

Coupon Effects and the Pricing of Japanese Government Bonds: An Empirical Analysis

YOUNG HO EOM, MARTI G. SUBRAHMANYAM, AND JUN UNO

YOUNG HO EOM is with the Federal Reserve Bank of New York and Yonsei University in Korea.

MARTI G. SUBRAHMANYAM is the Charles E. Merrill professor of finance and international business at the Stern School of Business at New York University in New York, New York.

JUN UNO is general manager of NIKKEI-QUICK Information Technology Company Ltd. in Japan.

The term structure of interest rates describes the relationship between the yield to maturity of zero-coupon Treasury bonds (the spot interest rate) and the maturity of the bonds. Estimation of the term structure of interest rates implied by coupon Treasury bonds is important for analysts and participants in the securities markets for a variety of reasons.

For investors and traders, the term structure at any given time provides a basis for identifying over- and undervalued bonds, for hedging portfolios, and for evaluating investment performance. The term structure of interest rates also serves as a (default-free) benchmark for the pricing of defaultable financial assets such as corporate bonds, as well as derivative securities such as interest rate swaps, caps, and floors. For policymakers, the term structure of interest rates is a standard input in monetary policy analysis, since it reflects the current expectations of market participants about the evolution of future interest and inflation rates.

In modeling the term structure of interest rates, a common (or a "representative" agent's) discounting function is often postulated for the valuation of future cash flows. The reason is that, theoretically speaking, in a frictionless market, the term structure of interest rates does not depend on any particular investor's preferences for consumption over future periods. This is due to the arbitrage mechanism by which investors

can take long and short positions to profit from differences, if any, in the implied valuation of cash flows across securities. Consequently, in equilibrium, there is a common, marketwide, valuation function for future cash flows across investors.¹

This theoretical argument is at odds with the predictions of traditional hypotheses about the term structure of interest rates such as the market segmentation hypothesis and the preferred habitat theories, both of which imply that each bond is somewhat unique and appeals to a particular investor clientele. Such clientele hypotheses rest on the assumption that market frictions such as differential taxation of income and capital gains, transaction costs, constraints on short sales, and regulatory restrictions inhibit the arbitrage mechanism, causing investors to choose to hold bonds whose characteristics best match their own preferences.

Even in the presence of market frictions, however, the "representative" term structure of interest rates may still serve as a good summary of market information for practical purposes. This would be valid as long as the prices of bonds in the equilibrium with and without frictions closely resemble each other.

This article examines several issues relating to the modeling of the term structure of interest rates and the pricing of Japanese government bonds (JGBs) issued by the Japanese Treasury during the period 1990-

1996. The JGB market is one of the largest government bond markets in the world, second only to the U.S. Treasury market. The outstanding amount of JGBs was about \$2 trillion at year-end 1996. (The comparable figure for the U.S. Treasury market was \$5.3 trillion.)

Despite its size, the JGB market is known to have several anomalous features that inhibit its efficiency, of which two are significant: illiquidity of bonds other than the benchmark bond, and a "coupon effect," whereby bonds are priced differently from the present value of their future cash flows, depending on their coupon rates.² These features are attributable to market frictions such as taxes and transaction costs (including the costs of establishing a short position), as well as regulatory restrictions on the financial institutions that are major participants in the JGB market, all of which reduce the general efficiency of the market.

For example, Kikugawa and Singleton [1994] use data for the period from 1990 through 1992 to conclude that there are substantial coupon effects in the JGB market. They suggest that the term structure models that have been widely applied to other Treasury markets such as the U.S. and the U.K. should be used with care in the JGB market, in view of various institutional considerations.

Recently, efforts have been made by the regulatory authorities to increase market efficiency through changes in both the primary and the secondary segments of the market. As a result, the JGB market has undergone a structural change in its trading patterns, which should have an effect on the term structure.³

It is of interest, therefore, to investigate how the term structure in Japan has changed over time in the recent period of regulatory liberalization, and to explore key features to be considered in modeling the term structure in the JGB market. Our purpose is twofold:

1. To examine the structural changes in the JGB market in the 1990s and assess their impact on JGB prices.
2. To analyze the main features of the term structure of interest rates in Japan during the 1990s, and to explore the possibility of applying multifactor term structure models in the yen market.

I. INSTITUTIONAL CHARACTERISTICS OF THE JGB MARKET

The JGB market, like other treasury bond markets, has several frictions that cause individual bonds to

be priced differently from the "average" pricing in the market indicated by the term structure of interest rates. Two of these frictions can be clearly related to the institutional characteristics of the JGB market: the benchmark and coupon effects.

The benchmark effect arises due to the fact that a substantial part of the market trading volume is concentrated in a single bond. As a result, the demand for this benchmark bond and hence its price incorporates the liquidity offered by the bond, in addition to the factors that influence the price of any other government bond.

The coupon effect can be attributed to tax, accounting, and regulatory factors that cause investors to value bonds differently, depending on whether their coupon is high or low.

The Benchmark Phenomenon in the JGB Market

During most of the period since the inception of the JGB market in 1966, a substantial proportion of the trading volume in the secondary market has been concentrated in one bond, known as the benchmark bond. More recently, this proportion has been steadily declining.

There is neither a systematic basis nor a formal process for how a particular bond is selected to be a benchmark bond, although benchmark bonds have three major characteristics: 1) a relatively large notional amount outstanding, 2) a remaining time to maturity that is long, typically at least nine years, and 3) a coupon rate close to the current par bond yield.⁴

Historically, these characteristics have been closely related to the needs of the major participants who underwrite and distribute JGBs in the market. Prior to 1977, the major Japanese banks were licensed by the Bank of Japan to be part of the underwriting syndicate, and were each allocated part of every new issue of JGBs. They were not allowed to resell these bonds in the market, with the implicit understanding that the Bank of Japan would purchase them from the underwriters after a period of one year.

As increasing budget deficits in Japan led to large amounts of bonds being issued over time, the underwriting banks came to be exposed to substantial interest rate risk. In 1977, financial institutions in the underwriting syndicate were allowed to sell JGBs in the market after a minimum holding period of one year. The holding period was further shortened to 100 days in 1981.

The interest rate risk management problem existed even after the restriction on resale was lifted in the mid 1980s, because there was still about a one-month period between the auction and the actual issue of a bond. Since "when-issued" (essentially forward) trading was not permitted, unlike in the U.S., the institutions remained subject to substantial interest rate exposure.

A major concern for Japanese financial institutions engaging in bond dealing was to manage their asset-liability mix and hedge the interest rate risk of their bond holdings, within the institutional constraints. It was necessary, therefore, to identify a hedge instrument that offered reasonable liquidity. Consequently, the trading of JGBs was concentrated in a particular bond with a maturity of at least nine years, because a large proportion of JGBs issued were of ten-year maturity. Thus, a bond with coupon and duration characteristics similar to those of a majority of the bonds in the current portfolios of the financial institutions became an ideal hedge instrument for adjusting the interest rate risk of their holdings.

Since institutions frequently traded a particular bond to adjust their interest rate risk, a large outstanding amount of the bond was important in order to reduce the market price impact of their transactions.⁵ The preference for bonds selling at a price close to par is partly related to the desire to have a hedging vehicle whose characteristics match those of recently issued bonds and partly to the "coupon effect," discussed later.

In general, in most markets, the bond yields of illiquid bonds tend to be higher than those of liquid bonds. For example, the liquidity differences between "on-the-run" and "off-the-run" bonds in the U.S. Treasury market have a significant price effect, as noted by Elton and Green [1997].

In related evidence, Garbade [1984], Kamara [1994], and Amihud and Mendelson [1991] find that Treasury note yields are higher than Treasury bill yields of similar maturity, and suggest that the yield differential is a result of differences in liquidity. This phenomenon was especially important in the Japanese bond market until recently, since the liquidity effect was exacerbated due to the heavy concentration of trading in the benchmark bond. Boudoukh and Whitelaw [1991] report that the yield spread between a basket of "off-the-run" issues and various benchmark issues over time, averaged 40-60 basis points, with a spread as high as 100 basis points during 1986 and 1987.⁶

During the 1990s, however, the relative trading volume of the benchmark bond and its premium have undergone significant changes. As a result of less concentration of trading in the benchmark bond, the benchmark premium has gradually contracted, and the liquidity premium of the most recent benchmark bond (#182) is almost non-existent.⁷

Exhibits 1 and 2 summarize the characteristics of benchmark bonds and their relative volumes in the 1990s, based on monthly trading volume in the over-the-counter market.⁸ The table and the figure show a clear downward trend of the relative trading volume of

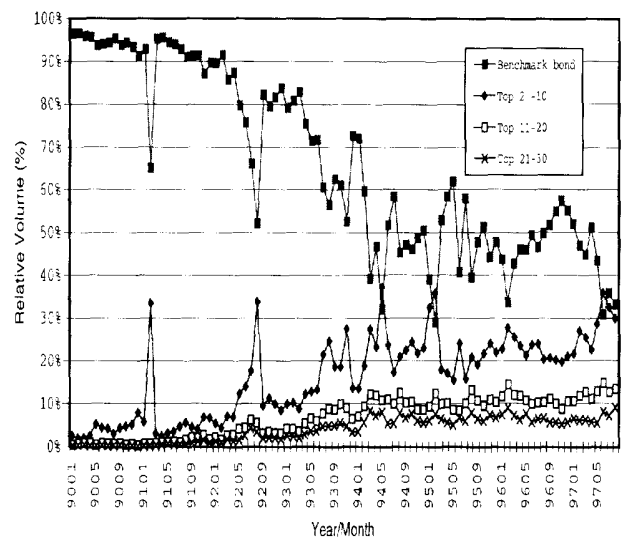
EXHIBIT 1

Characteristics of Benchmark Bonds in JGB Market

Issue	Coupon (%)	Amount Outstanding (million yen)	Period From-To (Months)	Time to Maturity (Year)	Relative Volume Average (Min-Max) (%)
119	4.8	1,851,800	Nov 89-Feb 91 (16)	9.6-8.3	94.6 (91-97)
129	6.4	2,300,000	Feb 91-Aug 92 (20)	9.1-7.6	88.2 (66-96)
145	5.5	4,068,000	Sep 92-Nov 93 (15)	9.5-8.3	70.8 (53-84)
157	4.5	5,002,000	Dec 93-May 94 (6)	9.5-9.1	54.4 (39-72)
164	4.1	1,004,000	Jun 94-Feb 95 (9)	9.5-8.8	47.8 (39-58)
174	4.6	1,294,000	Mar 95-Feb 96 (12)	9.5-8.6	49.3 (39-62)
182	3.0	2,464,000	Mar 96-Sept 97 (19)	9.5-7.7	46.7 (31-57)

The relative volume of individual benchmark JGBs is computed as a proportion of total JGB volume (relative volume = OTC volume of a JGB/total OTC volume of all JGBs). Source: Nakagawa [1992] and statistics published by Japan Bond Trading Co. Ltd.

EXHIBIT 2 Relative Trading Volume of Benchmark JGBs



the benchmark bond, from an average of 94.6% for bond #119 during the period November 1989–February 1991, to 46.7% for bond #182 during the period March 1996–September 1997.⁹ In individual months, the relative volume went from a high of 97% in February 1990 to a low of 31% in July 1997.

Consistent with this pattern, Exhibit 2 also shows that the relative trading volume of the non-benchmark bonds (bonds ranked 2–10 in trading volume) was under 5% in 1990, 20% in 1994, rising to 30% in 1997. Similarly, the bonds in ranks 11–20 and 21–30 are also traded in increasingly larger amounts relative to the whole market for JGBs in the over-the-counter market.

Exhibit 3 shows the relative volume of trading for the ten-year and six-year JGB issues. There were only two non-benchmark JGBs [#129 (5.1%) and #130 (1.4%)] that had more than 1% of the total volume in December 1990. In December 1996, sixteen non-benchmark JGBs had more than 1% of the relative trading volume.

Thus, overall, the importance of the benchmark bond, measured by its relative trading volume, has been declining during the 1990s, while the relative trading volume of non-benchmark bonds has been increasing during the same period.

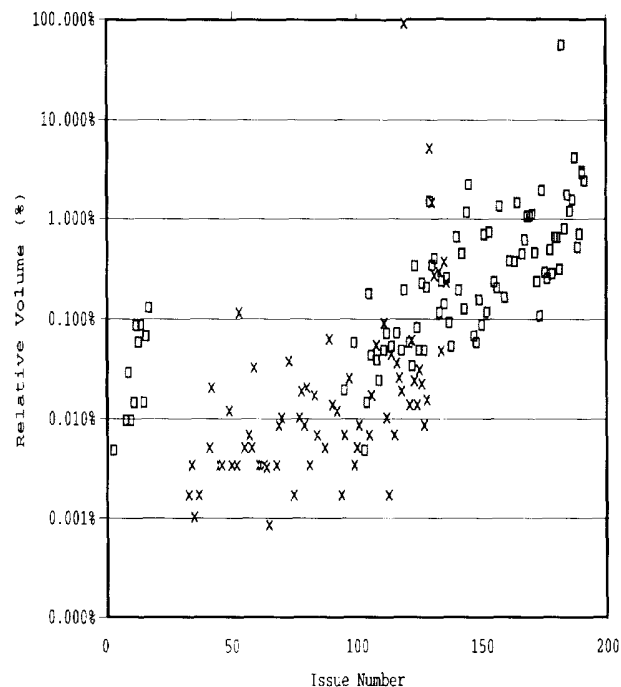
The reduced concentration of trading in the

benchmark bond in the 1990s is attributable to several structural changes in the primary and secondary markets. In April 1989, 40% of the face amount of new issues was issued by auction rather than by allocation, and after October 1990, the proportion determined by auction increased further to 60% of the total amount issued. Additional liquidity was provided to the market by the issue of JGBs of four-year and six-year maturities by the Japanese Treasury starting in November 1993 and February 1994, respectively. In addition, a medium-term JGB futures contract was introduced in February 1996.

These institutional changes reduced the high demand for the benchmark bond and increased the relative volume of trading of non-benchmark JGBs, because of reduced hedging needs for the underwriters' holdings of newly issued bonds.

In the secondary market, the liquidity of non-benchmark JGBs increased with the development of the securities lending business. Hence, it became possible to exploit arbitrage opportunities more easily and eliminate the relative mispricing of JGBs between the cash and

EXHIBIT 3 Relative Trading Volume of Individual JGBs



futures markets. In addition, the liquidity in the futures market, driven by low transaction costs, spilled over to the cash market for even non-benchmark bonds.

Coupon Effect

The other important feature of the JGB market is the "coupon" effect, which describes the preference of some Japanese investors for bonds with higher coupon payments, all other factors being the same. Instead of maximizing the total yield from a bond (including capital gains/losses), these investors are interested in earning a high current yield, measured by the ratio of the coupon to the market price. This motivation stems from regulatory and tax considerations.

For example, because Japanese insurance laws prohibited the payment of dividends on their policies out of capital gains from their investments, insurance companies tended to favor high-coupon bonds that yield income within the fiscal year.¹⁰ Similarly, Japanese banks sometimes act to boost their current income ratio by investing in high-coupon bonds, since the current income ratio is an issue of concern to Japanese bank regulators. Thus, higher-coupon bonds are priced "rich" relative to other bonds of similar maturity or duration, thereby providing a lower yield to maturity. Hence, there is an inverse relationship between the coupon of the bond and its yield to maturity.

In the U.S. and U.K. Treasury markets, the relationship between the coupon and the yield on bonds is usually the opposite — high-coupon bonds normally trade at higher yields than low-coupon bonds with the same maturity and duration. This effect can be attributed to institutional factors, primarily the differential taxation of capital gains and ordinary income. Investors have an incentive to invest in low-coupon bonds because of preferential tax treatment of capital gains in the U.S. and U.K.¹¹ Further, in these markets, investors with long-dated liabilities may also want to hold low-coupon bonds due to the risk associated with reinvesting the coupon income.

Thus, the coupon effect in the JGB market is mostly driven by the regulatory structure, while in the U.S. and U.K., it is driven by the asymmetric tax treatment of income versus capital gains, as well as by reinvestment risk considerations.

A variation of the coupon effect in the JGB market is the "par effect," resulting in investors showing a preference for recent bond issues with coupons close to

current market yields, i.e., bonds priced near par. This preference for "near-par" bonds is closely related to the historical cost accounting system of Japanese financial institutions.

For example, bonds priced above par are avoided by some institutions, such as public pension funds, that follow a buy-and-hold strategy, since their accounting rules do not allow for accumulation or amortization of premiums/discounts relative to par. Other public entities, such as insurance cooperatives for government institutions, have internal rules prohibiting the buying of securities that would lead to recorded losses on redemption.¹²

In principle, these anomalous features in the JGB market may have existed mainly due to market frictions. The magnitude of the benchmark effect and coupon effects would depend on the extent of market frictions, such as the costs of short-selling bonds and other transaction costs. In the 1990s, the costs involved with short sales have been declining. Lending of JGBs started concurrently with the lifting of restrictions on short sales in 1989. Since transactions were generally not collateralized due to interest rate regulations on cash-collateralized securities lending, the volume was initially relatively small, but grew over time.¹³

Another important friction in the JGB market relates to the settlement aspects of the tax system. For example, for tax-exempt entities and designated financial institutions including banks, government agencies, and dealers, there is no withholding tax on the coupon payments from bonds held in a registered form.¹⁴ Due to withholding tax, which complicates the payment of accrued interest for trade settlement, bonds are usually kept in clean-registered form for trading purposes, and non-financial institutions and non-residents commonly leave securities in the name of designated financial institutions (street name) until the coupon date.¹⁵ As a result, JGB trading is segmented by type of trades and holder, and arbitrage by non-residents, who are not exempt from withholding taxes, is restricted.

The securities transaction tax, which requires sellers to pay a certain percentage of the transaction value of the securities as tax, also affects the transaction costs of different segments of the market. For example, dealers pay a tax of 1/10,000 of the selling price, regardless of the length of holding period. Therefore, the transaction tax is effectively greater for bonds with a shorter maturity, and hence, dealers demand a higher bid-ask spread at the shorter end of the term structure to cover these tax costs.¹⁶

II. DATA AND ESTIMATION METHODS

Data

In order to estimate the term structure of interest rates implied by the JGB market, we collected weekly (Friday) closing prices on the Tokyo Stock Exchange of non-benchmark Japanese government bonds (JGBs) of different maturities. The source of the data is the NEEDS data base, maintained by the Nihon Keizai Shimbun.¹⁷

We chose to analyze ten-year JGBs due to limited availability of data on JGBs of different maturities. Only ten- and twenty-year (maturity at issue) JGBs are listed on the Tokyo Stock Exchange and, of these, only ten-year JGBs have a regular auction cycle. These data were available for the period April 27, 1990, through May 17, 1996, yielding a total of 26,685 observations.

The prices are based on "small-lot transactions" on the Tokyo Stock Exchange at 3:00 pm. Small-lot transactions are concentrated on the organized exchange, with prices set once a day at 3:00 pm, based on a call auction, and quoted in one sen (0.01 yen) increments. In the absence of small orders, major brokers are responsible for providing quotations based on the transaction prices in the over-the-counter (OTC) market.¹⁸

As in the case of the U.S. Treasury bond market, the drawback of using data from the exchange is that the relative trading volume is rather small.¹⁹ Thus, because of sparse trading on the exchange, the prices we obtain may not always be representative of market conditions, and hence may be a "noisy" representation of the true price.

The exchange quotations are unlikely to be biased, however, since trades on the exchange are conducted mainly to set an "authorized price." Only authorized prices on the Tokyo Stock Exchange are accepted for the settlement of futures contracts, as well as for financial and tax accounting. Hence, any systematic bias in the prices on the exchange compared to OTC market is likely to be arbitrated away.

Estimation of the Term Structure of Japanese Government Bonds

In a world without taxes, liquidity effects, and other frictions, the price of a bond on a coupon date is the present value of the promised future cash flows:²⁰

$$P_j(t, t + M_j) = \sum_{m=1}^{M_j} c_{jm} \exp(-i(t, t + m)m) + F_j \exp(-i(t, t + M_j)M_j) \quad (1)$$

$$= \sum_{m=1}^{M_j} c_{jm} D(t, t + m) + F_j D(t, t + M_j) \quad (2)$$

where $P_j(t, t + M_j)$ is the price at time t of bond j with a maturity M_j ; c_{jm} is its coupon at time m , $m = 1, 2, \dots, M_j$; and F_j is its face value paid at M_j . The continuously compounded spot interest rate between time t and $t + m$ is defined as $i(t, t + m)$. The discount or present value function is defined by $D(t, t + m) = \exp(-i(t, t + m)m)$. Hence, the term structure of interest rates at time t can be described by the present value function, or, equivalently, the vector of spot interest rates (or forward interest rates).

Market prices are available for bonds of only certain maturities, while spot rates are required for all maturities in order to define the whole term structure of interest rates. It is necessary, therefore, to fit an approximate discounting function for the spot or forward rates to obtain the rates for all possible maturities. The rationale for this approach is that a general functional specification of discount factors can explain all current bond prices as closely as possible. Further, this allows us to obtain flexible spot curves that are not dependent on a specific theory of the term structure of interest rates and thus can explain virtually all the common shapes that are encountered in the market.

There is a broad variety of approximating functions used in practice. A common approach uses polynomial functions based on cubic splines, originally proposed by McCulloch [1971, 1975], with variations such as exponential splines proposed by Vasicek and Fong [1982] and B-splines used by Lantier and Smoot [1989] and Steeley [1991].²¹ The spline methods require a large number of observations relative to other polynomial methods, since more coefficients have to be estimated.

For the estimation of the term structure of interest rates in the JGB market, we use the cubic B-spline method, a modification of the method proposed by McCulloch. The motivation for this modification of the cubic-spline method is the finding by Shea [1984] that some spline bases, such as those chosen by McCulloch [1971, 1975], can result in inaccuracies due to a nearly perfectly collinear regressor matrix. The essential features of the cubic B-spline method are discussed in the appendix.

A practical consideration in implementing spline methods is the "knot" placement scheme that specifies the number of knots and the partitioning of the knots. The number of knots (or points on the term structure between which the coefficients of the spline function are fixed) determines the variety of feasible shapes of the term structure permitted by the estimation procedure.

If too many knots are used, the resulting term structure may be too "wiggly," since it fits almost all the observed data points. On the other hand, the use of only a few knots produces a fit that may be too smooth, resulting in larger fitting errors. Thus, there is a trade-off between the smoothness of the function and the fitting errors. Hence, the choice of the number of knot points is analogous to the number of independent variables in a regression.

The other related issue is the partitioning of the knots. In Litzenberger and Rolfo [1984], three interior knots are placed at one year, five years, and ten years, while McCulloch [1975] places knots that yield an equal number of observations in each maturity interval. We use four interior knots at two years, four years, six years, and eight years over the maturity range of zero to ten years, since our observations are roughly equally spaced across maturities.

Estimation of Coupon Effects

There have been very few empirical studies of the coupon effects in the JGB market, and even these refer to the early 1990s.²² The coupon effects are unlikely to have been stable over time due to the major structural changes in 1990s, such as the deregulation of financial markets and institutions in Japan in general. Hence, it is necessary to examine the data for the whole period from 1990 until 1996 to investigate the coupon effect.²³

Despite the potential importance of coupon effects in the JGB market, it is difficult to parameterize their influence on bond prices. There are two possible explanations for the coupon effect: taxes and regulation. If the coupon effect is driven mostly by the asymmetric tax treatment of income and capital gains as in the U.S., we could measure it empirically using an explicit model of differential taxation.

The typical approach to the measurement of the tax effect is to assume that an equilibrium in the market exists with no clientele effect, so that outstanding bonds are priced correctly for every tax bracket. Under

this assumption, the discount factors (i.e., the term structure) for a given tax rate are estimated via a non-linear regression, as in the studies by Jordan [1984], Litzenberger and Rolfo [1984], and Green and Odegaard [1997].²⁴

If the coupon effect is not necessarily driven by tax considerations, it can be estimated by assuming a general relationship between the coupon and the bond price. In Vasicek and Fong [1982], for example, the price discount attributed to the coupon effect is assumed to be proportional to the current yield. Garbade [1984], on the other hand, uses piecewise linear terms based on the differential between the yield of the bond and par bond yield with the same maturity.

We model the relationship between the bond price and the coupon in a non-linear fashion. This choice is motivated by the fact that, in Japan, the coupon effects can be mainly attributed to non-tax considerations such as regulation and accounting practice. Furthermore, linear parameterization of the coupon effect is problematic, since there are two distinct types of coupon effects. The first is due to accounting and regulatory factors, inducing a demand for bonds with higher coupons, all else being equal. The other is due to the preference for bonds trading close to par.

To circumvent the difficulties in parameterizing non-present value factors on the JGB prices, we adopt an orthogonal series estimator based on Legendre polynomials. The flexibility of semi-non-parametric (or non-parametric) regression enables us to estimate the relationship without an a priori parametric model and could help in modeling simpler parametric formulations of the coupon effects.

Specifically, we estimate the functional form of non-present value factors such as current yield (defined as C/P) and the deviation from par defined as $(P - 100)/100$ where C is the coupon and P is the "dirty" market price (i.e., the price including accrued interest). Suppose that the regression function can be represented as an orthogonal series:

$$m(x) = \sum_{j=0}^{\infty} \beta_j \vartheta_j(x) \quad (3)$$

where $\{\vartheta_j\}_{j=1}^{\infty}$ is a basis of functions, and $\{\beta_j\}_{j=1}^{\infty}$ are the unknown coefficients. Since only a finite number of observations are available, the regression function is approximated by

$$\hat{m}(x) = \sum_{j=0}^k \hat{\beta}_j \vartheta_j(x) \quad (4)$$

In the estimation, we choose to use the Legendre polynomials that constitute an orthogonal system of functions on $[-1, 1]$.²⁵

III. EMPIRICAL RESULTS

Fitted Price Errors

The absolute fitted price error is defined as the absolute difference between the actual bond price including accrued interest (the "dirty" bond price) and the model bond price, $|P_{\text{actual}} - P_{\text{model}}|$ based on the 26,685 observations in our sample. The relative absolute fitted error is defined as the ratio of the absolute fitted price error to the JGB price, i.e., $|(P_{\text{actual}} - P_{\text{model}})/P_{\text{actual}}|$. The model prices are based on the cubic B-spline method, estimated for each trading day with four internal knots spaced equally between zero and ten-years.

Exhibit 4 summarizes the mean and standard deviation of the absolute fitted price errors and the relative absolute fitted price errors for the JGBs during the sample period. Panel A of the table provides the summary statistics of the absolute fitted price errors grouped by year, and Panel B by time to maturity of the bonds.

The table shows that the mean of the absolute fitted price error in the JGB market is on the order of 11 basis points, with a standard deviation of about 11 basis points, which is somewhat higher than the errors found in comparable studies in the U.S. Treasury bond market. For example, Bliss [1996] reports a mean absolute fitted price error of around 6-11 basis points, indicating the greater liquidity and the smaller effect of non-present value factors in the case of the U.S. Treasury bond market.

Thus, in spite of the fact that liquidity and coupon effects play a greater role in the JGB market, the absolute fitted price error in the JGB market is not all that different from the U.S. Treasury bond market. Overall, the cubic B-spline method yields reasonable estimates of the term structure of the JGBs.

Given the deregulatory trend in the 1990s, we would expect that pricing errors would decline over time, if liquidity and coupon effects have indeed diminished. Panel A shows that the mean and the stan-

dard deviation of the absolute fitted price error became smaller in recent years. They declined from around 14.4 basis points (with a standard deviation of 15.8 basis points) in 1990 to about 10.6 basis points (with a standard error of 8.7 basis points) in 1996, indicating that the effect of non-present value factors became somewhat muted over time.

Panel B of the table reports the mean and standard deviation of the absolute fitted price error of the JGBs grouped by time to maturity. The magnitude of the absolute fitted price error increased with the time to maturity, from 7.2 basis points (with a standard error of 6.6 basis points) for the short maturities to 13.1 basis points (with a standard error of 13.3 basis points) for the long maturities. This result is also consistent with Bliss [1996], who finds, in addition, that the poor fit for long-term bonds in the U.S. Treasury market is not sensitive to the choice of estimation method.

Empirical Results of the Coupon Effects

Exhibit 3 presents the statistics on the relative trading volume of individual JGBs during three different periods. It shows that in each period, the trading volume is positively related to the issue numbers of the bonds. The issue numbers of the bonds indicate the chronological sequence of the issues, and hence their age.

Thus, trading volume is inversely related to the remaining time to maturity.²⁶ This result is in line with the findings of Bliss [1996] and Sarig and Warga [1989] who also use the age of a bond as a proxy for its liquidity.

In the regression, the coupon yield measures the tax and regulatory effects discussed earlier in relation to the time to maturity and the liquidity effects. We use two sets of variables to measure the coupon effects caused by tax and regulatory factors. The first set is based on the relative price premium/discount of a bond, $(P - 100)/P$, where P is the market price of a bond including accrued interest, and measures the proportionate deviation from par.

Since premium and discount bonds may exhibit different behavior, we construct a piecewise linear relation by defining two variables, the relative discount of a bond $\text{Max}[(100 - P)/P, 0]$ and its relative premium $\text{Max}[(P - 100)/P, 0]$. Bonds selling at a premium would have a positive value for the first variable and a zero value for the second, while those selling at a discount would have the opposite. We also define a dummy variable whose value is equal to one if the bond is at a premium,

EXHIBIT 4

Summary Statistics of Fitted Price Errors of JGBs

Panel A. By Year

Year	Fitted Price Error			Relative Error	
	N	Mean	Std. Dev	Mean	Std. Dev
90	2940	0.14365	0.15783	0.15619	0.17679
91	4288	0.10702	0.10408	0.10959	0.10882
92	4430	0.12239	0.13483	0.11805	0.13100
93	4361	0.09791	0.09726	0.08928	0.08847
94	4515	0.11245	0.09182	0.10481	0.08674
95	4514	0.10002	0.07542	0.09091	0.06997
96	1637	0.10643	0.08662	0.09705	0.08219
Total	26685	0.11182	0.10994	0.10807	0.11125

Panel B. By Time to Maturity

Time Maturity	Fitted Price Error			Relative Error	
	N	Mean	Std. Dev	Mean	Std. Dev
0.0-2.0	5029	0.07234	0.06565	0.06897	0.06241
2.0-7.0	14009	0.11573	0.10546	0.11045	0.10436
7.0-10.0	7647	0.13059	0.13254	0.12946	0.13859

Data source: NEEDS data base of Nihon Keizai Shimbun. The fitted price is based on the B-spline extension of the McCulloch method with four knots placed at maturities of two, four, six, and eight years. The fitted price error (actual price minus estimated price) is based on the unit points of 100 of face value, and the relative error (actual price minus estimated price)/actual price is in percentage terms.

and zero otherwise.

The second explanatory variable we use to measure the coupon effect is the current yield, (C/P) where C is the coupon and P is the market price of the bond including accrued interest.

Exhibit 5 reports the empirical results for the linear regression of the relative fitted price errors on these explanatory variables. The regression results in the table indicate that bond characteristics such as time to maturity, the relative discount or premium, and the current yield of the bond are significant variables in explaining the relative fitted price errors. The signs of the coefficients, however, are sensitive to the choice of variables and are not always consistent with the discussion in the previous section.

The results may be due to the limitations of the linear specification used in the regression, when the

underlying structure may, in fact, be non-linear. Furthermore, the R-square of the regression for the total period is only 2.2% for the first subperiod, 1990-1992, and 10.8% for the second subperiod, 1993-1996. These R-squares are lower than those reported in Bliss [1996] with a similar set of explanatory variables for the U.S. Treasury markets.

Furthermore, contrary to our expectation that the importance of non-present value factors would diminish over time, the regression results for the subperiods indicate that R-squares are actually somewhat higher for recent years. The low R-square for the total period and the higher R-squares for recent years may be due to the fact that linear regression may not capture the complicated, perhaps non-linear, pricing effects of bond characteristics, or the presence of other explanatory variables.

In order to investigate the potential non-linearity in the relationship between the relative fitted price errors and the bond characteristics, we use non-linear regression based on the Legendre orthogonal polynomial series. As in the linear regression, we use the time to maturity (a proxy for liquidity) in addition to two sets of explanatory variables to measure the coupon effects: the current yield and the relative price premium/discount of a bond.²⁷

Exhibit 6 reports the results for the non-linear regression of the relative fitted pricing errors. The first feature of the results for this non-linear estimation is the improvement in the R-squares. For the subperiod 1990-1992, the R-square increases from 2.2% to 5.2%, based on the piecewise linear specification. The R-square significantly increases to 18.2% from 10.8% for the second subperiod, 1993-1996.

The other noteworthy feature in Exhibit 6 is that the relative premium/discount of a bond is more important in the second subperiod than in the first subperiod. In the second subperiod, this variable explains 7% of the variation in the relative fitted price errors, while it explains less than 1% in the first subperiod. The coefficients of the higher-order polynomials, other than the linear term, are statistically significant for both subperiods, confirming that the linear model used in Exhibit 5 may be severely misspecified.

The estimated functional relation based on the polynomials are presented in Exhibit 7. Exhibit 8 shows that the relative pricing errors for the 1990-1992 period have a "W"-shape with respect to the relative premium/discount of the bonds, and the pricing errors also

EXHIBIT 5

Linear Regressions of Relative Fitted Errors on Non-Present Value Factors

Variable	Subperiod 1 1990-1992			Subperiod 2 1993-1996		
	Const	0.00035 (5.56634)	-0.00017 (-0.72014)	-0.00018 (-0.70468)	0.00111 (10.68364)	-0.00752 (-9.07374)
Time	0.00002 (2.15448)	-0.00006 (-7.18051)	-0.00005 (-5.56624)	-0.00005 (-13.20791)	-0.00012 (-32.39296)	-0.00010 (-24.10262)
Dummy	-0.00014 (-2.18285)	0.00319 (11.222125)	0.00321 (10.61822)	-0.00024 (-2.40416)	0.010394 (12.50928)	0.00556 (5.27337)
Disc	-0.00206 (-3.46694)		-0.00059 (-0.99224)	-0.02729 (-11.78486)		-0.02047 (-7.45697)
Prem	-0.00229 (-3.48137)		-0.001959 (-2.99449)	-0.00366 (-20.56252)		-0.00225 (-12.27059)
CouponD		0.01610 (3.82942)	0.01573 (3.68954)		0.20689 (10.15295)	0.10392 (4.24086)
CouponP		-0.04043 (-14.45911)	-0.03983 (-14.13112)		-0.04001 (-30.08158)	-0.03471 (-25.00643)
R ²	0.00356	0.02091	0.02171	0.06956	0.09554	0.10774

Regression results of relative fitted price error (actual price minus estimated price)/actual price, in percentage points on time to maturity (Time), a dummy variable whose value is one if a bond is traded at a premium (Dummy); the relative discount of a bond ($\text{Disc} = \text{Max}[(100 - P)/P, 0]$); the relative premium of a bond ($\text{Prem} = \text{Max}[(P - 100)/P, 0]$); the relative coupon rate for discount bonds ($\text{CouponD} = C/P$); and the relative coupon rate for premium bonds ($\text{CouponP} = C/P$). T-statistics of the ordinary least squares estimates are in parentheses.

increase with coupon yield, confirming the importance of both factors in explaining the pricing errors.

For the second subperiod, however, the relative pricing errors have an inverted "U"-shape with respect to the relative premium/discount of the bonds, highlighting the importance of the par effect alone. For instance, Exhibit 8 shows that the pricing error on a par bond is positive, on average, while that of other bonds, selling at a premium or discount, is, on average negative.

Exhibit 9 also presents the estimated functional relation between the relative pricing errors and the explanatory variables, based on the non-parametric kernel estimation. The estimated results are quite similar to those based on the Legendre polynomials.

Given the results in Exhibits 5 and 6, we can conclude that coupon effects have been changing with the structural change in the 1990s, although there are strong coupon effects in the JGB market. For example, for the 1993-1996 period, a par bond had, on average, a 0.05% premium relative to a premium or discount bond, while the average premium was about 0.02%-0.03% for the 1990-1992 period. On the other hand, a high-

coupon bond had a significant premium as much as 0.08%-0.10% for the 1990-1992 period, while the premium was almost non-existent for the 1993-1996 period.

Thus, both the coupon and the par effects were important in explaining the fitted price errors in the early 1990s, but in the mid-1990s, only the par effect was pronounced.

Characteristics of the Estimated Spot Rates

The spot rates estimated from the JGBs are obtained from the estimated discount function on each trading date during the sample period, 1990-1996. One concern about using these spot rates may be that the rates are measured with "noise" induced by the coupon, par, and other effects. Our empirical analysis so far, though, indicates that the fitted price errors are not far off from those in the U.S. Treasury market and hence may throw light on the underlying structure of the JGB market.

To investigate this further, we examine the impact of these effects on the spot rates using factor analysis. We

EXHIBIT 6

Non-Linear Regressions of Relative Fitted Price Errors on Non-Present Value Factors

Variable	Subperiod 1			Subperiod 2		
	1990-1992			1993-1996		
Const	0.00893 (-1.23391)	0.04181 (8.80024)	0.01554 (2.00307)	0.00609 (1.58892)	0.08784 (41.48370)	0.05367 (13.60645)
Time	0.00284 (3.75608)	-0.00953 (-10.93928)	-0.00707 (-7.49334)	-0.00431 (-11.09444)	-0.01399 (-37.44112)	-0.01172 (-27.46601)
P1(P-100/100)	0.03129 (2.98538)		0.00552 (0.53208)	0.00309 (0.49746)		-0.01259 (-2.10527)
P2(P-100/100)	-0.01701 (-1.67291)		0.00303 (0.30400)	-0.06478 (-10.69149)		-0.03513 (-6.06385)
P3(P-100/100)	0.002383 (0.25933)		-0.00563 (-0.62687)	-0.03262 (-5.44594)		-0.05089 (-9.01374)
P4(P-100/100)	0.01514 (2.41195)		0.03849 (6.15670)	-0.02955 (-8.27206)		-0.02432 (-7.07024)
P5(P-100/100)	0.00572 (1.07318)		0.00703 (1.35063)	0.00643 (1.75727)		-0.01767 (-5.06708)
P1(C/P)		-0.00193 (-0.30983)	-0.00408 (-0.65300)		-0.10032 (-34.89167)	-0.09744 (-33.04696)
P2(C/P)		-0.06227 (-10.30635)	0.06697 (-11.05682)		0.02827 (11.00192)	0.01730 (6.50656)
P3(C/P)		0.062969 (11.22572)	0.063383 (11.34187)		-0.02467 (-9.69467)	-0.02456 (-9.64937)
P4(C/P)		-0.009316 (-2.28890)	-0.00647 (-1.58201)		0.06814 (32.46731)	0.06666 (31.67195)
P5(C/P)		-0.00586 (-1.54770)	-0.01190 (-3.08229)		-0.02457 (-12.24533)	-0.01728 (-8.44969)
R ²	0.00486	0.04361	0.05231	0.07012	0.16292	0.18159

Regression results of the relative fitted price error, (actual price minus estimated price)/actual price, in percentage points on the time to maturity (Time); normalized Legendre polynomials of "par-ness" of a bond $(P - 100)/100$; and normalized Legendre polynomials of current income (coupon/price) of a bond. T-statistics of the ordinary least squares estimates are in parentheses.

would expect that if the spot rates are estimated with significant error due to the specific characteristics of individual bonds, the total variation in the spot rates that can be explained by common factors should be relatively low.

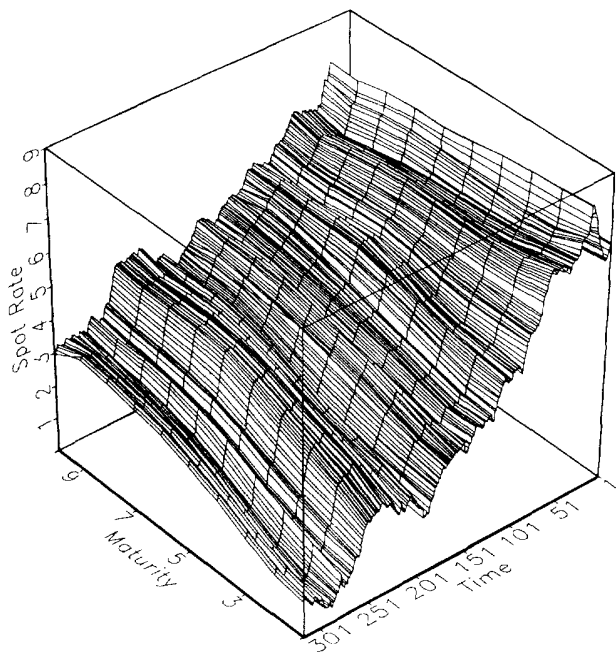
Panel A in Exhibit 10 provides the summary statistics of the estimated spot rates in terms of levels. The term structure of interest rates for the sample period is, on average, upward-sloping as indicated by the means. Although the autocorrelation coefficients of the spot rates are close to one, the standard deviations decline with maturity, indicating that the spot rates may be mean-reverting.

Panel B of Exhibit 10 provides the same statistics

in terms of first differences of the spot rates. One notable feature in the data is that the term structure of autocorrelations of the first-differenced series declines with maturities. For example, for the one-year spot rate, the autocorrelation is 0.264, while it is 0.024 for the ten-year. This feature is also found in U.S. Treasury yields, for example, in Balduzzi and Eom [1998]. The higher moments of the differenced JGB spot rates increase with maturity initially and then decrease for longer maturities. Panel C shows that the correlation coefficient between the weekly spot rates decline as the differences between the maturities increase.

One way to summarize the information embedded in the various spot rates is to identify the common

EXHIBIT 7 Estimated Spot Rates



factors driving these rates. This type of analysis is very useful in studying the shape of the term structure of interest rates and modeling the changes in the term structure to price and hedge interest rate-derivative securities. The correlation structure of the JGB spot rates can be examined through principal components analysis to identify the common factors that explain most of variations in the spot rates over time.

Studies of the U.S. Treasury bond market by several authors including Garbade [1984] and Litterman and Scheinkman [1991] show that the movements in bond yields can be explained well by a few common factors. Examples include the level, slope, and curvature of the term structure of interest rates.

We define a K-factor model to explain the changes in the spot rates.²⁸ Specifically, we assume that the change in spot rate of maturity j over a one-week interval is a weighted sum of K unobservable factors of the form:

$$\Delta y_{jt} = \beta_{j0} + \beta_{j1}f_{1t} + \beta_{j2}f_{2t} + \dots + \beta_{jK}f_{Kt} + \varepsilon_{jt} \quad (5)$$

where f_{it} , $j = 1, \dots, K$ represent the K common factors that influence JGB spot rates at time t .²⁹

Instead of choosing the number of factors on the

basis of statistical tests, we implement a three-factor model so as to make our analysis of the Japanese market comparable to other studies of the U.S. Treasury market by Garbade [1984] and Litterman and Scheinkman [1991]. Exhibit 11 presents the factor loadings of the first three factors. The factor loadings are estimated using ten spot rates ranging in maturity from one to ten years over the sample period.

As in the case of the studies of the U.S. Treasury market, the first three common factors can be identified as the shifts in the level, slope, and curvature of the yield curve. For example, a shock of one standard deviation in the first factor increases the one-year rate about 25 basis points and the five-year rate about 35 basis points. If

EXHIBIT 8 Coupon Effect: Non-Linear Regression

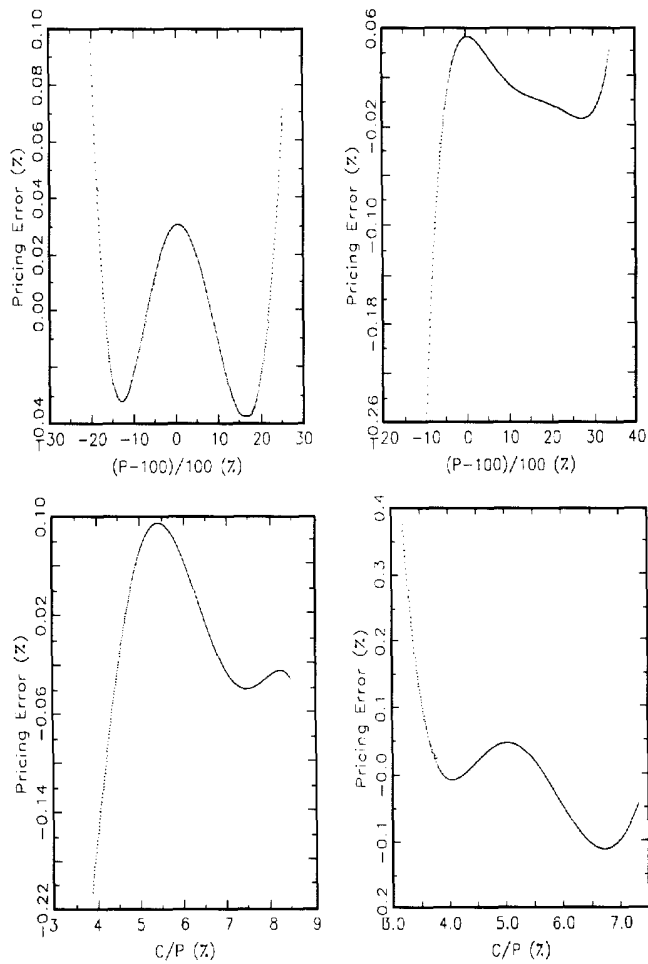
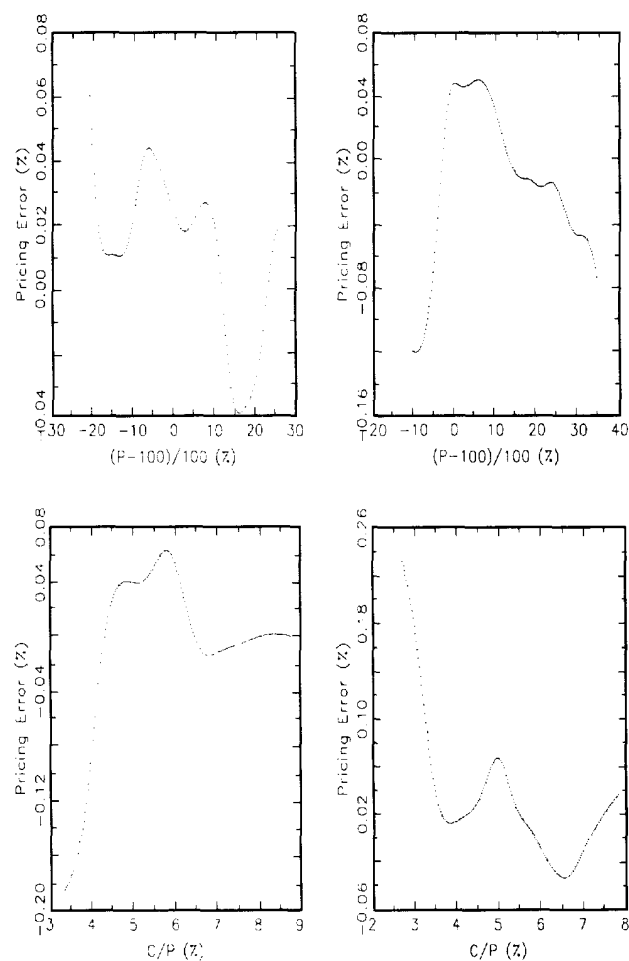


EXHIBIT 9

Coupon Effect: Nonparametric Kernel Regression



there is a shock of one standard deviation in the second factor, the one-year rate decreases by 40 basis points, while the ten-year rate increases by 40 basis points. The third factor, the curvature of the yield curve, changes the short and long end of the curve in a direction opposite to that for the medium-term yields.

As in the case of the U.S. Treasury bond market, the total variation explained by the first three factors is quite large. Exhibit 12 provides the results on the total proportion of the variance explained by the three factors and the proportion of the variance explained by each factor for different maturities. The first factor explains more than 95% of the variation for five-year rates, but it explains only about 70% of the variation for

both the one- and the ten-year rates. On the other hand, the second factor accounts for about 20% and 16%, respectively, of one- and ten-year rates, but does not explain much of the variation in medium-term yields. The third factor explains only a small proportion of the changes in the yields across all maturities.³¹

Overall, the empirical results indicate that the JGB spot rates are affected by a small number of systematic factors, although the spot rates may be measured with "noise" due to the presence of non-present value factors.

EXHIBIT 10

Summary Statistics of Spot Rates from Japanese Government Bond Market

Panel A. Level of Weekly Spot Rates

Maturity	Mean	St Dev	Skewness	Kurtosis	Auto-Correlation
2 year	3.81687	2.08378	0.37168	-0.96377	0.99263
3 year	3.98263	1.92804	0.33341	-0.93333	0.99254
4 year	4.20766	1.77436	0.31108	-0.94577	0.99246
5 year	4.47313	1.63425	0.28452	-1.01144	0.99223
7 year	4.84264	1.40385	0.28459	-0.94078	0.99057
10 year	4.97832	1.24818	0.21046	-0.95586	0.99013

Panel B. First Differences of Weekly Spot Rates

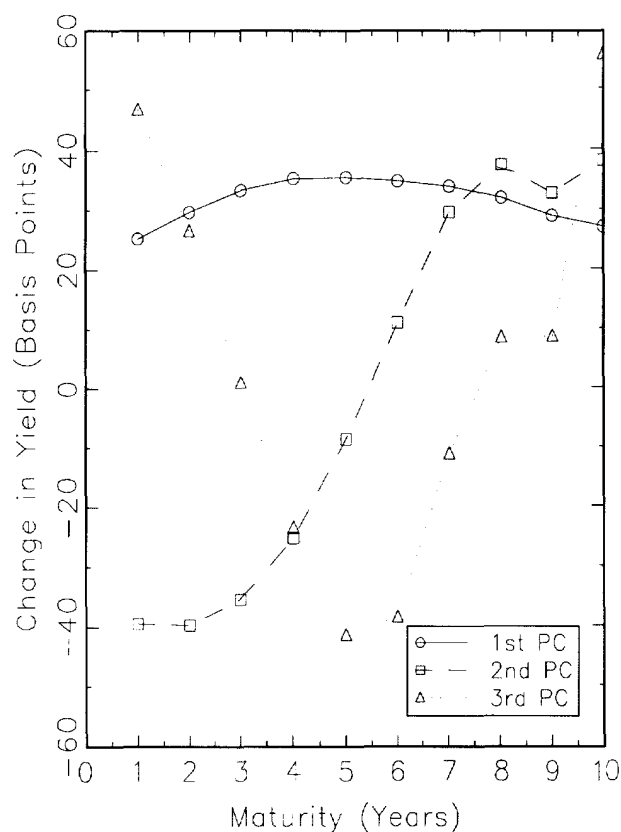
Maturity	Mean	St Dev	Skewness	Kurtosis	Auto-Correlation
2 year	-0.01818	0.10439	0.32670	0.94571	0.26439
3 year	-0.01634	0.11169	0.41056	1.38400	0.22545
4 year	-0.01455	0.11545	0.43915	1.29713	0.18650
5 year	-0.01334	0.11353	0.47884	1.05782	0.16310
7 year	-0.01241	0.11126	0.66719	1.44103	0.02976
10 year	-0.01135	0.10123	0.32137	0.55748	0.02410

Panel C. Correlation Coefficients between Weekly Spot Rates

Maturity	2-Year	3-Year	4-Year	5-Year	7-Year	10-Year
2-Year	1.000	0.969	0.915	0.870	0.732	0.605
3-Year	0.969	1.000	0.982	0.932	0.784	0.655
4-Year	0.915	0.982	1.000	0.972	0.837	0.697
5-Year	0.870	0.932	1.000	0.972	0.837	0.697
7-Year	0.732	0.784	0.837	0.910	1.000	0.882
10-Year	0.605	0.655	0.697	0.750	0.882	1.000

EXHIBIT 11

Factor Loadings Using Principal Components Analysis



IV. CONCLUSION

We examine the possibility of using polynomial methods to fit the term structure of interest rates using data from the Japanese government bond (JGB) market. Our analysis using data from the 1990s indicates that it is possible to closely fit the term structure of default-free rates, with fitting errors that are only slightly larger than those found in studies of the U.S. Treasury bond market. The pricing errors are largely explained by coupon, par, and maturity effects caused by various regulatory, accounting, and tax factors.

In particular, both the coupon and the par effects are important in explaining the errors in the early 1990s, while only the par effect is a relevant factor in the mid-1990s. This suggests that structural changes in the Japanese financial markets have reduced

EXHIBIT 12

Three-Factor Model of Spot Rates from the Japanese Government Bond Market

Maturity	Total Variance	Factor 1	Factor 2	Factor 3
1 year	94.41	68.72	19.67	6.01
2 year	97.19	79.03	16.56	1.59
3 year	98.35	86.83	11.52	0.00
4 year	97.70	91.24	5.42	1.04
5 year	99.20	95.21	0.65	3.33
6 year	98.10	94.07	1.09	2.93
7 year	98.80	90.56	7.98	0.25
8 year	97.69	84.09	13.44	0.14
9 year	96.20	83.55	12.46	0.18
10 year	93.93	70.10	16.24	7.58

Total variance explained by three factors for each maturity and the proportion of total variance in percentage explained by each factor for each maturity.

the importance of non-present value factors in explaining the pricing of government bonds.

The analysis indicates that three factors explain a substantial part of the variation in the spot rates on bonds, as in other Treasury markets. These factors can be identified as changes in the level, slope, and curvature of the yield curve. Thus, our research suggests that the multifactor models of the term structure that are widely used in the arbitrage-free pricing of derivatives can also be applied to Japanese fixed-income markets.

APPENDIX

The Cubic B-Spline Method

By Weierstrass's approximation theorem, every continuous function on a closed interval can be approximated uniformly to any prescribed accuracy by a polynomial. In approximating such a smooth function, spline functions are often used. Spline functions can be thought of as linear combinations of elementary polynomials or *bases* defined over the approximation interval.

McCulloch [1975] shows that a spline function consisting of piecewise cubic polynomials can be used by a least squares estimation to approximate the term structure of interest rates. The idea is to connect every adjoining pair of points by an individual piecewise cubic polynomial. The individual cubic functions are then joined together at the knot points into a smooth curve with the restriction that the first two derivatives of the two adjoining segments match at the knots. A numerically stable parameterization of a cubic spline is pro-

vided by a cubic B-spline basis.

The p -th order B-spline $Q_i^p(m)$ associated with the knots $\tau_1 \dots \tau_{i-p}$ is defined as

$$Q_i^p(m) = \sum_{j=1}^{i-p} \left(\prod_{\substack{j=1 \\ j \neq i}}^{i-p} \frac{1}{(\tau_j - \tau_i)} \right) (m - \tau_i)_+^{p-1} \quad -\infty < m < \infty$$

where $(m - \tau_i)_+ = \text{Max}[0, (m - \tau_i)]$. For example, if $p = 1$, $Q_i^1(m) = 1/(\tau_{i+1} - \tau_i)$, $\tau_i \leq m < \tau_{i+1}$, and zero otherwise. If $p = 2$, $Q_i^2(m) = 0$, $m \leq \tau_i$, $Q_i^2(m) = (m - \tau_i)/((\tau_{i+1} - \tau_i)(\tau_{i+2} - \tau_i))$, $\tau_i \leq m < \tau_{i+1}$, $Q_i^2(m) = (\tau_{i+2} - m)/((\tau_{i+2} - \tau_i)(\tau_{i+2} - \tau_{i+1}))$, $\tau_{i+1} \leq m < \tau_{i+2}$, and $Q_i^2(m) = 0$, $\tau_{i+2} \leq m$.

One important property of the B-splines is that B-splines are linearly independent on a certain interval, that is, $\{Q_i^p\}_{i=1}^{r-1-p+1}$ are linearly independent on $[\tau_i, \tau_r]$, and thus form a basis.

To further facilitate computational stability, we choose to approximate a function by normalized splines, because $Q_i^p(m)$ can be sometimes extremely large or extremely small. The normalized B-spline $N_i^p(m)$ associated with the knots $\tau_1 \dots \tau_{i+m}$ is defined as

$$N_i^p(m) = (\tau_{i+p} - \tau_i) Q_i^p(m)$$

If $p = 1$, $N_i^1(m) = 1$, $\tau_i \leq m < \tau_{i+1}$, and zero otherwise. If $p = 2$, $N_i^2(m) = 0$, $m \leq \tau_i$, $N_i^2(m) = (m - \tau_i)/(\tau_{i+1} - \tau_i)$, $\tau_i \leq m < \tau_{i+1}$, $N_i^2(m) = (\tau_{i+2} - m)/(\tau_{i+2} - \tau_{i+1})$, $\tau_{i+1} \leq m < \tau_{i+2}$, and $N_i^2(m) = 0$, $m_{i+2} \leq \tau$. A higher-order normalized B-spline $N_i^p(m)$ associated with the knots $\tau_1 \dots \tau_{i+m}$ can be easily computed by a recursion:

$$N_i^p(m) = \frac{(m - \tau_i) N_i^{p-1}(m)}{(\tau_{i+p-1} - \tau_i)} + \frac{(\tau_{i+p} - m) N_{i+1}^{p-1}(m)}{(\tau_{i+p} - \tau_{i+1})}$$

A discount function can be represented as linear combinations of B-spline bases, defined over the interval $[0, M]$, where M is the maximum maturity of a bond in the sample. Namely, the discount function, $D(t, m)$, at time t with time to maturity, m , has a unique expansion of the form

$$D(t, m) = \sum_{i=1}^{p+K} \beta_{t,i} N_i^p(m), \quad \text{all } \tau_p \leq m < \tau_{p+K+1}$$

where $\{N_i^p\}_{i=1}^{p+K}$ are p -th order normalized B-splines associated with an extended partition $\{\tau_i\}_{i=1}^{2p+K}$; K is the number of knot points; and $\{\beta_{t,i}\}_{i=1}^{p+K}$ are the B-spline expansion coefficients at time t .

Note that the extended partition with $2p$ number of extra knots outside the interval $[\tau_p = 0, \tau_{p+K+1} = M]$ enables us to obtain the linear independence property of B-splines;

that is $\{N_i^p\}_{i=1}^{p+K}$ form a basis on $[\tau_p, \tau_{p+K+1}]$. Note also that, because of the unique connection between a function and its B-spline expansion coefficients, the information about $\{\beta_{t,i}\}_{i=1}^{p+K}$ suffices to describe the entire discount function.

The least squares estimates of $\{\beta_{t,i}\}_{i=1}^{p+K}$ at each time t are obtained from

$$\{\beta_{t,i}\}_{i=1}^{p+K} = \min \sum_{n=1}^{N_t} \epsilon_{j,t}^2$$

where $\epsilon_{j,t}$ is a fitting error of j bond at time t , defined as

$$\begin{aligned} \epsilon_{j,t} &= P_j^{\text{Actual}}(t, t + \tau_{M_j}) - \left[\sum_{m=1}^{M_j} c_{j,m} D(t, t + \tau_{M_j}) + F_j D(t, t + \tau_{M_j}) \right] \\ &= P_j^{\text{Actual}}(t, t + \tau_{M_j}) - \left[\sum_{m=1}^{M_j} c_{j,m} \sum_{i=1}^{p+K} \beta_{t,i} N_i^p(\tau_{M_j}) + F_j \sum_{i=1}^{p+K} \beta_{t,i} N_i^p(\tau_{M_j}) \right] \end{aligned}$$

There are several assumptions to be made in implementing spline methods so as to achieve desirable estimates of the term structure of interest rates. First, the discount function to be estimated can be defined either in terms of spot or forward interest rates. We choose to estimate the function in terms of spot interest rates.

Second, the order of the polynomial has to be chosen. Following McCulloch [1975] and others in the term structure literature, we use cubic splines, i.e., $p = 4$.

Third, the "knot" placement scheme, which specifies the number of knots and the partitioning of the knots, has to be defined. For instance, in Litzenberger and Rolfo [1984], three interior knots are placed at one year, five years, and ten years, while McCulloch [1975] places knots that yield an equal number of observations in each interval.

In general, there is a trade-off between minimum squared errors and the smoothness of an estimated function. If more knots are used, or higher-order polynomials are used, the function fits the local shape better, but the estimated term structure may be too "wiggly." The use of only a few knots or of low-order polynomials produces a fit that may be too smooth, resulting in larger fitting errors.

One may explicitly incorporate the "penalty function" of the smoothness explicitly in the estimation to choose the number of knots on a statistical basis. In this article, we use four equally spaced interior knots at two, four, six, and eight years over the maturity range of zero to ten years.

Specifically, we use the cubic B-spline, $p = 4$, to estimate B-spline expansion coefficients, $\{\beta_{t,i}\}_{i=1}^{p+K}$, five knots ($K = 5$) over the interval between 0 and ten years based on the the least squares estimation of errors between the actual price and the model price. By restricting $\beta_1 = 1$, we can ensure that $D(0) = 1$, since $N_1^p(0) = 1$ and $N_i^p(0) = 0$ for $i > 1$.

ENDNOTES

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¹If the cash flows are certain, perfect substitutes are available for individual bonds, and there are no capital constraints or other frictions, equilibrium does not exist, because arbitrageurs would want to take infinite positions to exploit profit opportunities. In the presence of frictions, however, the common marketwide valuation function reflects the shadow price of these constraints.

²For instance, securities with similar maturity and coupon characteristics have been shown to trade at different yields, a phenomenon that could be attributed to liquidity differences.

³To a certain extent, liquidity and coupon effects have also been important in other Treasury markets, including the U.S. and the U.K. For instance, some securities trade as "specials," commanding a lower yield simply because they are "on-the-run." In addition, in both these markets, high-coupon bonds normally trade at higher yields than low-coupon bonds with the same duration, due to tax considerations.

⁴See Boudoukh and Whitelaw [1991] and Nakagawa [1992] for a more detailed discussion.

⁵This hedging demand steadily declined with the increasing liquidity offered by an alternative hedging vehicle, the JGB futures contract, which started trading in 1985.

⁶Similar results were obtained for more recent years by Imamura [1995].

⁷This is based on conversation with market participants.

⁸There is no official record of which bond was the benchmark at a particular time. Given the relative trading volume of JGBs in the OTC market, one can identify particular JGBs as benchmark bonds, however. The data on trading volumes were obtained from Japan Bond Trading Co Ltd, a "brokers' broker" during the period 1990-1997.

⁹When a benchmark bond is replaced by a new bond, it usually takes only a couple of days for the change to be reflected in market trading volume. In order to take this change in the benchmark into account, we drop the first and last months of the period for computing the average relative volume.

¹⁰This regulation was gradually eased during the early

1990s and finally lifted with revision of the law in April 1996.

¹¹In the U.K., most investors in the gilt (government bond) market are taxed at their marginal rate of tax on ordinary income, but are exempt from taxation of capital gains. In the U.S., this tax bias is not as strong.

¹²The "par effect" is at odds with optimal bond trading, as proposed by Constantinides and Ingersoll [1984], who argue that investors can trade bonds optimally to minimize tax liabilities by dynamic trading, rather than buying and holding to maturity. As a result the price of a bond under dynamic trading is higher than the price under a buy-and-hold strategy, due to the "tax-timing option" embedded in the bond. In Japan, capital gains are taxed as total profit for taxable residents. In addition, unlike in the U.S., there is no amortization of the bond premium over the remainder of its time to maturity. Under this tax scenario, the optimal tax strategy would be to realize capital losses immediately and defer capital gains as far into the future as possible, making it preferable to buy bonds selling for a premium.

¹³The loan of securities was earlier recorded as a loan for the purpose of calculating the level of risky assets for capital adequacy requirements based on standards established by the Bank of International Settlements (BIS). Since abolition of the regulation in January 1996, however, the outstanding amount of government securities lent has grown to about 18 trillion yen by the end of July 1997.

¹⁴If a bond is registered with a withholding-tax-exempt entity on its coupon date, it is referred to as "clean-registered." For a "dirty-registered" bond, which is owned by taxable entities, tax on the coupon payment is withheld at 20% rate.

¹⁵In such cases, JGBs are cleared through physical delivery of transfer orders for registered names.

¹⁶The typical bid-ask spreads of JGBs are 0.5 bp for the benchmark, 0.5-1 bp for the seven- to ten-year, 2 bp for the two- to seven-year, 2-4 bp for the less than two-year maturities. The spreads for JGBs with a remaining life of more than seven years, which can be delivered into the futures contract, are at roughly the same levels as those of the benchmark JGBs.

¹⁷Ten-year JGBs are issued every month but usually mature on a quarterly cycle in March, June, September, and December each year. Of the amount issued, 60% is auctioned, with the remaining 40% allocated to the members of the underwriting syndicate.

¹⁸Large-lot transactions (over ten million yen) are not required to be traded on organized exchanges. When they are traded, orders are matched by a call auction at the market open and close, and continuous auctions are held for the rest of the session.

¹⁹The proportion of the total volume traded on the exchange was 3.9% in 1990, 2.6% in 1992, 1.2% in 1993, and 1.1% in 1994. (Source: Securities Markets in Japan, 1996).

²⁰Coupon payments in the JGB market are on a semi-

annual basis, with accrued interest calculated on an actual/365 basis.

²¹Other approaches include Bernstein polynomials employed by Schaefer [1981] and Laguerre polynomials suggested by Nelson and Siegel [1987].

²²Examples are Singleton [1994] and Kikugawa and Singleton [1994], who fit the JGB yield curve from 1990 to 1992 and find that there are substantial coupon and par effects on bond yields.

²³This is equally valid for the liquidity effect because the liquidity of non-benchmark JGBs has improved dramatically during this period (not addressed in this article).

²⁴An alternative approach would be to assume a market with clientele effects in which agents hold some bonds but not others. Such a market is characterized by a tax bracket-specific term structure. For instance, in a study of the U.K. gilt market, Schaefer [1981] uses the linear programming technique to identify optimal portfolios for investors with specific tax rates.

²⁵In the implementation, we use the normalized Legendre polynomials of order five. The first five Legendre polynomials are $P_1(x) = x/\sqrt{2/3}$; $P_2(x) = 1/2(3x^2 - 1)/\sqrt{2/5}$; $P_3(x) = 1/2(5x^3 - 3x)/\sqrt{2/7}$; $P_4(x) = 1/8(35x^4 - 30x^2 + 3)/\sqrt{2/9}$; and $P_5(x) = 1/8(63x^5 - 70x^3 + 15x)/\sqrt{2/11}$. To check the robustness of estimated results based on polynomials, we also use the Nadaraya-Watson non-parametric kernel regression with Gaussian kernels. See Härdle [1989] for a detailed description of the methodology.

²⁶Since bonds are issued at fairly regular intervals, the issue number of a bond is a fairly good measure of its chronological age.

²⁷Kikugawa and Singleton [1994] also use the coupon level and the degree to which a bond is trading away from par as the "adjustment" factors in modeling the JGB term structure.

²⁸Two alternative data inputs have been used in previous research. The first is the time series of returns on zero-coupon bonds of different maturities. The other is the changes in the spot rate themselves. The former is relevant to the needs of a bond portfolio manager, while the latter is in line with the models of the term structure that are used in derivatives pricing.

²⁹In matrix notation, we can write Equation (5) as

$$\Delta y_t = B F_t + \epsilon_t$$

where the unobservable factors are assumed to satisfy:

$$E(F_t) = 0, E(\epsilon_t) = 0, \text{Cov}(F_t, \epsilon_t) = 0$$

Assuming that K-factors are stationary with variance $\Phi = \text{var}(F_t)$, the variance-covariance structure of the change in spot rate, V , can be written as

$$V = B\Phi B' + \Omega$$

where $\Omega = \text{cov}(\epsilon_t)$. One way of identifying the model is through a principal components analysis that assumes that Ω is diagonal and $B'B = I_K$.

³⁰We also checked the sensitivity of the results to the subperiod considered and found that the specification is fairly robust.

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