr>
<tr>
<td></td>
<td>(-2.81)</td>
<td>(0.32)</td>
<td>(-1.10)</td>
<td>(-2.84)</td>
</tr>
<tr>
<td><strong>Dealers’ relative constraint</strong></td>
<td>1,055.8***</td>
<td>-578.5</td>
<td>180.3</td>
<td>1,164.9***</td>
</tr>
<tr>
<td><strong>× Investment Grade</strong></td>
<td>(2.88)</td>
<td>(-0.88)</td>
<td>(0.52)</td>
<td>(3.06)</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td></td>
<td></td>
<td></td>
<td>175,399.6*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(1.77)</td>
</tr>
<tr>
<td>$\pi_t \times$ Dealers’ relative constraint</td>
<td>20,943.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(1.05)</td>
</tr>
<tr>
<td>$\pi_t \times$ Investment Grade</td>
<td>-40,099.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-0.32)</td>
</tr>
<tr>
<td>$\pi_t \times$ Dealers’ relative constraint</td>
<td>-46,588.6*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>× Investment Grade</strong></td>
<td></td>
<td></td>
<td></td>
<td>(-1.88)</td>
</tr>
<tr>
<td>$N$</td>
<td>6,954,148</td>
<td>8,850,406</td>
<td>15,804,554</td>
<td>6,953,766</td>
</tr>
</tbody>
</table>
Table 8: **Forecast of the time to the next trade.** This table tests how the logarithm of my forecast of the time to the next trade depends on dealers’ inventory holding costs. The “Dealers’ relative constraint” is defined as $\frac{\sigma_d^2}{\sigma_c^2}$, the ratio of the conditional variances from GARCH(1,1) models of the equity returns of the US dealers listed in Table 1 ($\sigma_d^2$), and the insurance sector ($\sigma_c^2$). It is a proxy for the relative financing constraints that the dealers are suffering from, versus the customers, in the corporate bond market. Standard errors are clustered at the issuer level. $t$-statistics are in parentheses. I denote the statistical significance at a 1%, 5%, and 10% level with ***, ** and *, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Std. Error</th>
<th>$t$-stat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment Grade</td>
<td>7.7266***</td>
<td>(95.65)</td>
<td></td>
</tr>
<tr>
<td>High Yield</td>
<td>7.8088***</td>
<td>(79.01)</td>
<td></td>
</tr>
<tr>
<td>Dealers’ relative constraint</td>
<td>0.0012</td>
<td>(0.54)</td>
<td></td>
</tr>
<tr>
<td>Dealers’ relative constraint $\times$ Investment Grade</td>
<td>-0.0223***</td>
<td>(-9.23)</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>6,954,148</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 2: Aggregate US corporate bond holdings by Primary Dealers. This figure shows the time series of aggregate US corporate bond inventory held by primary government bond dealers, measured in market value, par value and % par of the market. By all 3 measures, there is a large decrease in inventory starting from October 2007, which coincided with a time when some dealing banks (Citi, Merrill Lynch, UBS) were experiencing large write-downs of their assets. A second decrease in inventory began in June 2011, during the sovereign debt crisis. Data is from The Federal Reserve Bank of New York’s website, which reports weekly holdings and trade volume, aggregated across all primary government bond dealers and across all bonds. Table 1 lists the names of primary government bond dealers, and the major dealers who trade on MarketAxess, an electronic limit order market for US corporate bonds. There is a large overlap in the two lists, so it seems reasonable that the aggregate inventory of primary government bond dealers is likely representative of the wider dealer market.
Figure 3: markup over time. This plot shows the time series of the median "markup", as a percentage of par, when the dealer is buying from a customer (circle) and when selling to a customer (cross). The markup is defined as the difference between a customer-dealer price and the inter-dealer price. The bid-ask spread is then the sum of the markup to buy and the markup to sell. In the financial crisis of 2007-09, as dealers sought to reduce their inventory, they priced bonds low to encourage customers to buy from them, and discourage them to sell to them. We see this effect by higher markups when dealers were buying, and lower when they were selling.
Figure 4: **Mean trade size over time.** This plot shows the time series of the mean trade size when the dealer is buying from a customer (B, blue circle), when selling to a customer (S, green cross), and for interdealer trades (D, red cross). During the financial crisis of 2007-09 average trade size has decreased for all 3 categories of trade.
Figure 5: Markup by trade size group. This figure shows the mean markup, as a percentage of par, split into 4 trade size buckets: retail trades, which are classified in the previous empirical literature as having a par value of less than $100,000; odd lot trades with a par value of at least $100,000 but less than $1 million; institutional trades of at least $1 million, which is a round lot in the US corporate bond market, but less than $10 million; and trades of at least $10 million which I denote as mega.
Figure 6: Markup by trade size. This figure shows the mean markup computed from my TRACE sample, as a percentage of par, by trade size which is plotted on a log scale. There is generally a negative relationship between the markup and trade size, whether the dealer is buying from, or selling to, a customer. But for small trades between $1,000 and $20,000 par, and large trades in excess of $500,000 par, the markup is increasing in trade size.
Figure 7: **Intraday volume.** This plot shows intraday volume by minute. There is a spike in customer volume just after 3pm, when Barclays announces the level of its daily index. This plot shows a "normal" day, but this spike is especially pronounced at the end of the month.
Figure 8: **Insurance sector’s risk appetite relative to dealers.** This figure shows the ratio of conditional variances of the major US corporate bond dealers to the insurance sector, as a measure of the insurance sector’s risk appetite relative to the dealers. This ratio is very high (much higher than one) during the financial crisis.
Figure 9: Nash bargaining in dealer-customer trade. The solid red and dashed blue lines represent the indifference curves for the dealer and customer, respectively. The region below (above) the solid (dashed) line represents the region where the dealer (customer) has a gain from trade. The intersection of these regions, where there are gains from trade to both dealer and customer, are shaded gray. The dot is plotted at the coordinates of the optimal quantity and total price (quantity × price/unit). The length of the arrows represent the gains from trade to the customer and dealer. Here the customer’s relative bargaining power, $\eta$ is $\frac{1}{3}$. 
Appendix A  Proofs

Appendix A.1 Value function conjecture

Assume the value function takes the following form, at each time $t$:

$$V_{t+1}(d_{t+1}, a_{t+1}, s_{t+1}) = \gamma_{t+1} d_{t+1} + \gamma_{t+1} a_{t+1} + (\gamma_{t+1} a_{t+1})^2 + \gamma_{t+1} s_{t+1}$$

We proceed by backwards induction on trade time, starting at the time of maturity and re-issuance of the risky asset.

Appendix A.1.1 Maturity and Re-issuance at time $T$

At time $T$ the asset matures, and all dealers receive payoff $V_{T_{\text{issuer}}}$ from the issuer for each unit they hold. The asset is reissued, and dealers can buy unlimited quantity $q_{T_{\text{issuer}}}$ at price per unit $p_{T_{\text{issuer}}}$, which depends only on the state at time $T$. She arrives at time $T$ with $a_{T}$ units of the risky asset, and $d_{T}$ in cash which includes interest accumulated for the period $(T-1,T]$, which is paid at $T$. At time $T$ her value function is given by:

$$V_{T_{T}}(d_{T}, a_{T}, s_{T}) = \max_{q_{T_{\text{issuer}}}} \left\{ \begin{array}{l} s_{T}(1-R^{-1}) - f_{T}(a_{T}^d)^2 + a_{T}^d V_{T_{T}} \text{receive interest holding cost } \text{maturity} \text{re-issuance} \\ - p_{T_{\text{issuer}}} q_{T_{\text{issuer}}} \end{array} \right\} + \delta E_T \left[ V_{T+1} \left( R \left( \frac{d_{T}^d}{T} - \frac{d_{T}^d(1-R^{-1}) - f_{T-1}(a_{T}^d)^2 + a_{T}^d V_{T_{T}}}{T_{T_{\text{issuer}}} q_{T_{\text{issuer}}}} \right), q_{T_{\text{issuer}}}, s_{T+1} \right) \right]$$

The first order condition with respect to $q_{T_{\text{issuer}}}$ yields the dealer’s holding at time $T$:

$$a_{T+1}^d = q_{T_{\text{issuer}}} \frac{p_{T_{\text{issuer}}} (1 + (\delta R) E_T[s_{T+1}^a]) - \delta E_T [\gamma_{T+1}^a]}{2 \delta E_T [\gamma_{T+1}^{aa}]}$$
The second order condition with respect to $q^{\text{issuer}}_T$ confirms this is a maximum as long as $E_T [\gamma^{aa}_{T+1}] < 0$. I will later show that it is. Plugging this optimal quantity back in to the value function:

$$V^d_T (s^d_T, a^d_T, s_T) = \left(1 - R^{-1} + \delta E_T [\gamma^a_T] \right) s^d_T + V^\text{issuer}_T \left(1 + \delta R E_T [\gamma^a_T] \right) a^d_T - \left(1 + \delta R E_T [\gamma^a_T] \right) f_T (a^d_T)^2$$

$$+ q^{\text{issuer}}_T \left( \delta q^{\text{issuer}}_T E_T [\gamma^{aa}_{T+1}] + \delta E_T [\gamma^{aa}_{T+1}] - \left(1 + \delta R E_T [\gamma^{a}_{T+1}] \right) p^{\text{issuer}}_T \right) + \delta E_T [\gamma_{T+1}]$$

$$\gamma_T = \delta \left(E_T [\gamma_{T+1}] - (a^d_{T+1})^2 E_T [\gamma^{aa}_{T+1}] \right)$$

**Appendix A.1.2 Inter-dealer trade**

At each trade time $t = ..., -1, 0, 1, ...$ all dealers can access the inter-dealer market, buying quantity $q^i_t$ at price $p^i_t$. The inter-dealer market is assumed to be perfectly competitive, and so the dealer takes the price as given. She arrives at time $t$ with $a^d_t$ units of the risky asset, and $s^d_t$ in cash which includes interest accumulated for the period $(t-1, t]$, which is paid at $t$. Notice that $R (s^d_t - s^d_t (1 - R^{-1})) = s^d_t$, so $s^d_t (1 - R^{-1})$ is the interest paid for this period. At time $t$ her value function is given by:

$$V^d_t (s^d_t, a^d_t, s_t) = \max_{q^i_t} \left\{ s^d_t (1 - R^{-1}) - f_t (a^d)^2 - p^i_t q^i_t + \delta E^d_t \left[ R \left( s^d_t - s^d_t (1 - R^{-1}) - f_t (a^d)^2 - p^i_t q^i_t \right), a^d_t + q^i_t, s_{t+1} \right] \right\}$$

The first order condition with respect to $q^i_t$ yields the dealer’s holding after inter-dealer trading at time $t$:

$$a^d_{t+1} = a^d_t + q^i_t = \frac{p^i_t \left( 1 + \delta R E^d_t [\gamma^a_T] \right) - \delta E^d_t [\gamma^{aa}_{T+1}]}{2 \delta E^d_t [\gamma^{aa}_{T+1}]}$$

(1)

The second order condition with respect to $q^i_t$ confirms this is a maximum as long as $E^d_t [\gamma^{aa}_{T+1}] < 0$. I will later show that it is. Plugging this optimal quantity back in to the value function:
\[ V_t^d(S_t^d, a_t^d, s_t, q_t^c) \]

\[ = \frac{S_t^d(1 - R)}{1 - \delta} - f_t(a_t^d)^2 - p_t^d(a_{t+1}^d - a_t^d) + \delta E_t^d \left( R \left( S_t^d - (1 - R) - f_t(a_t^d)^2 - p_t^d(a_{t+1}^d - a_t^d) \right) \right) \]

\[ \left( 1 - \frac{R}{1 - \delta} \right) \left( 1 + \delta E_t^d \gamma_{t+1}^d \right) S_t^d + p_t^d \left( 1 + \delta RE_t^d \gamma_{t+1}^s \right) a_t^d - \left( 1 + \delta RE_t^d \gamma_{t+1}^s \right) f_t(a_t^d)^2 + a_{t+1}^d \left( \delta a_{t+1}^d E_t^d \gamma_{t+1}^a + \delta E_t^d \gamma_{t+1}^a - \left( 1 + \delta RE_t^d \gamma_{t+1}^s \right) p_t^d \right) + \delta E_t^d \gamma_{t+1}^a \]

### Appendix A.1.3 Customer-dealer trade

The dealer trades with customer \( j \) at each trade time with probability \( \pi_j^c \). We can think of this probability as the joint probability of the dealer and customer meeting, and there being a gain from trade.

**The dealer’s gain from trade**

Suppose a dealer meets a customer at time \( t \). The dealer’s value function \(^{13}\) if she buys optimal quantity \( q_t^c \) at average price per unit \( p_t^c \) is given by:

\[ \tilde{V}_t^d \left( S_t^d, a_t^d, s_t, q_t^c \right) = \frac{S_t^d(1 - R)}{1 - \delta} - f_t(a_t^d)^2 - p_t^d q_t^c + \delta E_t^d \left( V_{t+1}^d \left( R \left( S_t^d - (1 - R) - f_t(a_t^d)^2 - p_t^d q_t^c \right) a_t^d + q_t^c, s_{t+1} \right) \right) \]

The gain from the dealer buying quantity \( q_t^c \) at price \( p_t^c \) (as opposed to not trading, i.e. \( q_t^c = 0 \)) is given by:

\[ \text{Dealer’s gain from trade} = q_t^c E_t^d \left[ \delta \gamma_{t+1}^a + \delta \left( 2a_t^d + q_t^c \right) \right] \gamma_{t+1}^a - \left( 1 + \delta R \gamma_{t+1}^s \right) p_t^c \]

**The customer’s gain from trade**

There are a continuum of customers who live for one unit of time, and trade only at birth. Consider one customer with an endowment of (cash, risky

\(^{13}\)I denote it \( \tilde{V}(s, a, s, q) \) since \( V \) is a function of \( (s, a, s) \), but \( \tilde{V}(s, a, s, q) \equiv V(s, a) \) if \( q \) is optimal.
asset units) = (S^c_t, a^c_t) entering trade at time t. He is risk-neutral, with utility over post-trade wealth W given by \( u(W) = E[W] = S^c + a^c V^c \), where \( V^c \) is the expected payoff to the customer of one unit of the risky asset. His interest rate is \( R^c_t \). His gain from selling \( q^c_t \) units of the risky asset to the dealer at an average price \( p^c_t \) per unit is given by:

\[
\text{Customer's gain from trade} = \delta^c \left( R^c_t \left( \frac{S^c_t + p^c_t q^c_t}{2} \right) + (q^c_t - q^c_t) V^c_t \right) - \delta^c \left( R^c S^c_t + q^c_t V^c_t \right)
\]

where \( \tilde{V}^c_t = V^c_t / R^c_t \). Note that the customer’s gain from trade doesn’t depend on his pre-trade holdings in either cash or the risky asset. It also doesn’t depend on \( \pi \) directly since the customer only trades once, and so when trading with a dealer, he doesn’t consider their next meeting.

For both the customer and dealer to have strictly positive gains from trade, we need \( p^c_t \) and \( q^c_t \) to exist such that:

\[
q^c_t \tilde{V}^c_t < q^c_t p^c_t < \frac{\delta q^c_t E^d_t \left( \gamma^a_{t+1} \left( \frac{\gamma^a_{t+1}}{\gamma^a_{t+1}} + 2a^d_{t+1} + q^c_t \right) \right)}{1 + \delta R E_t \left( \gamma^a_{t+1} \right)}
\]

The RHS is a negative quadratic in \( q^c_t \), with roots at \( q^c_t = 0 \) and \( q^c_t = -E_t \left( \gamma^a_{t+1} + 2a^d_{t+1} \right) \), and whose slope at \( q^c_t = 0 \) is \( \frac{\delta E_t [\gamma^a_{t+1} + 2a^d_{t+1} + 2]}{1 + \delta R E_t [\gamma^a_{t+1}]} \). The LHS is linear in \( q^c_t \), with positive slope \( \tilde{V}^c_t \), also with its root at \( q^c_t = 0 \). Thus there are gains from trade as long as \( \frac{\delta E_t [\gamma^a_{t+1} + 2a^d_{t+1} + 2]}{1 + \delta R E_t [\gamma^a_{t+1}]} \neq \tilde{V}^c_t \). Figure 9 shows the regions where there are gains from trade, the equilibrium total price and quantity at which the dealer and customer will transact, and how the total gain from trade is split between them, given their relative bargaining power. We now derive the price and quantity at which they trade.

The bargaining game between the dealer and the customer

Let \( \eta \in [0, 1] \) and \( 1 - \eta \) be the relative bargaining power of the customer and dealer, respectively. They bargain over both price and quantity using Nash bargaining:

\[
\max_{p^c_t, q^c_t} q^c_t \left( p^c_t - \tilde{V}^c_t \right)^\eta \left( E^d_t \left[ \delta \gamma^a_{t+1} + \delta \left( 2a^d_t + q^c_t \right) \gamma^a_{t+1} - \left( 1 + \delta R \gamma^a_{t+1} \right) p^c_t \right] \right)^{1-\eta}
\]
The FOC wrt $q_t^c$ yields:

$$p_t^c = \frac{\delta E_t^d \left[ \gamma^a_{t+1} + (2a_t^d + (2 - \eta)q_t^c) \gamma^{aa}_{t+1} \right]}{1 + \delta R E_t^d \left[ \gamma^s_{t+1} \right]}$$  \hspace{1cm} (2)$$

The FOC wrt $p_t^c$ yields:

$$\Rightarrow p_t^c = \eta \left( \frac{\delta E_t^d \left[ \gamma^a_{t+1} + (2a_t^d + q_t^c) \gamma^{aa}_{t+1} \right]}{1 + \delta R E_t^d \left[ \gamma^s_{t+1} \right]} \right) + (1 - \eta) \tilde{V}_t^c$$  \hspace{1cm} (3)$$

Notice that $p_t^c = \eta V_t^d + (1 - \eta) \tilde{V}_t^c$, where $V_t^d$ is the price when the dealer’s gain is zero. Combining the FOCs (2) and (3), yields the dealer’s position after she trades with a customer at time $t$:

$$a_{t+1}^{d,c} \equiv a_t^d + q_t^c = \frac{(1 + \delta R E_t^d \left[ \gamma^s_{t+1} \right]) \tilde{V}_t^c - \delta E_t^d \left[ \gamma^a_{t+1} \right]}{2\delta E_t^d \left[ \gamma^{aa}_{t+1} \right]}$$  \hspace{1cm} (4)$$

Notice that the dealer’s new position after trading with the customer does not depend on her inventory before.

The dealer’s value function at time $t$, before she knows if she will trade with a customer, is given by:

$$V_t^{d,cj} \left( s_t^d, a_t^d, s_t \right) = \left( 1 - R^{-1} + \delta E_t^d \left[ \gamma^s_{t} \right] \right) s_t^d + \left( (1 + (\delta R) E_t^d \left[ \gamma^a_{t} \right]) \left( (1 - \eta_j) V_t^{c,j} + \delta \eta_j E_t^d \left[ \gamma^a_{t+1} \right] \right) a_t^d \right)$$

$$+ \left( \delta \eta_j E_t^d \left[ \gamma^a_{t+1} \right] - \left( 1 + (\delta R) E_t^d \left[ \gamma^a_{t+1} \right] \right) f_t \right) (a_t^d)^2 + E_t^d \left[ \gamma_{t+1} + a_t^d + (1 - \eta_j) \left( 1 + \delta R E_t^d \left[ \gamma^s_{t+1} \right] \right) V_t^{c,j} \right]$$

**Appendix A.2 Periods of no trading**

In periods when no dealer trades, neither with a customer, another dealer, or the issuer, she still gets paid interest, and pays the holding cost. Her value function is given by:

$$V_t^d \left( s_t^d, a_t^d, s_t \right) = \left( 1 - R^{-1} + \delta E_t^d \left[ \gamma^s_{t} \right] \right) s_t^d + \delta E_t^d \left[ \gamma^a_{t} \right] a_t^d + \left( \delta E_t^d \left[ \gamma^a_{t+1} \right] - f_t \left( 1 + \delta R E_t^d \left[ \gamma^s_{t+1} \right] \right) \right) (a_t^d)^2 + E_t^d \left[ \gamma_{t+1} \right]$$
Appendix A.3  Recursions

Appendix A.3.1  Maturity / reissuance

\[ \gamma^s_T = 1 - R^{-1} + \delta E_T [\gamma^s_{T+1}] \]
\[ \gamma^a_T = \left( 1 + \delta R E_T [\gamma^s_{T+1}] \right) V^\text{issuer}_T \]
\[ \gamma^{aa}_T = - \left( 1 + \delta R E_T [\gamma^s_{T+1}] \right) f_T \]
\[ \gamma_T = \delta \left( E_T [\gamma_{T+1}] - (q^\text{issuer}_T)^2 E_T [\gamma^{aa}_{T+1}] \right) \]

Appendix A.3.2  Inter-dealer trade

\[ \gamma^{s,i}_T = 1 - R^{-1} + \delta E_t^d [\gamma^s_{t+1}] \]
\[ \gamma^{a,i}_T = \left( 1 + \delta R E_t^d [\gamma^s_{t+1}] \right) p_{it} \]
\[ \gamma^{aa,i}_T = - \left( 1 + \delta R E_t^d [\gamma^s_{t+1}] \right) f_{it} \]
\[ \gamma^i_T = \delta \left( E_t^d [\gamma_{t+1}] - (a^d_{t+1})^2 E_t^d [\gamma^{aa}_{t+1}] \right) \]

Appendix A.3.3  Customer-dealer trade

\[ \gamma^{s,j}_t = 1 - R^{-1} + \delta E_t^d [\gamma^s_{t+1}] \]
\[ \gamma^{a,j}_t = \eta_j \delta E_t^d [\gamma^a_{t+1}] + \left( 1 + \delta R E_t^d [\gamma^s_{t+1}] \right) (1 - \eta_j) V^c_{t+1} \]
\[ \gamma^{aa,j}_t = \delta \eta_j E_t^d [\gamma^{aa}_{t+1}] - \left( 1 + \delta R E_t^d [\gamma^s_{t+1}] \right) f_t \]
\[ \gamma^j_t = E_t^d \left[ \delta \gamma_{t+1} - a^c_{t+1} \left( \delta \eta_j \left( \gamma^a_{t+1} + \gamma^{aa}_{t+1} a^c_{t+1} \right) + (1 - \eta_j) \left( 1 + \delta R E_t^d [\gamma^s_{t+1}] \right) V^c_{t+1} \right) \right] \]
Appendix A.3.4  No trading

\[ \gamma_t^S = 1 - R^{-1} + \delta E_t^d[\gamma_{t+1}^S] \]
\[ \gamma_t^a = \delta E_t^d[\gamma_{t+1}^a] \]
\[ \gamma_t^{aa} = \delta E_t^d[\gamma_{t+1}^{aa}] - f_t \left( 1 + \delta R E_t^d[\gamma_{t+1}^S] \right) \]
\[ \gamma_t = \delta E_t^d[\gamma_{t+1}] \]

Appendix A.4  Uncertainty about trade type

\[ V_t^d(s_t^d, a_t^d, s_t) = \pi_i V_t^{d,i} + \sum_j \pi_j^c V_t^{d,cj} + (1 - \pi_i - \pi^c) V_t^d \]

\[ = \left( 1 - R^{-1} + \delta E_t^d[\gamma_{t+1}^S] \right) s_t^d + \left( 1 + \delta R E_t^d[\gamma_{t+1}^S] \right) \left( \pi_i p_t^l + \sum_j \pi_j^c V_t^{c,j} \right) + \left( 1 - \pi_i - \pi^c + \sum_j \pi_j^c \eta_j \right) \delta E_t^d[\gamma_{t+1}^a] a_t^d \]

\[ + \left( - \left( 1 + \delta R E_t^d[\gamma_{t+1}^S] \right) f_t + \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi_j^c \eta_j \right) E_t[\gamma_{t+1}^{aa}] \right) (a_t^d)^2 + \pi_i \gamma_t^i + \sum_j \pi_j^c \gamma_t^{c,j} + (1 - \pi_i - \pi^c) \gamma_t \]

Appendix A.5  Recursions

\[ \gamma_t^S = 1 - R^{-1} + \delta E_t[\gamma_{t+1}^S] \]

\[ \Rightarrow \gamma^S = \frac{1 - R^{-1}}{1 - \delta} \]
So $\gamma^s$ is a constant.

\[
\gamma_t^a = \left(1 + \delta R \gamma^s\right) \left(\pi_i p_t^i + \sum_j \pi_j^c (1 - \eta_j) V_t^{c_j}\right) + \delta \left(1 - \pi_i - \pi^c + \sum_j \pi_j^c \eta_j\right) E_t [\gamma_{t+1}^{a}] = ... = \left(1 + \delta R \gamma^s\right) \left(\sum_{s=0}^{T-t-1} \left(\delta \left(1 - \pi_i - \pi^c + \sum_j \pi_j^c \eta_j\right)\right)^s \left(\pi_i E_t [p_{t+s}^i] + \sum_j \pi_j^c (1 - \eta_j) E_t[V_{t+s}^{c_j}]\right) + \left(\delta \left(1 - \pi_i - \pi^c + \sum_j \pi_j^c \eta_j\right)\right)^{T-t} E_t [V_{T}^{issuer}]\right)
\]

which only depends on the state at time $t$.

\[
\gamma_t^{aa} = -\left(1 + \delta R \gamma^s\right) f_t + \delta \left(1 - \pi_i - \pi^c + \sum_j \pi_j^c \eta_j\right) E_t [\gamma_{t+1}^{aa}] = ... = -\left(1 + \delta R \gamma^s\right) \sum_{s=0}^{T-t} \left(\delta \left(1 - \pi_i - \pi^c + \sum_j \pi_j^c \eta_j\right)\right)^s E_t [f_{t+s}]
\]

which only depends on the state at time $t$.

### Appendix A.6 Prices and quantities

#### Appendix A.6.1 Customer-dealer trade

The dealer’s post-trade inventory position, after buying $q_{t}^c$ units of the risky asset from a customer, is given by:

\[
a_{t+1}^{d,c} = a_t^d + q_t^c = \left(1 + \delta R E_t^d [\gamma_t^s]\right) \tilde{V}_t^{c} - \delta E_t^d [\gamma_{t+1}^{a}] \over 2\delta E_t^d [\gamma_{t+1}^{aa}]
\]

\[
= \delta E_t^d \left[\sum_{s=0}^{T-t-2} \left(\delta \left(1 - \pi_i - \pi^c + \sum_j \pi_j^c \eta_j\right)\right)^s \left(\pi_i p_{t+s+1}^i + \sum_j \pi_j^c (1 - \eta_j) V_{t+s+1}^{c_j}\right) + \left(\delta \left(1 - \pi_i - \pi^c + \sum_j \pi_j^c \eta_j\right)\right)^{T-t-1} E_t^d [V_{T}^{issuer}]\right] - V_t^{c}
\]

\[
= \delta E_t^d \left[\sum_{s=0}^{T-t} \left(\delta \left(1 - \pi_i - \pi^c + \sum_j \pi_j^c \eta_j\right)\right)^s f_{t+s+1}\right] \over 2\delta E_t^d \left[\sum_{s=0}^{T-t-1} \left(\delta \left(1 - \pi_i - \pi^c + \sum_j \pi_j^c \eta_j\right)\right)^s f_{t+s+1}\right]
\]
The dealer’s new position after trading with the customer does not depend on her inventory before.

**Appendix A.6.2 Economic decomposition of customer-dealer price**

Dealer’s gain from buying $q^c_t$ units at price per unit $p^c_t$:

$$
g^d_t = q^c_t E_t \left[ \delta \gamma^a_{t+1} + \delta \left( 2a^d_t + q^c_t \right) \gamma^a_{t+1} - (1 + \delta R \gamma^g) p^c_t \right]
\quad (1 + \delta R \gamma^g) \left( -p^c_t q^c_t + \frac{\delta E_t \left[ \gamma^a_{t+1} + (2a^d_t + q^c_t) \gamma^a_{t+1} \right]}{1 + \delta R \gamma^g} \right)
$$

The customer-dealer price is $\eta V^d + (1 - \eta) V^c$ where $V^d$, the dealer’s valuation, is characterized as the customer-dealer price when the dealer’s bargaining power, $1 - \eta$, is zero.

$$
p^c_t = \eta \left( \frac{\delta E_t \left[ \gamma^a_{t+1} + (2a^d_t + q^c_t) \gamma^a_{t+1} \right]}{1 + \delta R \gamma^g} \right) + (1 - \eta) V^c_t
$$

Using the recursions for the value function coefficients, the dealer’s gain becomes:

$$
g^d_t(a^d_t, p^c_t, q^c_t) = (1 + \delta R \gamma^g) \left( -p^c_t q^c_t + \frac{\delta E_t \left[ \gamma^a_{t+1} + (2a^d_t + q^c_t) \gamma^a_{t+1} \right]}{1 + \delta R \gamma^g} \right)
\quad (1 + \delta R \gamma^g) \left( -p^c_t q^c_t - \delta \left( 2a^d_t + q^c_t \right) q^c_t E_t[\gamma^a_{t+1}] + \delta \pi_t E_t[p^c_{t+1}] q^c_t \right)
\quad + \delta \left( (1 - \pi - \pi^c) \delta E_t \left[ \gamma^a_{t+1} + (2a^d_t + q^c_t) \gamma^a_{t+1} \right] q^c_t \right)
\quad + \sum_j \pi^c_j E_t \left[ \eta_j \left( \frac{\delta E_{t+1} \left[ \gamma^a_{t+2} + (2a^d_t + q^c_t) \gamma^a_{t+2} \right]}{1 + \delta R \gamma^g} \right) + (1 - \eta_j) V^c_{t+1} \right] q^c_t
$$
We get the recursion $g_t^d (a_t^d, p_t^c, q_t^c)$

\[
= (1 + \delta R^\gamma) \left( -p_t^c q_t^c - \delta \left( 2a_t^d + q_t^c \right) E_t[f_{t+1}] + \delta \pi_t E_t[p_{t+1}^i] q_t^c + \delta \left( 1 - \pi_i - \pi^c \right) \delta p_t^c q_t^c + \sum_j \pi_j^c E_t \left[ p_{t+1}^j (a_t^d, q_t^c) q_t^c \right] \right) + \delta (1 - \pi_i - \pi^c) E_t \left[ g_{t+1}^d (a_t^d, p_t^c, q_t^c) \right]
\]

Iterating:

\[
g_t^d (a_t^d, p_t^c, q_t^c) = (1 + \delta R^\gamma) q_t^c \sum_{s=0}^{T-t-2} (\delta(1 - \pi_i - \pi^c))^s \left( -p_t^c - \delta \left( 2a_t^d + q_t^c \right) E_t[f_{t+s+1}] + \delta \pi_t E_t[p_{t+s+1}^i] + \delta \left( 1 - \pi_i - \pi^c \right) \delta p_t^c + \sum_j \pi_j^c E_t \left[ p_{t+s+1}^j (a_t^d, q_t^c) \right] \right) + \delta (1 - \pi_i - \pi^c) E_t \left[ g_{T-t}^d (a_t^d, q_t^c, p_t^c) \right]
\]

using the boundary condition from the last trade before the bond matures:

\[
g_{T-t}^d (a_t^d, q_t^c, p_t^c) = (1 + \delta R^\gamma) \left( -p_t^c q_t^c + \delta E_{T-t-1}[V_T^{\text{issuer}}] q_t^c - \delta E_{T-t-1}[f_T](2a_t^d + q_t^c) q_t^c \right)
\]

\[
g_t^d (a_t^d, p_t^c, q_t^c) = (1 + \delta R^\gamma) q_t^c \sum_{s=0}^{T-t-1} (\delta(1 - \pi_i - \pi^c))^s \left( -p_t^c - \delta \left( 2a_t^d + q_t^c \right) E_t[f_{t+s+1}] + \delta \pi_t E_t[p_{t+s+1}^i] + \delta \left( 1 - \pi_i - \pi^c \right) \delta p_t^c + \sum_j \pi_j^c E_t \left[ p_{t+s+1}^j (a_t^d, q_t^c) \right] \right) + \delta (1 - \pi_i - \pi^c) E_t \left[ V_T^{\text{issuer}} \right]
\]

The dealer’s valuation is characterized as the price that makes this gain zero. $V_t^d (a_t^d, q_t^c)$

\[
= -\delta \left( 2a_t^d + q_t^c \right) \sum_{s=0}^{T-t-1} (\delta(1 - \pi_i - \pi^c))^s E_t[f_{t+s+1}] + \delta \sum_{s=0}^{T-t-2} (\delta(1 - \pi_i - \pi^c))^s \left( \pi_t E_t[p_{t+s+1}^i] + \sum_j \pi_j^c E_t \left[ p_{t+s+1}^j (a_t^d, q_t^c) \right] \right) + \delta (1 - \pi_i - \pi^c) E_t \left[ V_T^{\text{issuer}} \right]
\]

From earlier, we know we can write $p_t^c (a_t^d, q_t^c) = (2a_t^d + q_t^c) x_t + y_t$. Let $q_t$ denote the optimal quantity the dealer would buy if she met a customer at time $t$. Let $q_{t+s+1}$ be the optimal quantity the dealer would buy if she met a customer at time $t + s + 1$, assuming her last trade was buying $q_t$ at time $t$.
t. Cash gain from trade at time \( t + s + 1 \), if dealer had traded at time \( t \):

\[
\text{Cash gain} = -q_{t+s+1}^c \times p_{t+s+1}^c \left( a_t^d + q_t^c, q_{t+s+1}^c \right) - \left( -q_t^c + q_{t+s+1}^c \right) \times p_{t+s+1}^c \left( a_t^d, q_t^c + q_{t+s+1}^c \right)
\]

\( q_t^c \times p_{t+s+1}^c \left( a_t^d, q_t^c \right) \)

Let \( q_t^c \) denote the optimal quantity the dealer would buy if she meets a customer at time \( t \). Let \( q_{t+s+1}^c \) and \( q_{T}^{\text{issuer}} \) denote the optimal quantity the dealer would buy if she meets a dealer or customer at time \( t + s + 1 \), or issuer at time \( T \) respectively, assuming her last trade was buying \( q_t^c \) at time \( t \). If a dealer has inventory \( a_t^d \), I can show that her gain from trade, \( g_t^d (a_t^d, p_t^c, q_t^c) \), from buying \( q_t^c \) units at price per unit \( p_t^c(a_t^d, q_t^c) \), is:

\[
\frac{-p_t^c q_t^c}{\text{buy } q_t^c \text{ units from time-} t \text{ customer}}
\]

\[
\left( (a^d + q^c)^2 - (a^d)^2 \right)^{T-t-1} \sum_{s=0}^{T-t-1} \left( \delta \left( 1 - \pi_i - \pi_c \right) \right) E_t [f_{t+s+1}]
\]

discounted holding costs on \( (a^d + q^c) \) units instead of \( a^d \), until next expected trade

\[
\frac{T-t-2}{\delta} \sum_{s=0}^{T-t-2} \left( \delta \left( 1 - \pi_i - \pi_c \right) \right)^s \left( \pi_i E_t [p_{t+s+1}^i] \left( -q_{t+s+1}^c - \left( -\left( q_{t+s+1}^c + q_t^c \right) \right) \right) + \sum_{j} \pi_j E_t \left[ -q_{t+s+1}^c + q_t^c \right] \right)
\]

\( \text{trade at } t \text{ and } t+s+1 \)

\( \text{trade at } t+s+1 \text{ only} \)

\[
\left( \delta \left( 1 - \pi_i - \pi_c \right) \right)^{T-t-1} \delta \left( -q_{T}^{\text{issuer}} + q_t^c \right) E_t [V_t^{\text{issuer}}]
\]

\( \text{buy } a_t^{\text{issuer}} \text{ units instead of } (q_{T}^{\text{issuer}} + q_t^c) \text{ from issuer at time } T \text{, if dealer doesn't meet a counterparty in the previous } T-t-1 \text{ rounds of trading} \)

The customer-dealer price can be expressed as a weighted average of the dealer’s and customer’s valuations, where the weights are given by their relative bargaining power:

\[
p_t^c = \eta V_t^d + (1 - \eta) V_t^c
\]
where $V^d_t$ is the customer-dealer price such that the dealer’s gain from trade at time $t$ is zero. The price per unit can thus be written recursively:

$$p^c_t = \eta \left(-\delta \left(2a^d_t + q^c_t\right) \sum_{s=0}^{T-t-1} \left(\delta(1 - \pi_i - \pi^c)\right)^s E_t[f_{t+s+1}] \right)$$

holding costs on $(a^d + q^c)$ units instead of $a^d$, until next trade

$$+ \delta \sum_{s=0}^{T-t-2} \left(\delta(1 - \pi_i - \pi^c)\right)^s \left(\pi_i E_t[p^i_{t+s+1}] + \sum_j \pi^c_j E_t \left[-q^c_{t+s+1} \cdot p^c_{t+s+1} \left(a^d_t + q^c_t, q^c_{t+s+1}\right) - \left(-q^c_t + q^c_{t+s+1}\right) \cdot p^c_{t+s+1} \left(a^d_t, q^c_t + q^c_{t+s+1}\right)\right] / q^c_t \right)$$

buy $q^c_{t+s+1}$ units from time-$(t+s+1)$ counterparty, instead of $q^c_{t+s+1} + q^c_t$. The counterparty is a dealer with probability $\pi_i$, and customer $j$ with probability $\pi^c_j$

$$+ \left(\delta(1 - \pi_i - \pi^c)\right)^{T-t-1} \delta E_t \left[V^\text{issuer}_T\right]$$

buy $q^\text{issuer}_T$ units instead of $\left((q^\text{issuer}_T + q^c_t)\right)$ from issuer at time $T$, if dealer doesn’t meet a counterparty in the previous $T-t-1$ rounds of trading

$$+(1 - \eta)V^c_t$$
Appendix A.6.3  Inter-dealer trade

The dealer’s post-trade inventory position, after buying \( q^i_t \) units of the risky asset from the inter-dealer market, is given by:

\[
\begin{align*}
\Delta a^d_{t+1} &= a^d_t + q^i_t = \\
&= \frac{p^i_t \left( 1 + \delta R E_t^d | _{t+1} \right) - \delta E_t^d \left[ \gamma_{t+1}^a \right]}{2 \delta E_t^d \left[ \gamma_{t+1}^{aa} \right]} \\
&= \frac{\delta E_t^d \left[ \sum_{s=0}^{T-t-2} \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right) s \left( \pi_i p^i_{t+s+1} + \sum_j \pi^c_j \left( 1 - \eta_j \right) V^c_{t+s+1} \right) + \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right) \left( T-t-1 \right) V^{issuer}_T \right] - p^i_t}{2 \delta E_t^d \left[ \sum_{s=0}^{T-t-1} \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right) s f_{t+s+1} \right]}
\end{align*}
\]

Market-clearing in the inter-dealer market states that net trading must be zero. Let \( a^D_t \) denote the aggregate inventory of dealers who trade in the inter-dealer market at time \( t \).

\[
\begin{align*}
a^D_t &= \pi_i a^d_{t+1} \\
&= \pi_i \left( \frac{\delta E_t^d \left[ \sum_{s=0}^{T-t-2} \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right) s \left( \pi_i p^i_{t+s+1} + \sum_j \pi^c_j \left( 1 - \eta_j \right) V^c_{t+s+1} \right) + \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right) \left( T-t-1 \right) V^{issuer}_T \right] - p^i_t}{2 \delta E_t^d \left[ \sum_{s=0}^{T-t-1} \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right) s f_{t+s+1} \right]} \right)
\end{align*}
\]

This gives a recursion for the inter-dealer price, which can be solved by backwards induction from the trade before the maturity of the asset.

\[
\begin{align*}
p^i_t &= \delta E_t^d \left[ \sum_{s=0}^{T-t-2} \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right) s \left( \pi_i p^i_{t+s+1} + \sum_j \pi^c_j \left( 1 - \eta_j \right) V^c_{t+s+1} \right) + \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right) \left( T-t-1 \right) V^{issuer}_T \right] \\
&\quad - \frac{2 \delta a^D_t}{\pi_i} E_t^d \left[ \sum_{s=0}^{T-t-1} \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right) s f_{t+s+1} \right]
\end{align*}
\]
Appendix A.6.4 Issuance

The dealer’s inventory position after maturity of one risky asset, and issuance of a new one, is given by:

\[ a^d_{T_1+1} = q^\text{issuer}_{T_1} = \frac{\pi^\text{issuer} (1 + (\delta R) E_T[\gamma^S_{T+1}] - \delta E^T[T_{T+1}]}{2 \delta E_T[\gamma^a_{T+1}]}

= \delta E_{T_1} \left[ \sum_{s=0}^{T_2-T_1-1} \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right)^s \left( \pi_i p_{T_1+s+1} + \sum_j \pi^c_j (1 - \eta_j)V^c_{T_1+s+1} \right) + \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right)^{T_2-T_1-1} V^\text{issuer}_{T_2} \right] - p^\text{issuer}_{T_1}

= \frac{2 \delta E_{T_1} \left[ \sum_{s=0}^{T_2-T_1-1} \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right)^s f^T_{T_1+s+1} \right]}{2 \delta E_{T_1} \left[ \sum_{s=0}^{T_2-T_1-1} \left( \delta \left( 1 - \pi_i - \pi^c + \sum_j \pi^c_j \eta_j \right) \right)^s f^T_{T_1+s+1} \right]}
A.7 Markup

The dealer’s holding after inter-dealer trading is given by:

\[ a_{t+1}^{d,i} = a_t^{d,i} + q_t^i = \frac{p_t^i (1 + \delta R \gamma_t^S) - \delta E_t^d [\gamma_{t+1}]^{a_t}}{2 \delta E_t^d [\gamma_{t+1}]^{a_t}} \]

The dealer’s holding after customer-dealer trading is given by:

\[ a_{t+1}^{d,c} = a_t^{d,c} + q_t^c = \frac{V_t^c (1 + \delta R \gamma_t^S) - \delta E_t^d [\gamma_{t+1}]^{a_t}}{2 \delta E_t^d [\gamma_{t+1}]^{a_t}} \]

The customer-dealer price is given by:

\[ p_t^c = \eta \left( \frac{\delta E_t^c [\gamma_{t+1}]^{a_t} + \delta (2a_t^{d,c} + q_t^c) E_t^d [\gamma_{t+1}^{a_t}]}{1 + \delta R \gamma_t^S} \right) + (1 - \eta) V_t^c \]

The inter-dealer price can be written as:

\[ p_t^i = \frac{\delta E_t^d [\gamma_{t+1}]^{a_t} + 2 \delta (a_t^{d,i} + q_t^i) E_t^d [\gamma_{t+1}^{a_t}]}{1 + \delta R \gamma_t^S} \]

\[ = V_t^c + \frac{2 \delta ((a_t^{d,i} + q_t^i) - (a_t^{d,c} + q_t^c)) E_t^d [\gamma_{t+1}^{a_t}]}{1 + \delta R \gamma_t^S} \]

The markup when the dealer is selling to the customer is thus given by:

\[ p_t^c - p_t^i = \left( \frac{V_t^c - \eta \delta q_t^c E_t^d [\gamma_{t+1}^{a_t}]}{1 + \delta R \gamma_t^S} \right) - \left( V_t^c + \frac{2 \delta ((a_t^{d,i} + q_t^i) - (a_t^{d,c} + q_t^c)) E_t^d [\gamma_{t+1}^{a_t}]}{1 + \delta R \gamma_t^S} \right) \]

\[ = \delta \eta q_t^c - 2 \left( \frac{a_t^{d,c} + q_t^c}{\text{mean inventory}} \right) - \left( \frac{a_t^{d,i} + q_t^i}{\text{mean inventory}} \right) \]

\[ \delta \left( 1 - \pi_i - \sum_j^{T-t-1} \pi_j (1 - \eta_j) \right)^u E_t[f_{t+u+1}] \]
where $T$ is the maturity of the bond, $\pi_i$ the probability each dealer will trade in the inter-dealer market each round, and $\pi_j^c$ the probability that she will trade with customer $j$ who has bargaining power $\eta_j$.

### A.8 Approximating the markup for estimation

From equation (5) the markup per unit for a dealer with inventory $a^{d,c}_t$, buying $q^c_t$ units of the asset from a customer with bargaining power $\eta$, at time $t$ is approximately:

$$\pi_i^t - p_i^c \approx (2 - \eta)q^c_t \sum_{u=1}^{T-t-1} \left( \beta_u + \beta_{f,u}f_t \right) \left( c_u + \beta_{i,u}\pi_i + \sum_j \beta_{c,j,u}\pi_j^c \right)$$

Assume further that $\pi_j^c = k\pi_i$ for all $j$, for some $k > 0$. Then the probability of trading with a customer or in the inter-dealer market is $\pi \equiv \pi_i + \sum_j \pi_j^c = (kJ + 1)\pi_i$. It can be shown that:

$$\sum_{u=1}^{T-t-1} \left( \beta_u + \beta_{f,u}f_t \right) \left( c_u + \beta_{i,u}\pi_i + \sum_j \beta_{c,j,u}\pi_j^c \right) = \beta_0 + \beta\pi + \beta_{f,f}f_t + \beta_{f,\pi}\pi f_t$$

where $\beta_0, \beta_f > 0$ and $\beta\pi, \beta_{f,\pi} < 0$.

$$\beta_0 \equiv \sum_{u=1}^{T-t-1} c_u \beta_u > 0$$

$$\beta\pi \equiv \frac{1}{1 + kJ} \sum_{u=1}^{T-t-1} \beta_u \left( \beta_{i,u} + k \sum_j \beta_{c,j,u} \right) < 0$$

$$\beta_f \equiv \sum_{u=1}^{T-t-1} c_u \beta_{f,u} > 0$$

$$\beta_{f,\pi} \equiv \sum_{u=1}^{T-t-1} \beta_{f,u} \left( \beta_{i,u} + k \sum_j \beta_{c,j,u} \right) < 0$$

The markup to buy becomes:

$$p_i^t - p_i^c \approx (2 - \eta)q^c_t \left( \beta_0 + \beta\pi + \beta_{f,f}f_t + \beta_{f,\pi}\pi f_t \right)$$