Understanding Tail Risk\textsuperscript{1}

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\textsuperscript{1}Based on work with Nic Kozeniauskas, Julian Kozlowski, Anna Orlik and Venky Venkateswaran.
Why Study Information Frictions?

- Every expectation, mean, variance, covariance is conditioned on some information set. Information assumptions pervade every model.
- Preferences, technologies and budget constraints have been exhaustively studied. Information is less explored.
- The finance sector is all about collecting, processing, transmitting and selling data (information).
- Problem: We can’t observe information. How to discipline?

Solutions:
- Count news stories, analyst reports, textual analysis, big data processing.
- Model information choice: Information differs from preferences because we have some control over what we learn.
- Treat agents like econometricians (today). They see what we see.
Introduction to Tail Risk

- Asset prices reflect risk. Risk depends on underlying variance of an outcome and on how much one knows.
- Many models shock tail risk, uncertainty, firm-specific risk … But where do these come from?

- Tail Risk: \( \text{Prob}[y_{t+1} < \alpha | I_t] \)
- Uncertainty: Stdev of a forecast (error) conditional on \( I_t \).
  \[
  U_t = \text{Std}[y_{t+1} | I_t] = \sqrt{E \left[ (y_{t+1} - E (y_{t+1} | I_t))^2 | I_t \right]}
  \]
- Firm-specific (micro) risk: \( \int (y_{it} - \bar{y}_t)^2 di. \)
- Higher-order uncertainty: \( \int (E[y_t | I_{it}] - \bar{E}[y_t])^2 di. \)
Two Possible Sources of Shocks

1. Actual variance of some data-generating distribution changes.
   - Jurado, Ludvigson, Ng (2015): Find two large increases in macro variance.
   - Tail risk? Hard to measure changes.

2. Conditional variance changes because our beliefs about the distribution change.
   - Why would beliefs change if the true distribution is the same?
   - We must not know the distribution and learn about it.
How Do We Learn About Distributions?

1. A Bayesian parametric approach

2. A classical econometrics, non-parametric approach.

In both cases,

- We’ll use macro data and standard econometric tools to estimate a distribution and then re-estimate it each period with new data. Our agents do the same.
- Changes in variance and tails of this distribution are a key source of shocks.
Key feature: Agents estimate tail probabilities.

A normal distribution fixes these $\rightarrow$ no $U_t$ action.
Need parameters that govern higher moments (skewness).
  - Can capture skewness in GDP data (-0.3)
  - Key for our forecasts to resemble SPF forecast data

Solution: Take a linear hidden state model (Kalman filter system) and do an exponential twist.

A form of g-and-h transformation used in statistics for Bayesian distribution fitting (Headrick '10).
Forecasting Exercise

- We estimate this:

\[
y_t = c + b \exp(-S_t - \sigma \varepsilon_t)
\]

\[
S_t = \rho S_{t-1} + \sigma^S \xi_t
\]

where \( \varepsilon_t \) and \( \xi_t \sim iid \ N(0, 1) \). \( y_t = GDP \) growth.

Use real-time GDP data (1968-2013, Philly Fed) to estimate.

- Begin with prior beliefs estimated from 1947-68 data.

- Observe each quarter of data and apply Bayes’ Law.
  - Metropolis-Hastings + change-of-measure
    \( \rightarrow \) distributions of parameters.

- How big are uncertainty changes? \( U_t = Std[y_{t+1} | I_t] \).
I'm uncertain about the uncertainty of the economy. This, I am certain of...
Result 1: Large Uncertainty Shocks

Parameter learning + Skewness = Large uncertainty shocks.
What Explains Large Shocks? Tail Risk.

Tail Risk_t \equiv \text{Prob}[y_{t+1} \leq -6.8\% | y^t] \quad (1\text{-in-100 year event})

\text{Correlation}(BSw, U_t) \text{ is } 75\% \quad (\text{both detrended}).

Most changes in uncertainty come from re-estimating tail risk.
Why Is Tail Risk Volatile?

- Extreme event probabilities are very sensitive to small revisions in skewness.
- Skewness keeps fluctuating because it is hard to learn.
Tail Risk amplifies uncertainty in bad times

Skewness can be represented as a concave function of a normal.

Skewness, which governs tail risk, amplifies macro uncertainty in bad states.
Tail Risk Also Creates Forecast Bias

\[ E[y_{t+1}|y^t, \theta] \] is mean GDP growth = 2.68%.
\[ E[y_{t+1}|y^t] \] is average growth forecast = 2.29% in data, = 2.27% in model.

**Lemma:** Suppose \( y = g(x) \) where \( g \) is concave and \( x \sim N(\mu, \sigma) \). \( \mu \) and \( \sigma \) are unknown, with unbiased beliefs. Then mean > forecast.

When we estimate parameter uncertainty and skewness, we match the bias in professional forecasts.
Approach 2: Non-parametric, Classical Estimation

Consider an iid shock, \( \phi_t \), with unknown pdf \( g \)

Information set: finite history of shock realizations \( \{ \phi_{t-s} \}_{s=0}^{n_t-1} \)

The Gaussian kernel density estimator

\[
\hat{g}_t(\phi) = \frac{1}{n_t \kappa} \sum_{s=0}^{n_t-1} \Omega \left( \frac{\phi - \phi_{t-s}}{\kappa} \right)
\]

Key property: Beliefs are martingales

\[
E_t[\hat{g}_{t+1}|I_t] \approx \hat{g}_t \quad \rightarrow \quad \text{Persistence}
\]

Next: use this mechanism to create persistence (long run risk).
Tail Risk Stayed High

Why do some recessions have persistent effects?

- Because they cause us to re-assess macro risk.

Suggestive evidence from financial markets: A tail risk index

Note: Constructed from out-of-the-money put options on S&P 500
Economic Model - based on Gourio (AER, 2012)

Representative household with Epstein-Zin preferences over, $C_t - \zeta \frac{L_t^{1+\gamma}}{1+\gamma}$

A continuum of firms, indexed by $i$

- **Production:** $y_{it} = A k_{it}^\alpha l_{it}^{1-\alpha}$

- **Aggregate capital quality shocks:** $k_{it} = \phi_t \hat{k}_{it}$, $\phi_t \sim g(\cdot)$ iid

- **Idiosyncratic shocks (iid):** $\Pi_{it} = v_{it} [y_{it} + (1-\delta)k_{it}]$, $\int v_{it} di = 1$

Debt has a tax advantage and a default cost.

Labor hired in advance, before observing shocks.
Capital Quality Shocks

Key feature: Increase left tail risk, post-2009

We do:
- Calibrate model,
- feed in this data through 2007, normalize ‘07 outcome to 0.
- observe effects of 08-09 shocks,
- take random draws from the 2009 distribution (report avg outcome).
Without belief revisions, a steady recovery to initial level
Conclusions

- Obviously, no one knows the true distribution of shocks. Simple, disciplined tools to replace rational expectations hypothesis.

- New data permanently reshapes our assessment of macro risks, especially tail risks because data on tails is scarce.

- Changes in tail risk provide a unified theory of uncertainty, risk, sentiment shocks and belief biases.

  → A new persistence mechanism / source of long-run risk.
  → A new source of price fluctuations
  → A new risk factor

How much of business cycle fluctuations, asset pricing puzzles or other phenomena could learning about tail risk explain?