Knowing What Others Know:
Coordination Motives in Information Acquisition

Additional Notes

Christian Hellwig
University of California, Los Angeles
Department of Economics

Laura Veldkamp
New York University
Stern School of Business

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General Equilibrium Foundations for Planning Model

In this appendix, we derive the objective function (equation 4) of our dynamic planning and price adjustment model from a fully specified dynamic general equilibrium model. The model we discuss here is similar to Hellwig (2004), with the main difference being the updating of information by firms. Consider a standard model of incomplete nominal adjustment with monopolistic firms, along the lines of Blanchard and Kiyotaki (1987), with nominal prices being preset, conditional on available information, before markets open. Time is discrete and infinite. There is a measure 1 continuum of different intermediate goods, indexed by $i \in [0, 1]$, each produced by one monopolistic firm using labor as the unique input into production. There is a final consumption good, which is produced by a perfectly competitive final goods sector using the continuum of intermediates according to a Dixit-Stiglitz CES technology with constant returns to scale. On the consumption side, there is an infinitely-lived representative household, with preferences defined over the final consumption good and labor supply in each period. The household faces a Cash-in-Advance constraint, and has to finance consumption out of the current period’s nominal balances. Each period is separated into two stages: in the first stage, a nominal shock is realized in the form of a stochastic lump sum transfer to the representative household. At this point, each intermediate goods producers decides whether or not to pay a fixed labor cost to ‘plan’, by which he updates his information set in way that we will describe below. Intermediate firms then set prices on the basis of their information sets. In the second stage, markets open. Intermediates are traded at
the posted prices, and intermediate producers hire labor to satisfy the demand for their products at the posted prices. The wage rate and the final goods price adjust to clear the labor, goods and money markets.

**Household Preferences:** The representative household’s preferences over final good consumption and labor supply \( \{C_{t+\tau}, n_{t+\tau}\}_{\tau=0}^{\infty} \) are given by

\[
U_t = \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} \beta^\tau (\log C_{t+\tau} - n_{t+\tau}) \right]
\]  

where \( \beta \in (0, 1) \) denotes the discount rate and \( \mathbb{E}_t (\cdot) \) denotes the household’s expectations as of date \( t \). The household’s objective is to maximize (1) subject to its sequence of flow budget constraints, for \( \tau = 0, 1, \ldots \)

\[
P_{t+\tau} C_{t+\tau} + M_{t+\tau}^d = W_{t+\tau} n_{t+\tau} + M_{t+\tau-1}^d + T_{t+\tau} + \Pi_{t+\tau}
\]  

where \( M_{t+\tau}^d \) denotes the household’s demand for nominal balances, \( P_{t+\tau} \) the price of the final consumption good, \( W_{t+\tau} \) the nominal wage rate, \( T_{t+\tau} \) a stochastic monetary transfer the household receives at the beginning of each period, and \( \Pi_{t+\tau} \) the aggregate profits of the corporate sector, which are rebated to the household. Wage payments and corporate profits are transferred to the household at the end of each period. In addition, the household has to satisfy a Cash-in-Advance constraint and finance its purchases of the consumption good out of its nominal balances after receiving the monetary transfer; i.e. for \( \tau = 0, 1, \ldots \)

\[
P_{t+\tau} C_{t+\tau} \leq M_{t+\tau-1}^d + T_{t+\tau}
\]  

The nominal money supply is stochastic, with the government making a lump sum transfer \( T_{t+\tau} = M_s^t - M_{t+\tau-1}^s \) to the representative household at the beginning of each period. Specifically, \( m_t \equiv \log M_s^t \) follows a random walk, \( m_t = m_{t-1} + \mu_t. \mu_t \sim \mathcal{N}(0, \sigma^2) \) is i.i.d. over time. In each period, the household chooses final good consumption \( C_t \), labor supply \( n_t \), and money demand \( M_{t}^d \) to maximize (1), subject to the constraints (2) and (3). Finally, \( \gamma^{-1} = \beta e^{\frac{1}{2} \sigma^2} < 1 \). This assumption guarantees that the Cash-in-Advance constraint is binding in every state and date:

**Lemma 1** The Cash-in-Advance constraint is always binding, and the household’s optimal consumption is given by

\[
C_t = \frac{M_s^t}{P_t}.
\]  

The equilibrium wage rate satisfies

\[
W_t = \gamma M_s^t.
\]
**Proof.** Let $\theta_t$ denote the Lagrange multiplier on the period $t$ Cash-in-Advance constraint, and $\chi_t$ the Lagrange multiplier on the period $t$ budget constraint. The first-order conditions for the representative households problem are:

\[
\frac{1}{P_tC_t} = \chi_t + \theta_t \quad \chi_t = \beta E_t (\chi_{t+1} + \theta_{t+1}) \quad W_t \chi_t = 1
\]

We check that the proposed solution $C_t = M_t$ satisfies these first-order conditions. To see that it does, notice that $\chi_t = \beta E_t \left( \frac{1}{P_t} + \frac{1}{P_{t+1} \chi_{t+1}} \right) = \frac{1}{M_t} \beta E_t (e^{-\mu_{t+1}}) = \frac{1}{\gamma M_t} > 0$, and $\theta_t = \frac{1}{M_t} - \frac{1}{\gamma M_t}$, which is positive whenever $\gamma^{-1} \equiv \beta E_t (e^{-\mu_{t+1}}) = \beta e^{\frac{1}{2} \sigma^2} < 1$. Therefore, $\gamma^{-1} < 1$ is sufficient for the Cash-in-advance constraint to hold every period. But then, the wage equation follows immediately.

**Final Good Producers:** A large number of final goods producers uses the intermediate goods to produce the final output according to a constant returns to scale technology, which is given by the CES aggregator

\[
C_t = \left[ \int_0^1 (c_i^\theta) \frac{\theta - 1}{\theta} di \right]^{\frac{1}{\theta - 1}}. \quad (6)
\]

Final goods producers maximize profits, taking as given the market prices of intermediate and final goods. For a total demand $C_t$ of the final good by the household, a final goods price $P_t$, and input prices $P_{it}$, the demand for intermediate good $i$ by the final good sector is given by

\[
c_i^t = c (P_{it}) = C_t \left( \frac{P_{it}}{P_t} \right)^{-\theta} . \quad (7)
\]

The final goods price $P_t$ is given by the Dixit-Stiglitz aggregator

\[
P_t = \left[ \int_0^1 (P_{it})^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \quad (8)
\]

**Intermediate Good Producers:** Each intermediate good is produced by a single monopolistic firm using labor as the only input into production, according to a technology with decreasing returns to scale. In order to produce $y$ units of good $i$, firm $i$ needs to hire $n(y)$ units of labor, where $n(y)$ is given by

\[
n(y) = \frac{1}{\delta} y^\delta, \quad (9)
\]

with $\delta > 1$. $i$'s nominal profits $\pi_i^t$ (not including information costs), as a function of its price $P_{it}$, are given by:

\[
\pi_i^t = P_{it} c (P_{it}) - W_t n \left( c (P_{it}) \right), \quad (10)
\]

\footnote{Alternatively, the production function $g(n)$ is given by $g(n) = [\delta n^{1/\delta}]$}
where \( c(p_i^t) \) denotes the stochastic demand firm \( i \) faces for its product, given by (7).

In each period, intermediate goods producers first decide whether or not to pay a fixed cost of \( C > 0 \) units of labor to ‘plan’. By planning, firms update their information set to incorporate all new information since the last date at which the firm planned. Formally, if firm \( i \) last planned in period \( s < t \), then its information set from the previous period is \( \mathcal{I}_{i-1}^t = \mathcal{I}_s \equiv \{m_{s-\tau}\}_{\tau=0}^\infty \). If this firm plans in the current period, its current period information set is \( \mathcal{I}_i^t = \mathcal{I}_t \), if not, its information set remains \( \mathcal{I}_i^t = \mathcal{I}_{i-1}^t = \mathcal{I}_s \). In particular, this assumption implies that firms do not incorporate any intermediate observations of wage rates, demand realizations or prices into their beliefs inbetween planning dates.

**Equilibrium Pricing and Planning Decisions:** After taking its planning decision, each firm decides what prices \( p_i^t \) to set; \( p_i^t \) must be measurable with respect to \( \mathcal{I}_i^t \). Let \( E_i^t(\cdot) \) denote the agents’ expectation conditional on \( \mathcal{I}_i^t \). We discount the firms’ nominal profits according to the household’s expected marginal utility of money. At the stage of price-setting, firms therefore set \( p_i^t \) to maximize

\[
E_i^t\left( \beta E_t \left( \frac{1}{C_{t+1} P_{t+1}} \right) \pi_i^t \right) = E_i^t \left( \frac{\pi_i^t}{W_t} \right) = E_i^t \left( \frac{1}{\gamma M_t} \left[ P_i^t c(P_i^t) - W_t n(c(P_i^t)) \right] \right) \\
= E_i^t \left( \frac{1}{\gamma M_t} \left[ P_i^t M_t \left( \frac{P_i^t}{P_t} \right)^{-\theta} - \gamma M_t \frac{1}{\delta} \left( \frac{M_t}{P_t} \left( \frac{P_i^t}{P_t} \right)^{-\theta} \right)^\delta \right] \right) \\
= E_i^t \left[ \frac{1}{\gamma} \left( \frac{P_i^t}{P_t} \right)^{1-\theta} - \frac{1}{\delta} \left( \frac{M_t}{P_t} \right)^\delta \left( \frac{P_i^t}{P_t} \right)^{-\theta \delta} \right]. \tag{11} \]

Therefore, a firm that last planned in period \( s \leq t \) sets \( P_{t,s} \) as its optimal price in period \( t \), where

\[
P_{t,s} = \left( \frac{\theta \gamma}{\theta - 1} \right)^{1-\theta \delta \gamma / \delta} \left[ E_s \left( M_t^\delta P_t^{\delta(\theta-1)} \right) \right] \left[ E_s \left( P_t^{\theta-1} \right) \right] \]

\( E_s \) denotes the firm’s expectations conditional on \( \mathcal{I}_s \). The firm’s expected profits (not including information costs) in period \( t \) are

\[
E_s \left( \frac{\pi_i^t}{W_t} \right) = \left[ \frac{\theta \gamma}{\theta - 1} - \frac{1}{\delta} \right] \left( \frac{\theta \gamma}{\theta - 1} \right)^{-\theta \delta \gamma / \delta} \left[ E_s \left( M_t^\delta P_t^{\delta(\theta-1)} \right) \right] \left[ E_s \left( P_t^{\theta-1} \right) \right] \left[ E_s \left( P_t^{\theta-1} \right) \right] \]

Under full information, the optimal price in period \( t \) is

\[
P_t^* = \left( \frac{\theta \gamma}{\theta - 1} \right)^{1-\theta \delta \gamma / \delta} M_t^{\delta / \delta} P_t^{\delta(\theta - 1) / \delta} \]
and full information profits are

\[
\frac{\pi_i^t (P^*_t)}{W_t} = \left[ \frac{\theta}{\theta - 1} - \frac{1}{\delta} \right] \left( \frac{\theta \gamma}{\theta - 1} \right)^{-\frac{\delta \gamma}{1 - \theta + \theta \delta}} \left( \frac{M_t}{P_t} \right)^{(1 - \theta) \delta}.
\]

Taking the log of \(P^*_t\), we find

\[
p_t^* \equiv \log P^*_t = (1 - r) \log \left( \frac{\theta \gamma}{\theta - 1} \right)^{\frac{1}{\delta}} + (1 - r) m_t + r \log P_t,
\]

which, up to a constant, gives us exactly the linear relation in eq. 5 of the main text, with

\[
r = \frac{(\delta - 1)(\theta - 1)}{1 - \theta + \theta \delta}.
\]

Finally, let \(\lambda_{t,s}\) denote the measure of firms in period \(t\) who last planned in period \(s \leq t\). \(\{\lambda_{t,s}\}\) is updated from the planning decisions exactly as described in the text, and the price index \(P_t\) satisfies

\[
P_t = \left\{ \sum_{s \leq t} \lambda_{t,s} P_{t,s}^{1 - \theta} \right\}^{\frac{1}{1 - \theta}}.
\]

Therefore, once we replace the firm’s reduced form objective (eq. 4 in the main text) by

\[
\sum_{t=0}^{\infty} \beta^t \left[ \frac{\pi_i^t (P^*_t)}{W_t} - \frac{\pi_i^t (P^t)}{W_t} \right] + D^t C
\]

and take into account that the price index is computed according to (12), our equilibrium definition applies identically to this environment. 2

**Second-order approximation.** To complete our discussion, we show that we can approximate the firms’ objective quadratically as in eq. 4 in the text, the price index by a simple arithmetic mean, when \(\sigma\) is small. Clearly, when shocks are small, \(p_t \equiv \log P_t \approx \sum_{s \leq t} \lambda_{t,s} \log P_{t,s}\), that is, the log of price index (a hyperbolic mean) is approximated by the arithmetic mean of the log price changes. We therefore approximate the loss in (13) by the quadratic loss in eq. 4 of the main text.

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2The planning cost is modelled as a labor cost. The above expression is derived from multiplying \(C\) with \(W_t\) and dividing by the firm’s stochastic discount factor, which again is \(W_t\) in the present model.
Now,

\[
\frac{\pi_i^t (P_i^*)}{W_t} - \frac{\pi_i^t (P_i^t)}{W_t} = \frac{1}{\gamma} \left( P_i^* \right)^{1-\theta} \left[ 1 - \left( \frac{P_i^t}{P_i^*} \right)^{1-\theta} \right] - \frac{1}{\delta} \left( \frac{M_t}{P_i^t} \right) \left( P_i^* \right)^{-\theta\delta} \left[ 1 - \left( \frac{P_i^t}{P_i^*} \right)^{-\theta\delta} \right]
\]

\[= \frac{1}{\gamma} \left( \frac{\theta\gamma}{\theta - 1} \right)^{\frac{1-\theta}{1-\theta+\delta\gamma}} \left( \frac{M_t}{P_i^t} \right)^{\frac{(1-\theta)\delta}{1-\theta+\delta\gamma}} \left[ 1 - \left( \frac{P_i^t}{P_i^*} \right)^{1-\theta} \right]
\]

\[\approx \frac{\theta}{\theta - 1} \left[ 1 - \left( \frac{P_i^t}{P_i^*} \right)^{1-\theta} \right] - \frac{1}{\delta} \left[ 1 - e^{(1-\theta)x_i^t} \right] - \frac{1}{\delta} \left[ 1 - \left( \frac{P_i^t}{P_i^*} \right)^{-\theta\delta} \right]
\]

Thus, the firm’s objective is quadratic in \( \log P_i^t - \log P_i^* \). Using a second-order Taylor expansion around \( x_i^t = 0 \), we have

\[
\frac{\theta}{\theta - 1} \left[ 1 - \left( \frac{P_i^t}{P_i^*} \right)^{1-\theta} \right] - \frac{1}{\delta} \left[ 1 - \left( \frac{P_i^t}{P_i^*} \right)^{-\theta\delta} \right] \approx \frac{1}{2} \theta \left[ 1 - \theta + \delta \theta \right] (x_i^t)^2
\]

Thus, the firm’s objective is quadratic in \( \log P_i^t - \log P_i^* \). It mirrors the objective in the planning model (equation 4 of the paper), up to a linear transformation. The multiplier in that linear transformation does include a term \( \frac{M_t}{P_i^t} \), equal to real consumption, which fluctuates. But the fluctuations are small, on the order of 2% up or down in a business cycle. This is not large enough to induce firms to change their planning calculations in a noticeable way, over time. Furthermore, it would not distort the strategic motives to plan, in any period. Therefore, the quadratic objective function, with constant coefficient is a good second-order approximation to the firm’s true objective.

References
