

# Arbitrage

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# Outline

- Arbitrage definitions
- Arbitrage pricing
- Arbitrage pricing with transactions costs
- Real-world arbitrage trading

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# Arbitrage Definitions

- In finance *theory* an “arbitrage” is defined as:
  - a trading strategy that generates a completely riskless profit, that is,
  - a trading strategy that generates a positive cash flow at some time and non-negative cash flows at all times.
- On Wall Street “arbitrage” also often refers to
  - a trading strategy that is expected to make a profit
  - This is also called a “statistical arbitrage” (stat. arb.)
- In this class, we follow the finance-theory tradition: an arbitrage generates a riskless profit

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## Arbitrage Pricing

- Important insight in finance:
  - there cannot be arbitrage opportunities
  - if there were, arbitrageurs would trade aggressively to exploit the arbitrage.
  - called the “No-Arbitrage Condition”
- Perhaps surprisingly, using this restriction alone we can
  - compute restrictions on security prices
  - compute explicitly prices of derivatives.

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## Implications of No Arbitrage

1. If two securities have the same payoffs, they must have the same price
2. If a portfolio has the same payoff as a security, the price of the security must be equal to the price of the portfolio
  - In that case, the portfolio is called a *replicating portfolio*
3. If a self-financing trading strategy has the same final payoff as a security, the price of the security must be equal to the cost of the strategy
  - This is called a *dynamic hedging strategy*

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## Example 1: Arbitrage Pricing

- Suppose there are two kinds of zero-coupon bonds, “CATs” and “TIGRs,” both paying a face value of \$100 in 1 year.
- Suppose that the price of CATs is \$98.
- What must the price of TIGRs be?
- How could you make an arbitrage if TIGRs were trading at \$97?

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## Example 2: Arbitrage Pricing with a Replicating Portfolio

- Suppose
  - a zero-coupon bond that matures 1 year from today costs \$98
  - a zero-coupon bond that matures 2 years from today costs \$96
  - a zero-coupon bond that matures 3 years from today costs \$93
- What must be the price of a coupon bond with a 10% coupon rate?
- How could you make an arbitrage if the coupon bond were trading at \$120?

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7

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## Example 3: Arbitrage Pricing with Dynamic Hedging

- Suppose
  - a zero-coupon bond that matures 1 year from today costs \$98
  - 1 year from today, a zero-coupon bond that matures 2 years from today also costs \$98
- What must be the price of a zero-coupon bond that matures 2 years from now?
- How could you make an arbitrage if the bond were trading at \$97?

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8

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## Arbitrage Pricing with Transactions Costs

- If there are transactions costs, it is more difficult to make an arbitrage trading strategy
- Therefore, we cannot determine prices exactly using the No-Arbitrage Condition
- But, we can find an *interval* in which the price must be.
- Said differently, we can find an *upper- and lower bound* for the price.

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9

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## Example 4: Arbitrage Pricing with Transactions Costs

- Suppose there are two zero-coupon bonds, CATs and TIGRs, both paying a face value of \$100 in 1 year.
- The price of CATs is \$98
- Supposing the cost of shorting is \$1, and the cost of buying is 0, what is, respectively, the highest and lowest possible price of TIGRs?
- What if the cost of shorting is \$0.50 and the cost of buying is \$0.50?

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10

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## Real-World Stat. Arb. Strategies

- Relative mis-pricings and convergence trades 
  - index arbitrage
  - fixed-income securities, e.g.
  - on-the-run vs. off-the-run Treasuries
- Special situations
  - devaluations of currency
  - mergers and acquisitions (“risk arb.”)
  - IPOs, carve-outs, and spin-offs
  - announcements (e.g. of earnings or macro news)

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