Security Design with Status Concerns

Suleyman Basak
London Business School and CEPR
E-mail: sbasak@london.edu

Dmitry Makarov
Higher School of Economics
E-mail: dmakarov@hse.ru

Alex Shapiro
New York University
E-mail: ashapiro@stern.nyu.edu

Marti Subrahmanyam
New York University
E-mail: msubrahm@stern.nyu.edu

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Abstract

This paper provides a status-based explanation for convertible securities. We consider a dynamic setting in which a status-driven entrepreneur with a project idea decides what security to issue to finance the project and how to manage the project over time. We solve the problem analytically and find that the optimal security is considerably similar to a convertible security. Our model can also explain why relatively riskier companies, such as start-ups or small firms, resort to convertible securities more often, as well as why these securities differ in their conversion ratios. The dynamic nature of our model is key to our results whereas asymmetric information and ensuing agency conflicts play no role, which makes our explanation of convertible securities notably different from existing ones.

JEL Classifications: G32, C61, G24, D86.

Keywords: Security Design, Status Concerns,Convertible Securities, Internal Financing, External Financing, Hybrid Securities.
1 Introduction

Financial instruments play a fundamental role in the economy by facilitating interaction between entrepreneurs, those with project ideas, and financiers, those who wish to invest their resources. An appropriately designed financial instrument, or a security, allows an entrepreneur to access financier’s resources in exchange for providing the financier with a claim on future profits of the project. There is a voluminous security design and financial contracting literature examining how the choice of security depends on various salient features underlying security issuance. However, while there is substantial evidence that individuals in general, and entrepreneurs in particular, care about status, little is known about the effects of status concerns on optimal security design. This is somewhat surprising given how extensively status concerns have been studied in other areas of economics and finance. This paper contributes towards filling this gap.

We consider a security design problem in a dynamic complete information setting in which an entrepreneur with status concerns issues a security to finance her project. Solving the model analytically, we find that the optimal security closely resembles a convertible security. Our paper thus provides a status-based explanation for convertible securities, which are notable for combining features of both equity and debt. In contrast to existing explanations discussed below which focus on agency conflicts arising under asymmetric information, ours is the first, to our knowledge, that highlights a role for these securities unrelated to mitigating such conflicts. Our model can also explain why it is riskier firms, such as start-ups or small firms, that are more likely to issue such securities. Methodologically, we believe that our framework is general enough to be applied to examining status concerns in other settings.

It has long been recognized that people care about their status in society, and in particular about financial status (Frank (1985), Heffetz and Frank (2011)). Informally, how much someone cares about status is likely to be related to how actively she pursues opportunities

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1 To demonstrate this, we discuss in Remark 1 of Section how our analysis can be adopted to study the optimal choice of compensation schemes in employer-employee interactions in the presence of status concerns. It appears that the resulting model can potentially explain compensation packages consisting of a fixed wage and a performance-based bonus, which are widely used in reality.

2 In this paper, we do not consider non-monetary factors that may affect one’s status in reality, such as education or occupation. Given this, one may think of our work as focusing on financial status. There could be interesting dynamic interactions between different types of status concerns. For example, Focke, Maug, and Niesen-Ruenzi (2016) find that CEOs of prestigious firms earn less, and one explanation is that CEOs may expect a higher financial status in the future after accumulating enough work experience at a high-status company. Exploring such dynamic trade-offs between different status types is beyond the scope of this paper.
that can propel her to a higher status, and by this measure entrepreneurs’ concern for status appears to be rather pronounced. There is considerable evidence supporting this point. According to the 2011 High Impact Entrepreneurship Global Report, a comprehensive cross-country study of entrepreneurship, the idea that successful entrepreneurs have high status has wide support among both entrepreneurs and non-entrepreneurs. Becker, Murphy, and Werning (2005) argue that entrepreneurship as an activity is especially appealing in countries in which entrepreneurial success leads to high status. Begley and Tan (2001) provide empirical support for this argument. It is generally accepted that another specific feature of entrepreneurs, besides status concerns, is their willingness to take risks. Begley and Boyd (1987) find that status concerns (in their language, “need for achievement”) and risk-taking propensity are two of the three features distinguishing entrepreneurs from the rest (the third feature is tolerance of ambiguity).

We model status concerns of an entrepreneur following a seminal insight of Friedman and Savage (1948). They point that the two characteristics—status concerns and risk-taking propensity—could actually be closely related to each other, giving rise to a preference-based approach to modelling status concerns. Suppose that one’s wealth is close to but lower than the threshold level exceeding which leads to high status. Then, one may be willing to pursue high-risk strategies because the ensuing higher volatility of wealth implies a higher chance to end up above the threshold. On the other hand, when status change is unlikely, which is the case when wealth is sufficiently lower or higher than the threshold, one exhibits the standard aversion to risk. To generate such pattern of risk-taking, Friedman and Savage propose preferences that are convex for intermediate wealth levels corresponding to *middle status*, but are concave for low and high wealth levels corresponding to *low* and *high status*, respectively. Subsequent research formally shows how convexities in preferences and the ensuing risk-taking incentives may arise endogenously (Patel and Subrahmanyam (1978), Gregory (1980), Robson (1992), Vereshchagina and Hopenhayn (2009), among others).

We develop a continuous-time framework in which an entrepreneur dynamically manages a project. Her goal is to maximize an objective function that embeds incentives to take risks when wealth is in the middle status region, and so high status is in sight. Formally, the objective function has a convexity over middle status wealth levels. The project value is modeled as a dynamic random process whose characteristics are controlled by the entrepreneur. In particular, she dynamically chooses a parameter reflecting the novelty of the product being
developed, whereby a higher product novelty corresponds to a higher mean growth rate and volatility of the project value. We adopt the dynamic framework not only for generality but, importantly, because our main results cannot be obtained in a static setting, as explained below. For the purpose of building intuition, we start with the internal financing case, in which the entrepreneur manages the project without relying on outside funding, and so does not issue a security. We then consider the external financing case, our main focus, in which the entrepreneur finances the project by selling to the financier a security, which is a claim on the project’s future value. The financier does not have status concerns, and so is risk-averse at all wealth levels. She buys the security if it provides her with the required reservation level of expected utility.

We provide an explicit expression for the product novelty in the internal financing case, uncovering that the entrepreneur continuously alters the novelty as the project evolves over time. In particular, if at any point in time the project value enters the middle status region, and so high status is in sight, the entrepreneur substantially increases the product novelty and, as a result, the project risk—the desired effect due to status concerns. Status considerations become weaker if the project value moves away from the middle status region, and accordingly the entrepreneur lowers the project risk by reducing the product novelty. We see that the status-driven entrepreneur is actively involved in managing the project, making full use of the dynamic setting. In contrast, the entrepreneur without status concerns maintains a constant product novelty at all times. These results verify that our status specification is indeed in line with the patterns of risk-taking associated with status concerns discussed above.

We solve analytically for the optimal security in the external financing case and find that it is considerably similar to a convertible security, featuring both equity- and debt-like components. Comparing this to the optimal security without status concerns, which is equity-like, we see that it is the debt-like component that emerges due to status concerns. The reason is that similarly to internal financing, with external financing status concerns create incentives for the entrepreneur to take on considerable risk when high status is within reach.

This prompts the entrepreneur to introduces a debt-like segment to insulate the risk

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3We also find that the status-induced increase in the project riskiness can be more pronounced under either type of financing. Under a plausible calibration, we show that the entrepreneur adopts a riskier strategy with external financing when the project is relatively young, whereas for more mature projects the risk is higher in the internal financing case.
averse financier from this risk. The security has to provide the financier with the required reservation utility, and so when designing it the entrepreneur has to cater to the financier’s risk preferences.

We note that this economic mechanism relies in an important way on the dynamic nature of the entrepreneur’s behavior—the entrepreneur may or may not increase the project riskiness in the future, and the debt-like segment in the convertible security protects the financier in those future states of the world in which the increase occurs. We complement this heuristic argument for the importance of the dynamics by showing formally that the optimal security in a static version of the model is different from a convertible security. In particular, this optimal security does not have a debt-like segment, but instead has a segment resembling a short equity position, or a negative stake in the project. Just as one can reduce her exposure to project risk by offering a positive stake in the project to another individual (the classical idea of risk sharing), one can equally increase her risk exposure by offering a negative stake. Thus, introducing a short-equity segment in the security is the entrepreneur’s way of taking risks in a static setting in which the product novelty is fixed and so cannot be used to modify the risk profile.

Going back to the dynamic results, we find that the higher is the project volatility, the more the optimal security resembles a convertible security. Hence, our model predicts that convertible securities are likely to be more prevalent among more volatile firms. This result is consistent with the evidence that convertible securities are mainly used by start-ups and small companies, which are known to be relatively volatile (the evidence is discussed in Section 5). When the project volatility is high, there is a greater need to protect the financier from the status-driven risky behavior of the entrepreneur, and so the flatter is the middle debt-like segment of the optimal security. Thus, it gets closer to the slope of the middle segment of an actual convertible security, which is fully flat.

Our model also provides a possible explanation, based on risk aversion, of why convertible securities have different conversion ratios. A conversion ratio determines what share of the project the financier will have if she chooses to convert the security into equity. We find that the more risk averse the financier is, the lower is the optimal security’s conversion ratio. On the contrary, the more risk averse the entrepreneur is, the higher is the conversion ratio. A higher conversion ratio implies that the financier will hold a larger share of the project if she
converts, implying a higher exposure to the project risk. A more risk averse entrepreneur prefers a lower project risk, and the optimal security reflects this preference through a lower conversion ratio. Analogous reasoning explains a positive relation between the entrepreneurs’ risk aversion and the conversion ratio.

The term “entrepreneur” in this paper can also refer to an established company that issues a security. Our analysis seems applicable in this context as well. First, companies’ important decisions, such as security issuance, are ultimately made by CEOs, and CEOs are likely to have pronounced status concerns. In addition to an obvious point that someone with little concern for status is not likely to become a CEO in the first place, there is also evidence direct evidence supporting this point. This is also consistent with survey findings that wealthier people tend to care more about status (McBride (2001), Dynan and Ravina (2007)). Second, our approach to modelling status concerns is supported by extensive research on organizational economics initiated by the influential work of Cyert and March (1963). This research challenges the view that all complex interactions within companies can be reduced to the standard assumption of profit maximization. It is argued that real companies, when deciding how much risk to take, consider their current performance relative to a certain aspiration level, a target that a company tries to achieve (see Audia and Greve (2006) and the literature review therein). A common argument in this literature is that “managers seem to feel that risk taking is more warranted when faced with failure to meet targets than when targets were secure,” and that “executives ... would not take risks where a failure could jeopardize the survival of the firm” (March and Shapira (1987)). This pattern—taking risks when below but near the target and avoiding risks when either above or well behind the target—mirrors the idea of Friedman and Savage used in this paper.

Our paper contributes to the literature aiming to explain the use of convertible securities. A common theme of existing works is that convertible securities help to mitigate various agency problems, which typically arise under asymmetric information. In particular, convertible securities are shown to mitigate the asset substitution problem (Green (1984)), window-dressing behavior (Cornelli and Yosha (2003)), inefficient investment (Schmidt (2003)), the underinvestment problem (Lyandres and Zhdanov (2014)), and other asymmetric informa-

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4Shemesh (2014) presents evidence that the CEOs’ risk-taking behavior is affected by status concerns. Other works exploring the behavior of CEOs through the prism of status concerns include Wade, Porac, Pollock, and Graffin (2006), Malmendier and Tate (2009), and Goel and Thakor (2010).
tion problems (Constantinides and Grundy (1989), Stein (1992), Repullo and Suarez (2004), Hellmann (2006), Chakraborty and Yilmaz (2011)). Our analysis shows that convertible securities also have an economic role when agency conflicts are absent, as is the case in our model.

Ours and the above studies are part of a broader security design and financial contracting literature that seeks to rationalize the multitude of financial contracts used in reality. Excellent reviews of this literature are provided in Allen and Winton (1995), Hart (2001), Biais, Mariotti, and Rochet (2013), and Sannikov (2012). Kaplan and Stromberg (2003) compare financial contracting models’ ability to explain contracts used in reality. Several studies in this area, notably Cadenillas, Cvitanic, and Zapatero (2007) and Bolton and Harris (2013), consider, like us, settings without agency problems, but they do not explain the use of convertible securities.

Our work also contributes to the growing literature investigating the role of status concerns in various areas of economics and finance. Recent examples include Ball, Eckel, Grossman, and Zame (2001), Becker, Murphy, and Werning (2005), Moldovanu, Sela, and Shi (2007), Auriol and Renault (2008), Besley and Ghatak (2008), Wendner and Goulder (2008), Roussanov (2010), Dijk, Holmen, and Kirchler (2014), Georgarakos, Haliassos, and Pasini (2014), and Hong, Jiang, Wang, and Zhao (2014).

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the internal financing case, where the entrepreneur finances the project herself. Section 4 characterizes the optimal security in the external financing case. Section 5 examines the properties of the optimal security. Section 6 concludes. Appendix A presents all proofs, Appendix B examines settings with alternative preference specifications, and Appendix C studies security design in a static framework.

2 Model

We start with describing some key elements of our model, after which we provide all the details. We consider a dynamic continuous-time economy with full information and no agency problems. (We discuss the results of a static analysis at the end of Section 4). There are two agents in the economy: an entrepreneur with status concerns who has a project idea, and a
financier who has the resources to finance the project. When the entrepreneur has enough own resources to launch and develop the project, she does not interact with the financier. When her own resources are insufficient, the entrepreneur issues a security to the financier so as to receive funding today in exchange for a claim on the project’s future profits. We refer to these two cases as internal financing and external financing cases, respectively.

The main goal of this paper is to study the implications of status concerns for security design, and our main focus is to examine the optimal security in the external financing case. We also examine the internal financing case because, besides being of independent interest for understanding how status concerns affect project development, it allows us to describe in a clear way the economic mechanisms at play underlying the optimal security.

The future project value is uncertain and follows a random process whose parameters are controlled by the entrepreneur. For the external financing case to be non-vacuous, we assume the financier cannot manage the project herself without the entrepreneur, or technically speaking, the financier is not able to trade the project risk. Hence, the financier faces an incomplete market for this risk; for the entrepreneur, on the other hand, the market is complete. Some of the above features make our framework similar to that of Cadenillas, Cvitanic, and Zapatero (2007), as elaborated on in Section 2.3.

As already discussed in the Introduction, the entrepreneur in our model may refer to various participants of financial markets seeking financing for their projects, such as a start-up firm or an established company, private or public. Analogously, the financier is also a broad term here that refers to any entity that can provide financing, such as a venture capitalist, an angel investor, a bank, or the financial market. Accordingly, the term “security” may mean either a non-tradable (private) financial agreement or a publicly traded financial instrument. Finally, when we say that the project is financed or funded by the financier, we do not only mean the monetary contribution. The financier’s involvement often includes sharing her experience and expertise, giving access to her network of contacts, and so on. It is widely believed that such non-monetary forms of support have a substantial value.

We now provide a detailed description of the model.
2.1 Project Value Dynamics

We consider an entrepreneur with a unique project idea that she develops over time. We model this by positing that she dynamically chooses the process $\phi$ controlling the evolution of the project value $V$:

$$dV_t/V_t = \phi_t \mu dt + \phi_t \sigma d\omega_t,$$

where $\mu, \sigma > 0$ are the project mean growth rate and volatility, respectively, and $\omega$ is a standard Brownian motion representing the uncertainty in the economy. Specification (1) formalizes the idea of “nothing ventured, nothing gained.” Specifically, if the entrepreneur wants to raise the project’s expected growth rate (first term), she needs to increase the parameter $\phi$, implying that she is also raising the project’s riskiness (second term).

To elaborate further, when working on a project, the entrepreneur develops a certain product that she plans to sell at a profit in the market. In the process of development, she can dynamically choose how novel the product is going to be relative to existing ones. The more novel the product is, the higher are the expected future profits due to lower competition, also raising the expected growth rate of the project value. At the same time, the future demand for novel products is less predictable, implying a higher project riskiness. Given this, we refer to the parameter $\phi$ as the product novelty. One can note that a process similar to (1) often arises in dynamic asset pricing models, and it describes the wealth dynamics of an investor. In that context, the parameter $\phi$ corresponds to the choice of leverage. This analogy between the two settings proves useful in that it enables us to develop an approach to solving the model (see the Appendix).

2.2 Status Concerns

A key novelty of this paper, as compared to the existing security design literature, is that the entrepreneur is driven by the desire to achieve a higher financial status, a well-documented feature of human behavior that we refer to as status concerns (see footnote 2). As elaborated in the Introduction, we model status concerns in line with the classical insight of Friedman

5Technically, the entrepreneur can choose a negative product novelty $\phi_t < 0$ as we impose no restrictions on $\phi$. However, doing so reduces the expected project value in the future, which is not in the interests of the entrepreneur. In all model solutions presented later in the paper, it is always the case that the product novelty is positive, though proving that this is true in general seems challenging.
and Savage (1948). The idea is that when one’s wealth is sufficiently high but not yet at a level associated with high status, one is willing to take risks to increase the probability of reaching high status. Otherwise, when a status change is not likely, which is the case when wealth is either sufficiently low or high, one exhibits the usual aversion to risk. Given this, we consider preferences that are concave for low wealth levels—the low status region, convex for intermediate wealth levels—the middle status region, and concave for high wealth levels—the high status region.

Given the well-documented tendency of a typical entrepreneur to take risks, preferences with convexities seem especially appropriate for modeling entrepreneurs’ behavior. As indicated by Becker, Murphy, and Werning (2005), “[S]tart-ups and other entrepreneurial efforts...are much more common and less well rewarded than would be expected from the usual assumptions of risk aversion and diminishing marginal utility of income.” The convexities and the ensuing risk-loving behavior need not necessarily be one’s inherent trait, but can arise endogenously due to various mechanisms even if one is inherently risk-averse, as shown by Patel and Subrahmanyam (1978), Gregory (1980), Robson (1992), and Vereshchagina and Hopenhayn (2009). Let us briefly describe one of these mechanisms, broadly following Patel and Subrahmanyam’s ideas, without taking a stand that it is more relevant than others in our context.

The traditional argument for a decreasing marginal utility relies on the divisibility of consumption goods. Under divisibility, one can consume the same set of goods regardless of one’s wealth, with a higher wealth level resulting simply in a higher consumption of each good. Consuming more of the same goods leads to satiation, and hence marginal utility decreases in wealth. Clearly, in reality, when one’s wealth increases she can well start consuming new types of goods that she could not afford before because they are both expensive and non-divisible. Examples are “status” goods such as a private jet, a yacht, a membership of elite golf clubs, and so on. When one switches from “low” to “high” status goods, the satiation mechanism is not at work, and so the marginal utility may be increasing in a region of wealth in which the switching occurs.

Accordingly, we posit that the entrepreneur’s utility function $u_{E}(\cdot)$ over her wealth $W_{E\tau}$ at some future date $\tau$ is
\[ u_E(W_{E\tau}) = \begin{cases} 
\frac{(W_{E\tau})^{1-\gamma_E}}{1-\gamma_E} & W_{E\tau} < L, \\
\frac{(W_{E\tau} - \alpha)^{1-\gamma_E}}{1-\gamma_E} + B, & W_{E\tau} \geq L,
\end{cases} \] (2)

where \( \gamma_E, L > 0, \alpha \in [0, L], \) and \( B = (L^{1-\gamma_E} - (L - \alpha)^{1-\gamma_E})/(1 - \gamma_E) \geq 0 \) ensures continuity of preferences. The parameter \( \alpha \) represents the status concerns—the higher \( \alpha \) is, the stronger is the entrepreneur’s desire to achieve high status, and so the more pronounced is the convexity region in the utility. The special case of \( \alpha = 0 \) corresponds to a standard CRRA utility function with no status concerns. Figure 1 presents typical shapes of preferences with status concerns. Going from left to right in Figure 1, we first have the low-status region of wealth in which the utility is concave, then the middle-status region with convex utility, and finally the high-status region with concave utility. The position of the middle-status region is determined by the parameter \( L \); henceforth, we refer to \( L \) as the status level (of wealth). The parameter \( \gamma_E \) represents the entrepreneur’s risk aversion when her wealth is in the low or high-status region. Our subsequent results are robust to alternative preference specifications. In general, there can be several wealth thresholds going over which increases a person’s status, while in our setting there is a single threshold wealth level. Our approach corresponds to situations in which the entrepreneur, when designing the security, is concerned only with reaching the next higher status level and is not affected by even higher levels.

The other agent is a financier who can provide funds and other resources required to launch and develop the project. We assume that the financier is not willing to take risks to increase her financial status, and so has a standard CRRA utility function \( u_F(\cdot) \) over her wealth \( W_{FT} \) at some future date \( T \):

\[ u_F(W_{FT}) = \frac{(W_{FT})^{1-\gamma_F}}{1-\gamma_F}, \] (3)

where \( \gamma_F > 0 \) is the financier’s relative risk aversion.

\[ ^6\text{To demonstrate this, in Appendix B we solve our model under two different concave-convex-concave preference specifications, and find that our main predictions remain valid. Moreover, our results are not driven by the kink in the utility function} \] (2) \( \text{(seen in Figure 1), as we explain in the proof of Proposition 1 in Appendix A.} \)

\[ ^7\text{We thank Andrew Winton for this observation.} \]
2.3 External and Internal Financing

The main goal of this paper is to analyze the external financing case in which the entrepreneur can develop the project only if the financier agrees to fund it. In return for funding, the entrepreneur offers the financier a state-contingent claim, or a security, represented by a function $W_{FT}(V_T)$. The security specifies the amount $W_{FT}$ that the financier will receive at date $T$ for each possible realization of the project value $V_T > 0$. We do not impose any restrictions on the function $W_{FT}(V_T)$, and so the set of admissible securities consists of virtually all possible securities. As we abstract away from agency problems, the entrepreneur cannot distort or misreport the true project value $V_T$ and cannot refuse to pay the full amount $W_{FT}(V_T)$. Moreover, the entrepreneur cannot run with the money before paying back the financier, which we model by assuming $\tau > T$, meaning that the entrepreneur’s horizon $\tau$ is longer than that of the financier $T$.

The financier agrees to finance the project if her expected utility under the security offered to her is not lower than her (commonly known) reservation utility $\bar{u}_F$, which reflects the amount of resources she plans to devote to the project (money, time, effort), as well as factors such as her outside investment opportunities and bargaining power. We assume that this reservation utility is not prohibitively high from the entrepreneur’s perspective, and so

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8In some studies, the function $W_{FT}(V_T)$ is alternatively referred to as a sharing rule, as it specifies how the project value will be shared between the involved parties. We also note that our results would not change if the financing offer were initiated by the financier.
the financing transaction between the entrepreneur and the financier takes place. At the payoff date $T$, the entrepreneur pays the financier the required amount out of the project value and continues managing the project until her horizon $\tau$, at which time she consumes the project value $V_{\tau}$.

The optimal security $W_{FT}^*(V_T)$ and the optimal product novelty process $\phi_t^*, t \in [0, \tau)$ are such that the financier accepts the security, and the corresponding time-$\tau$ project value $V_{\tau}^*$ maximizes the entrepreneur’s expected utility (2), as formalized in Definition 1.

**Definition 1** In the external financing case, the optimal security, $W_{FT}^*(V_T)$, and the product novelty process, $\phi_t^* > 0$, $t \in [0, \tau)$, are determined as the solution to the problem

$$\max_{\phi_t, W_{FT}} E[u_E(V_{\tau})]$$

subject to

$$dV_t = V_t \phi_t \mu dt + V_t \phi_t \sigma d\omega_t - W_{FT} dI_t,$$

$$E[u_F(W_{FT})] \geq \bar{u}_F,$$

where $I_t$ is a step function $I_t \equiv 1_{\{t=T\}}$.

It also proves valuable to examine the internal financing case, in which the entrepreneur develops the project without external help. Definition 2 describes the corresponding optimization problem of the entrepreneur.

**Definition 2** In the internal financing case, the entrepreneur’s dynamic choice of the product novelty, $\phi_t^* > 0$, $t \in [0, \tau)$, is given by the solution to the problem

$$\max_{\phi_t} E[u_E(V_{\tau})]$$

subject to

$$dV_t = V_t \phi_t \mu dt + V_t \phi_t \sigma d\omega_t.$$

Having presented our model, we now relate it to the framework of Cadenillas, Cvitanic, and Zapatero (2007). Like these authors, we examine the optimal security in a setting with full information and no agency problems. The main difference between Cadenillas et al. and
our work is that they do not consider status concerns, our main focus. Moreover, there is no costly effort in our model, i.e., the entrepreneur does not derive disutility from running the project, whereas this feature is present in Cadenillas et al. (and also in a large body of studies in this area). While the assumption that work reduces one’s utility is appropriate in various situations, we believe that not including this feature is less problematic in our particular setting. Indeed, real-life entrepreneurs commonly enjoy working on their projects, and typically devote their full effort to them without perceiving it as a utility-reducing activity. This argument is also made in Schmidt (2003): “Typically, the problem is not to get the entrepreneur to work hard enough, but rather to induce her to allocate her effort efficiently.” Finally, in our model, the entrepreneur’s choice of project volatility determines the project’s mean growth rate, whereas in Cadenillas et al. the two are chosen separately. Because, unlike these authors, we do not have costly effort, allowing the entrepreneur to choose the two parameters separately would lead to her choosing an infinite mean growth rate in our setting.

3 Project Development with Internal Financing

In this Section, we investigate how the entrepreneur develops the project in the internal financing case, when she does not need to attract funding from the financier. The mechanisms underlying the entrepreneur’s optimal behavior in this case are key to understanding the structure of the optimal security (Section 4). Moreover, the ensuing analysis allows us to verify that the entrepreneur’s behavior under status concerns is in line with the risk-taking patterns proposed by Friedman and Savage (1948) and prevalent among entrepreneurs and companies, as discussed in the Introduction.

Proposition 1 presents the optimal product novelty and the corresponding project value in closed form. It turns out that the economic state variable in the solution is the state-price process \( \xi_t \) given by \( \xi_t = \exp(-\mu(\omega_t - \mu t/(2\sigma))/\sigma) \), as is typically the case in asset pricing models. A low \( \xi_t \) is associated with good states (high realization of uncertainty \( \omega_t \)), and a high \( \xi_t \) is associated with bad states (low \( \omega_t \)).
Proposition 1 Under internal financing, the optimal product novelty is given by

\[ \phi_t^* = \frac{\mu}{\sigma^2 V_t} \left[ K_{1t}(y\xi_t)^{-1/\gamma_E} + \frac{\alpha}{K_{3t}} n \left( \frac{\ln \frac{B}{\alpha y\xi_t} - K_{2t}}{K_{3t}} \right) \right], \quad (7) \]

and the optimal project value, \( V_t^* \), is given by

\[ V_t^* = K_{1t}(y\xi_t)^{-1/\gamma_E} + \alpha N \left( \frac{\ln \frac{B}{\alpha y\xi_t} - K_{2t}}{K_{3t}} \right), \quad (8) \]

where \( N(\cdot) \) and \( n(\cdot) \) are the standard normal cumulative distribution function and probability density function, respectively, \( y \) is given in the Appendix, the constant \( B \) is as defined in equation (2), and the deterministic quantities \( K_{1t}, K_{2t}, \) and \( K_{3t} \) are given by

\[ K_{1t} \equiv e^{(1-\gamma_E)(\tau-t)\mu^2/(2\gamma_E^2 \sigma^2)}, \quad K_{2t} \equiv (\tau-t)\mu^2/(2\sigma^2), \quad K_{3t} \equiv \sqrt{\tau-t}\mu/\sigma. \quad (9) \]

Figure 2 plots the optimal product novelty (7) and project value (8) when the entrepreneur has status concerns \( \alpha > 0 \) (solid lines) and in the benchmark case of no status concerns \( \alpha = 0 \) (dashed lines). From panel (a), we see a sharp distinction between the cases of status concerns and no status concerns in terms of how actively the entrepreneur manages the project. In particular, the status-driven entrepreneur adjusts the product novelty depending on the project value, whereas without status concerns she simply chooses a constant product novelty. Recalling the earlier discussion of the three status regions (Section 2.2), we place \( L \) and \( \bar{L} \) onto the \( x \)-axis in panel (a) and \( y \)-axis in panel (b) to mark the boundaries of these regions, so that the low, middle, and high-status regions correspond to, respectively, \( V_t < L \), \( L \leq V_t \leq \bar{L} \), and \( V_t > \bar{L} \)\(^{11}\).

Panel (a) reveals that with status concerns, the entrepreneur takes substantial risks by increasing the novelty in the middle status region as she, driven by the convex part of her preferences, attempts to increase the chances to achieve high status by making the project riskier. In the low and high status regions, on the other hand, the status change is unlikely,

\(^{10}\)The parameter values are \( \gamma_E = 3, \alpha = 0.5 \) for the solid line and \( \alpha = 0 \) for the dashed line, \( L = 2, V_0 = 3, \mu = 0.1, \sigma = 0.8, t = 3.5, \tau = 4 \), and so \( B = 0.0972 \).

\(^{11}\)While the entrepreneur’s status is realized at her horizon \( \tau \) (when her consumption takes place), the entrepreneur can compute her expected status at any prior date \( t < \tau \). Accordingly, we refer to the region \( V_t < L \) as the low-status region because, when \( V_t < L \), the entrepreneur expects to have low status at date \( \tau \), and analogously for the two other regions.
Figure 2: Optimal Product Novelty and Project Value with Internal Financing. Panel (a) depicts the time-\( t \) optimal product novelty, \( \phi^*_t \), and panel (b) depicts the optimal project value, \( V^*_t \), in the internal financing case. In both panels, the solid line corresponds to the case of status concerns and the dashed line to the case of no status concerns.

and so the entrepreneur reduces the project novelty to levels close to that without status concerns. These results confirm that the way we model status concerns is indeed consistent with Friedman and Savage (1948).

A natural but important implication of a status-driven increase in the product novelty in the middle status region is an increased volatility of the project value as compared to the case without status concerns. Indeed, panel (b) reveals that with status concerns the project value (solid line) is more sensitive to the realization of uncertainty \( \xi \) in the middle status states of the world \( [\xi^b, \xi^f] \) than without status concerns (dashed line) in the corresponding middle status states of the world \( [\xi^b, \xi^f] \)). This mechanism, which we further comment on in Section 5 (see Figure and its discussion), plays a key role in determining the structure of the optimal security in the external financing case.

4 Optimal Security with External Financing

In this Section, we consider the external financing case and characterize analytically the optimal security issued by the entrepreneur. Depending on her current wealth, our entrepreneur
can either be seeking risks as she tries to achieve a higher status or averse to risks when status concerns are weak. Studying models with preferences embedding both types of risk attitude and developing the (non-standard) techniques to solve them have proved valuable in various areas of finance. However, to our knowledge there are no models with such preferences in the security design and financial contracting literature. Our paper is a first step in that direction.

Proposition 2 The optimal security $W_{FT}^*(V_T)$ is given parametrically through a pair of functions $(W_{FT}(x), V_T(x))$ where the parameter $x$ varies from 0 to $+\infty$. The functions $W_{FT}(x)$ and $V_T(x)$ are

$$W_{FT}(x) = (\bar{u}_F(1 - \gamma_F))^{-1/(\gamma_F-1)} e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F},$$

$$V_T(x) = K_{1T} g(x)^{-1/\gamma_E} + \alpha N \left( \frac{\ln(B/\alpha) - \ln g(x) - K_{2T}}{K_{3T}} \right) + (\bar{u}_F(1 - \gamma_F))^{-1/(\gamma_F-1)} e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F},$$

where the function $g(x)$ is implicitly given by

$$K_{1T} K_{3T} g(x)^{(\gamma_E-1)/\gamma_E} + \gamma_E B n \left( \frac{\ln(B/\alpha) - \ln g(x) + K_{2T}}{K_{3T}} \right) = z x.$$  

In the above, $N(\cdot)$ and $n(\cdot)$ are the standard normal cumulative distribution function and probability density function, respectively, the constant $B$ is as given in equation (2), the quantities $K_{1T}, K_{2T},$ and $K_{3T}$ are as given in Proposition 1, and $z$ is provided in the Appendix.

Panel (a) of Figure 3 plots the optimal security when status concerns are present (solid line) and absent (dotted line). We see that the status-driven entrepreneur finances the project by offering to the financier a security that is considerably similar to a convertible security, which has the payoff profile depicted in panel (b). Absent status concerns ($\alpha = 0$, dotted line in panel (a)), the optimal security is similar to equity. A key feature of a con-


\[12\] The parameter values are $\alpha = 1.5$ for the solid line and $\alpha = 0$ for the dashed line, $\gamma_F = 3$, $u_F = -0.5$, $T = 3$, and the other parameters are as in Figure 2.

\[13\] The optimal security is identical to equity when the entrepreneur’s and financier’s risk aversions are the same (as in Figure 3). When they are different, the optimal security’s payoff profile is not given by a straight line, and so is not identical to equity. However, as argued at the end of this Section, real entrepreneurs are not likely to issue a non-linear security defined by a complicated formula, and instead are likely to resort to one of the standard securities whose payoff profile is as close as possible to the optimal one. Given this, even when the two risk aversions are different, the entrepreneur is still likely to issue an equity. To account for this discussion, in the paper we often refer to the optimal security as equity-like, as a way to indicate that it is not necessarily identical to equity.
Figure 3: Optimal Security and Convertible Security. Panel (a) depicts the optimal security $W^*_F$ in the external financing case of our model, while panel (b) depicts the payoff profile of an actual convertible security. In panel (a), the solid line corresponds to the case of status concerns and the dashed line to the case of no status concerns.

The existing explanations of convertible securities, discussed in the Intro-

\[14\] For a more detailed description of a convertible security and its payoff profile, see for example Section 24-6 in Brealey, Myers, and Allen (2010). Hereafter, we use a generic term “convertible security,” rather
duction, center around agency problems under asymmetric information. Our paper makes a novel point that convertible securities also have an economic role beyond mitigating conflicts between agents.

From panel (a), we see that a concern for status causes the entrepreneur to introduce a debt-like segment into an equity-like security, which she would choose without status concerns. The entrepreneur knows that she is going to increase the project risk via a higher product novelty in the middle status region, as explained in Section 3, and includes the debt-like segment as a way to insulate the financier from this risk. Recall that the financier does not have status concerns, and so would find an equity-like security unattractive as it would pass on to its holder the higher risk due to the entrepreneur’s status-driven actions. Aware of the need to make the security attractive so that the financier’s participation constraint is satisfied, the entrepreneur offers a convertible security with the debt-like segment—promising a payoff insensitive to project fluctuations—for middle status project values when the entrepreneur intends to take considerable risks.

Looking at the two panels of Figure 3, we see that our optimal security (panel (a)) does not perfectly replicate an actual convertible security (panel (b)). A difference that may not be visually obvious is that the slopes of the two A-B segments are different: it is 45 degrees for an actual convertible and lower than 45 degrees for our optimal security. The 45-degree line means that the security issuer pays the full project value $V_T$ to the holder. In our setting, this means that the entrepreneur would be left with zero wealth, which is not allowed by the entrepreneur’s preference specification. A lower than 45 degrees slope of A-B ensures positive wealth. Despite the differences, we view our model as providing an explanation for convertible securities. For a given truly optimal security, the security used in the real transaction is likely to be one of the standard securities, or a portfolio thereof, with payoff structure as close as possible to the optimal one. For example, this can be because

\[15\] Though in Section 3 the increase in project risk in the middle status region is established in the internal financing case, we formally verify that the analogous incentives are present under external financing. The analysis is in Section 5.

\[16\] There are ways to extend our model to address this issue. One can assume that the entrepreneur has other sources of income (e.g., housing wealth, savings), in which she would not be left with zero wealth if she chose to pay the full project value. Alternatively, one could consider preferences that are well-defined at zero wealth. We leave these extensions for future work.
parties involved in real transactions prefer to deal with familiar securities whose properties are well understood. Following this reasoning, real status-driven entrepreneurs are expected to finance their projects by convertible securities as they appear to be the best match for our optimal security among the set of standard securities.

4.1 Optimal Security in a Static Setting

To further elaborate on the economic mechanism underlying our main implications and the necessity for a dynamic setting, we here consider a simpler static security design setting with status concerns and investigate whether such a setting could also explain the use of convertible securities. Before addressing this question formally, we start with an intuitive reasoning to justify the importance of a dynamic setting. Recall that the entrepreneur introduces a debt-like segment into an equity-like security to protect the financier from a possible future increase in the project riskiness, which is a dynamic feature. Indeed, this increase will happen only under a certain realization of uncertainty, if the project value dynamics brings it into the middle status region. This interplay between the product novelty dynamics and the structure of the security cannot be at work in a static setting in which the product novelty is constant over time.

We solve a static version of our model which is the same as presented in Section 2 with the only difference being that the product novelty parameter $\phi$ is constant. The full analysis is provided in Appendix C, and we here only present and discuss the main result. Figure 4 plots a typical shape of the optimal security.
From Figure 4, we see that the static optimal security has equity-like segments but does not have a debt-like segment, making it distinctly different from a convertible security. Instead of the debt-like segment in the dynamic optimal security occurring for middle status project values, the static optimal security has a segment corresponding to a negative, or a short, equity position. The entrepreneur still wants to take risks in the middle status region driven by the convex part of her preferences, and unable to do so through increasing the product novelty, she offers the financier a negative stake in the project in this region. Analogously to the classical risk sharing result, that offering a positive stake in the project to another agent allows one to share risks and thus reduce her own risk exposure, offering a negative stake allows one to increases her risk exposure.

5 Properties of the Optimal Security

In this Section, we examine how the shape of the optimal security is affected by the model parameters. This allows us not only to understand the security’s properties and relate them to empirical evidence, but also to show that our key findings are robust to different parametrizations.\footnote{We have additionally examined the model predictions under a large number of other parametrizations to ensure that our main results are not driven by a specific choice of parameter values. Given that the results are the same across all parametrizations—that the optimal security is similar to a convertible security—we below report only the representative results of this analysis instead of all of them.}

Among the model parameters, one—the project volatility parameter $\sigma$—stands out as being particularly relevant for relating our model to empirical evidence on the use of convertible securities.\footnote{Other key parameters of our model are related to agents’ preferences. However, existing theoretical works on convertible securities typically disregard the role of preferences by assuming risk neutrality. As empirical research tends to be driven by theory, the possible linkages between the use of convertible securities and the preference parameters of the agents issuing or buying such securities remain largely unexplored empirically.} Issuing convertible securities is a predominant form of financing for—relatively more volatile—start-up firms (Sahlman (1990), Gompers (1999), Kaplan and Stromberg (2003)), while for—relatively less volatile—established publicly traded companies, convertible securities do not constitute the main source of external financing (which are equity and debt). A similar relation between the use of convertibles and volatility is observed when one considers only publicly traded companies. For example, summarizing the relevant evidence, Brennan and Schwartz (1988) point out that “companies issuing convertible bonds tend to be characterized by higher market and earnings variability, higher business and or financial
risk.” A similar observation is made in Brealey, Myers, and Allen (2010), who note that “convertibles tend to be issued by the smaller and more speculative firms,” as well as in Noddings, Christoph, and Noddings (2001).19

\[
\begin{align*}
V_T & \quad \text{Financier’s payoff} \\
\gamma_F & \quad \text{high project volatility } \sigma \\
\gamma_E & \quad \text{low project volatility } \sigma
\end{align*}
\]

Figure 5: Effect of Project Volatility on Optimal Security. This figure depicts the optimal security for relatively high project volatility (solid line) and relatively low project volatility (dashed line).

We find that our model is consistent with these findings. Towards this, in Figure 5 we depict the optimal security for varying levels of the project volatility parameter \( \sigma \). The figure reveals that, as the volatility increases (going from a dashed to a solid line), the similarity between the optimal security and a convertible security becomes more pronounced, in that the middle segment becomes flatter and so tends to the fully flat corresponding segment of an actual convertible security. To understand why, recall that the role of the middle segment is to insulate the financier from an increase in the project risk in the middle status region. The higher \( \sigma \) is, the riskier is the project value other things equal (as seen from (1)), and so the higher is the need to protect the financier against the risk, resulting in a flatter middle segment for a higher volatility parameter.

19Noddings, Christoph, and Noddings (2001) consider publicly traded companies in the U.S. that have issued either convertible debt or convertible preferred stocks. Out of the companies using convertible debt, 58% are small-cap companies, 27% are middle-cap companies, and 15% are large-cap companies. For companies using convertible preferred stocks, the corresponding numbers are 47%, 39%, and 14%. This evidence, combined with the well-documented regularity that smaller companies tend to be more volatile than larger ones, suggests that more volatile companies are more likely to issue convertible securities.

20In Figure 5, \( \sigma = 0.2 \) for the dashed line and \( \sigma = 0.8 \) for the solid line. In panel (a) of Figure 6, \( \gamma_E = 3 \) for the dashed line and \( \gamma_E = 5 \) for the solid line; in panel (b), \( \gamma_F = 3 \) for the dashed line and \( \gamma_F = 5 \) for the solid line. In panel (a) of Figure 7, \( L = 2 \) for the dashed line and \( L = 2.5 \) for the solid line; in panel (b),
Figure 6: Effect of Risk Aversion on Optimal Security. Panel (a) depicts the optimal security when the entrepreneur is relatively more risk averse (solid line) and relatively less risk averse (dashed line). Panel (b) depicts the optimal security when the financier is relatively more risk averse (solid line) and relatively less risk averse (dashed line). The parameter values are provided in footnote 20.

We next explore how the optimal security depends on the entrepreneur’s and the financier’s risk aversion, \( \gamma_E \) and \( \gamma_F \), respectively. The resulting optimal securities are depicted in Figure 6. From panel (a), we see that a higher level of risk aversion for the entrepreneur leads to a steeper payoff profile of the optimal security. When she is more risk averse, the entrepreneur prefers a security that implies a lower variability of her wealth. A steeper payoff profile of the security allows the entrepreneur to shift the wealth variability from herself to the financier. Analogous reasoning explains why a higher level of risk aversion for the financier implies a flatter payoff profile of the optimal security, as depicted in panel (b). In both panels, the most notable effect is that the slope of the right-most segment of the optimal security changes. As highlighted earlier (discussion of segment C-D in panel (b) of Figure 3), this slope is linked to the conversion ratio of a convertible security. Therefore, our analysis reveals that risk aversion heterogeneity, across both entrepreneurs and financiers, is a possible explanation for convertible securities having different conversion ratios in reality.

The optimal security also depends on the status level (of wealth) \( L \) and the financier’s reservation utility \( \tilde{u}_F \). What constitutes low and high status could well differ across individu-

\[ u_F = -1 \text{ for the dashed line and } u_F = -0.5 \text{ for the solid line.} \]

The other parameter values are as in Figure 3.
als, and by varying the status level we can look at entrepreneurs with different perceptions of status. By varying the financier’s reservation utility, we explore how the optimal security depends on the distribution of bargaining power between the entrepreneur and the financier—an increase in the reservation utility of the financier corresponds to an increase in her bargaining power. The results are depicted in panels (a) and (b) of Figure 7, respectively. As discussed in Section 4, the middle segment of the optimal security corresponds to the middle status project values. Accordingly, as the status level $L$ increases, the middle segment occurs for higher project values, as seen in panel (a). When the financier’s reservation utility $\bar{u}_F$ increases, she needs to be offered a higher expected payment to finance the project. This is why the higher $\bar{u}_F$ is, the higher is the payoff of the optimal security for each project value $V^*_T$, as seen in panel (b).

Finally, in our earlier discussions, we have relied on the conjecture that in the external financing case the entrepreneur dynamically manages the product novelty $\phi$ in a way analogous to the internal financing case, which is examined in Section 3. To verify this conjecture, we plot in Figure 8 the product novelty with external financing.

We see that the entrepreneur’s behavior is similar in the external (solid lines in all panels) and internal (dashed lines) financing cases. Because with external financing a certain
Figure 8: Product novelty under external financing. Panel (a) depicts the optimal product novelty, $\phi_t^*$, in the internal (dashed line) and the external (solid line) financing cases for a relatively young project, $t = 1$. Analogously, panel (b) describes the product novelty for a relatively mature project, $t = 2$. The parameter values are as in Figure 3.

amount is paid to the financier out of the project value, the middle status region occurs for higher project values, and this is why the two humps are located at different positions on the x-axis. A less obvious result is that an increase in the product novelty may be more or less pronounced with external financing (panels (a) and (b), respectively), depending on the remaining time to the payoff date $T$. As the entrepreneur is able to share the project risk with the financier in the external financing case, she opts for a higher product novelty as compared to the internal financing case when she bears all the risk herself. This explains panel (a). However, as time passes and the payoff date approaches, maintaining the high product novelty becomes costlier as it leads to the high riskiness of the payoff to the financier, which negatively affects her expected utility. This is why in panel (b) the entrepreneur lowers the product novelty below the level under internal financing.

Remark 1. Status-based explanation for compensation with fixed and performance-related components. We believe that the model developed in this paper can be adopted to provide a status-based explanation for compensation packages with both fixed and performance-related components, which are widely used in practice. Towards this, one can consider a model that builds on the above analytical framework but adopts it to a different context. Here is a brief outline of the model. There is an employer who needs to hire a worker to manage a certain project. The employer cares about status as in our main analysis, thereby having preferences featuring convexity; the worker has no status concerns, having standard
concave preferences. The employer offers the worker a compensation package that maximizes the employer’s expected utility while providing the worker with a certain reservation utility. One aspect in which this modified model is different is that the person who will manage the project, the worker, receives the offer, whereas in the main model the person who will manage the project, the entrepreneur, makes the offer. However, as noted in footnote 8, this does not affect our main implications, and the shape of the optimal compensation package in the modified model will be similar to that of a convertible security, as depicted by the solid line in Figure 3(a). Looking at this figure, if the outcome of the worker’s actions is average, she receives an almost fixed wage (segment $B-C$ in Figure 3(a)), whereas if the outcome is sufficiently high she additionally receives a performance-dependent bonus (segment $C-D$). If her work is unsuccessful, she may be fired or the company may default, implying compensation below the promised fixed wage (segment $A-B$).

6 Conclusion

This paper develops a dynamic security design framework in which a status-driven entrepreneur dynamically manages a project. We characterize analytically the entrepreneur’s behavior in the internal financing case, where the project is financed by the entrepreneur, and in the external finance case, where the entrepreneur issues a security to finance the project. In the external financing case, our main focus, we show that the optimal security is considerably similar to a convertible security, and that this result relies on the dynamic behavior of the entrepreneur. We also provide a possible explanation of why convertible securities are mainly used by relatively volatile firms (start-ups, small firms), and also why convertible securities have different conversion ratios.

While in our model managing a project involves a single choice, the product novelty, in reality doing so involves multiple activities, such as research, development, marketing, examining the business models of competitors, etc. The entrepreneur has to allocate available resources across these activities. To account for this, one could generalize our framework by assuming that the project value is driven by multiple random factors, and that the entrepreneur chooses her loadings on these factors. We believe that our main insights would remain valid for this generalization. In our model the entrepreneur is not allowed to redeem
the security before its maturity, whereas some real-world convertible securities include call provisions allowing the issuers to do so. Incorporating this feature into our model could be an interesting generalization. Finally, as noted in footnote 16, it would be of interest to extend our model by assuming that the entrepreneur has other sources of income besides the project.
Appendix A: Proofs

Proof of Proposition 1. In proving Propositions 1-2 and B1-B2, we employ martingale methods in a continuous-time complete market setting. These methods are particularly popular in portfolio choice and asset pricing models. Accordingly, to make our analysis easier to relate to such familiar models, we consider an investment problem that is methodologically analogous to the problem faced by our entrepreneur. Specifically, consider an investor who dynamically allocates her wealth between two assets, cash and a risky asset following a geometric Brownian motion with a mean return $\mu$ and volatility $\sigma$. If we denote the investor’s time-$t$ wealth by $V_t$ and the wealth share invested in the risky asset by $\phi_t$, then, as is well-known in the continuous-time finance literature, the dynamic process for wealth $V_t$ is given by the process $[1]$. Hence, the investor with the same preferences as the entrepreneur optimally chooses the same risky wealth share as the optimal product novelty chosen by the entrepreneur $[21]$

As discussed in the portfolio choice literature (see, e.g., Carpenter (2000), Basak, Pavlova, and Shapiro (2007)), to solve our non-concave optimization problem in Definitions 1 and 2, we convert it into an equivalent concave problem by concavifying the entrepreneur’s preferences. To do so, we replace the convex part of the utility function (corresponding to middle status) with a linear segment $a + b \cdot W_{E_T}$ that is tangent to both the low-status segment of the utility function (top line in specification $[2]$) and the high-status segment (bottom line in $[2]$). Denoting the tangency points by $L$ and $T$, respectively, the parameters $a$ and $b$ of the linear segment are obtained by solving the following system of equations:

---

$[21]$ Though we employ the investment setting because of its familiarity, one can think of examples where the entrepreneur’s job is indeed similar to that of an investor. In particular, this would be the case if we considered a project consisting of several sub-projects with different expected growth rates and levels of riskiness. The entrepreneur would need to choose dynamically how to allocate her resources between these sub-projects, and would therefore, similarly to an investor, form a (optimal) portfolio of sub-projects. Aghion and Stein (2008) formally explore a similar idea by considering a model in which a firm manager allocates one unit of effort between two strategies, one increasing sales growth and the other improving profit margins. In their model, there are no status concerns and the manager does not issue financial securities, which are the key ingredients of our work.
\begin{align*}
\frac{L^{1-\gamma_E}}{1-\gamma_E} &= a + bL, \\
\frac{(\bar{L} - \alpha)^{1-\gamma_E}}{1-\gamma_E} + B &= a + b\bar{L}, \\
L^{-\gamma_E} &= b, \\
(\bar{L} - \alpha)^{-\gamma_E} &= b.
\end{align*}

(A1)

The first and second equations in this system ensure that the concavified utility function is continuous at the points $L$ and $\bar{L}$. The third and fourth equations ensure that the utility function is smooth at $L$ and $\bar{L}$. Solving the system, we obtain

\begin{align*}
a &= \frac{\gamma_E(B/\alpha)^{1-1/\gamma_E}}{1-\gamma_E}, \\
b &= B/\alpha, \\
L &= (B/\alpha)^{-1/\gamma_E}, \\
\bar{L} &= (B/\alpha)^{-1/\gamma_E} + \alpha.
\end{align*}

(A2)

Hence, the concavified utility function of the entrepreneur is

\begin{align*}
u_E(W_{E\tau}) &= \begin{cases} \\
(W_{E\tau})^{1-\gamma_E} & W_{E\tau} < (B/\alpha)^{-1/\gamma_E}, \\
W_{E\tau}(B/\alpha) + \frac{\gamma_E(B/\alpha)^{1-1/\gamma_E}}{1-\gamma_E} & (B/\alpha)^{-1/\gamma_E} \leq W_{E\tau} \leq (B/\alpha)^{-1/\gamma_E} + \alpha, \\
\frac{(W_{E\tau} - \alpha)^{1-\gamma_E}}{1-\gamma_E} + B & W_{E\tau} > (B/\alpha)^{-1/\gamma_E} + \alpha.
\end{cases}
\end{align*}

(A3)

Before proceeding with the solution, let us clarify the point made in footnote 6 that the presence of the kink in the utility function (2) does not affect our results. Because of the concavification, the shape of the utility function (2) between the wealth levels $L < L$ and $\bar{L} > L$ is essentially irrelevant. Hence, the kink at status level $L$ can be smoothed so that the concavified utility function (A3)—and hence all our results—remain unchanged.

Given the utility function (A3), the first-order condition with respect to time-$\tau$ project value $V_{\tau}$, after some manipulation, is

\begin{align*}
V_{\tau}^* &= \begin{cases} \\
(y_{\xi_{\tau}})^{-1/\gamma_E} & y_{\xi_{\tau}} > B/\alpha, \\
(y_{\xi_{\tau}})^{-1/\gamma_E} + \alpha & y_{\xi_{\tau}} \leq B/\alpha,
\end{cases}
\end{align*}

(A4)

where $y$ is the Lagrange multiplier computed from the condition that the project value $V_{\tau}^*$
is feasible: $E[\xi_t V_t^*] = V_0$, and $\xi$ is as given in Section 3. To compute the optimal time-$t$ project value $V_t^*$ for $t < \tau$, we use the fact that the process for $\xi_t V_t^*$ is a martingale, and so $V_t^* = E_t[\xi_t V_t^*]/\xi_t$. Substituting herein expression (A4), we obtain

$$V_t^* = E_t[(y\xi_t)^{-1/\gamma_E}\xi_t]/\xi_t + \alpha E_t[\xi_t 1_{\{\xi_t \le B/\alpha\}}]/\xi_t. \quad (A5)$$

To compute the two expectations in (A5), we use the fact that the state-price process $\xi_t$ is lognormally distributed, implying that $\xi_t^{1-1/\gamma_E}$ is also lognormally distributed with mean growth rate $(1 - \gamma_E)\mu^2/(2\gamma_E^2\sigma^2)$, and so

$$E_t[(y\xi_t)^{-1/\gamma_E}\xi_t]/\xi_t = (y\xi_t)^{-1/\gamma_E} e^{(1-\gamma_E)(\tau - T)\mu^2/(2\gamma_E^2\sigma^2)}. \quad (A6)$$

As $\xi_\tau$ is lognormally distributed, its truncated expected value can also be computed explicitly (e.g., Chapter 19 in Greene (2011)):

$$E_t[\xi_\tau 1_{\{\xi_\tau \le B/\alpha\}}]/\xi_t = N\left(\frac{\ln B - (\tau - t)\mu^2/(2\sigma^2)}{\sqrt{\tau - t}\mu/\sigma}\right). \quad (A7)$$

Substituting (A6) and (A7) into (A5) yields (8).

Applying Itô’s Lemma to (8), after some algebra, we obtain the diffusion term in the dynamic process for $V_t^*$ as

$$- (\xi_t\mu/\sigma) \frac{\partial V_t^*}{\partial \xi_t} d\omega_t = (\xi_t\mu/\sigma) \left[\frac{K_1(t)}{\gamma_E} (y\xi_t)^{-1/\gamma_E} + \frac{\alpha}{K_3(t)} n \left(\frac{\ln B - K_2(t)}{K_3(t)}\right)\right] d\omega_t. \quad (A8)$$

From (1), the diffusion term is $\phi^*_t \sigma V_t^* d\omega_t$, which after equating with (A8) and rearranging yields (7).

**Q.E.D.**

**Proof of Proposition 2.** During the time period $(T, \tau]$, after the financier is paid, the setting is analogous to that with internal financing, which means that the entrepreneur’s behavior is as characterized in Proposition 1. Given this, we compute the entrepreneur’s time-$T$ indirect utility function $v_E(W_{ET}) = E_T[u_E(V_t^*)]$ by using the optimality condition (A4) and taking into account that the time-$T$ project value $V_t^*$ has to be feasible given the time-$T$ project value $V_\tau = W_{ET}$. This implies that the Lagrange multiplier $y$ is now implicitly given by $E_T[\xi_\tau V_\tau^*] = \xi_\tau W_{ET}$. In particular, the entrepreneur’s time-$T$ indirect utility is given
by

\[
v_E(W_{ET}) = E_T \left[ \frac{(y_\xi)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E} 1_{\{y_\xi > B/\alpha\}} \right] + E_T \left[ \frac{(y_\xi)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E} + B \right] 1_{\{y_\xi \leq B/\alpha\}}
\]

\[
= \frac{(y_\xi)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E} K_{1T} + B * N \left( \frac{\ln(B/\alpha) - \ln(y_\xi) + K_2T}{K_{3T}} \right),
\]

where \( y \) is defined implicitly by

\[
W_{ET} = E_T[\xi_\tau (y_\xi)^{-1/\gamma_E} 1_{\{y_\xi > B/\alpha\}}]/\xi_T + E_T[\xi_\tau ((y_\xi)^{-1/\gamma_E} + \alpha) 1_{\{y_\xi \leq B/\alpha\}}]/\xi_T
\]

\[
= (y_\xi)^{-1/\gamma_E} K_{1T} + \alpha N \left( \frac{\ln(B/\alpha) - \ln(y_\xi) - K_2T}{K_{3T}} \right).\]

In what follows, we will also need the expression for the marginal indirect utility function \( v'_E(\cdot) \). Equations \( \text{(A9)} \) and \( \text{(A10)} \) define the indirect utility as a composite function \( v_E(y(W_{ET})) \), which means that

\[
\frac{dv_E}{dW_{ET}} = \frac{dv_E}{dy} \frac{dy}{dW_{ET}}.
\]

Computing the two derivatives on the right-hand side of \( \text{(A11)} \) from \( \text{(A9)} \) and \( \text{(A10)} \), respectively, we get

\[
\frac{dv_E}{dy} = -K_{1T} y^{-1/\gamma_E} \xi_T^{(\gamma_E-1)/\gamma_E} - B * n \left( \frac{\ln(B/\alpha) - \ln(y_\xi) + K_2T}{K_{3T}} \right) / (yK_{3T}),
\]

\[
\frac{dy}{dW_{ET}} = \left( -K_{1T} \xi_T^{-1/\gamma_E} y^{-1/\gamma_E} - \alpha n \left( \frac{\ln(B/\alpha) - \ln(y_\xi) - K_2T}{K_{3T}} \right) / (yK_{3T}) \right)^{-1}.
\]

Substituting \( \text{(A12)} \) and \( \text{(A13)} \) into \( \text{(A11)} \) and rearranging yields the marginal indirect utility

\[
\frac{dv_E}{dW_{ET}} = \frac{K_{1T} K_{3T} (y_\xi)^{(\gamma_E-1)/\gamma_E} + \gamma_E B * n \left( \frac{\ln(B/\alpha) - \ln(y_\xi) + K_2T}{K_{3T}} \right)}{K_{1T} K_{3T} (y_\xi)^{-1/\gamma_E} + \gamma_E \alpha n \left( \frac{\ln(B/\alpha) - \ln(y_\xi) - K_2T}{K_{3T}} \right)}.
\]

Though it is straightforward to prove that \( v_E(\cdot) \) is increasing, establishing that \( v_E(\cdot) \) is concave appears daunting given its complicated functional form. However, we have verified the concavity for a large number of model calibrations. We proceed by treating \( v_E(\cdot) \) as an increasing concave function, with \( v'_E(\cdot) > 0 \) and \( v''_E(\cdot) < 0 \).

Taking into account that the entrepreneur’s wealth at time T, after she pays the financier, is \( W_{ET} = V_T - W_{FT} \), we can equivalently write the entrepreneur’s optimization problem
(provided in Definition 1) as

$$\max_{\phi_t, W_{FT}} E[u_E(V_T - W_{FT})]$$ \hspace{1cm} (A15)

subject to  

$$dV_t = V_t\phi_t \mu dt + V_t\phi_t \sigma d\omega_t,$$ \hspace{1cm} (A16)

$$E[u_F(W_{FT})] \geq \bar{u}_F.$$ \hspace{1cm} (A17)

The first-order conditions for (A15), given the financier’s utility function (3), are

$$u'_E(V^*_T - W^*_{FT}) = z\xi_T,$$ \hspace{1cm} (A18)

$$-u'_E(V^*_T - W^*_{FT}) = z_1(W^*_{FT})^{-\gamma_F},$$ \hspace{1cm} (A19)

where $z$ and $z_1$ are the Lagrange multipliers associated with the constraints (A16) and (A17), respectively. From (A18) and (A19), we obtain

$$W^*_{FT} = (-z\xi_T/z_1)^{-1/\gamma_F}.$$ \hspace{1cm} (A20)

Substituting (A20) into (A17) yields

$$\bar{u}_F = \frac{(-z/z_1)^{1-1/\gamma_F}}{1 - \gamma_F} E[\xi_T^{1-1/\gamma_F}] = \frac{(-z/z_1)^{1-1/\gamma_F}}{1 - \gamma_F} e^{(1-\gamma_F)\mu^2 T/(2\gamma_F^2\sigma^2)}.$$ \hspace{1cm} (A21)

From (A21), the ratio $-z/z_1$ is

$$-z/z_1 = (\bar{u}_F(1 - \gamma_F))^{\gamma_F/(\gamma_F-1)} e^{\mu^2 T/(2\gamma_F^2\sigma^2)}.$$ \hspace{1cm} (A22)

Substituting (A22) into (A20) yields

$$W^*_{FT}(\xi_T) = (\bar{u}_F(1 - \gamma_F))^{-1/(\gamma_F-1)} e^{-\mu^2/(2\gamma_F^2\sigma^2)} \xi_T^{-1/\gamma_F}.$$ \hspace{1cm} (A23)

The optimal security specifies the entrepreneur’s payoff as a function of the project value $V^*_T$, and so for each state of the world $\xi_T$ we need to compute the corresponding project value
\( V^*_T(\xi_T) \). From (A18), (A10) and (A23), \( V^*_T \) is given by

\[
V^*_T(\xi_T) = K_1T(y\xi_T)^{-1/\gamma_F} + \alpha N \left( \frac{\ln(B/\alpha) - \ln(y\xi_T) - K_2T}{K_3T} \right) + (\bar{u}_F(1 - \gamma_F))^{-1/(\gamma_F - 1)} e^{-\mu^2/(2\gamma_F \sigma^2)} \xi_T^{-1/\gamma_F},
\]

where \( y \) is computed from (A14) by equating the right-hand side of (A14) to \( z\xi_T \). The Lagrange multiplier \( z \) is such that the time-\( T \) project value \( V^*_T(\xi_T) \) is feasible given the initial value \( V_0 \):

\[
E[\xi_T V^*_T(\xi_T)] = V_0.
\]

Equations (A23) and (A24) then provide the parametric characterization of the optimal security \( W^*_F(V_T) \), as stated in Proposition 2.

\textit{Q.E.D.}
Appendix B: Alternative specifications of status concerns

In this Appendix, we characterize analytically the optimal security under two alternative formulations of status concerns. In Section B1, we consider a setting with a multiplicative status concern specification, unlike the additive specification in the main analysis. In Section B2, we allow for different risk aversions in low and high-status regions.

B1. Multiplicative Status Specification

From numerous studies in finance and economics, it is understood that, when one modifies a standard utility function to account for a certain aspect of human behavior, model implications may well differ depending on whether the modification is additive or multiplicative. A prominent example is the extensive habit formation literature, in which models with multiplicative habits (e.g., Abel (1990)) often generate different predictions from those with additive habits (e.g., Campbell and Cochrane (1999)). Given this, one may wonder whether our main predictions are robust to an alternative multiplicative status specification. In this Section, we examine this question and demonstrate that our main results and insights remain equally valid.

We consider a setting that is the same as that described in Section 2, the only change being that the entrepreneur’s utility function \( u_E(\cdot) \) is now given by

\[
U_E(W_{E_T}) = \begin{cases} 
W_{E_T}^{1-\gamma_E} & W_{E_T} < L, \\
\frac{W_{E_T}^{1-\gamma_E}}{1-\gamma_E} & W_{E_T} \geq L,
\end{cases}
\]

(B1)

where \( B = (1-\alpha) L^{1-\gamma_E} / (1-\gamma_E) \). All the parameters in (B1) have the same interpretations as they have in (2). The utility function (B1) is concave-convex-concave when \( \alpha > 1 \); its shape is similar to that of (2) (see Figure 1). The case of \( \alpha = 1 \) corresponds to the (globally concave) CRRA utility function with no status concerns. Proposition B1 characterizes the optimal security; the proof is provided at the end of this subsection.

**Proposition B1** The optimal security \( W_{FT}^*(V_T) \) for the multiplicative status specification (B1) is given parametrically through a pair of functions \( (W_{FT}(x), V_T(x)) \), where \( x \) varies
from 0 to $+\infty$. The two functions are

\[
W_{FT}(x) = (\bar{u}_F(1 - \gamma_F))^{-1/(\gamma_F - 1)} e^{-\mu^2/(2\gamma_F^2 \sigma_F^2)} x^{-1/\gamma_F},
\]

\[
V_T(x) = g(x)^{-1/\gamma_E} K_{1T} N \left( \frac{-\ln b + \ln g(x) - K_4}{K_{3T}} \right) + \alpha^{1/\gamma_E} g(x)^{-1/\gamma_E} K_{1T} N \left( \frac{\ln b - \ln g(x) + K_4}{K_{3T}} \right)
\]

\[
+ (\bar{u}_F(1 - \gamma_F))^{-1/(\gamma_F - 1)} e^{-\mu^2/(2\gamma_F^2 \sigma_F^2)} x^{-1/\gamma_F},
\]

where the function $g(x)$ is implicitly given by

\[
g(x)^{-1/\gamma_E} K_{1T} K_{3T} C(g(x), 1 - \gamma_E) - B \ast n \left( \left( \frac{\ln b + \ln g(x) - K_4}{K_{3T}} \right) \right)
\]

\[
\frac{g(x)^{-1/\gamma_E} K_{1T} K_{3T} C(g(x), 1)}{g(x)^{-1/\gamma_E} K_{1T} K_{3T} C(g(x), 1)} = z x.
\]

In the above, $C(g(x))$ is

\[
C(g(x), \beta) = -N \left( \left( \frac{-\ln b + \ln g(x) - K_4}{K_{3T}} \right) \right) + n \left( \left( \frac{-\ln b + \ln g(x) - K_4}{K_{3T}} \right) \right)
\]

\[
- \alpha^{1/\gamma_E} N \left( \left( \frac{\ln b + \ln g(x) + K_4}{K_{3T}} \right) \right) - \alpha^{1/\gamma_E} n \left( \left( \frac{\ln b + \ln g(x) + K_4}{K_{3T}} \right) \right).
\]

$N(\cdot)$ and $n(\cdot)$ are the standard normal cumulative distribution function and probability density function, respectively, $B$ is as in equation (B1), $b$ is given in equation (B8), the constant $K_4$ is given by

\[
K_4 \equiv (1 - 2\gamma_E)(\tau - T)\mu^2/(2\gamma_E \sigma^2),
\]

$K_{1T}, K_{2T},$ and $K_{3T}$ are as given in Proposition 1, and $z$ is given by $E[\xi_T V^*_T(z \xi_T)] = V_0$.

Figure [B1] depicts the optimal security (panel (a)), and examines its properties with respect to the project volatility (panel (b)), entrepreneur’s risk aversion (panel (c)), and financier’s risk aversion (panel (d)). We see that the results are analogous to those obtained

\[
22\text{The parameter values are } \gamma_E = 3, \alpha = 15, L = 2, B = 1.74, V_0 = 3, \mu = 0.1, \sigma = 0.8, T = 3.5, \tau = 4, \gamma_F = 3, \text{ and } u_F = -0.5. \text{ In panel (b), } \sigma = 0.2 \text{ for the dashed line and } \sigma = 0.8 \text{ for the solid line. In panel (c)},
\]

\[34\]
Figure B1: Optimal Security and its Properties under Multiplicative Status. Panel (a) depicts the optimal security for the multiplicative status specification. Panel (b) depicts the optimal security for relatively high project volatility (solid line) and relatively low project volatility (dashed line). Panel (c) depicts the optimal security when the entrepreneur is relatively more risk averse (solid line) and relatively less risk averse (dashed line). Panel (d) depicts the optimal security when the financier is relatively more risk averse (solid line) and relatively less risk averse (dashed line).

under the additive status specification—see Figures 3, 5, 6(a), and 6(b), respectively.

Proof of Proposition B1. Because the steps of the proof are similar to those used in the proofs of Propositions 1 and 2, we provide only brief elaborations throughout the proof below.

Parameters $a$ and $b$ of the concavifying line $a + b * W_{E \tau}$ and the tangency points $L$ and $\overline{L}$ are computed from the system

$\gamma_E = 3$ for the dashed line and $\gamma_E = 5$ for the solid line. In panel (d), $\gamma_F = 3$ for the dashed line and $\gamma_F = 5$ for the solid line.
\[
\frac{L^{1-\gamma_E}}{1-\gamma_E} = a + bL, \\
\frac{\alpha + B}{1-\gamma_E}L = a + bL, \\
L^{-\gamma_E} = b, \\
\alpha L^{-\gamma_E} = b,
\]

solving which yields

\[
a = \frac{B}{1 - \alpha^{1/\gamma_E}}, \quad b = \left( \frac{(\gamma_E - 1)B}{\gamma_E(\alpha^{1/\gamma_E} - 1)} \right)^{\gamma_E/(\gamma_E - 1)}, \\
L = \left( \frac{(\gamma_E - 1)B}{\gamma_E(\alpha^{1/\gamma_E} - 1)} \right)^{1/(1-\gamma_E)}, \quad \alpha = \left( \frac{(\gamma_E - 1)B}{\gamma_E(\alpha^{1/\gamma_E} - 1)} \right)^{1/(1-\gamma_E)}. \tag{B8}
\]

The concavified utility function of the entrepreneur is

\[
\begin{align*}
\dot{u}_E(W_{ET}) &= \begin{cases} 
(W_{ET})^{1-\gamma_E} / (1-\gamma_E) & \text{for } W_{ET} < L, \\
1 - \gamma_E & \text{for } L \leq W_{ET} \leq \alpha, \\
(W_{ET})^{1-\gamma_E} / (1-\gamma_E) + B & \text{for } W_{ET} > \alpha.
\end{cases}
\end{align*}
\]

\tag{B9}

Using the first-order condition

\[
V^{*}_\tau = \begin{cases} 
(y\xi_\tau)^{-1/\gamma_E} & \text{for } y\xi_\tau > b, \\
(y\xi_\tau/\alpha)^{-1/\gamma_E} & \text{for } y\xi_\tau \leq b.
\end{cases} \tag{B10}
\]

in which \( y \) satisfies \( E_T[\xi_\tau V^{*}_\tau] = \xi_\tau W_{ET} \), we compute the indirect utility function \( v_E(W_{ET}) = E_T[u_e(V^{*}_\tau)] \):

\[
v_E(W_{ET}) = \frac{(y\xi_\tau)^{\gamma_E^{-1}}}{1-\gamma_E}K_{1T}N \left( \frac{-ln b + ln(y\xi_\tau) - K_4}{K_{3T}} \right) + \frac{\alpha^{1/\gamma_E}(y\xi_\tau)^{\gamma_E^{-1}}}{1-\gamma_E}K_{1T}N \left( \frac{ln b - ln(y\xi_\tau) + K_4}{K_{3T}} \right) + B*N \left( \frac{ln b - ln(y\xi_\tau) + K_{2T}}{K_{3T}} \right),
\]

\tag{B11}
where $y_{\xi T}$ is given by

$$W_{ET} = (y_{\xi T})^{-1/\gamma_E} K_1 T N \left( \frac{-\ln b + \ln (y_{\xi T}) - K_4}{K_3 T} \right) + \alpha^{1/\gamma_E} (y_{\xi T})^{-1/\gamma_E} K_1 T N \left( \frac{\ln b - \ln (y_{\xi T}) + K_4}{K_3 T} \right).$$

Differentiating (B11) and (B12) and rearranging, we obtain, respectively,

$$\frac{dv_E}{d(y_{\xi T})} = (y_{\xi T})^{-1/\gamma_E} K_1 T C(y_{\xi T}, 1 - \gamma_k) - \frac{B \ast n ((\ln b - \ln (y_{\xi T}) + K_2 T)/K_3 T)}{y_{\xi T} K_3 T},$$

(B13)

$$\frac{d(y_{\xi T})}{W_{ET}} = \left( (y_{\xi T})^{-1-1/\gamma_E} K_1 T C(y_{\xi T}, 1) \right)^{-1}.$$

(B14)

where $C(\cdot, \cdot)$ is given in (B5). Multiplying (B13) and (B14) yields, after some simple algebra, the marginal indirect utility

$$\frac{dv_E}{dW_{ET}} = (y_{\xi T})^{(\gamma_E - 1)/\gamma_E} K_1 T K_3 T C(y_{\xi T}, 1 - \gamma_k) - \frac{B \ast n ((\ln b - \ln (y_{\xi T}) + K_2 T)/K_3 T)}{(y_{\xi T})^{-1/\gamma_E} K_1 T K_3 T C(y_{\xi T}, 1)}.$$

(B15)

As is the case for the indirect utility function (A9), establishing analytically that $v_E(\cdot)$ given in (B15) is an increasing concave function does not appear possible. Therefore, we have verified numerically that this is the case for a large number of model calibrations.

The entrepreneur solves the optimization problem (A15) in which the indirect utility $v_E(\cdot)$ is now given by (B15). Modifying the solution of this problem presented in Proposition 2 appropriately so as to account for the different $v_E(\cdot)$ yields the optimal security presented in Proposition B1.

Q.E.D.

**B2. Different High and Low-Status Risk Aversions**

In this Section, we extend our main setting to incorporate different attitudes towards risk depending on status. This analysis is motivated by recent works in which an individual’s risk aversion differs between low and high wealth levels. Examples are Ait-Sahalia, Parker, and Yogo (2004) and Wachter and Yogo (2010). These studies show that allowing for differences in risk aversion is important when fitting the data.\(^{23}\)

\(^{23}\)Ait-Sahalia, Parker, and Yogo (2004) and Wachter and Yogo (2010) consider preferences defined over two goods, basic and luxury, where each type of good is associated with its own risk aversion parameter. In addition to controlling risk aversion, the relation between these parameters determines whether the utility function is homothetic or not. The two works show that the nonhomotheticity of the utility function—which obtains when the risk aversions are different—improves the fit to the data.
Accordingly, we assume that the entrepreneur’s utility function $u_E(W_{E\tau})$ is

$$u_E(W_{E\tau}) = \begin{cases} 
\frac{(W_{E\tau})^{1-\gamma_{E\ell}}}{1-\gamma_{E\ell}} & W_{E\tau} < L, \\
\frac{(W_{E\tau} - \alpha)^{1-\gamma_{Eh}}}{1-\gamma_{Eh}} + B & W_{E\tau} \geq L,
\end{cases}$$

(B16)

where $\alpha \in [L - L^{\gamma_{E\ell}/\gamma_{Eh}}, L)$, $\gamma_{E\ell}, \gamma_{Eh}, L > 0$, and $B = L^{1-\gamma_{E\ell}}/(1-\gamma_{E\ell}) - (L-\alpha)^{1-\gamma_{Eh}}/(1-\gamma_{Eh})$. As compared to specification (2), the novelty here is the presence of two risk aversion parameters, $\gamma_{E\ell}$ and $\gamma_{Eh}$, acting in low and high-status regions, respectively. The special case of $\alpha = L - L^{\gamma_{E\ell}/\gamma_{Eh}}$ corresponds to (globally) concave preferences without status concerns, whereas $\alpha > L - L^{\gamma_{E\ell}/\gamma_{Eh}}$ corresponds to a concave-convex-concave utility function capturing status concerns. The other parameters in (B16) have the same interpretations as the corresponding parameters in (2).

**Proposition B2** The optimal security $W^*_{FT}(V_T)$ under specification (B16) is given parametrically through a pair of functions $(W_{FT}(x), V_T(x))$, where the parameter $x$ varies from 0 to $+\infty$. The two functions are

$$W_{FT}(x) = (\bar{u}_F(1 - \gamma_F))^{-1/(\gamma_F - 1)} e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F},$$

(B17)

$$V_T(x) = g(x)^{-1/\gamma_{E\ell}} K_\ell \left( 1 - N \left( \frac{\ln b - \ln g(x) z + K_4}{K_3T} \right) \right) + g(x)^{-1/\gamma_{Eh}} K_{1h} N \left( \frac{\ln b - \ln g(x) + K_4}{K_{3T}} \right)$$

$$+ \alpha N \left( \frac{\ln b - \ln g(x) - K_{2T}}{K_{3T}} \right) + (\bar{u}_F(1 - \gamma_F))^{-1/(\gamma_F - 1)} e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F},$$

(B18)

where $g(x)$ is implicitly given by
\[
\begin{align*}
&\left[-g(x)\frac{3}{\gamma_{Eh}}K_{1h}\left(1 - N\left(\frac{\ln b - \ln g(x) + K_{4l}}{K_{3T}}\right)\right) + g(x)\frac{3}{\gamma_{Eh}}K_{1h}\left(\frac{\ln b - \ln g(x) + K_{4l}}{K_{3T}}\right) \right. \\
&\left. - g(x)\frac{3}{\gamma_{Eh}}K_{1h}\frac{3}{\gamma_{Eh}}N\left(\frac{\ln b - \ln g(x) + K_{2T}}{K_{3T}}\right) \right] = zx. (B19)
\end{align*}
\]

In the above, the constant \( b \) is implicitly given by

\[
\frac{\gamma_{Eh}b^{(\gamma_{Eh} - 1)/\gamma_{Eh}}}{1 - \gamma_{Eh}} - \frac{\gamma_{Eh}b^{(\gamma_{Eh} - 1)/\gamma_{Eh}}}{1 - \gamma_{Eh}} - \alpha b + B = 0, \quad (B20)
\]

\( K_{2T} \) and \( K_{3T} \) are as given in Proposition 1, \( K_{1l}, K_{1h}, K_{4l}, \) and \( K_{4h} \) are given by

\[
K_{1l} \equiv e^{(1-\gamma_{E})\frac{(\tau - T)\mu^2}{2\gamma_{E}^2 \sigma^2}}, \quad K_{1h} \equiv e^{(1-\gamma_{E})\frac{(\tau - T)\mu^2}{2\gamma_{E}^2 \sigma^2}},
\]

\[
K_{4l} \equiv (1 - 2\gamma_{E})\frac{(\tau - T)\mu^2}{2\gamma_{E}^2 \sigma^2}, \quad K_{4h} \equiv (1 - 2\gamma_{E})\frac{(\tau - T)\mu^2}{2\gamma_{E}^2 \sigma^2}, \quad (B21)
\]

and \( z \) is given by \( E[\xi_TV^*_T(z\xi_T)] = V_0 \).

The proof of Proposition B2 is provided at the end of this subsection. Figure B2 examines how varying the high-status risk aversion affects the optimal security. We see that the most pronounced effect is the changing slope of the right-most segment of the optimal security. As discussed in the main text, this slope reflects the conversion ratio implied by a convertible security. Hence, the existence of convertible securities with different conversion ratios can be explained by the differences in high-status risk aversion across entrepreneurs. When the entrepreneur’s high-status risk aversion is relatively high, and when she has high status, she makes the financier’s payoff for high project values more sensitive to the project value so as to reduce the sensitivity of her own wealth. This is why the slope of the right-most segment increases in high-status risk aversion. The behavior of the optimal security with respect to the other parameters is analogous to that examined in the main paper, and omitted here.
Figure B2: Effect of High-status Risk Aversion on the Optimal Security. The figure depicts the optimal security for relatively low high-status risk aversion $\gamma_{eh} = 3$ (solid line) and relatively high high-status risk aversion $\gamma_{eh} = 5$. The other parameter values are as in Figure 3.

Proof of Proposition B2. As in the proof of Proposition B1, we comment on the derivations only briefly because the logic of the proof is analogous to that in the proofs of Propositions 1 and 2.

Parameters $a$ and $b$ of the concavifying line $a + b \cdot W_{ET}$ and the tangency points $\underline{L}$ and $\overline{L}$ are given by

$$
\frac{\underline{L}^{1-\gamma_{E}}}{1-\gamma_{E}} = a + b \underline{L},
$$

$$
\frac{(\overline{L} - \alpha)^{1-\gamma_{Eh}}}{1-\gamma_{Eh}} + B = a + b \overline{L},
$$

(B22)

$$
\underline{L}^{-\gamma_{E}} = b,
$$

$$
(\overline{L} - \alpha)^{-\gamma_{Eh}} = b.
$$

Solving this system of equations, we find that the slope $b$ is given by (B20). The first-order condition is

$$
V^*_{\tau} = \begin{cases} 
(y_{\xi \tau})^{-1/\gamma_{E\ell}} & y_{\xi \tau} > b, \\
(y_{\xi \tau})^{-1/\gamma_{Eh}} + \alpha & y_{\xi \tau} \leq b,
\end{cases}
$$

(B23)

where $y$ satisfies $E_{T}[\xi_{\tau}V^*_{\tau}] = \xi_{T}W_{ET}$. Using (B23), we compute the indirect utility function
\( v_E(W_{ET}) = E_T[u_E(V^*_T)] \) as

\[
v_E(W_{ET}) = \left( \frac{(y\xi_T)^{(\gamma_E - 1)/\gamma_E}}{1 - \gamma_E} \right) K_U \left( 1 - N \left( \frac{\ln b - \ln(y\xi_T) + K_u}{K_{3T}} \right) \right) + \left( \frac{(y\xi_T)^{(\gamma_E - 1)/\gamma_E}}{1 - \gamma_E} \right) K_{1h} \nu N \left( \frac{\ln b - \ln(y\xi_T) + K_{4h}}{K_{3T}} \right) + B \nu N \left( \frac{\ln b - \ln(y\xi_T) + K_{2T}}{K_{3T}} \right),
\]

(B24)

and \( y\xi_T \) is defined implicitly by

\[
W_{ET} = \left( y\xi_T \right)^{-1/\gamma_E} K_U \left( 1 - N \left( \frac{\ln b - \ln(y\xi_T) + K_u}{K_{3T}} \right) \right) + \left( y\xi_T \right)^{-1/\gamma_E} K_{1h} \nu N \left( \frac{\ln b - \ln(y\xi_T) + K_{4h}}{K_{3T}} \right) + \alpha N \left( \frac{\ln b - \ln(y\xi_T) - K_{2T}}{K_{3T}} \right).
\]

(B25)

Computing from (B24) and (B25) the derivatives \( \frac{dv_E}{d(y\xi_T)} \) and \( \frac{d(y\xi_T)}{W_{ET}} \), respectively, and multiplying them yields the marginal indirect utility that appears on the left-hand side of equation (B19). Modifying the solution of the entrepreneur’s optimization problem (A15) presented in Proposition 2 appropriately to account for the different indirect utility—(B24) instead of (A9)—yields the optimal security presented in Proposition B2.

Q.E.D.
Appendix C: Optimal Security a Static Setting with Status Concerns

In this Appendix, we study the security design problem with status concerns in a static setting. Our goal is to investigate whether such a simpler static model could also explain the use of convertible securities. A static version of our model is obtained by assuming that the product novelty parameter $\phi$ is constant. Interestingly, relative to our main dynamic model, the static framework turns out to be neither more tractable nor does it generate a convertible security as the optimal one. This Appendix, together with the discussion in Section 3.1, provides the details.

Regarding the tractability, the static setting with status concerns turns out to be less tractable in the sense that we are able to solve it analytically only under two conditions. First, we need to assume that the product novelty parameter $\phi$ is higher than a certain threshold level. Second, it is only for the multiplicative preference specification analyzed in Appendix B1 that we are able to obtain analytical solutions. We first present and discuss the analytical solution of the static model under the two conditions. We then provide a numerical solution of the static setting without these conditions to verify that they do not affect our conclusions. In what follows, the two analyses are entitled “Tractable Analysis” and “Numerical Analysis,” respectively.

Tractable Analysis

We consider an economy analogous to that presented in Section 2 but assume that the entrepreneur’s preferences are given by the multiplicative specification (B1). The product novelty parameter $\phi$ is constant and, for tractability, exceeds a certain threshold (as explained in the proof of Proposition C1). Accordingly, the entrepreneur’s optimization problem is as given in Definition 1 of Section 2 but without the product novelty as a choice variable, and so the only choice is what security to issue. In the numerical analysis below, we allow the entrepreneur to also choose the product novelty and find that this does not affect our results. Proposition characterizes the optimal security.

Proposition C1 The optimal security $W^*_{FT}(V_T)$ in the static setting is given implicitly by
where the function \( f(\cdot) \) is

\[
f(x) = x^{-\gamma_E} K_1 N \left( \frac{\ln(L/x) - K_2}{\phi^* \sigma \sqrt{\tau - T}} \right) - \frac{(x)^{-\gamma_E} K_1 n}{1 - \gamma_E} \left( \frac{\ln(L/x) - K_2}{\phi^* \sigma \sqrt{\tau - T}} \right) / (\phi^* \sigma \sqrt{\tau - T}) \]

\[+ \alpha x^{-\gamma_E} K_1 N \left( \frac{-\ln(L/x) + K_2}{\phi^* \sigma \sqrt{\tau - T}} \right) + x^{-\gamma_E} K_1 n \left( \frac{-\ln(L/x) + K_2}{\phi^* \sigma \sqrt{\tau - T}} \right) / (\phi^* \sigma \sqrt{\tau - T}) \]

\[+ Bn \left( \frac{-\ln(L/x) + K_3}{\phi^* \sigma \sqrt{\tau - T}} \right) / (x \phi^* \sigma \sqrt{\tau - T}). \tag{C2}\]

In the above, the constants \( K_1, K_2, \) and \( K_3 \) are given by

\[
K_1 \equiv e^{(1-\gamma_E)(\phi^* \mu - \gamma_E (\phi^*)^2 \sigma^2 / 2) (\tau - T)}, \quad K_2 \equiv (\phi^* \mu + (0.5 - \gamma_E) (\phi^*)^2 \sigma^2) (\tau - T), \quad K_3 \equiv (\phi^* \mu - \phi^* \sigma^2 / 2) (\tau - T), \tag{C3}\]

and the optimal product novelty \( \phi^* \) and the Lagrange multiplier \( z \) are computed as given in the proof.

Figure 4 in Section 4.1 plots the optimal security under the same parameter values as in Figure B1.

**Numerical Analysis**

We resort to numerical analysis to show that the structure of the optimal security in the static setting remains the same when we adopt the additive status specification, as in the main body of the paper, as well as allow the entrepreneur to choose an arbitrary project value parameter \( \phi \). For a constant \( \phi \), the project value process is a geometric Brownian motion and so its realization at time \( T \) can take any value from a continuous set \((0, +\infty)\). Given the (uncountably) infinite number of realizations of the project value, coupled with non-standard preferences, solving this model even numerically appears challenging. We therefore discretize the project value process so as to obtain a binomial tree. This is a standard approach to approximating a geometric Brownian motion, and is discussed in detail in Cox, Ross, and Rubinstein (1979).

We have solved the model numerically under different parametrizations, and the results...
are similar. Figure C1 depicts a typical shape of the optimal security. Given the discretization, we note that the optimal security’s payoff profile is not a continuous function but is a function defined on a discrete finite set of future project values. The profile presented in the Figure is simply obtained via linear interpolation. We see that the optimal security is similar to that presented in Figure 4 in Section 4.1.

![Figure C1: Optimal Security in a Binomial Static Setting.](image)

The model parameters are as in Figure 3. Given the parameters of the project value process, we build the corresponding binomial tree with a quarterly time step.

**Proof of Proposition C1.** Given that $\phi$ remains unchanged after time 0, the project value $V$ follows a geometric Brownian motion process

$$dV_t/V_t = \phi \mu dt + \phi \sigma d\omega_t.$$  

Hence, sitting at time $T$, the logarithm of time-$\tau$ project value is distributed as

$$\ln V_\tau \sim N(\ln V_T + (\phi \mu - \phi^2 \sigma^2 / 2)(\tau - T), \phi^2 \sigma^2 (\tau - T)).$$
Using this expression, we compute the entrepreneur’s time-T indirect utility function \( v_E(V_T) = E_T[u_E(V_T^*)] \), where \( V_T \) is time-T project value after the financier is paid. This yields

\[
v_E(V_T) = \frac{(V_T)^{1-\gamma_E}}{1-\gamma_E} K_1 N \left( \frac{\ln(L/V_T) - K_2}{\phi \sigma \sqrt{\tau - T}} \right) + \alpha \frac{(V_T)^{1-\gamma_E}}{1-\gamma_E} K_1 N \left( \frac{-\ln(L/V_T) + K_2}{\phi \sigma \sqrt{\tau - T}} \right) \\
+ \frac{BN}{(\frac{-\ln(L/V_T) + K_3}{\phi \sigma \sqrt{\tau - T}})}
\]  

(C4)

where \( K_1, K_2, \) and \( K_3 \) are given in (C3). The optimal security is the solution of the problem:

\[
\max_{W_{FT}} E[v_E(V_T - W_{FT}(V_T))], \\
E[u_F(W_{FT})] \geq \bar{u}_F.
\]

We have numerically examined the shape of the function \( v_E(V_T) \) for a large number of model parametrizations and have established that it is concave if the product novelty \( \phi \) exceeds a certain threshold (whose value depends on the parametrization), and is concave-concave-convex otherwise. We are only able to solve this problem analytically in the standard case of a concave function \( v_E(V_T) \), and so to achieve this we assume that the product novelty exceeds the threshold.

The first-order condition is

\[-v'_E(V_T - W_{FT}) = z W_{FT}^{-\gamma_F}. \]  

(C5)

Differentiating \( v_E(\cdot) \) given in (C4), we obtain that the marginal indirect utility is \( v'_E(x) = f(x) \), where \( f(\cdot) \) is as given in (C2), and so the optimal security (C1) obtains. The Lagrange multiplier \( z \) is such that the financier’s expected utility is equal to her reservation utility \( \bar{u}_F \).

\[Q.E.D.\]
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