# Technical Notes

## New Financial Instruments for Hedging Changes in Volatility

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With the introduction of stock index futures and options as well as bond futures and options, investors can hedge against market volatility and interest rate volatility. But investors are still exposed to the risk of *changes* in volatilities. Market volatility can change in response to changes in macroeconomic factors such as inflation, unemployment or economic policy, or in response to changes in the volatility of equity of specific firms, due to shifts in capital structure or news about performance.

The history of U.S. stock indexes since the beginning of the century shows that volatility has exhibited substantial instability.[1] During the 1970s, for example, the annualized standard deviation of stock returns ranged from 10 to 40 per cent. In the '80s, before October 1987, volatility decreased to the 10 to 20 per cent range.

According to a report from BARRA, the 1973–75 period could be characterized as volatile, the 1976–79 period as quiet, the 1980–82 period as more volatile, the 1983–84 period as quiet and the 1985–86 period as more volatile.[2] From January 1973 to September 1987, daily average volatility was 1.15 per cent (18 per cent annually, using 250 trading days). In October 1987, daily volatility jumped to 5.87 per cent (90 per cent annualized). Even if October 19th is excluded, daily volatility was still 3.51 per cent, compared with the second-highest volatility, in October 1974, of 2.02 per cent.

A similar picture is obtained from the time series of volatility implied by option prices. In September 1986, for example, the implied volatility of the stock market jumped from about 15 per cent in the first week to 25 per cent in the second.[3] This amounts to a 60 per cent change within a few days. During October 1987, implied volatility increased from 20 to over 100 per cent on October 20 and declined to 30 to 40 per cent thereafter.[4]

Volatility changes are also apparent in the bond and foreign currency markets. From August 1986 to January 1987, for example, the implied volatility of options on the Swiss franc moved between 10.5 and 16 per cent. The historical (12-week rolling) volatility of 10-year Treasury notes moved in the range of 5 to 30 per cent in the 1982–87 period.[5]

Following the market crash, volatility increased, and the volume of trading in futures and options shrank considerably. Exchanges and institutions have expressed fears that the public may shy away from investing in risky assets because of the perception of enhanced riskiness.

While there are efficient tools for hedging against general changes in overall market directions, so far there are no effective tools available for hedging against *changes* in volatility. It should be noted that the percentage change in *volatility* is much greater than the change in the *level* of stock indexes. We therefore propose the construction of three volatility indexes on which cash-settled options and futures can be traded. One index would depict volatility in the equity market, the second volatility in the bond market and the third volatility in the foreign exchange market. "Volatility options" and "volatility futures" would expand the investment opportunities available to investors and provide efficient means to hedge against changes in volatilities.

### Constructing a Volatility Index

Our volatility index, to be named Sigma Index (SI), would be updated frequently and used as the underlying asset for futures and options. There are many ways to construct such an index. It could be based on the standard deviation obtained from historical observations (with more weight given to recent observations). It could be based on implied volatilities from options that have just traded. Or we could use a combination of historical and implied volatilities to provide some balance between long and short-run trends.

Admittedly, no volatility index can represent the volatility exposures of all market participants. Therefore, no volatility option or futures can provide a perfect hedge for all. But, because various volatility measures are highly correlated, we believe that most potential users would find the instruments on a volatility index useful, even if the index does not perfectly match their needs.

A volatility index would play the same role as the market index plays for options and futures on the index. In line with conventional stock indexes, each percentage point of standard deviation would be equivalent to 10 index points. For example, a standard deviation of 15 per cent (on an annual basis) would translate into an index level of 150 points. (Equivalently, each percentage point of standard deviation would equal $100.)

1. Footnotes appear at end of article.

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The following strategies illustrate the advantages of using options and futures to hedge volatility. The list of strategies is by no means exhaustive.

### Selling a Futures Contract on Volatility

Assume an investor anticipates a 2 per cent decrease in market volatility, from 17 to 15 per cent. If he wants to act on his conviction, he should sell a straddle (i.e., a put and a call). But in selling the straddle, he is incurring the risk of a potential large increase or decrease in stock prices, as well as the risk of a change in volatility. He will suffer a loss, for example, if price takes a large jump while volatility stays at 17 per cent.

With access to a volatility index, our investor can place a bet on a volatility decrease by selling a volatility-based instrument—a futures contract on a volatility index, for example.[6] Unlike a straddle, this contract will not gain in value if, for example, the underlying asset appreciates on a given day while volatility remains unchanged. If volatility, as measured by the Sigma Index, decreases from 170 to 150, the contract seller will gain \$200 [(170 − 150) × \$10], but he will be exposed to an adverse change in volatility. (The opposite is, of course, true for a buyer of a volatility futures contract.[7])

A volatility futures contract may also be used to hedge positions that are "short" volatility. A seller of a straddle, for example, loses if volatility goes up from 15 to 17 per cent. He can hedge his position by buying a volatility futures contract.

### Buying a Call Option on Volatility

Assume an investor buys a one-month, at-the-money call on the SI, with a striking price of 150. If, at maturity, the standard deviation is 16.9 per cent, the SI will have a value of 169, and the cash flow to the owner of the option will be (169 − 150) times \$10, or \$190 (assuming that each index point equals \$10). If the SI is 185 at maturity, the option's value is \$350 [(185 − 150) × 10]. However, if the value of the SI at maturity is 150 or less, the option will expire worthless.

An investor can buy a call on the SI if she believes that market volatility will increase during the life of the option. If her income would be adversely affected by increased market volatility, buying a call option on volatility provides insurance. Buying a call option on the SI allows the investor to participate in increased market volatility while limiting the investor's risk to the buyer's initial investment.

### Buying a Sigma Call to Hedge Volatility Risk

A covered volatility position consists of buying a Sigma call option and at the same time taking a position in index options. The Sigma call options eliminate or reduce the index option's exposure to volatility risk.

Assume an investor is short an at-the-money straddle (his position is delta-neutral). The value of the straddle, with 15 per cent volatility and one month to expiration, is \$500. If volatility rises to 16 per cent, the value of the straddle increases to \$550. The writer of the straddle has suffered a \$50 loss, even if the market has not moved.[8]

To hedge this risk, the writer of the straddle can buy a Sigma call with a strike of 150. The value of the call will change by \$50 when SI moves from 150 to 160 (because it is at the money), and the change will compensate the investor for the loss from writing the straddle. Of course, the investor's insurance premium is the cost of buying the Sigma call.

### Buying a Sigma Put to Hedge Volatility Risk

Assume an investor is long an at-the-money straddle (he has a delta-neutral position). The value of the straddle, with 15 per cent volatility and one month to expiration, is \$500. If volatility drops to 14 per cent, the value of the straddle decreases to \$450. The buyer of the straddle suffers a \$50 loss, even if the market has not moved. To hedge this risk, the buyer of the straddle can buy a Sigma put with a strike of 150. The value of the put will change by \$50 (because it is at the money) and will thus compensate the investor for the loss from the straddle.

### Buying a Sigma Call to Hedge a Ratio-Bullish Call Spread

Assume an investor is long one at-the-money call and short three out-of-the-money calls (in a delta-neutral position). The investor expects the market to go up to a certain level, but not to explode. He is, however, running the risk that volatility will increase and his losses from the short calls will exceed his gains from the one long call. By buying a Sigma call, he can reduce his exposure to a loss due to an increase in volatility.

### Valuation of Volatility Options

Below, we show how a volatility option can be evaluated. We also show that the volatility option may be unique in the sense that it cannot be replicated by conventional index or equity options.

Assume that volatility changes in a known way. When stock prices go up, volatility tends to decrease; when stock prices go down, volatility tends to increase.[9] Figure A describes the price behavior of a stock over three periods. Initially, the price of the stock is \$100; at time 1, it can increase to \$130 or decrease to \$70. If, at time 1, the stock is at \$130, it can increase in value by a factor of 1.2, to \$156, or decrease by a factor of 0.8, to \$104. If the value of the stock is \$70 at time 1, however, it faces greater volatility, increasing by a factor of 1.5, to \$105, or decreasing by 0.5, to \$35.

As a measure of volatility, we use the difference between the "up" (u) and "down" (d) factors for the stock at each state.[10] The initial volatility is 0.6 (1.3 − 0.7). It moves to 0.4 (1.2 − 0.8) if the stock reaches \$130 at time 1 and to 1.0 (1.5 − 0.5) if the stock goes to \$70. Figure B depicts the "tree" of this volatility measure (the difference between the up and down factors). Without any loss of generality, all the differences are multiplied by 100 so that the volatility index, denoted by F, has the same order of magnitude as the stock price S. Because the uncertainty about next period's volatility is resolved at each state, the volatility "tree" consists of two periods only.

Assume that a call option on the volatility index is being offered. We assume that the call has two periods to expiration, and that the striking price is 60. At time 2, therefore, the call has a positive value only if F equals 80 or 120.

To price the volatility option, we use the Cox-Ross-Rubinstein technique of pricing an option when the underlying distribution is binomial.[11] At time 2, when the stock price is either \$156 or \$104, the corresponding values of F are 40 [100 × (1.2 − 0.8)] and 60 [100 × (1.3 − 0.7)]. The value of the call is therefore zero in both states. As a result, the value of the volatility call at time 1, when S equals \$130, is also zero.

At maturity, the volatility option is worth 20 when S equals 105 (and F equals 80) or 60 when S equals \$35 (and F equals 120). Because these two stock prices branch out from a stock price of \$70 at time 1, a hedge portfolio at time 1 can be constructed with one volatility option and a proportion, $\alpha$, of the underlying share such that, at time 2:

$$20 + \alpha\, 105 = 60 + \alpha\, 35.$$

**Figure A** The Distribution of Stock Prices over Three Periods

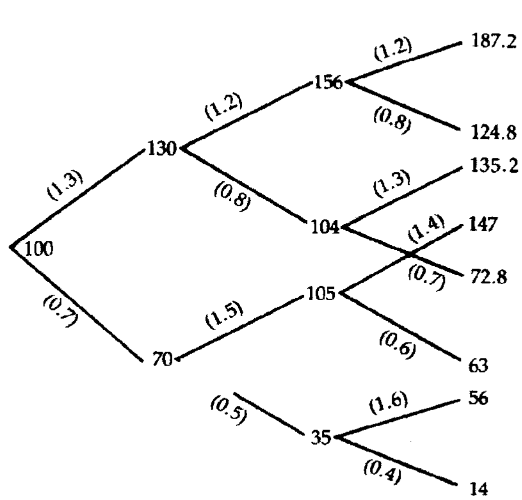
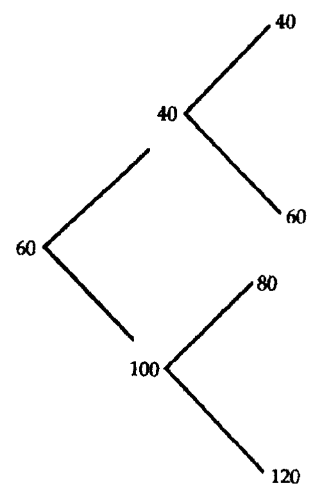



**Figure B** The Distribution of the Range Factors (the differences between "up and down") from Figure A Multiplied by 100




The hedge ratio, $\alpha$, is equal to 4/7. The hedge portfolio should yield the default-free interest rate. Thus, if the default-free interest rate is 10 per cent for the period:

$$(VC_{12} + (4/7)\,70)\,1.1 = 20 + 105\,(4/7) = 80$$

or

$$VC_{12} = (80/1.1) - 40 = 32.7,$$

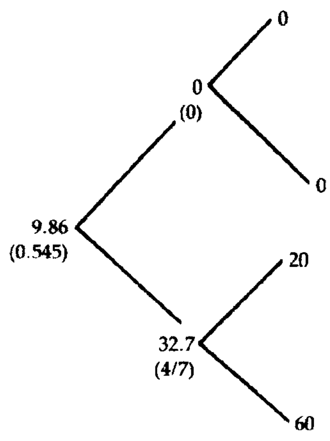
where $VC_{12}$ is the value of the volatility call at time 1, state 2.

Using a similar procedure, hedging the volatility option at time 0 against the underlying stock, and remembering that $VC_{11} = 0$, we get the no-arbitrage value for the volatility call at time 0, $VC_0 = 9.86$ (with the hedge ratio equal to 0.545). Figure C depicts the "tree" for the volatility call value.

From the example it can be seen that no simple option on the stock price itself has a payoff similar to that of the volatility option. A two-period call on S with a striking price equal to 104 would have a positive payoff in states 1 and 3 only, while the volatility option has a positive payoff in states 3 and 4 only. A put on S with a striking price of 105 would have a positive payoff in states 2 and 4 only. Even a straddle with a striking price of 105 for the call and 104 for the put will have a payoff profile different from that of the volatility option, with positive payoffs only at states 1 and 4.

Some traders use option-spread strategies to cope with possible changes in volatility. For example, a writer of an at-the-money straddle may buy an out-of-the-money straddle, creating a butterfly spread.

**Figure C** Values of a Volatility Option with a Striking Price of 60 Maturing at Time 2 (hedge ratios in parentheses)




This strategy will provide the trader with certain payoffs that minimize the risk of volatility changes. However, the option on volatility, coupled with the short straddle position, provides a different payoff scheme, protecting against an increase in volatility.

In principle, it is possible to adopt a dynamic strategy that uses the straddle itself in hedging the change in volatility. Such a strategy, however, suffers from problems similar to those of other dynamic strategies (e.g., portfolio insurance)—namely, discrete adjustments, transaction costs and frequent monitoring needs.

Ross has shown that, if the payoff on the underlying instrument is different for each state of nature, a combination of simple call and put options can create any desired payoff over the states of nature.[12] In the context of our example, where, at time 2, the payoffs on the stock are different over the four possible states, the payoff of the volatility option can be replicated by creating a portfolio consisting of specific quantities of the stock and three different options (with, say, striking prices equal to 146, 104 and 103). In this case, the volatility option is not unique and can be duplicated with conventional instruments. However, a volatility option is likely to be cheaper to trade, especially if the alternative options have to be replicated.

By changing our example slightly, we can show that the volatility option is in some cases unique. Assume that the stock reaches a price of $70 at time 1 and can move to either $104 or $35 in time 2. Under this assumption, at time 2 there are two states at which the stock is worth $104. Simple options cannot separate between two states if they have identical payoffs.[13] Therefore, the volatility option illustrated in the example cannot be replicated by a combination of the underlying stock, calls and puts. In such a case, the volatility option is unique. At the expiration of the volatility option, we capture the volatility of the stock prices, independent of the level of stock prices at that time.

A second conclusion to be drawn from the example is that the volatility call can be priced under certain assumptions, as is the case for the simple options. Admittedly, for more complex situations, we cannot provide an analytical solution for the value of the volatility option. Still, numerical solutions can be developed.

## Conclusion

The large swings in volatility over the past few years, especially since the October crash, have underlined the need for financial instruments for hedging changes in volatility. We suggest the creation of exchange-traded futures and options on a volatility index. Investors could establish long or short positions on volatility by trading volatility futures and limit or expand their volatility positions by using volatility options.

To price volatility options, we must know the process generating the distribution of returns on the stock market index. The Black-Scholes option-pricing model assumes the distribution for the underlying stock to be log-normal and the volatility to be constant. But the evidence indicates that volatility tends to change, sometimes rather dramatically. For the simple case in which the stock index follows a binomial process, volatility options can be priced and hedge ratios can be calculated. For more complicated processes, an analytical solution may not be derivable, but numerical methods can still be used to evaluate the options.

While we have based most of our discussion on stock market volatility, our claims are also valid for the foreign exchange and bond markets. Corporations and financial institutions involved in the foreign exchange markets, for example, are affected by exchange rate volatility; a jump in volatility may have adverse consequences. For them, foreign exchange volatility futures and options can provide proper hedging.

## Footnotes

1. See, for example, K. R. French, G. W. Schwert and R. F. Stambaugh, "Expected Stock Returns and Volatility," *Journal of Financial Economics*, September 1987.
2. "BARRA Special Report on . . . The Crash of October 1987" (BARRA, Berkeley, CA, December 1987).
3. See "Stock Index Options and Futures Market

Analysis" (Goldman Sachs & Co., New York, March 1986).

4. See "Stock Index Weekly Report" (Financial Futures Department, Kidder Peabody & Co., New York, November 1987).
5. See Boyce, Hughs, Niculescu and Waldman, "The Implied Volatility of Fixed Income Markets" (Salomon Brothers Inc, New York, January 1988).
6. It is possible to hedge changes in volatility using a ratio spread with other options. This, however, would require dynamic hedging in options, usually a very expensive strategy.
7. He is protected against volatility changes but not against isolated "jumps" that may not be associated with volatility changes.
8. An announcement of a forthcoming OPEC meeting could have such an effect on the volatility of oil prices.
9. This phenomenon has been reported on many occasions.
10. The variance of the return at each point is given by:

$$V^2 = p(1 - p)(u - d)^2,$$

where p and (1 − p) are the probabilities of the market going up and down, respectively. Hence the volatility measure should be given by $(u - d)\sqrt{p(1 - p)}$. If $p = 1 - p = 0.5$, then the volatility is $0.5(u - d)$. In our simplified example, we approximate the volatility by $(u - d)$. Because the probabilities do not affect the pricing, and there are no effects of scale, our example is valid in illustrating the pricing mechanism.
11. J. Cox, S. Ross and M. Rubinstein, "Options Pricing: A Simplified Approach," *Journal of Financial Economics*, September 1979.
12. S. A. Ross, "Options and Efficiency," *Quarterly Journal of Economics*, February 1976.
13. *Ibid.*

# Profitability of a Trading Strategy Based on Unexpected Earnings

*by John C. Alexander, Delbert Goff and Pamela P. Peterson, Florida State University*

Numerous academic studies have concluded that investors can reap superior performance by trading on the basis of the unexpected earnings contained in public announcements.[1] The observance of superior risk-adjusted performance from trading on the basis of publicly available information appears to be contrary to market efficiency—hence, the unexpected earnings anomaly.

This article examines the benefits from trading securities on the basis of standardized unexpected earnings after an earnings announcement. Our analysis differs from that of other researchers in three ways. First, we use holding-period returns, which reflect a buy-and-hold strategy, instead of cumulative returns, which assume daily portfolio rebalancing. Second, we examine a more recent time period—a 30-quarter period that extends beyond the time at which the unexpected earnings anomaly was first documented. Finally, we employ a trading rule to assess the benefits from trading on the basis of unexpected earnings, taking into account alternative transaction costs.

## Method

To determine whether an investor actually can earn abnormal returns on the basis of unexpected earnings, we compared holding-period returns on common stocks that have been subject to good news with those on common stocks that have been subject to bad news. Unexpected earnings were based on a simple forecasting model:[2]

$$EPS_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 D1_t + \beta_4 D2_t + \beta_5 D3_t + \mu_t, \quad (1)$$

where

- $EPS_t$ is the earnings per share for the security during quarter t;
- $t$ indicates the quarter, capturing the trend in earnings per share over time;
- $t^2$ is the quarter indicator squared, capturing changes in the trend of earnings per share over time;
- $D1_t$ indicates whether quarter t is the second quarter (1 for the second quarter, 0 otherwise);
- $D2_t$ indicates whether quarter t is the third quarter (1 for the third quarter, 0 otherwise);
- $D3_t$ indicates whether quarter t is the fourth quarter (1 for the fourth quarter, 0 otherwise);
- $\beta$s are the estimated coefficients; and
- $\mu_t$ is the disturbance term.

We estimated Equation (1) over 16 quarters and used the coefficients obtained to predict the earnings per share (EPS) for the 17th quarter:

$$\widetilde{EPS}_{17} = \tilde{\beta}_0 + \tilde{\beta}_1 t + \tilde{\beta}_2 t^2 + \tilde{\beta}_3 D1_t + \tilde{\beta}_4 D2_t + \tilde{\beta}_5 D3_t, \quad (2)$$

1. Footnotes appear at end of article.