Portfolio Selection with Multiple Risky Securities.

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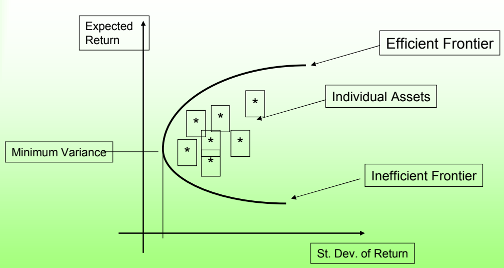
Outline

- Investment opportunity set
  – with many risky assets
  – with many risky assets and a risk-free security
- Optimal portfolio choice and two-fund separation
- Diversifiable and non-diversifiable risk

Investment Opportunity Set with Many Assets



Optimal Portfolio Selection with Many Risky Assets and a Risk-Free

1. Create the set of possible mean-SD combinations from different portfolios of risky assets
2. Find the "tangency portfolio," that is, the portfolio with the highest Sharpe ratio:

$$SR_i = \frac{E[R_i] - R_f}{\sigma_i}$$

3. Choose the combination of the tangency portfolio and the risk-free asset to suit your risk-return preferences.

Two-Fund Separation

- All investors hold combinations of the same two "mutual funds":
  - The risk-free asset
  - The tangency portfolio
- An investor's risk aversion determines the fraction of wealth invested in the risk-free asset
- But, all investors should have the rest of their wealth invested in the tangency portfolio.

Excell Example: Portfolio Optimizer

- Portfolio selection with 5 risky assets and 1 riskless asset.
- Why does security 2 have a large portfolio weight?
- Why do all securities have positive portfolio weights?
- Double mean return of security 1:
  - What is the effect on MVP?
  - What is the effect on the tangency portfolio?
- Importance of correlation: ρ_{45}=0, 0.7, 0.9.
  - How do portfolio weights depend on this correlation?

Risk Reduction in Diversified Portfolios

- Suppose we start with a typical US stock.
- Now suppose we add stocks to the portfolio, all stock positions equally weighted.
- (The best mix is not, in general, equally weighted -- but this is illustrative way of making a general point.)

Risk in Equally-Weighted Portfolios: Independent Returns

- Suppose we have an equally weighted portfolio (holding weights 1/N) of N <u>independent</u> stocks.
- The variance of the portfolio return is

$$\sigma_p^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2$$

$$= \frac{1}{N}\begin{bmatrix}\text{average}\\ \text{variance}\end{bmatrix}$$

- As the number of assets increase, the risk is diversified away. (The insurance principle.)

Risk in Equally-Weighted Portfolios: The General Case

- Suppose we have an equally weighted portfolio (holding weights 1/N) of N stocks.
- The variance of the portfolio return is:

$$\sigma_p^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2 + \frac{2}{N^2}\sum_{i=1}^{N}\sum_{j>i}^{N}\text{cov}(R_i, R_j)$$

$$= \frac{1}{N}\left[\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2\right] + \left[1-\frac{1}{N}\right]\left[\frac{1}{N(N-1)/2}\sum_{i=1}^{N}\sum_{j>i}^{N}\text{cov}(R_i, R_j)\right]$$

$$= \frac{1}{N}\begin{bmatrix}\text{average}\\ \text{variance}\end{bmatrix} + \left[1-\frac{1}{N}\right]\begin{bmatrix}\text{average}\\ \text{covariance}\end{bmatrix}$$

Risk in Equally-Weighted Portfolios: The General Case

- What happens when N goes to infinity?
- Variance of portfolio return

  -> average covariance of returns
- Risk of portfolio -> non-diversifiable risk

Classifications of Risk

- Part that cannot be diversify away:
  - `covariance risk', 'systematic risk' or 'non-diversifiable risk'
  - E.g. market risk, macroeconomic risk, industry risk
- Part that can be diversified away (in a large portfolio):
  - 'idiosyncratic risk', 'non-systematic risk', 'diversifiable risk' or 'unique risk'
  - E.g. Individual company news

Diversifiable vs. Non-Diversifiable Risk

- When held in a portfolio some of the risk of a stock disappears.
- Or, the risk contribution the stock makes to the portfolio is LESS than the risk of the stock if held in isolation:

$$\begin{pmatrix}\text{total risk in}\\ \text{a stock}\end{pmatrix} = \begin{pmatrix}\text{non - diversifiable}\\ \text{risk}\end{pmatrix} + \begin{pmatrix}\text{diversifiable}\\ \text{risk}\end{pmatrix}$$

- Investors want to be compensated for holding which kind of risk?

Percentage Risk Reduction

- What is the percentage reduction in risk we should expect from adding stocks to our portfolio?
- (In the graph 100% represents the typical risk of a US stock)