The Capital Asset Pricing Model (CAPM)

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Outline

- Key questions:
  - What is the *equilibrium required return, E(R),* of a stock?
  - What is the *equilibrium price* of a stock?
  - Which *portfolios* should investors hold in equilibrium?
- Answer: CAPM
  - Assumptions
  - Results:
    - Identify the tangency portfolio in equilibrium
    - Hence, identify investors' portfolios
    - Derive equilibrium returns (and hence prices)

CAPM: Introduction

- Equilibrium model that
  - predicts optimal portfolio choices
  - predicts the relationship between risk and expected return
  - underlies much of modern finance theory
  - underlies most of real-world financial decision making
- Derived using Markowitz's principles of portfolio theory, with additional simplifying assumptions.
- Sharpe, Lintner and Mossin are researchers credited with its development.
- William Sharpe won the Nobel Prize in 1990.

CAPM Assumptions

- Stylized Assumptions:
  1. The market is in a competitive equilibrium;
  2. Single-period investment horizon;
  3. All assets are tradable;
  4. No frictions;
  5. Investors are rational mean-variance optimizers with
  6. homogeneous expectations
- Some assumptions can be relaxed, and CAPM still holds.
- An important approximation of reality in any case.
- If many assumptions are relaxed, generalized versions of CAPM applies. (Topic of ongoing research.)

1: The market is in a competitive equilibrium

- Equilibrium:
  - Supply = Demand
  - Supply of securities is fixed (in the short-run).
  - If Demand > Supply for a particular security, the excess demand drives up price and reduces expected return.
  - (Reverse if Demand < Supply)
- Competitive market:
  - Investors take prices as given
  - No investor can manipulate the market.
  - No monopolist

2: Single-period horizon

- All investors agree on a horizon.
- Ensures that all investors are facing the same investment problem.

3: All assets are tradable

- This includes in principle:
  - All financial assets (including international stocks)
  - Real estate
  - Human capital
- This ensures that every investor has the same assets to invest in:
  - all the assets in the world, the "market portfolio"

4: No frictions

- No taxes
- No transaction costs (no bid-ask spread)
- Same interest rate for lending and borrowing
- All investors Investors can borrow or lend unlimited amounts. (No margin requirements.)

5-6: Investors are rational mean-variance optimizers with homogeneous expectations

- Investors choose efficient portfolios consistent with their risk-return preferences
- Investors have the same views about expected returns, variances, and covariances (and hence correlations).

What is the Equilibrium Tangency Portfolio?

- Recall from portfolio theory:
  - All investors should have a (positive or negative) fraction of their wealth invested in the risk-free security, and
  - *The rest of their wealth is invested in the tangency portfolio.*
  - *The tangency portfolio is the same for all investors* (homogeneous expectations).
- In equilibrium, supply=demand so:
  - **the tangency portfolio must be the portfolio of all existing risky assets, the "market portfolio" !!**

The Market Portfolio

- p_i = price of one share of risky security i
- n_i = number of shares outstanding for risky security i
- M = Market Portfolio. The portfolio in which each risky security i has the following weight:

$$\omega_{iM} = \frac{p_i \times n_i}{\sum_i p_i \times n_i}$$

$$= \frac{\text{market capitalization of security } i}{\text{total market capitalization}}$$

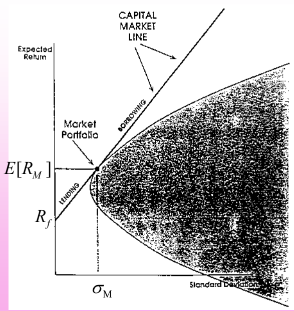
In words, the <u>market portfolio</u> is the portfolio consisting of all assets (everything!). However, you also invest in the market portfolio if you buy a few shares of every security, *weigthed by their market cap*.

The Capital Market Line (CML)

- Recall: The CAL with the highest Sharpe ratio is the CAL with respect to the tangency portfolio.
- In equilibrium, the market portfolio is the tangency portfolio.
- The market portfolio's CAL is called the <u>Capital Market Line (CML)</u>.
- The CML gives the risk-return combinations achieved by forming portfolios from the risk-free security and the market portfolio:

$$E(R_p) = R_f + \frac{[E(R_M) - R_f]}{\sigma_M} \sigma_p$$

The E(R)-SD Frontier and The Capital Market Line



The Required Return on Individual Stocks

- CAPM is most famous for its prediction concerning the relationship between risk and return for individual securities:

$$E[R_i] = R_f + \beta_i \cdot \lfloor E[R_M] - R_f \rfloor$$

$$\text{where } \beta_i = \frac{\text{cov}[R_i, R_M]}{\sigma_M^2}$$

- The model predicts that expected return of an asset is linear its 'beta'.
- This linear relation is called the Security Market Line (SML).
- The beta measures the security's sensitivity to market movements.

Deriving CAPM Equation using FOC

- The market portfolio is the tangency portfolio and therefore it solves:

$$\max_{w_1,\dots w_n \in \mathbf{R}} SR_p = \frac{E(R_p) - R_f}{\sigma_p}$$

$$\text{where } E(R_p) = \sum_i w_i E(R_i) + (1 - \sum_i w_i) R_f$$

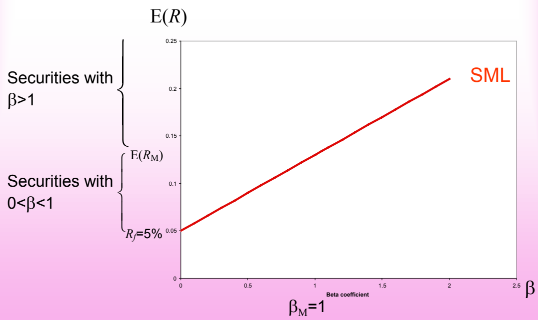
$$\sigma_p = \sqrt{\sum_{i,j} w_i w_j \text{cov}(R_i, R_j)}$$

- The first-order condition (FOC) is:

$$0 = \frac{\partial}{\partial w_i} SR_p \Big|_{w=w^M} \qquad \text{that is,}$$

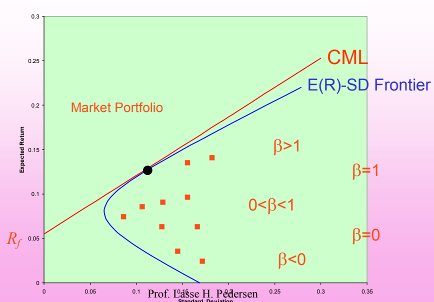
$$0 = (E(R_i) - R_f)\sigma_M - (E(R_M) - R_f)\frac{\text{cov}(R_i, R_M)}{\sigma_M}$$

Security Market Line (SML)

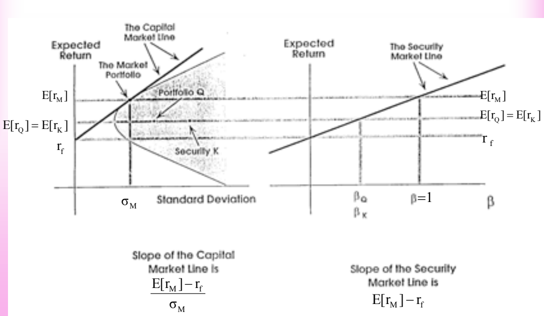


E(R)-SD Frontier and the Betas

- The graph relates the location of the individual securities with respect to the M-SD frontier to their betas.



The Capital Market Line and the Security Market Line



Systematic and Non-Systematic Risk

- The CAPM equation can be written as:

$$R_i = R_f + \beta_i \cdot (R_M - R_f) + error_i$$

$$\text{where } \beta_i = \frac{\text{cov}[R_i, R_M]}{\sigma_M^2}$$

$$\text{E}(error_i) = 0$$

$$\text{cov}[error_i, R_M] = 0$$

- This implies the total risk of a security can be partitioned into two components:

$$\underbrace{\sigma_i^2}_{\substack{\text{var}(R_i)\\ \textit{total}\\ \textit{risk}}} = \underbrace{\beta_i^2 \sigma_M^2}_{\substack{\textit{market}\\ \textit{risk}}} + \underbrace{\bar{\sigma}_i^2}_{\substack{\text{var}(error_i)\\ \textit{idiosyncratic}\\ \textit{risk}}}$$

Systematic and Non-Systematic Risk: Example

- ABC Internet stock has a volatility of 90% and a beta of 3. The market portfolio has an expected return of 14% and a volatility of 15%. The risk-free rate is 7%.
- What is the equilibrium expected return on ABC stock?
- What is the proportion of ABC Internet's variance which is diversified away in the market portfolio?

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \bar{\sigma}_i^2$$

$$(0.9)^2 = 3^2 \times 0.15^2 + \bar{\sigma}_i^2$$

$$\bar{\sigma}_i^2 = 0.6075 \qquad (\bar{\sigma}_i = 0.779)$$

$$\text{Hence } \frac{0.6075}{(0.9)^2} = 75\% \text{ of variance is diversified away}$$

Systematic and Non-Systematic Risk

- β_i measures security i's contribution of to the total risk of a well-diversified portfolio, namely the market portfolio.
- Hence, β_i measures the non-diversifiable risk of the stock
- Investors must be compensated for holding non-diversifiable risk. This explains the CAPM equation:

$$\text{E}(R_i) = R_f + \beta_i\, [\,\text{E}(R_\text{M}) - R_f\,], \quad i = 1,\ldots,\text{N}$$

Risk Premium

- Recall the SML: $E(R_i) = R_f + \beta_i [E(R_M) - R_f]$
- Stock *i*'s systematic or market risk is: β_i
- Investors are compensated for holding systematic risk in form of higher returns.
- The size of the compensation depends on the *equilibrium risk premium*, $[E(R_M) - R_f]$.
- The equilibrium risk premium is **increasing** in:
  1. the variance of the market portfolio
  2. the degree of risk aversion of average investor

Estimating Beta

An Example:

Some Betas, June 1996

Battle Mountain Gold Company	.40
Boeing Corporation	.90
Bristol-Myers Squibb	.95
California Water Company	.45
Caterpillar Inc.	1.20
Coca-Cola	.95
Dow Chemical	1.15
Exxon Corporation	.65
The Gap, Inc.	1.45
General Electric	1.15
Harley-Davidson	1.65
Idaho Power Company	.65
Intel Corporation	1.35
Kaufman & Broad Home	1.65
Kellogg	1.00
Merrill Lynch & Company	1.90
Oshkosh B'Gosh (clothing mfg.)	.60
Outback Steakhouse	2.10
Procter & Gamble	1.05
Ralston Purina	.90
Telefonos de Mexico	1.35
Tootsie Roll Industries	.75
Toys 'Я' Us	1.45
Western Digital	1.85

Many institutions estimate beta's, e.g.:

- Bloomberg
- Merrill Lynch
- Value Line
- Yahoo

Estimating Beta by Linear Regressions (OLS)

- CAPM equation: $E[R_i] - R_f = \beta_i \left[E[R_m] - R_f \right]$
- Get data on "excess returns":

$$R_i^e(t) = R_i(t) - R_f \qquad R_M^e(t) = R_M(t) - R_f$$

- where R_f is the risk-free rate from time t-1 to time t.
- Estimate β_i by running the regression:

$$R_i^e(t) = \alpha_i + \beta_i R_m^e(t) + error_i(t)$$

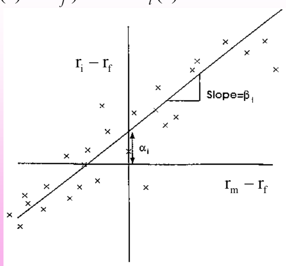
- Typically, 60 months of data are used.

Security Characteristic Line (SCL)

The SCL is the "regression line":

$$R_i(t) - R_f = \alpha_i + \beta_i (R_M(t) - R_f) + error_i(t)$$

Note:
CAPM implies $\alpha_i = 0$

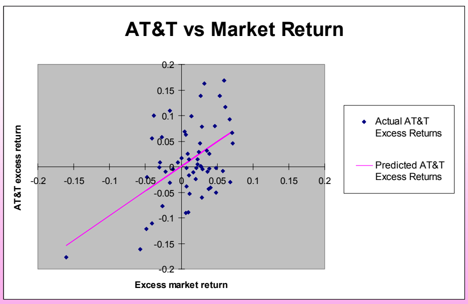


Estimating Beta: Real Life Example, AT&T

- Take 5 years (1994-1998) of monthly data on AT&T returns, S&P500 returns and 1 month US T-bills.
- Construct excess returns
- Run the regression, for instance using Excel:
  - apply *Tools, Add-ins, Analysis ToolPak*
  - use *Tools, Data Analysis, Regression*
- The result is in the spreadsheet "betareg.xls"
- Excel Regression output:

	Coefficients	*SE*	*t Stat*	*P-value*
Intercept	0.0007	0.0091	0.0748	0.9406
X Variable 1	0.9637	0.2172	4.4366	0.0000

Estimating Beta: Real-Life SCL for AT&T



Applications of the CAPM

- Portfolio choice
- Shows what a "fair" security return is
- Gives benchmark for security analysis
- Required return used in capital budgeting to
  - compute NPV of risky project
  - or "hurdle rate" for IRR
- Evaluation of fund manager performance.

Stock Selection and Active Management

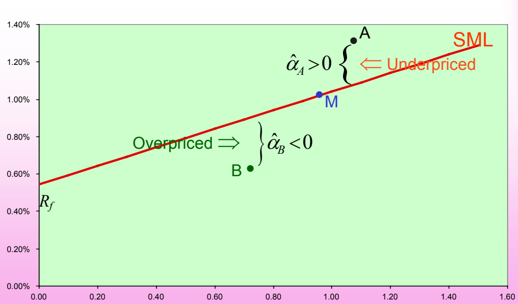
- One possible benchmark for *stock selection* is to find assets that are cheap relative to CAPM (or more advanced models).
- A security's *alpha* is defined as:

$$\alpha_i = E[R_i] - R_f - \beta_i \cdot \lfloor E[R_M] - R_f \rfloor$$

$$\text{where } \beta_i = \frac{\text{cov}[R_i, R_M]}{\sigma_p^2}$$

- Some fund managers try to buy positive-alpha stocks and sell negative-alpha stocks.
- CAPM predicts that all alpha's are zero.

Stock Selection



Active and Passive Strategies

- An “active” strategy tries to beat the market buy stock picking, by timing, or other methods
- But, CAPM implies that
  - security analysis is not necessary
  - every investor should just buy a mix of the risk-free security and the market portfolio, a “passive” strategy.

Summary

- The CAPM follows from equilibrium conditions in a frictionless mean-variance economy with rational investors.
- Prediction 1: Everyone should hold a mix of the market portfolio and the risk-free asset. (That is, everyone should hold a portfolio on the CML.)
- Prediction 2: The expected return on a stock is a linear function of its beta. (That is, stocks should be on SML.)
- The beta is given by: $$\beta_i = \frac{\text{cov}[R_i, R_M]}{\sigma_M^2}$$
- A stock’s beta can be estimated using historical data by linear regression. (That is, by estimating the SCL.)